Towards an experimental violation of a motional-state Bell's inequality using ultracold helium

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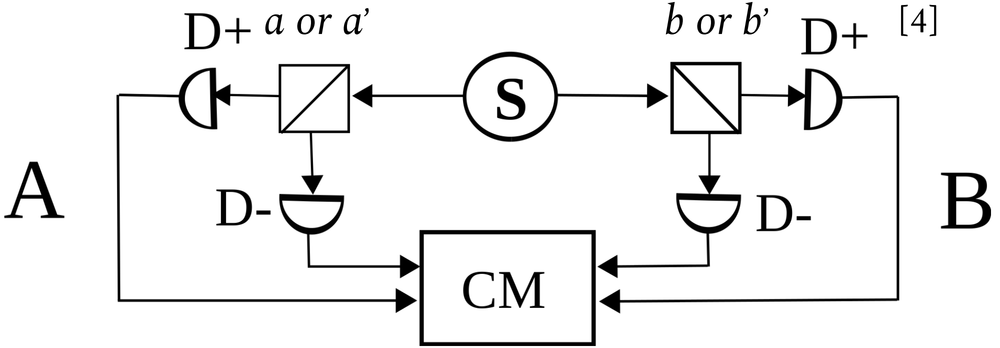
What is a Bell inequality?

- Quantum mechanics implies "action at a distance" is possible [1]
- Prompts the idea of supplementing quantum mechanics with "local-hidden-variable" theories to "fix" quantum mechanics [2]
- Constraints of such a theory lead to Bell inequalities: a set of conditions that all possible local-hidden-variable theories must obey [3]

[1] R. Horodecki, P. Horodecki, M. Horodecki, and K. Horodecki, Quantum entanglement, RevModPhys.81, 865 (2009)
[2] D. Bohm, A Suggested Interpretation of the Quantum Theory in Terms of "Hidden" Variables. I, Phys. Rev. 85, 166 (1952); A Suggested Interpretation of the Quantum Theory in Terms of "Hidden" Variables. II, PhysRev.85, 180 (1952)
[3] J. S. Bell, On the Einstein Podolsky Rosen paradox, PhysicsPhysiqueFizika.1.195 (1964)

CHSH Bell Inequality

Proof sketch:



$$a,\ a' = \pm 1 \quad b,\ b' = \pm 1$$

$$\therefore a - a' = 0 \ \text{ or } \ a + a' = 0$$

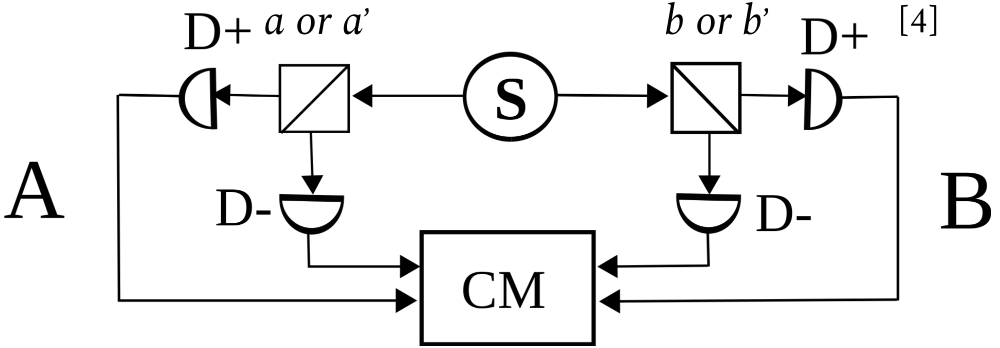
$$\text{define } S \equiv ab + a'b + ab' - a'b' = (a + a')b + (a - a')b' = \pm 2$$

$$\Rightarrow |\langle S \rangle| \leq \langle |S| \rangle = 2$$

$$\therefore |\langle ab \rangle + \langle a'b \rangle + \langle ab' \rangle - \langle a'b' \rangle| \leq 2$$

[4] George Stamatiou 2008, *Scheme of a "two-channel" Bell test*
[5] J. F. Clauser, M. A. Horne, A. Shimony, and R. A. Holt, Proposed experiment to test local hidden-variable theories, PhysRevLett.23.880 (1969)

CHSH Bell Inequality



Modeling this system using quantum mechanics we find the upper bound is

$$\Rightarrow |\langle S \rangle| \leq 2\sqrt{2}$$ [6]

which hence violates the tenets of local realism.

[6] Cirel'son, B. S. (March 1980). "Quantum generalizations of Bell's inequality". Letters in Mathematical Physics. 4 (2): 93–100.

Our Goal:
A Motional-state Bell inequality test with ultracold atoms

What is it?

- **Motional:** using the states of motion, in this case momentum.
- **Ultracold Atoms:** Specifically helium-4 atoms

Why do we want to do it?

- No Bell violation has been conducted with massive particles using external degrees of freedom, such as momentum.
- Such a test would give us insight into how quantum systems' behaviour scales with gravity.

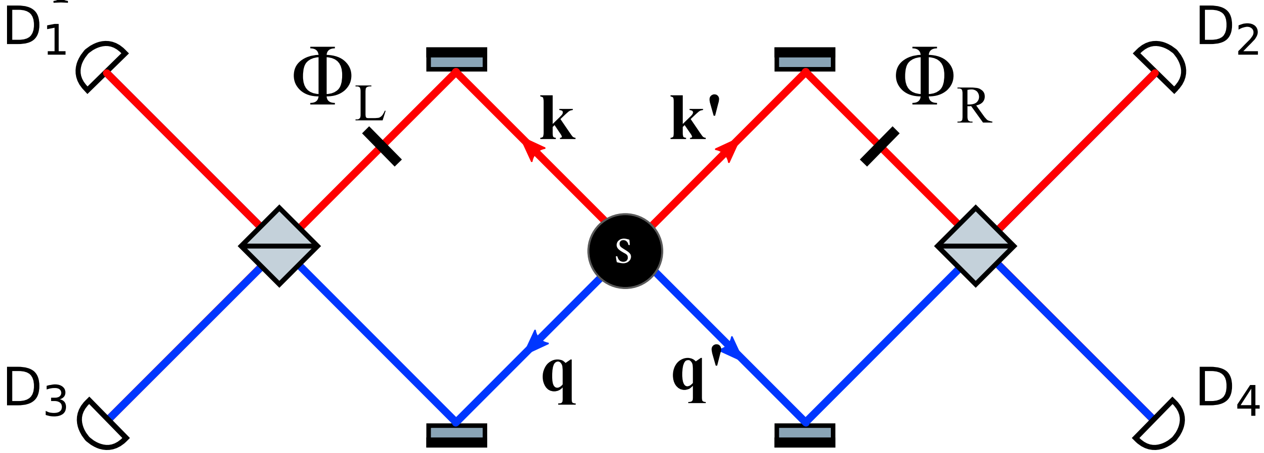
[7] R. J. Lewis-Swan and K. V. Kheruntsyan, Proposal for a motional-state Bell inequality test with ultracold atoms,
[8] Thomas, K. F., Henson, B. M., Wang, Y., Lewis-Swan, R. J., Kheruntsyan, K. V., Hodgman, S. S., and Truscott, A. G. (2022). A matter wave Rarity-Tapster interferometer to demonstrate non-locality. *arXiv preprint arXiv:2206.08560.*

How will we do it?

- Method/scheme: Rarity-Tapster interferometer [9]
- Platform: ultracold helium apparatus [10]
- Entanglement source: *s*-wave scattering halo of ultracold helium [11]
- Bragg pulses: our substitute for mirrors and beam splitters

[9] J. G. Rarity and P. R. Tapster, Experimental violation of Bell's inequality based on phase and momentum,
[10] R. Dall and A. Truscott, Bose–Einstein condensation of metastable helium in a bi-planar quadrupole Ioffe configuration trap,
[11] V. Krachmalnicoff, J.-C. Jaskula, M. Bonneau, V. Leung, G. B. Partridge, D. Boiron, C. I. Westbrook, P. Deuar, P. Zí n, M. Trippenbach, and K. V. Kheruntsyan, Spontaneous Four-Wave Mixing of de Broglie Waves: Beyond Optics,

Rarity-Tapster interferometer



$$C_{ij} = \left\langle \hat{N}_i \hat{N}_j \right\rangle$$

$$E(\Phi_L, \Phi_R) = \frac{C_{12} + C_{34} - C_{14} - C_{23}}{C_{12} + C_{34} + C_{14} + C_{23}}$$

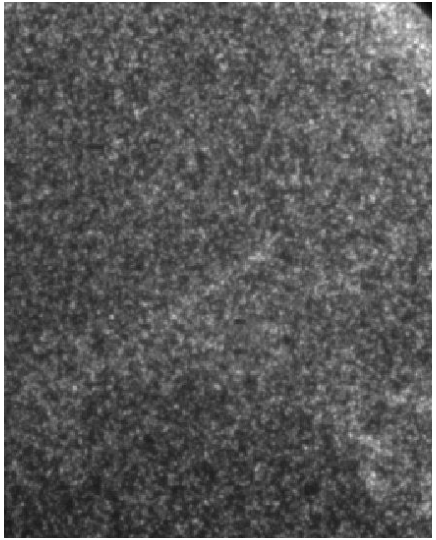
$$S_{CHSH} = |E(\Phi_L, \Phi_R) - E(\Phi_L, \Phi'_R) + E(\Phi'_L, \Phi_R) + E(\Phi'_L, \Phi'_R)| \leq 2, \ \forall \{\Phi_L, \Phi_R, \Phi'_L, \Phi'_R\}$$

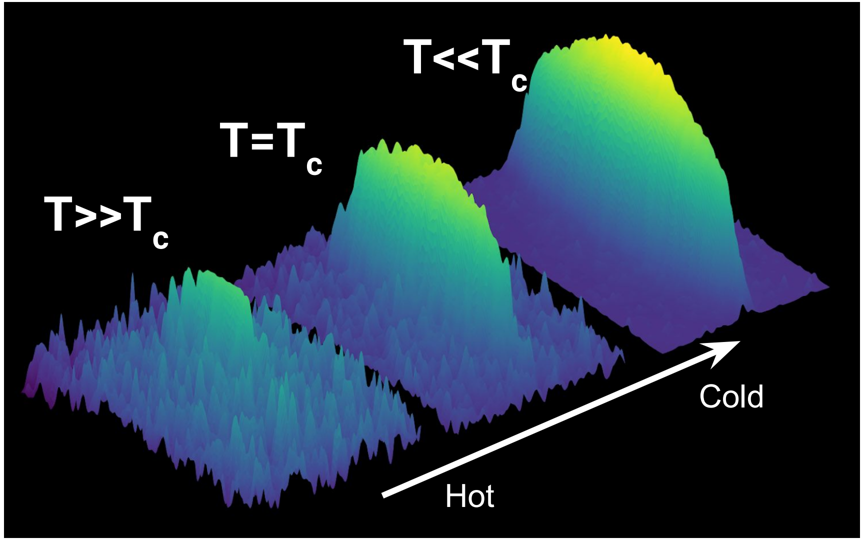
[9] J. G. Rarity and P. R. Tapster, Experimental violation of Bell's inequality based on phase and momentum,

Metastable Helium Bose-Einstein Condensate

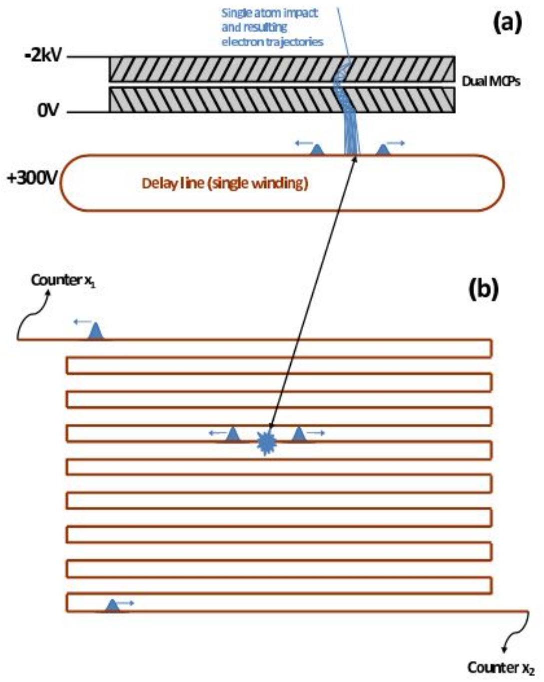
Bose-Einstein Condensates (BECs): Macroscopic Quantum Object

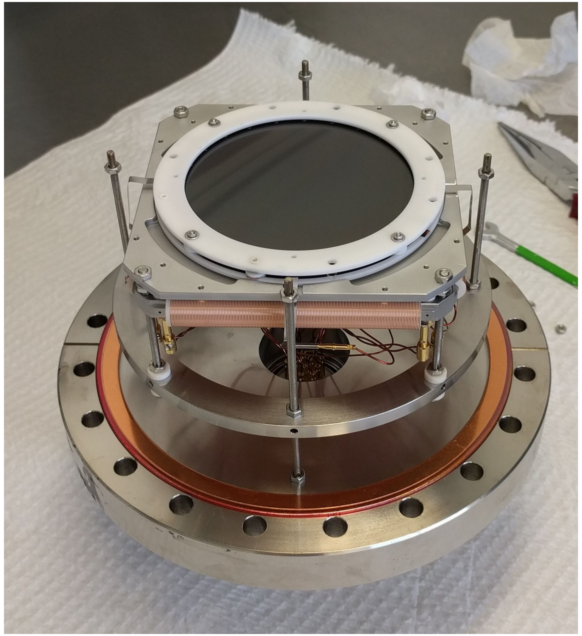
Metastable: 2^3S_1 state, 8000 second lifetime

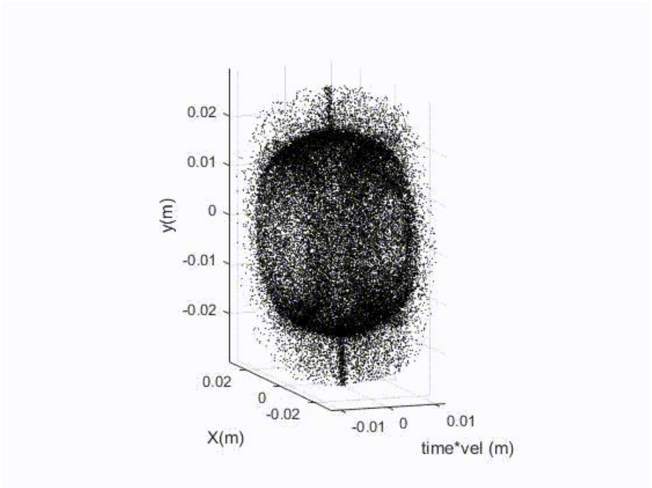




Multichannel Plate with Delay Line Detector







Metastable allows single particle detection

Source of entanglement: *s*-wave scattering halo

Full State

$$|\Psi\rangle = \bigotimes_{\mathbf{k}} \sqrt{1-\lambda^2} \sum_{n=0}^{\infty} \lambda^n \, |n, n\rangle_{\mathbf{k}-,\mathbf{k}+}$$

Reduced (Four-mode) state (assuming $\lambda \ll 1$)

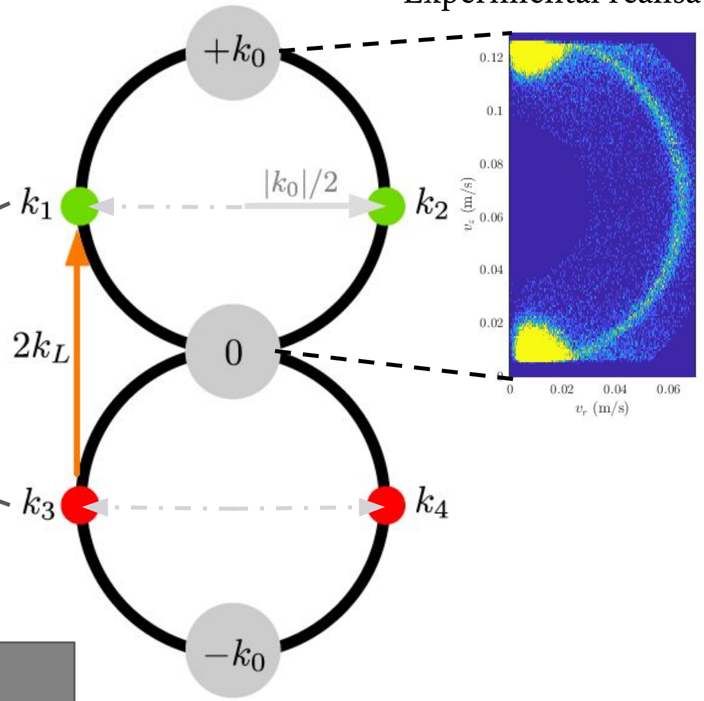
$$|\Psi\rangle_{12,34} \approx (1-\lambda^2)\left(|00,00\rangle_{12,34} + \lambda^2|00,11\rangle_{12,34} + \lambda^2|11,00\rangle_{12,34}\right)$$

$$\sim \frac{1}{\sqrt{2}}(|00,11\rangle_{12,34} + |11,00\rangle_{12,34})$$

(Excluding vacuum state)

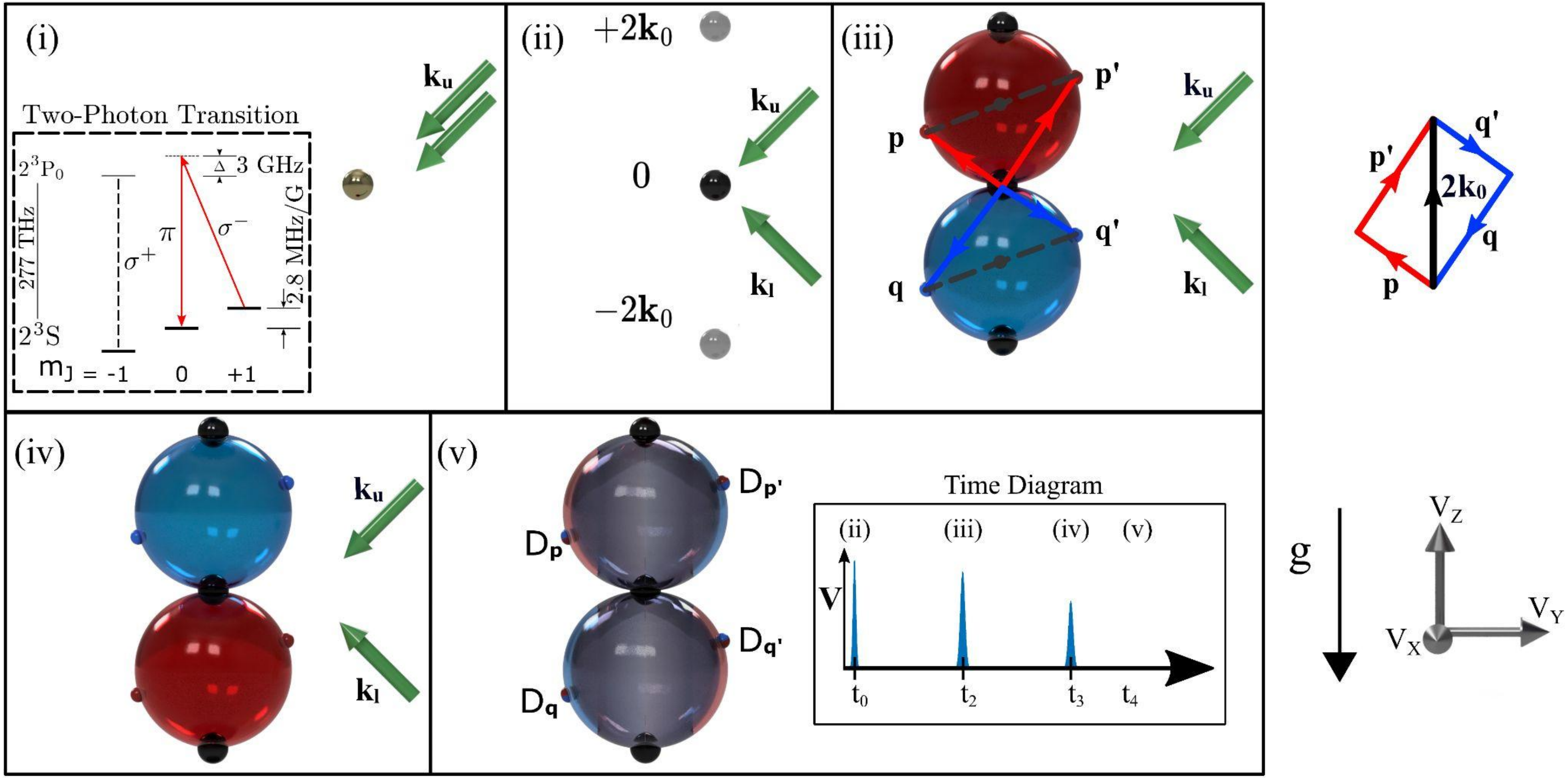
Prototypical Bell state

Experimental realisation



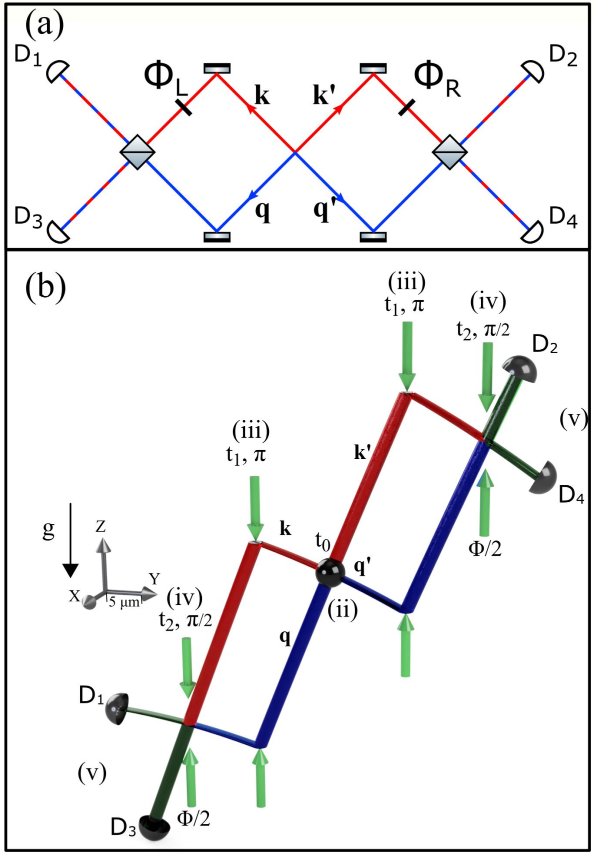
[12] Robert Lewis-Swan 2021, *Two halo Rarity-Tapster schematic*

Experiment Overview (Momentum Space)



[8] Thomas, K. F., Henson, B. M., Wang, Y., Lewis-Swan, R. J., Kheruntsyan, K. V., Hodgman, S. S., and Truscott, A. G. (2022). A matter wave Rarity-Tapster interferometer to demonstrate non-locality. *arXiv preprint arXiv:2206.08560.*

Position Space Picture



What do we expect to see?

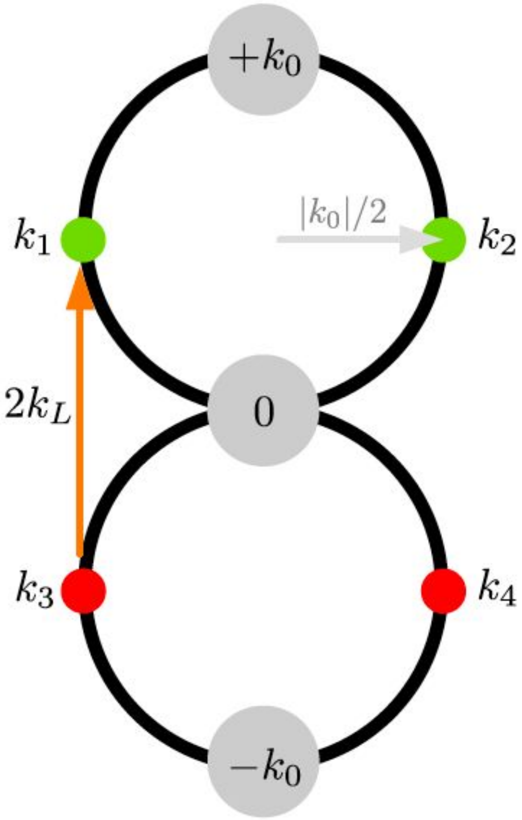
Two relevant parameters

$$C_{\text{between}} = C_{14} = C_{23}$$
$$C_{\text{same}} = C_{12} = C_{34}$$

Using a Gaussian analytic model, we find at the end of the interferometer

Correlation height

$$\frac{C_{\text{between}}}{N^2} = 1 + \frac{h}{16}\left[1 + \cos\left(\Phi_L + \Phi_R\right)\right] \alpha_x \alpha_y \beta_z \prod_d \lambda_d^{-2},$$
$$\frac{C_{\text{same}}}{N^2} = 1 + \frac{h}{16}\left[1 - \cos\left(\Phi_L + \Phi_R\right)\right] \alpha_x \alpha_y \beta_z \prod_d \lambda_d^{-2}.$$



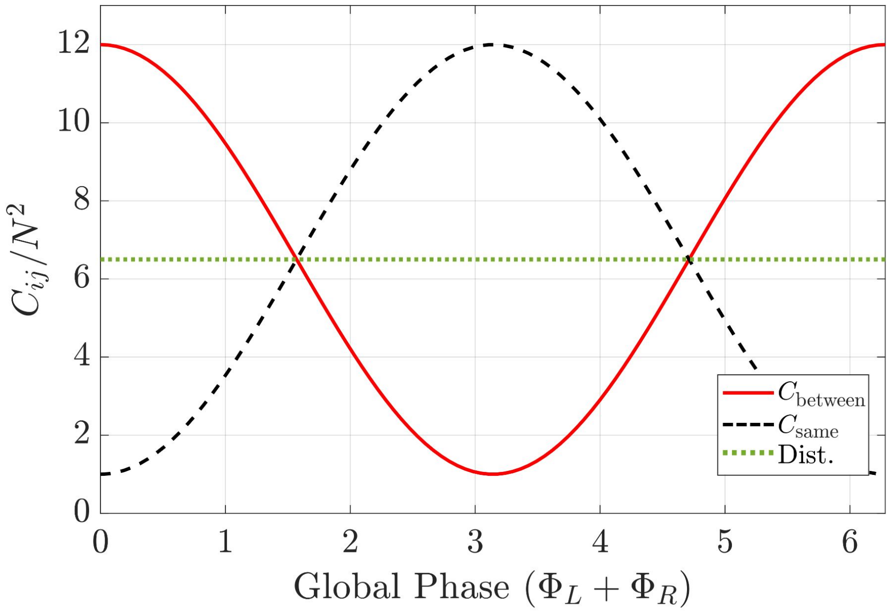
Finite correlation length, and spatial overlap effects

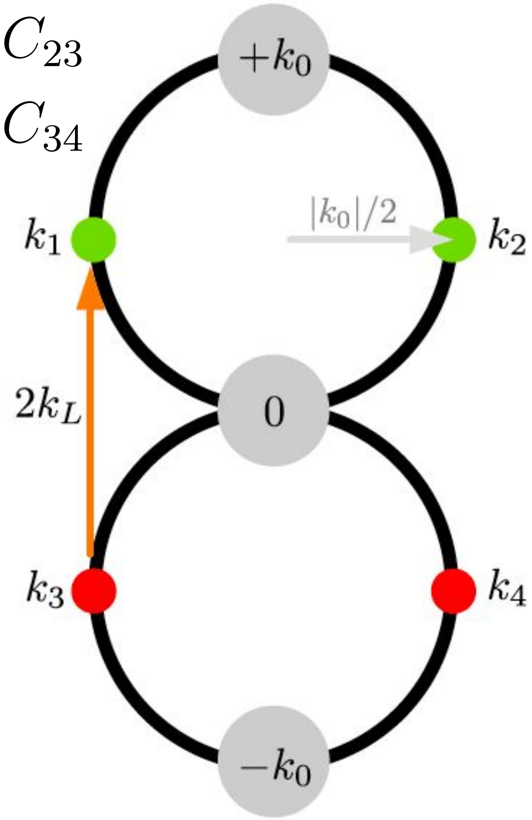
[8] Thomas, K. F., et al. (2022). A matter wave Rarity-Tapster interferometer to demonstrate non-locality. *arXiv preprint arXiv:2206.08560.*

What do we expect to see?

Quantum interference

$$C_{\text{between}} = C_{14} = C_{23}$$
$$C_{\text{same}} = C_{12} = C_{34}$$





Correlation amplitude of scattering halo

$$g^{(2)}(\Delta k) = \frac{\left\langle \hat{N}_k \hat{N}_{-k+\Delta k} \right\rangle}{\left\langle \hat{N}_k \right\rangle \left\langle \hat{N}_{-k+\Delta k} \right\rangle}$$

asuming $\left\langle \hat{N}_k \right\rangle \approx N \ \forall k$

and using the <u>two mode squeezed state</u>

$$|\Psi\rangle = \bigotimes_{\mathbf{k}} \sqrt{1-\lambda^2} \sum_{n=0}^{\infty} \lambda^n |n,n\rangle_{\mathbf{k}-,\mathbf{k}+}$$

gives

$$g^{(2)}(0) = h + 1 \sim 1 + \frac{1}{N} \quad \Rightarrow h \sim \frac{1}{N}$$

For Bell violation (ignoring all other effects):

$$\therefore g^{(2)}(0) > 2\sqrt{2} + 3$$ [7]

(or)

$$N < \frac{(\sqrt{2} - 1)}{2}$$

[7] R. J. Lewis-Swan and K. V. Kheruntsyan, Proposal for a motional-state Bell inequality test with ultracold atoms,

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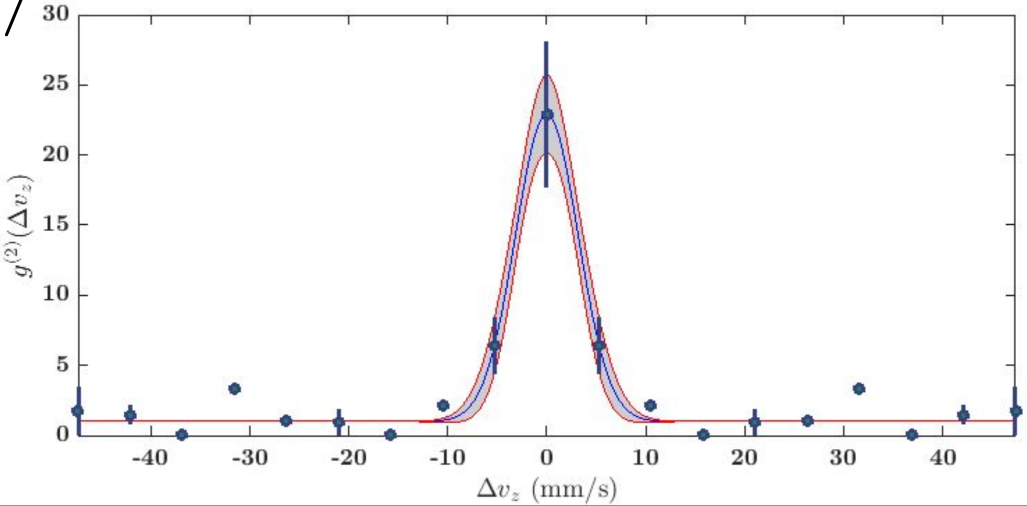
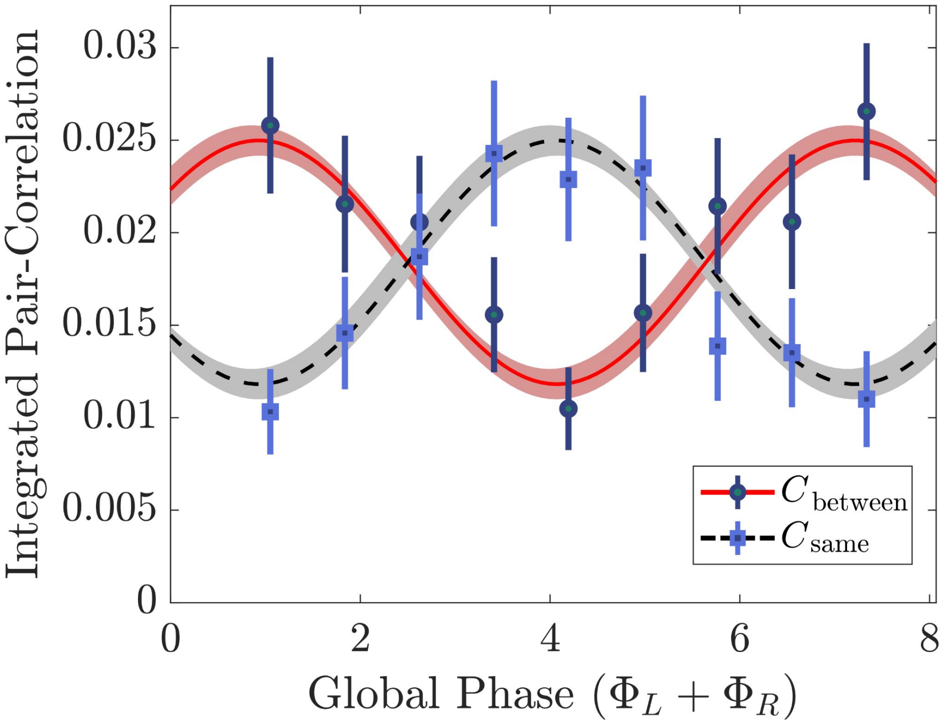
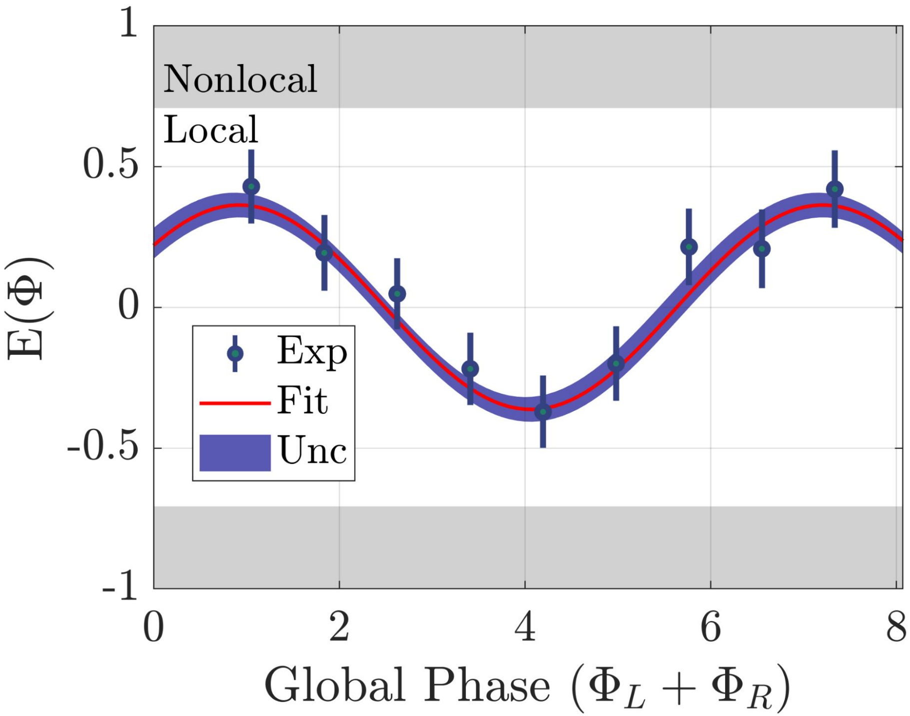
$$g^{(2)}(0) > 2\sqrt{2} + 3$$ [7]



[7] R. J. Lewis-Swan and K. V. Kheruntsyan, Proposal for a motional-state Bell inequality test with ultracold atoms,

Results (Rarity-Tapster)





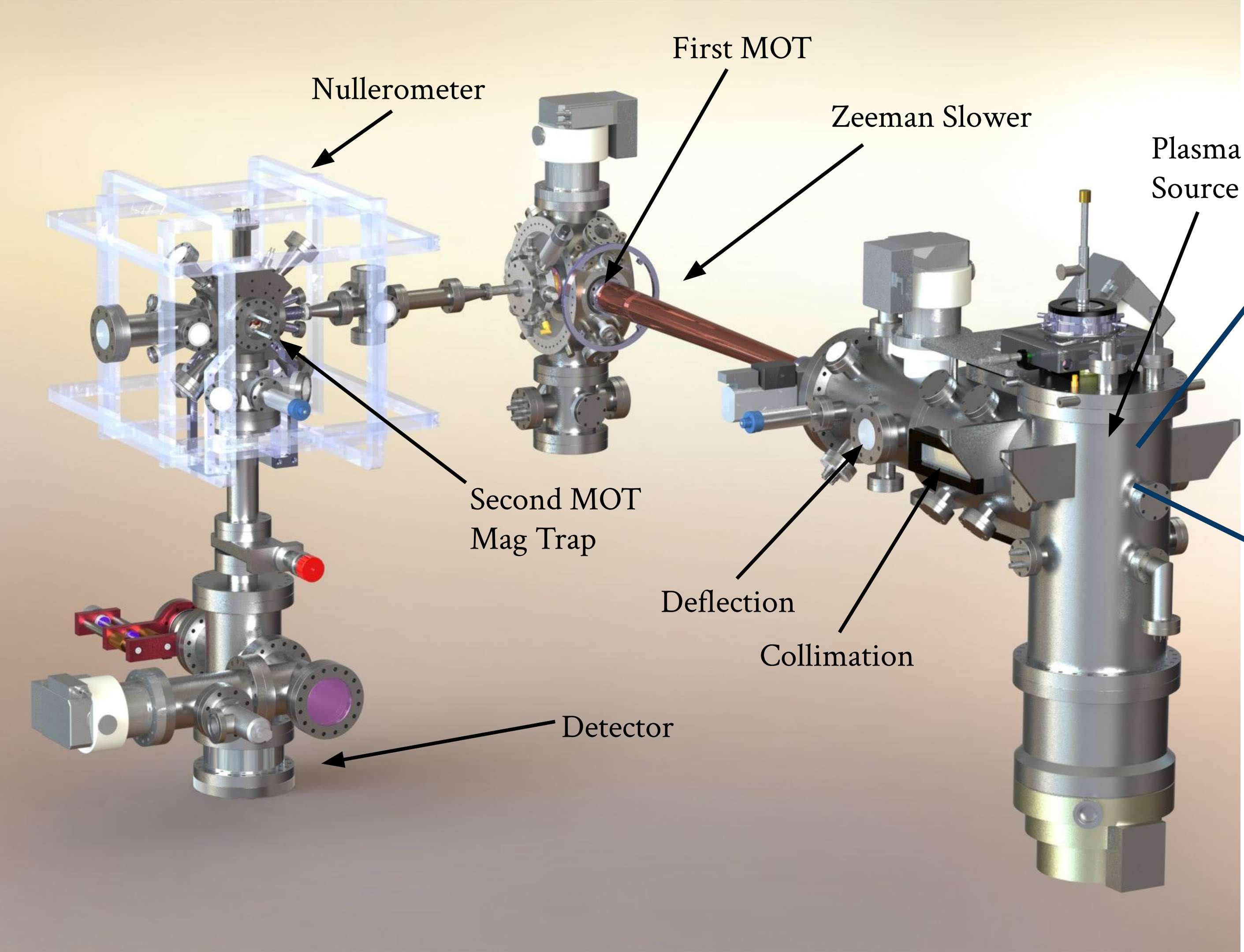
Each point is ~2000 realisations (or 17 hours of run time)

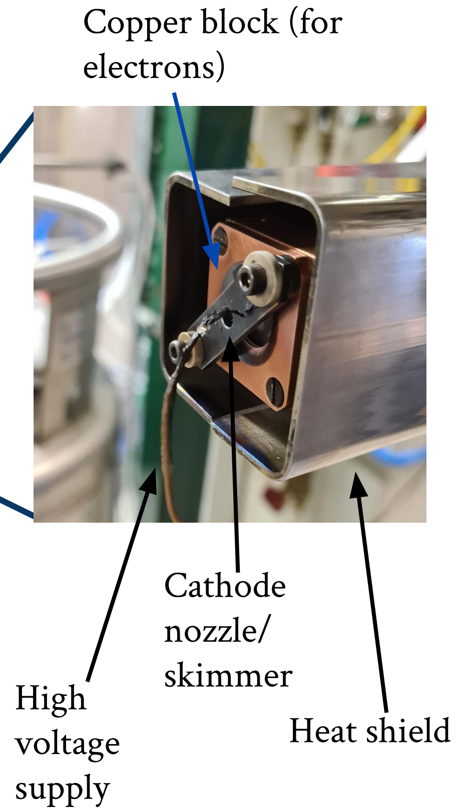
Future plans

- Further improve interference curve
- Implement phase beam for full Rarity-Tapster Bell test
- Perform a quantum eraser

Appendix

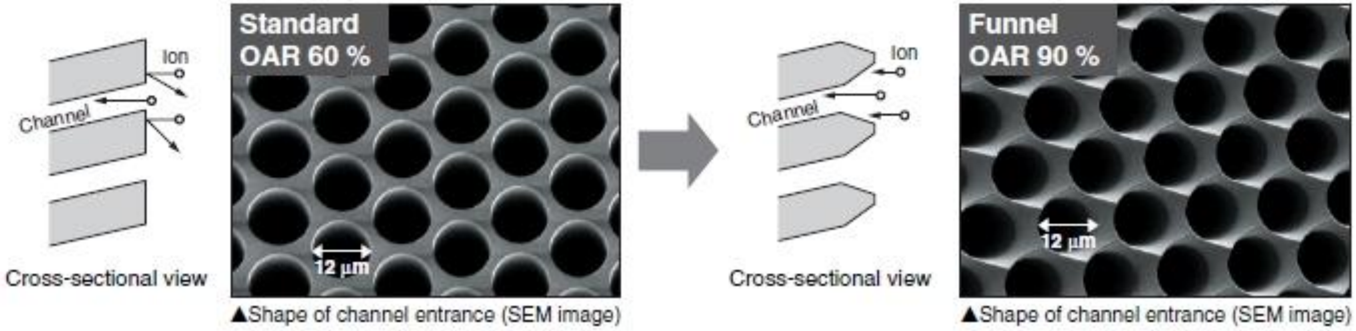
Our apparatus





Funnel Type MCP

Significantly higher quantum efficiency

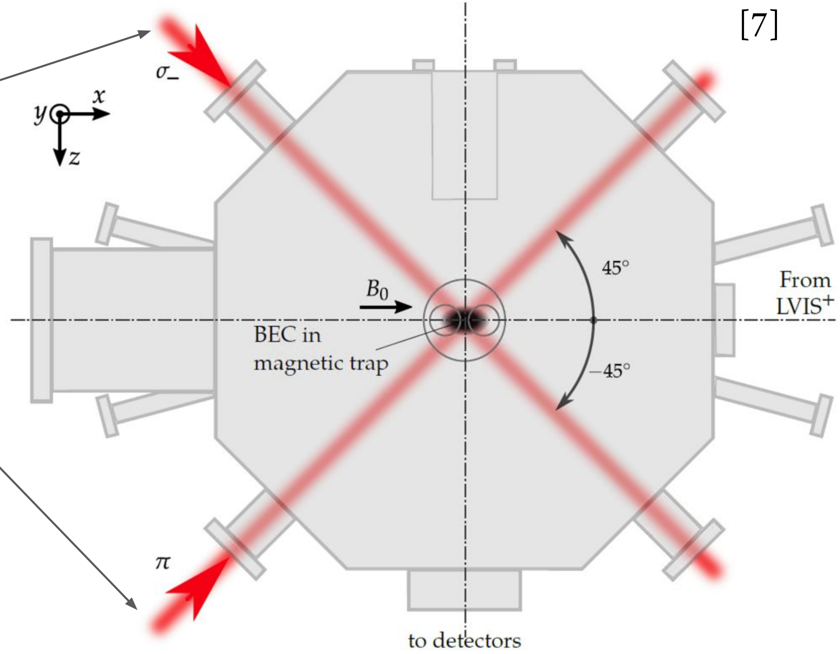


The data rate of the relevant parameters scale with the square of quantum efficiency

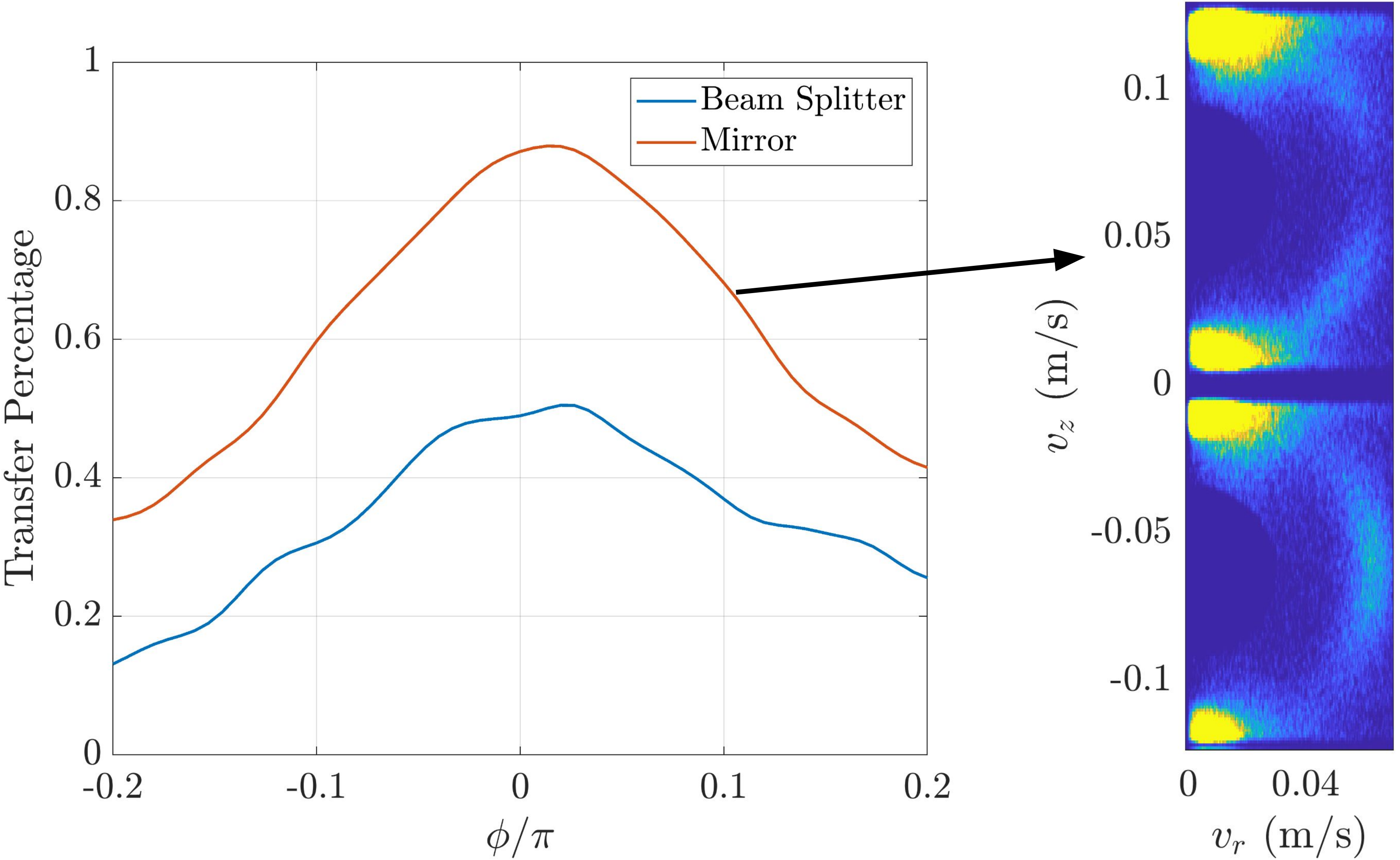
Bragg pulses

Schematic of beams:

Intensity of beams AOM controlled



[7] R. I. Khakimov 2016, *Source of correlated atom pairs*, Figure 5.2: Schematic of the Bragg/Raman beams entering the vacuum chamber.



Beam Splitter
Mirror
Transfer Percentage
ϕ/π
v_z (m/s)
v_r (m/s)
1
0.8
0.6
0.4
0.2
0
-0.2
-0.1
0
0.1
0.2
0.1
0.05
0
-0.05
-0.1
0
0.04

Quantum Correlator (E) dependence on integration size