Enhanced laser frequency suppression for LISA using arm and cavity locking systems

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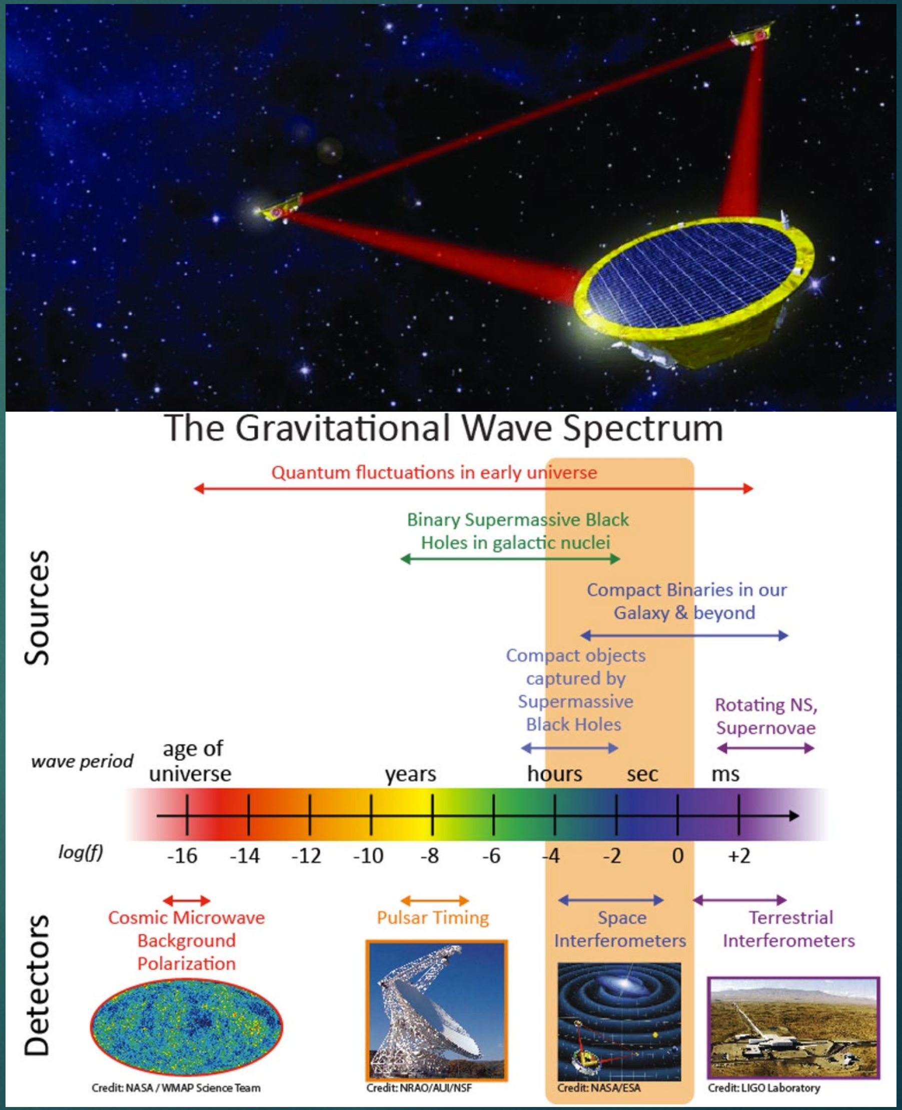
Background

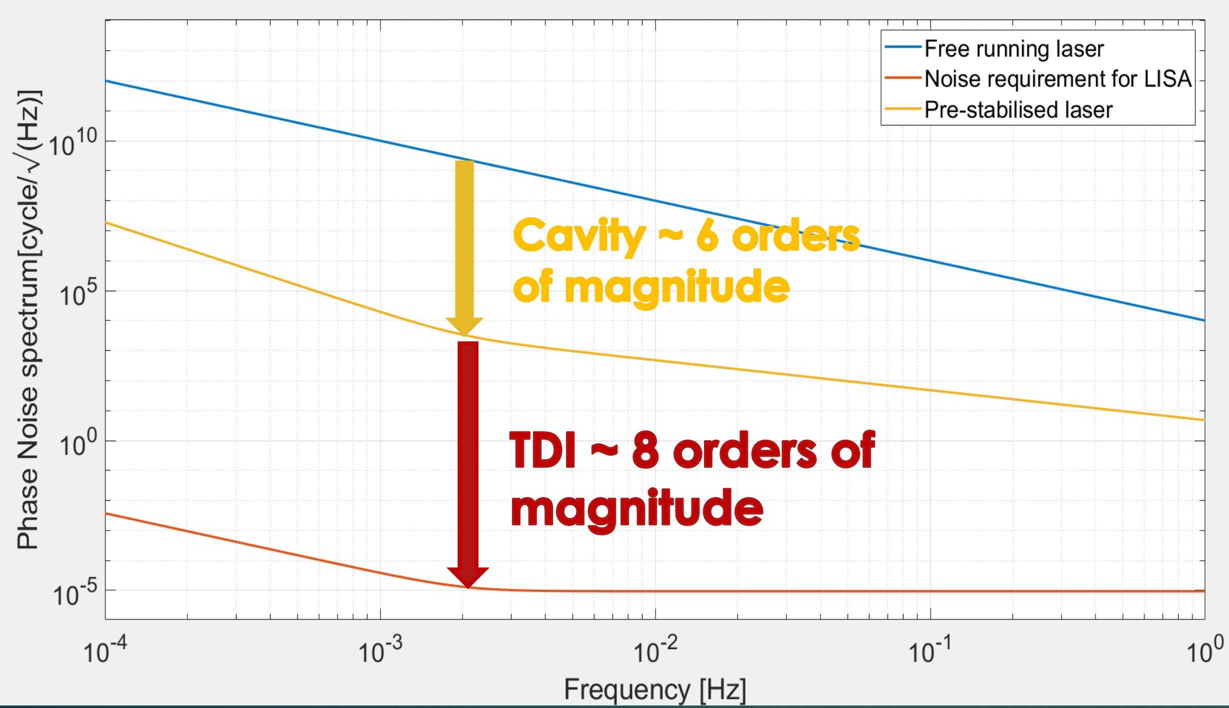
- LISA – Laser Interferometer Space Antenna [1]
  - Space-based Gravitational Wave detector
  - Three spacecrafts separated by 2.5 million kms
- Detection done in low frequency band
  - From 0.1 mHz to 1 Hz (signals between 1 – 10,000s) with a sensitivity of 10 $\mathrm{pm}/\sqrt{\mathrm{Hz}}$
  - Away from terrestrial noise sources
  - Capability to detect Massive Black Hole mergers and signals for cosmology and new physics!!!

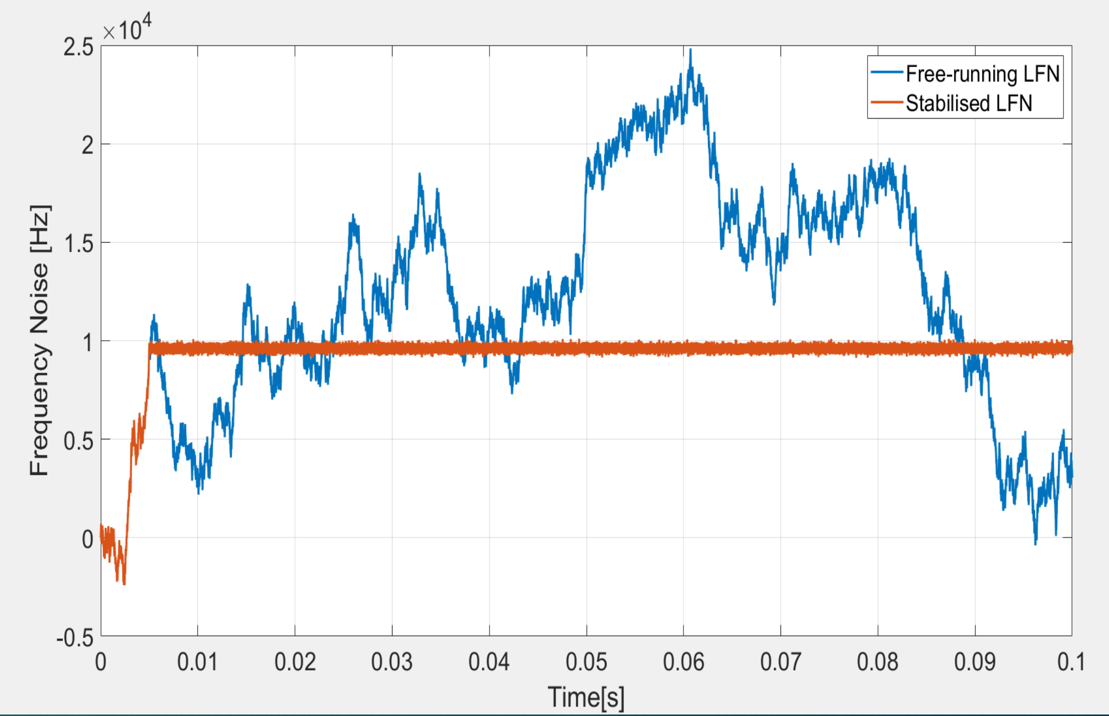


1. K. Danzmann et al. "LISA L3 mission proposal document", (2017)

Why should we care about stabilization

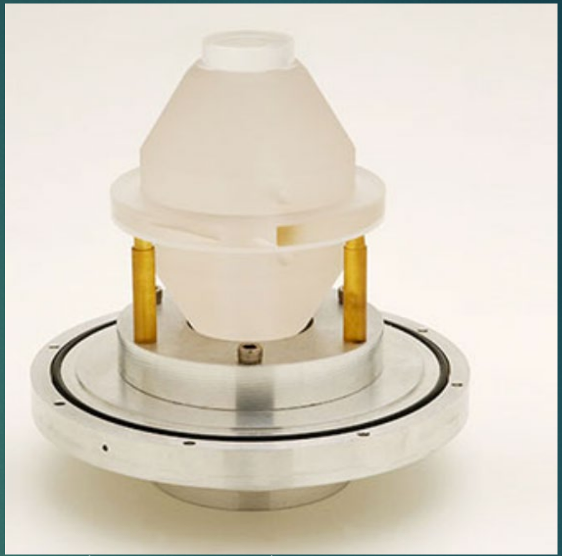
- Accuracy of measurements ← stability of the tool.
  - In laser interferometer, measurement accuracy ← laser frequency/phase noise.
- Laser stabilization for LISA
  - Requires 14 orders of suppression from free-running laser to meet LISA sensitivity of $10\ \mathrm{pm}/\sqrt{\mathrm{Hz}}$



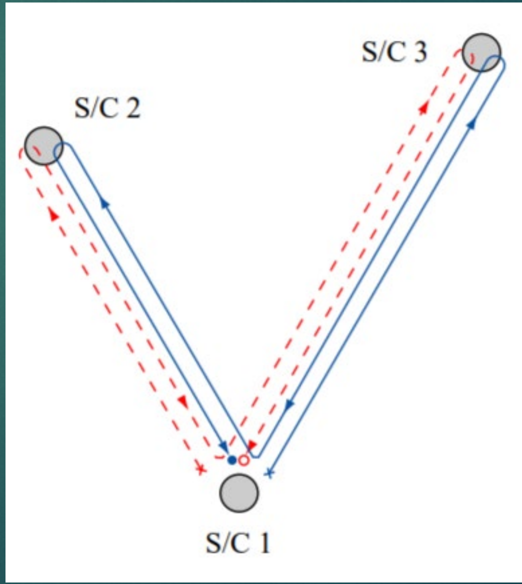


Laser stabilization in LISA

- Cavity locking – Technique to stabilize the laser with respect to an optical resonant cavity
  - Pound-Drever-Hall locking[2]
  - Linear error Transfer function within the linewidth of cavity
- Time-Delay Interferometry (TDI)
  - Post-processing technique that mimics an equal-arm Michelson response by applying appropriate delays to phase measurements.[3]
  - TDI-1, TDI-1.5, TDI-2



PC: Mid-plane cavity – Stable Laser Systems



2. E. D. Black. (2001). "An introduction to Pound–Drever–Hall laser frequency stabilization" (PDF). Am J Phys. 69 (1): 79–87
3. D.A. Shaddock, M. Tinto, F.B. Estabrook, J.W. Arm-strong Phys. Rev. D, 68 (2003), p. 061303(R)

Motivation and Outcomes

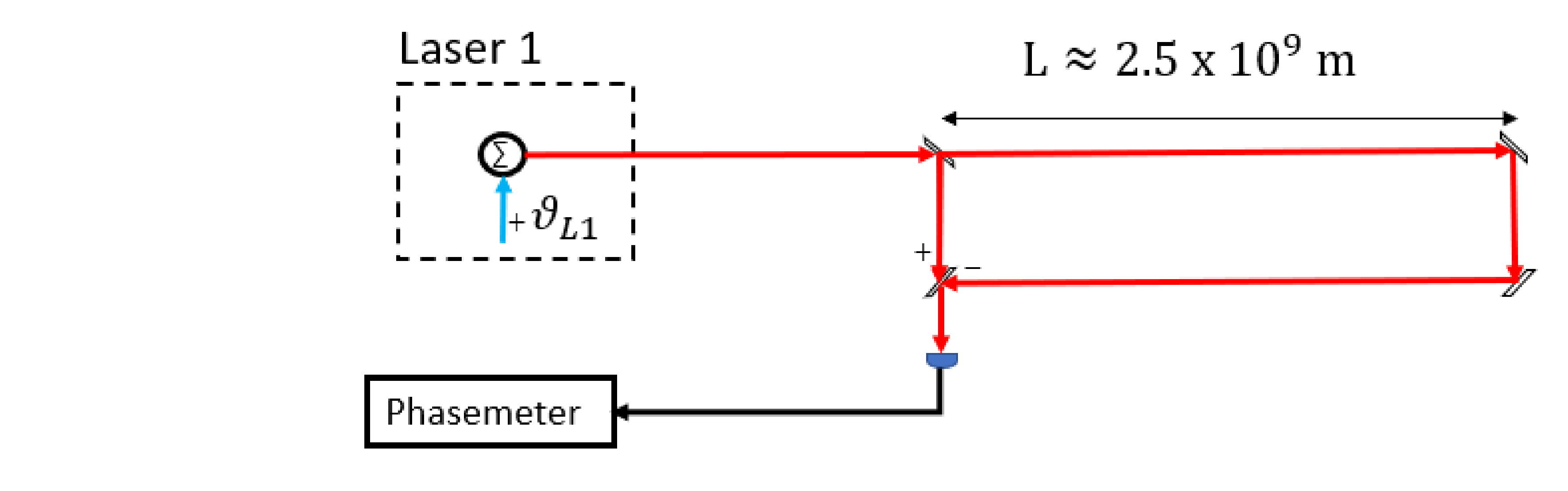
- TDI is a powerful technique (can suppress up to 8 orders), but it is difficult to verify on ground without the complexity of the system in space.
  - If TDI fails to meet the sensitivity requirements, there is a potential risk of losing out GW data.

Goal – Re-examine the arm locking stabilization to relax TDI requirements with no or minimal hardware changes to LISA baseline design.

Outcome: We successfully show a combination of arm and cavity locking, with only digital controllers, that can reduce the laser frequency noise by 3 orders of magnitude, giving LISA a large margin for TDI.

Arm locking

- Arm locking - Technique to stabilize the laser with respect to the arm length of the interferometer.



Arm locking

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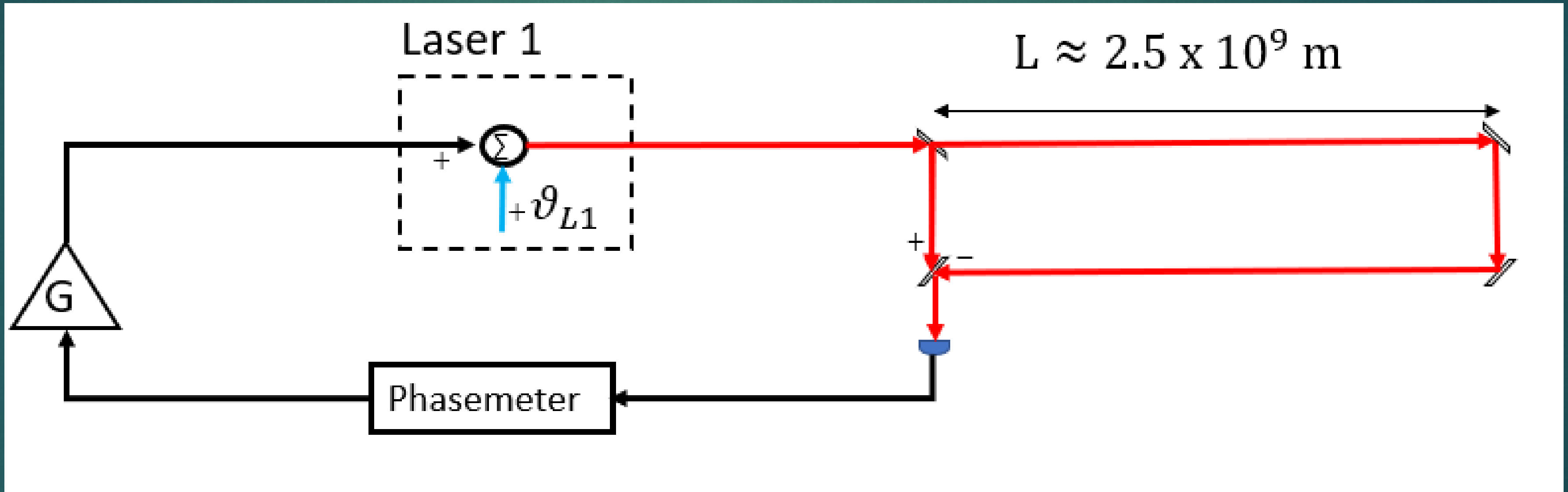
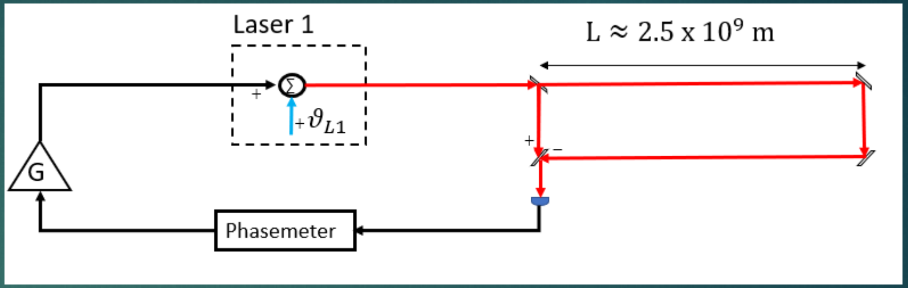


Arm locking

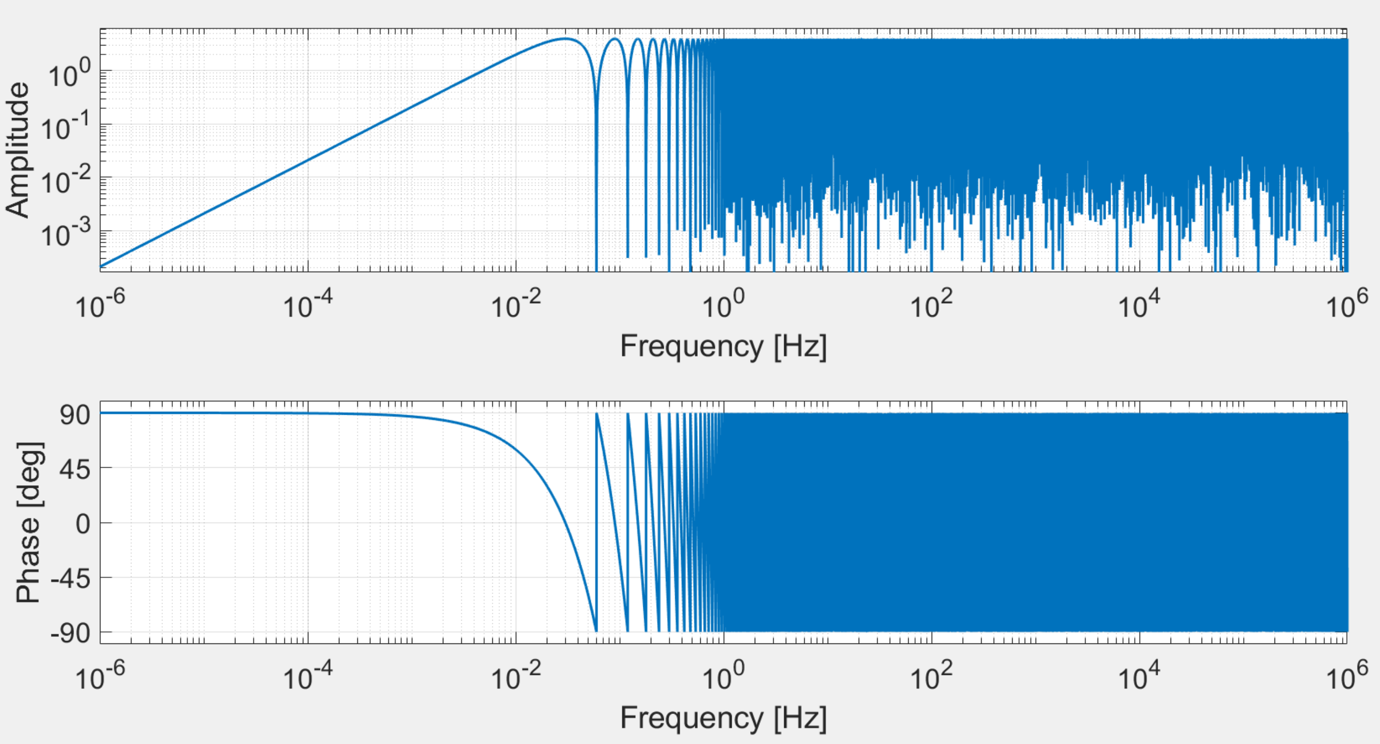


- Arm locking - Technique to stabilize the laser with respect to the arm length of the interferometer.
- Arm locking is already being achieved in LIGO [4]
- For LISA, there are a few significant differences
  - Part of the light takes ~17s to do a return trip back to the prompt signal
  - The spacecrafts are not fixed and move around in space
    - Up to 1% of the length -> 25,000 km at a maximum rate of 10m/s.

4. P.Fritschel, et al. "Readout and control of a power-recycled interferometric gravitational-wave antenna", App. Opt. 40, 4988 (2001).

Arm locking

- Different schemes of arm locking has been investigated[5-7] using one or both arms of the interferometer



$$P_s = 1 - e^{-s\tau}$$

Where τ is the round-trip time of the laser over the two spacecrafts

- Integration of arm locking with pre-stabilisation has been investigated [7-8].

5. B. S. Sheard, M. B. Gray, D. E. McClelland, and D. A. Shaddock, Phys. Lett. A 320, 9 (2003).
6. A. Sutton and D. A. Shaddock, Phys. Rev. D 78, 082001 (2008)
7. K. McKenzie, R. Spero and D. Shaddock, Phys. Rev. D 80, 102003 (2009)
8. D. A. Shaddock and et al, LISA Frequency Control WhitePaper, LISA Project technical note LISA-JPL-TN-823(2009)

Doppler frequency, laser pulling

- A technical challenge for Arm locking [7-8]
  - Received light is Doppler shifted by ~10 MHz due to relative speed of the spacecrafts.
  - Sensor has zero response at DC, and so the Doppler shifts must be cancelled

$$V_d = \frac{G}{1+GP_s} \approx \frac{1}{P_s} \approx \frac{1}{s} \text{ (at low frequencies and high gain)}$$

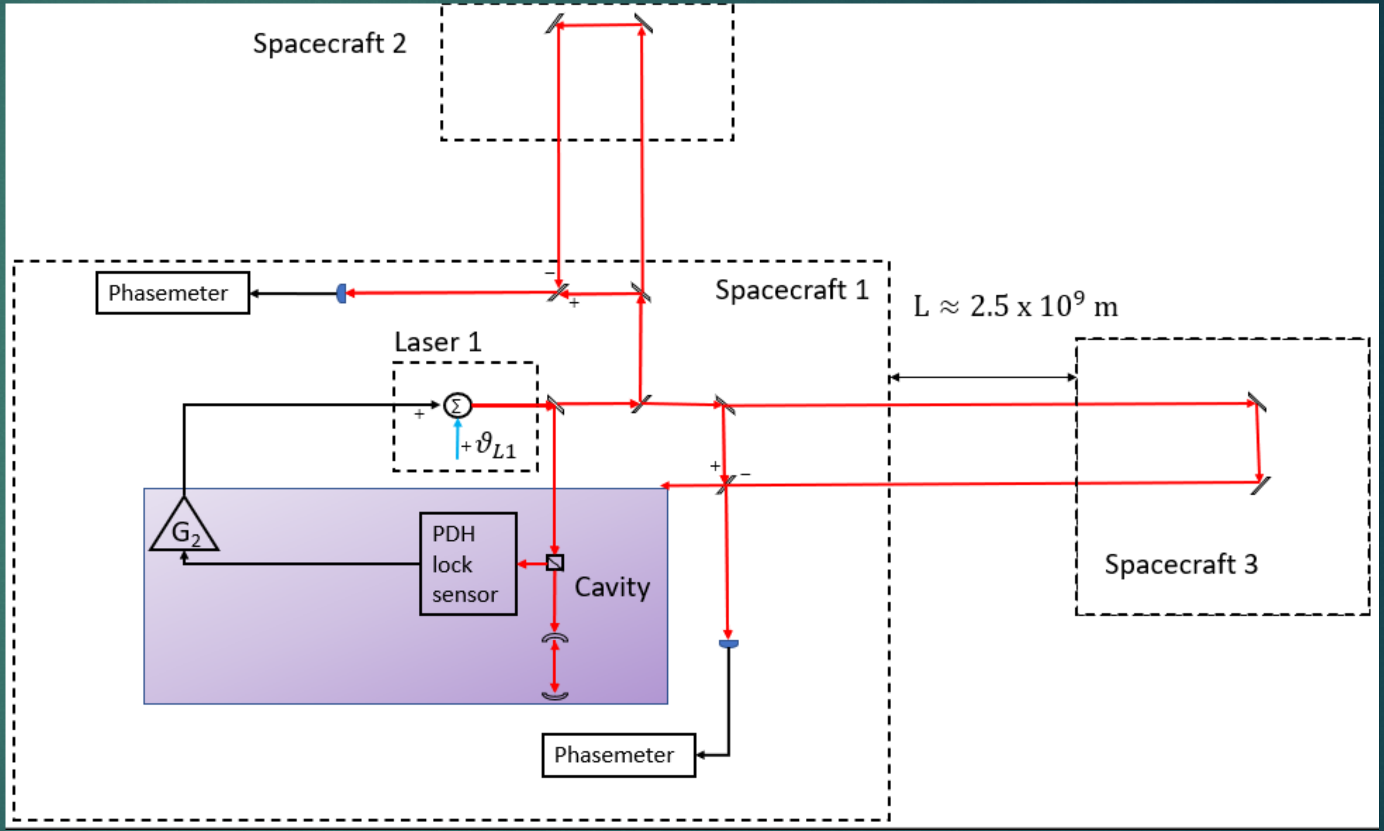
  - Error in the Doppler frequency knowledge will lead to **a ramp in laser frequency over time**, causing potential problems (like laser mode-hopping)
- No technique is compatible with the current LISA baseline design.
  - Requires additional modulation or tunable cavity length.[9-10]
  - Use the arm feedback to vary the resonance point of the cavity.

7. K. McKenzie, R. Spero and D. Shaddock, Phys. Rev. D 80, 102003 (2009)
8. D. A. Shaddock and et al, LISA Frequency Control WhitePaper, LISA Project technical note LISA-JPL-TN-823(2009)
9. L. Conti, M. D. Rosa, and F. Marin, J. Opt. Soc. Am. B 20, 462 (2003)
10. J.I. Thorpe, K. Numata, and J. Livas, Opt. Express 16,15980 (2008).

What are we doing?

LISA baseline stabilisation

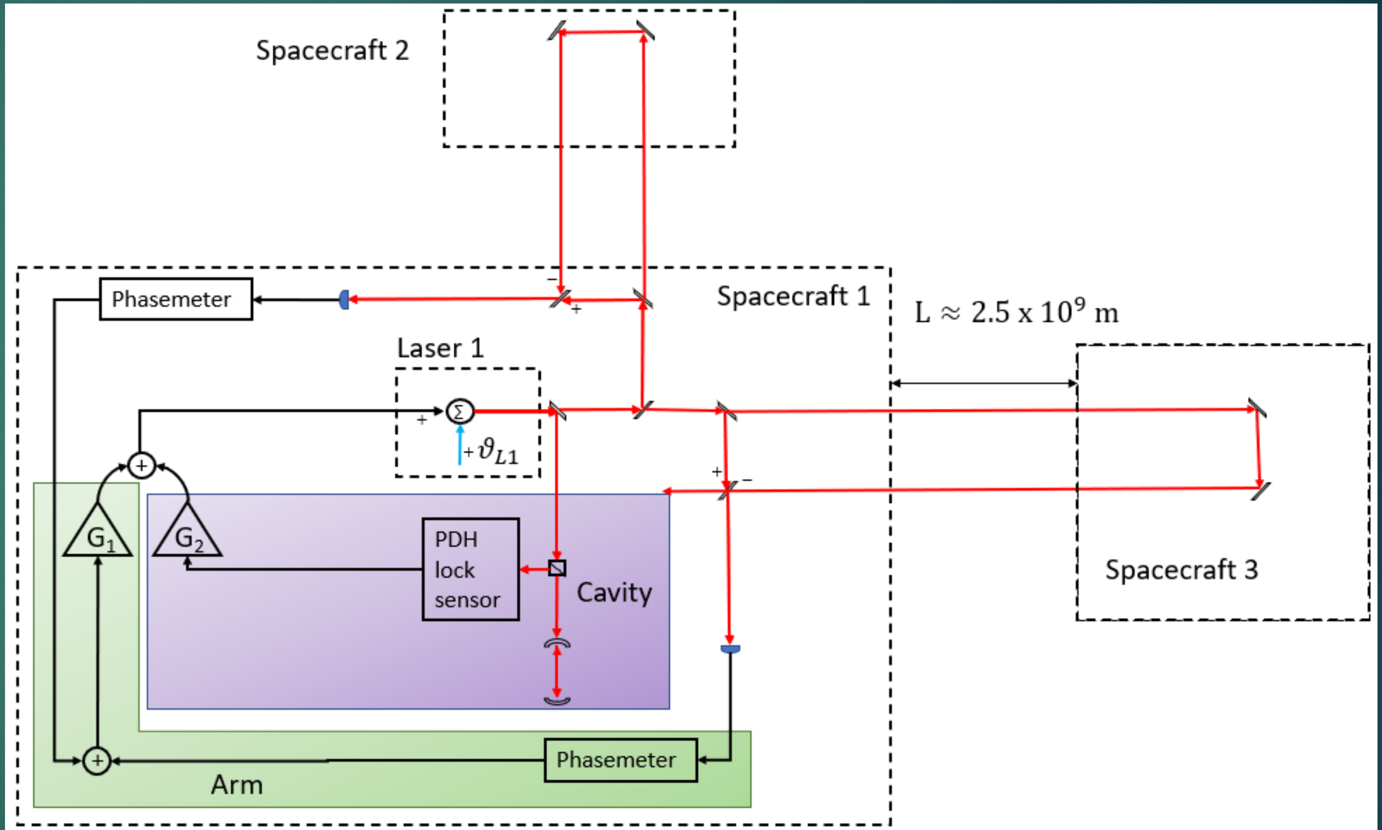
Pre-stablised laser with Fabry-Pérot cavity is sent to the spacecrafts and the phase is measured using Phasemeter.



What are we doing?

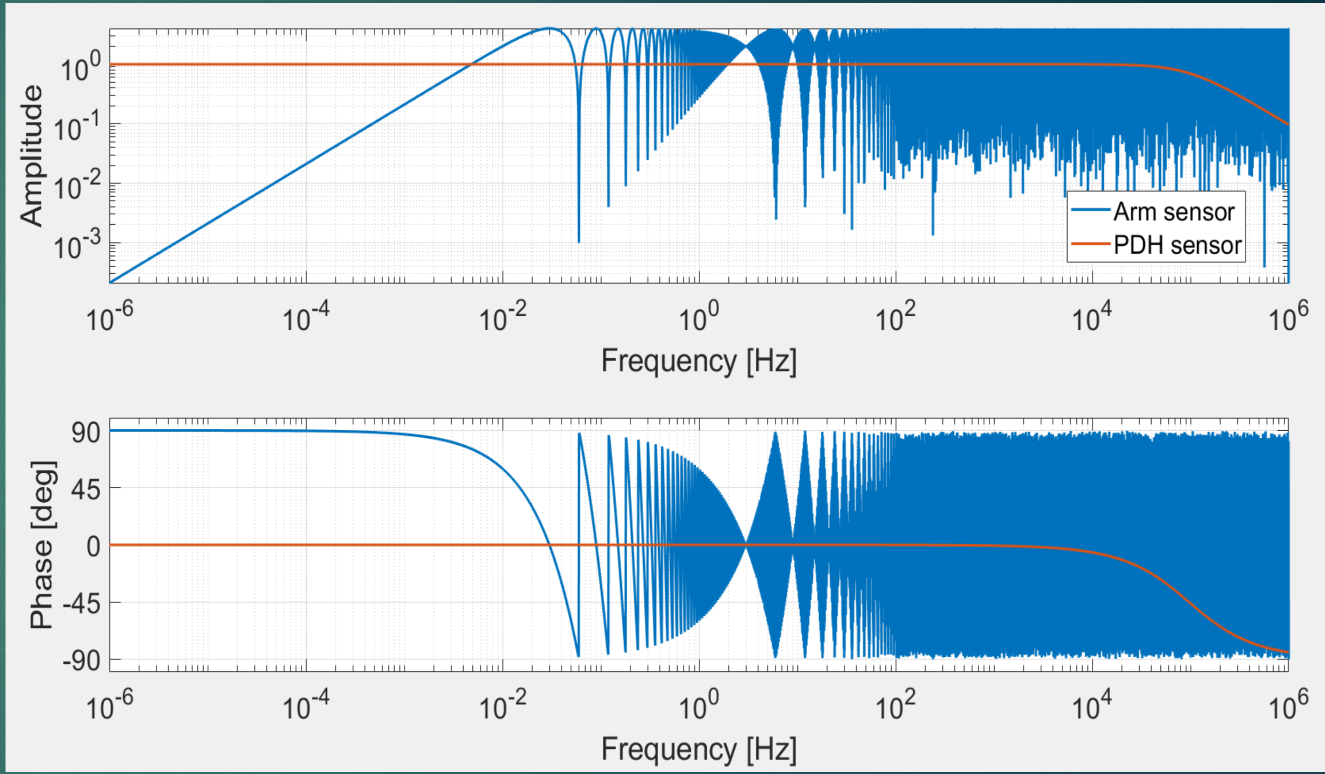
Combine both the arm sensor and the PDH sensor and feed it back to the laser.

We could utilize the best parts of both sensors simultaneously.



Controller Requirements

- The cavity response should be dominant at the unity gain frequency
- The arm response should be dominant in the mid frequency band(10^{-4} to 1 Hz)
- The cavity response should be dominant at lower frequencies (< 10^{-4} Hz)
  - Orbital dynamics dictate that the armlength variations are periodic with half-yearly and yearly period. [11]



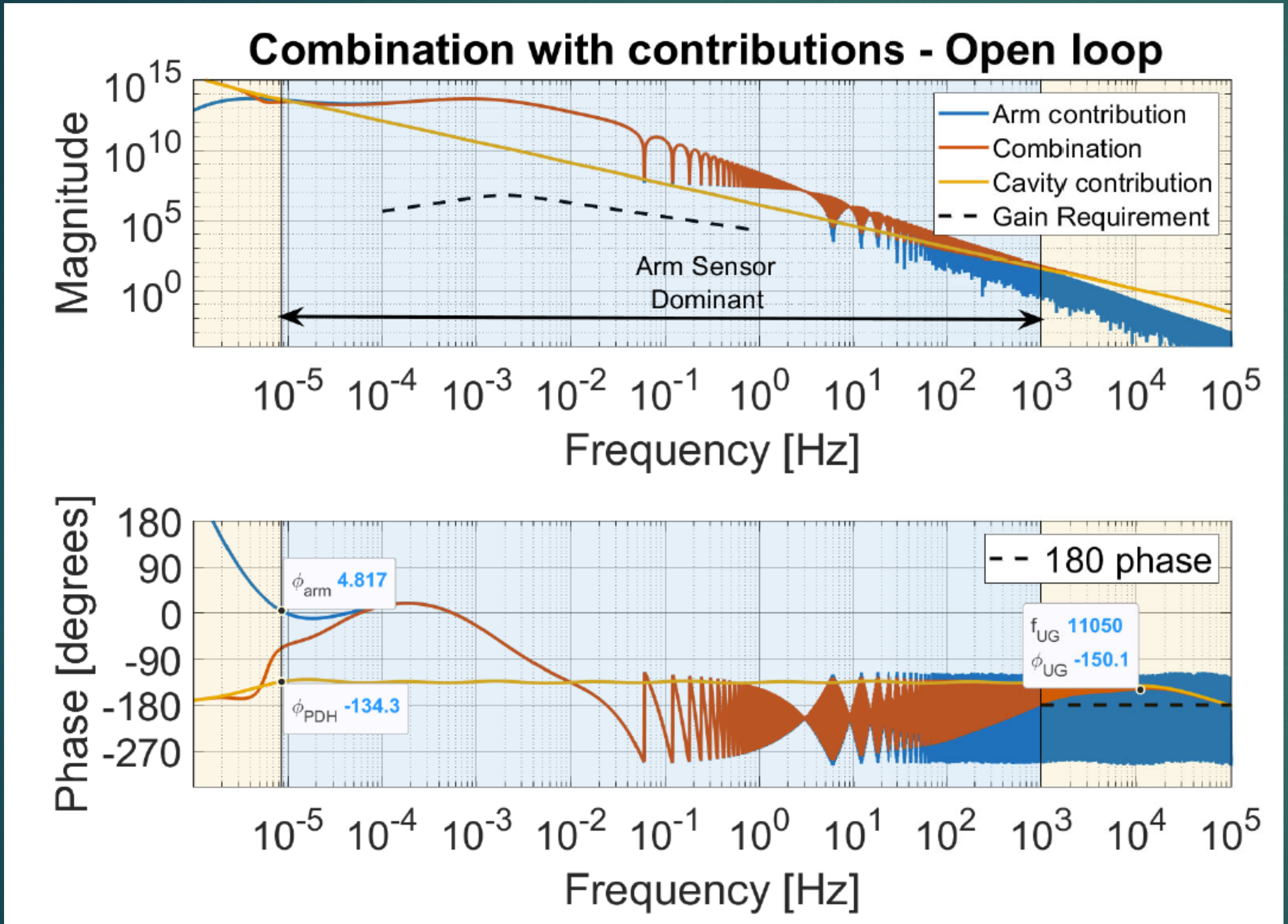
11. S. V. Dhurandhar, K. R. Nayak, S. Koshti, and J.-Y. Vinet, Classical Quantum Gravity 22, 481 (2005)
12. J.T. Valliyakalayil et al. Phys. Rev. D 105 062005 (2022)

Controller Requirements

- Make the controller more robust.
  - Due to requirement of PDH being dominant in UGF, we require that the UGF is at least a decade below the HWHM frequency (10 kHz).
  - The phase margin at unity gain crossings must be more than 30 degrees (open loop phase more than -150 degrees) => reduce any unstable behavior
- The doppler pulling should be at most 1/10th the linewidth
  - To ensure that the cavity remains locked in resonance.
  - The cavity in consideration has a FWHM frequency of 200 kHz, and so the Doppler pulling should be less than 20 kHz

12. J.T. Valliyakalayil et al. Phys. Rev. D 105 062005 (2022)

Controller Design Solution



Unity Gain Frequency ~11 kHz
Phase Margin at UGF ~ 30°
Phase margin at lower gain crossover point ~52°
Arm dominant from 10 μHz to 1 kHz.

The arm sensor has a controller with a slope of 2.3 while at low frequencies, it is AC coupled.

The cavity sensor has a controller with a slope of 1.5

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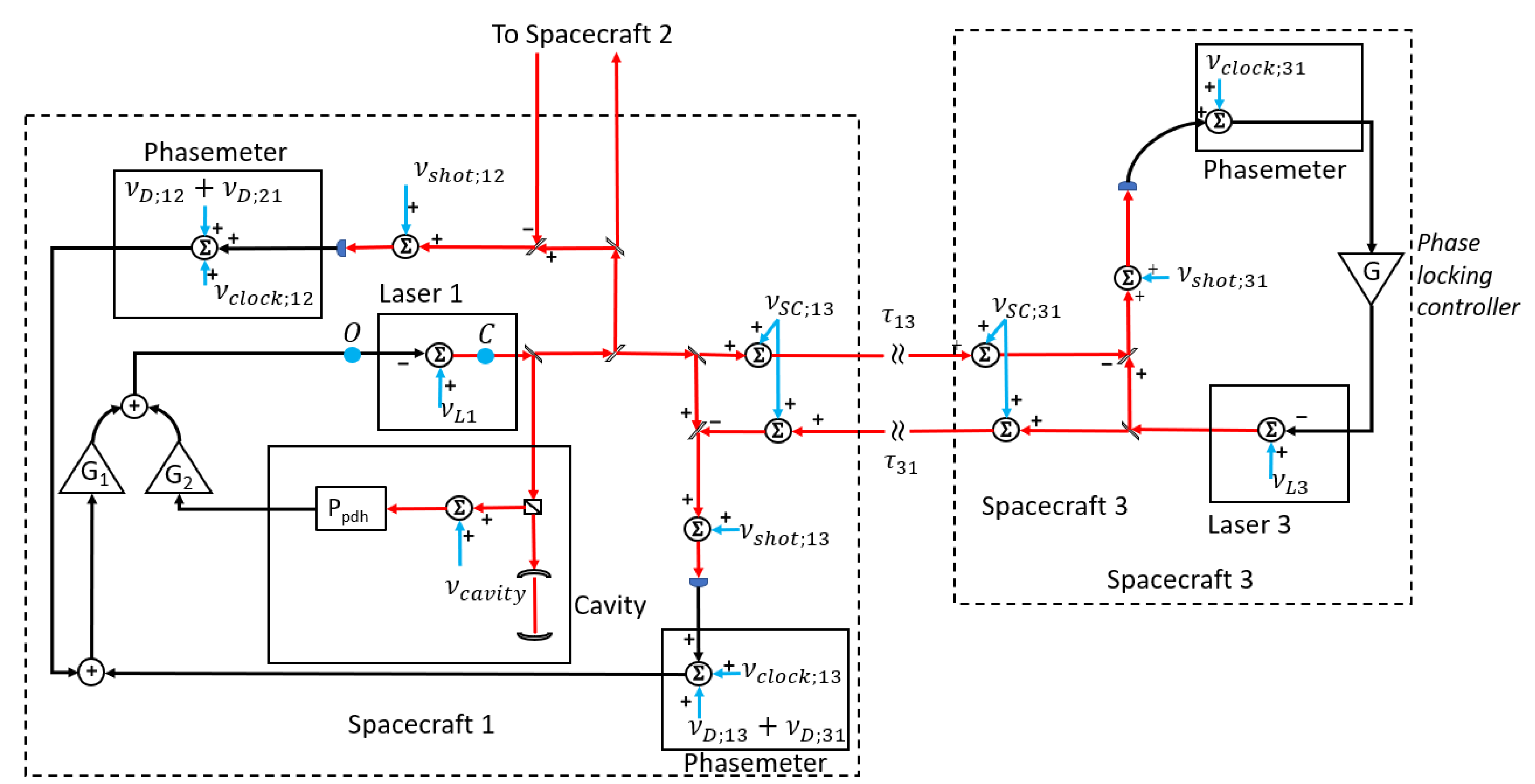
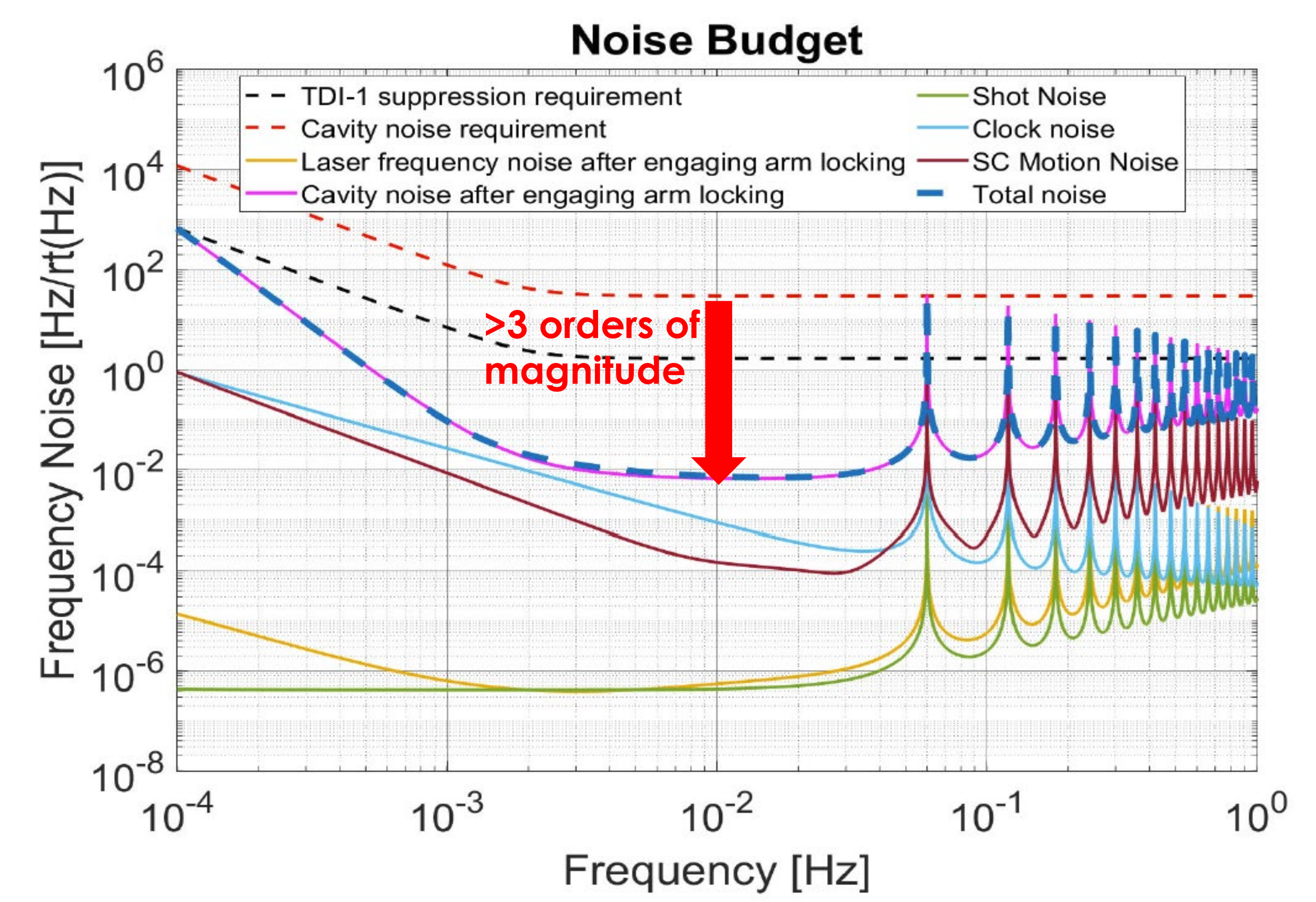
Noise sources



12. J.T. Valliyakalayil et al. Phys. Rev. D 105 062005 (2022)

Residual Noise requirements



GOAL REQUIREMENT

$$\nu_{TDI-1} = 1.7\frac{\text{Hz}}{\sqrt{\text{Hz}}}\sqrt{1+\left(\frac{2\text{mHz}}{\text{f}}\right)^4}$$

$$\nu_{cavity} = 30\frac{\text{Hz}}{\sqrt{\text{Hz}}}\sqrt{1+\left(\frac{2\text{mHz}}{\text{f}}\right)^4}$$

12. J.T. Valliyakalayil et al. Phys. Rev. D 105 062005 (2022)

Doppler pulling –Lock acquisition

- Due to the dominance of arm locking, Doppler pulling will persist and hence we look at the resultant pulling when the lock is initiated.
- Step response corresponding to turn on of the controllers.

$$\nu_C(t) \ = L^{-1}\left[\frac{\left(\left[\nu_{doppler;+}(s) - \nu_{doppler;lock}(s)\right]V(s)\right)}{s}\right]$$

$\nu_{\mathrm{doppler;+}}(\mathrm{s})$ is a toy model based on a combination of sinusoids of yearly and half-yearly period.

$$\nu_{doppler;+}(s) = \nu_1 \sin(\omega_1 t + \phi_1) + \nu_2 \sin(\omega_2 t + \phi_2)$$

$\nu_{doppler;lock}(s)$ can be either a second-order polynomial approximation

$$\nu_{doppler;lock}(s) = \nu_0 + \gamma_0 t + \frac{a_0 t^2}{2}$$

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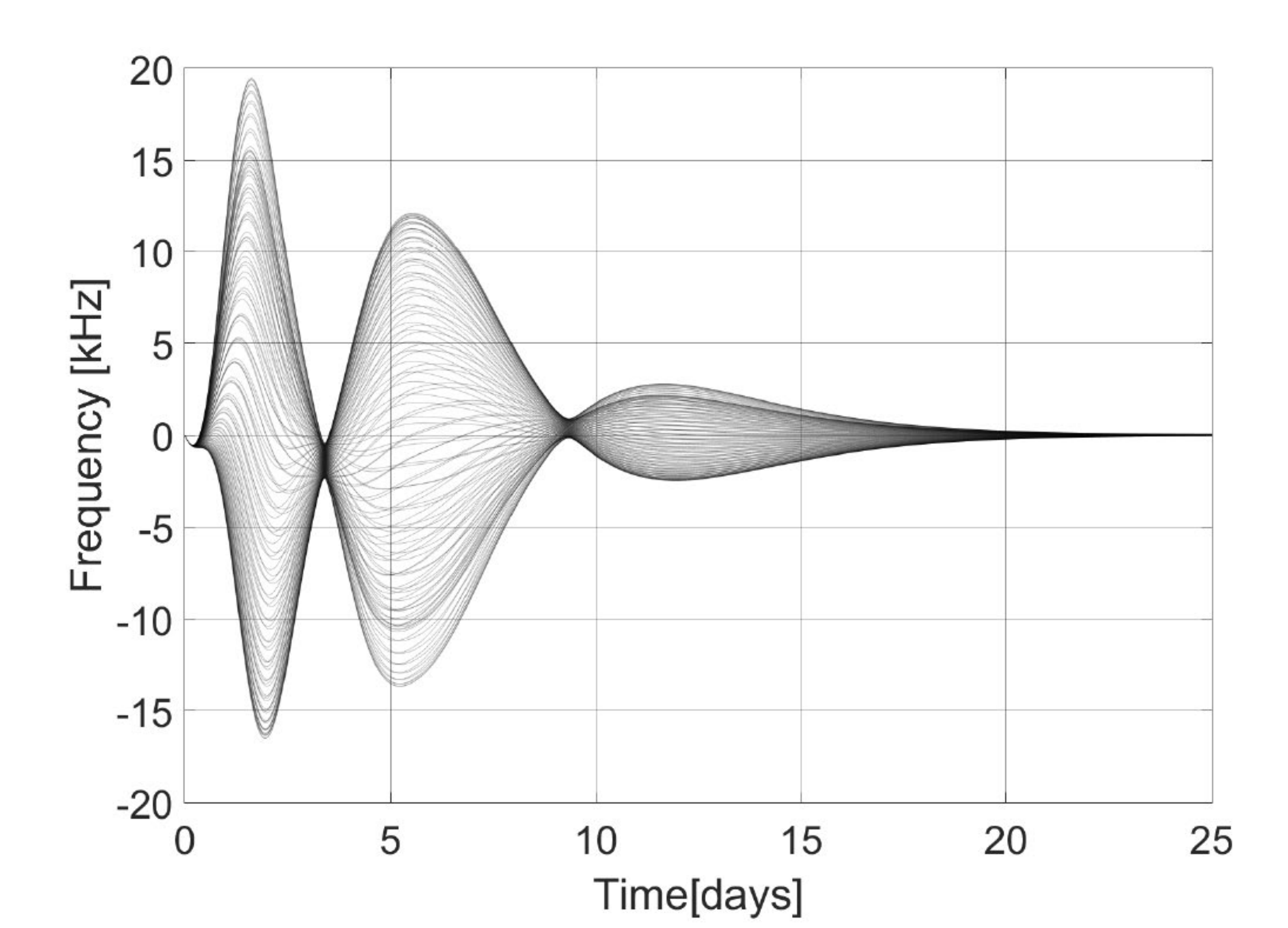
Doppler pulling – Lock acquisition



Cavity linewidth ~ 200 kHz

Doppler requirement < 20 kHz

$$\widetilde{\nu_0} < 10\ Hz$$
$$\widetilde{\gamma_0} < 60\ \mu Hz/s$$
$$\widetilde{\alpha_0} < 5\ nHz/s^2$$

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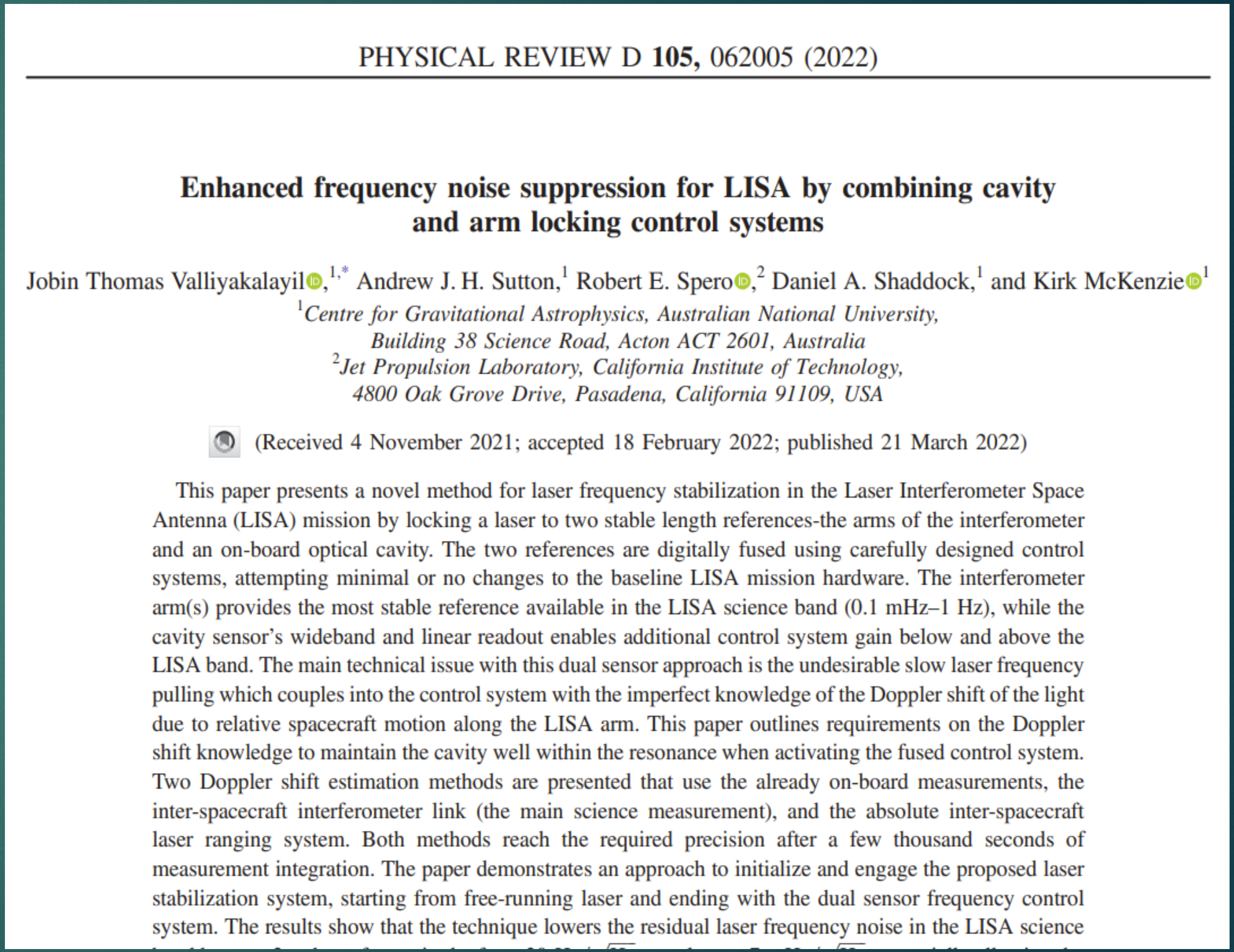
Arm locking paper

- Detailed analysis is provided in paper
- Proves new concept to combine existing cavity and laser hardware
  - Only a firmware upload for many benefits
- Includes Analytical description
- Includes Time domain simulation
- Proposes a lock acquisition scheme


PHYSICAL REVIEW D **105,** 062005 (2022)

Enhanced frequency noise suppression for LISA by combining cavity and arm locking control systems

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This paper presents a novel method for laser frequency stabilization in the Laser Interferometer Space Antenna (LISA) mission by locking a laser to two stable length references-the arms of the interferometer and an on-board optical cavity. The two references are digitally fused using carefully designed control systems, attempting minimal or no changes to the baseline LISA mission hardware. The interferometer arm(s) provides the most stable reference available in the LISA science band (0.1 mHz–1 Hz), while the cavity sensor's wideband and linear readout enables additional control system gain below and above the LISA band. The main technical issue with this dual sensor approach is the undesirable slow laser frequency pulling which couples into the control system with the imperfect knowledge of the Doppler shift of the light due to relative spacecraft motion along the LISA arm. This paper outlines requirements on the Doppler shift knowledge to maintain the cavity well within the resonance when activating the fused control system. Two Doppler shift estimation methods are presented that use the already on-board measurements, the inter-spacecraft interferometer link (the main science measurement), and the absolute inter-spacecraft laser ranging system. Both methods reach the required precision after a few thousand seconds of measurement integration. The paper demonstrates an approach to initialize and engage the proposed laser stabilization system, starting from free-running laser and ending with the dual sensor frequency control system. The results show that the technique lowers the residual laser frequency noise in the LISA science

Thank you for listening

Any Questions?

Supplemental – Transfer Functions

1. $$V(s) = \frac{-G_1(s)}{1 + G_1(s)P_+(s) + G_2(s)P_{pdh}(s)}.$$

2. $$\begin{aligned} LN(s) &= \frac{\nu_{C;L}(s)}{\nu_L(s)} \\ &= \frac{1}{1 + G_1(s)P_+(s) + G_2(s)P_{\text{pdh}}(s)}. \end{aligned}$$

3. $$\begin{aligned} TN(s) &= \frac{\nu_{C;cavity}(s)}{\nu_{cavity}(s)} \\ &= \frac{-G_2(s)P_{\text{pdh}}(s)}{1 + G_1(s)P_+(s) + G_2(s)P_{\text{pdh}}(s)}. \end{aligned}$$

Supplemental - Controller Design

- Controller 1 – Split in 3 Stages
  - Stage 1 – Provide an effective slope of $1/(f^{0.3})$ – accomplished with a low pass filter cascade.
  - Stage 2 – Provide a high pass filtering with corner frequency at around 1.29 mHz and 1.29 μHz. Total slope is 140db/decade.
  - Stage 3 – Provide a lag compensator for phase transition
- Controller 2 – Provide an effective slope of $f^{0.5}$
- Controller 3 – Provide an effective slope of $1/f^2$ – accomplished with a double integrator.

Or……

- Controller 1 – Split in 3 Stages
  - Stage 1 – Provide an effective slope of $1/(f^{2.3})$ – accomplished with a low pass filter cascade and two integrators.
  - Stage 2 – Provide a high pass filtering with corner frequency at around 1.29 mHz and 1.29 μHz. Total slope is 140db/decade.
  - Stage 3 – Provide a lag compensator for phase transition
- Controller 2 – Provide an effective slope of $1/f^{1.5}$ – accomplished with a low pass filter and a single integrator.

Supplemental - Noise Sources

- Laser frequency noise -> $\nu_L = \frac{30kHz}{f} \frac{Hz}{\sqrt{Hz}}$
- Cavity noise -> $\nu_{cavity} = 30 \frac{Hz}{\sqrt{Hz}} \sqrt{1 + \left(\frac{2mHz}{f}\right)^4}$
- Shot noise -> $\nu_{shot} = s(6.9\ x\ 10^{-5})\ \frac{Hz}{\sqrt{Hz}}$
- Spacecraft motion noise -> 1.5 $\frac{\sqrt{1+\left(\frac{2mHz}{f}\right)^4}}{\lambda} \frac{nHz}{\sqrt{Hz}}$
- Clock noise -> $\nu_{beatnote} * \frac{2.4 * 10^{-12}}{\sqrt{f}} \frac{Hz}{\sqrt{Hz}}$