

# Berezinskii-Kosterlitz-Thouless (BKT) transitions in a ferromagnetic superfluid

Andrew Underwood, Andrew Groszek, Xiaoquan Yu, Blair Blakie, Lewis Williamson



DODD-WALLS CENTRE  
for Photonic and Quantum Technologies



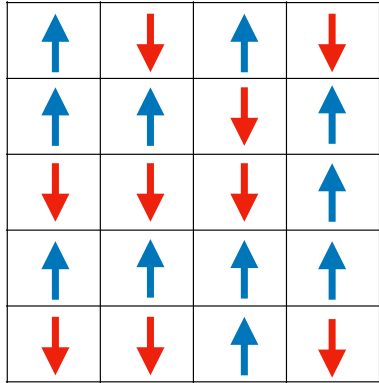
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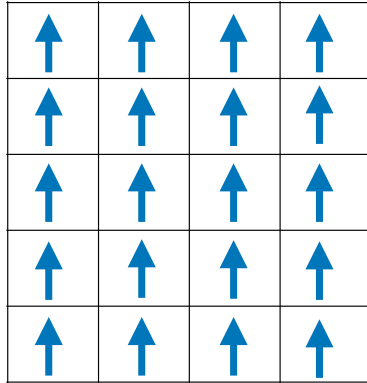
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$$T > T_c$$

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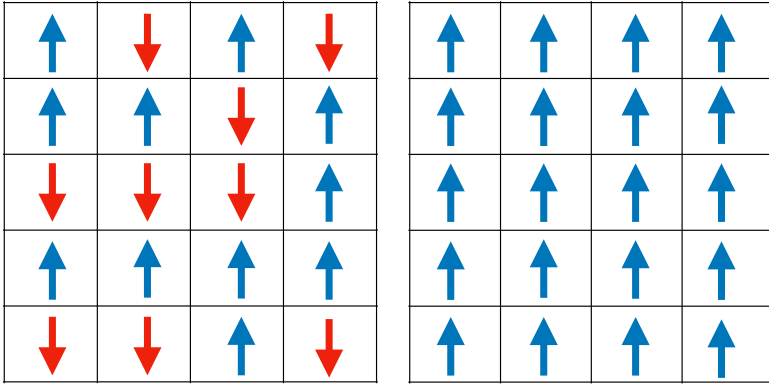
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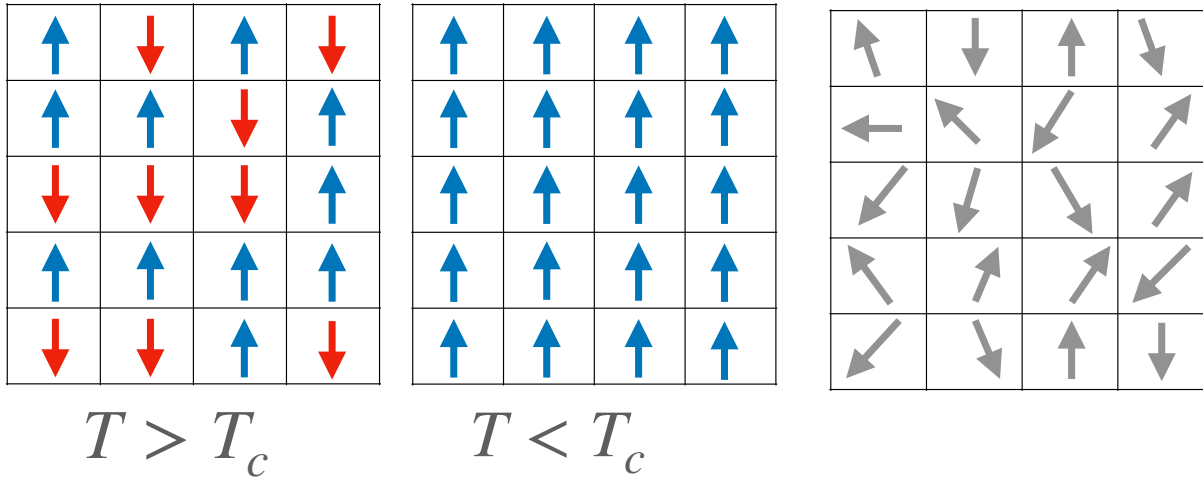


Lev Landau

Phase transitions involve changes of symmetry (order)

L. Landau Zh. Eksp. Teor. Fiz. 7 19-32 (1937)

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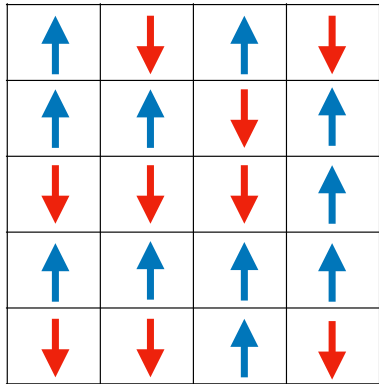


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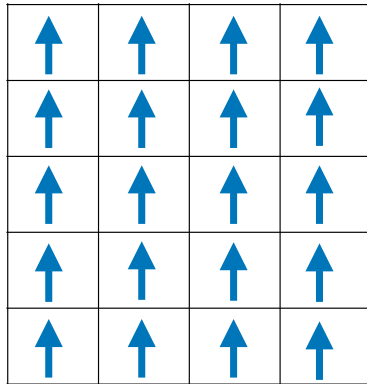
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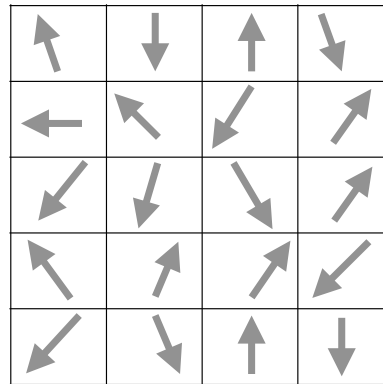
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2D systems with continuous symmetry show no long-range order

N. D. Mermin and H. Wagner Phys. Rev. Lett. **17** 1133-1136 (1966)

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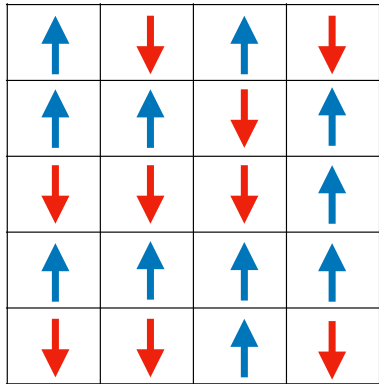


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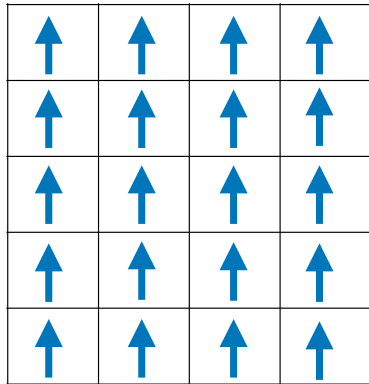
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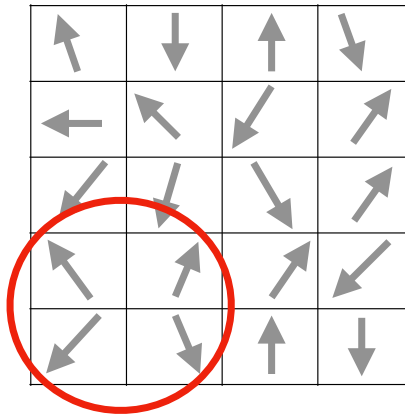
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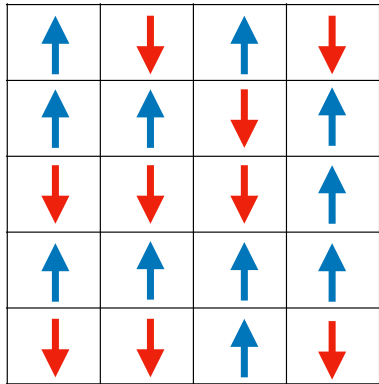


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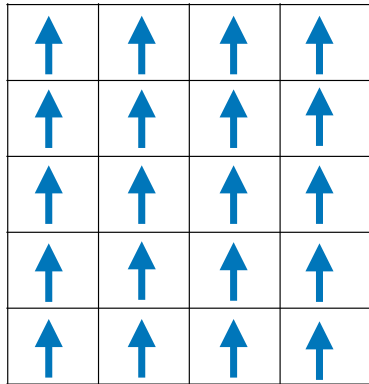
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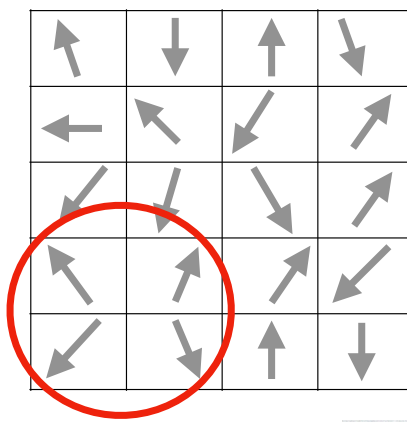
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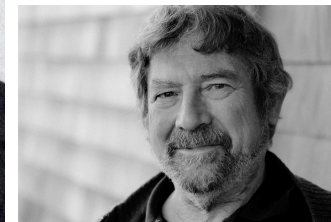
Topological phase transitions

V. L. Berezinskii Sov. Phys. JETP **32** 493-500 (1971); **34** 610-616 (1972)

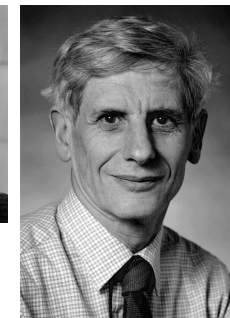
J. M. Kosterlitz and D. J. Thouless J. Phys. C **6** 1181-1203 (1973)



Vadim Berezinskii

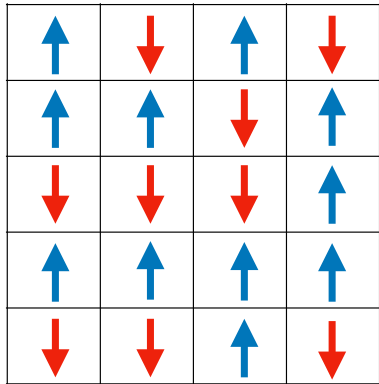


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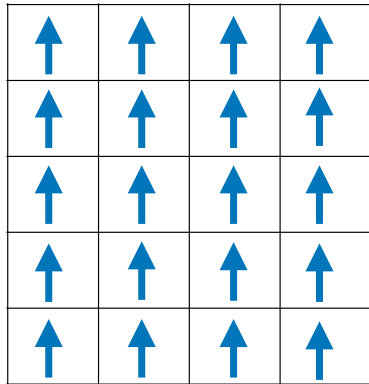


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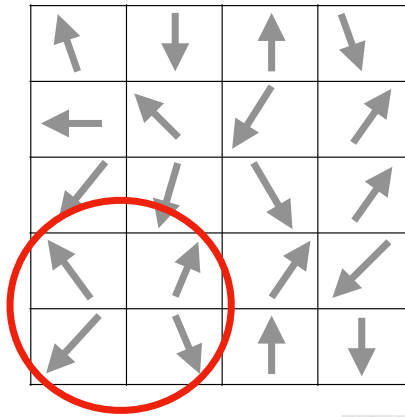
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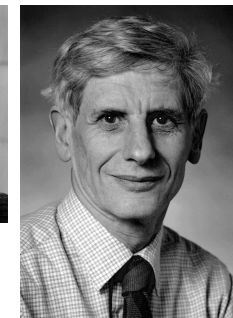
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Interacting systems with U(1) symmetries exhibit BKT transitions in 2D.  
Below the BKT temperature the system is superfluid.

# Examples

## Helium-4

D. J. Bishop and J. D. Reppy Phys. Rev. Lett. **40** 1727 (1978)

## Weakly interacting atomic BECs

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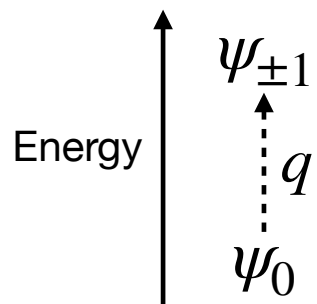
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Energy ↑  
 $\psi_{0,\pm 1}$

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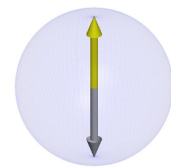
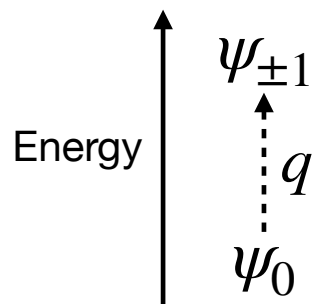
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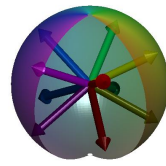


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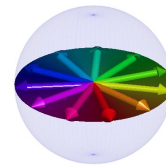
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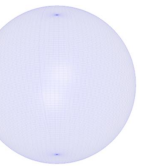
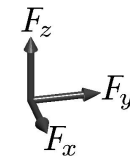
Easy-axis



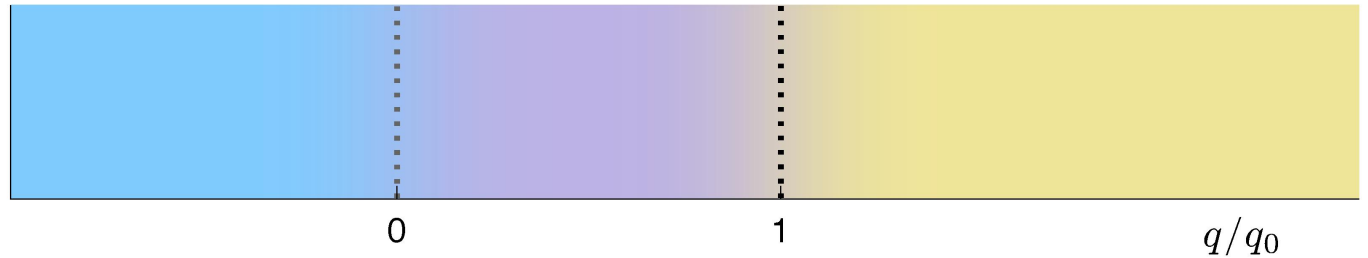
Isotropic



Easy-plane

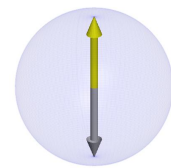
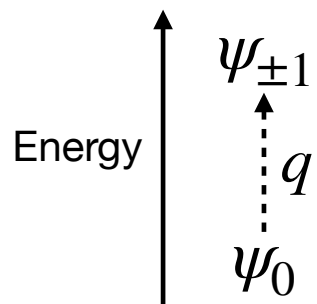


Polar

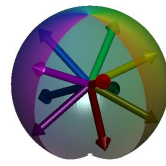


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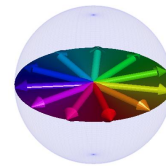
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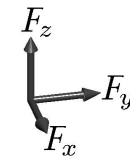
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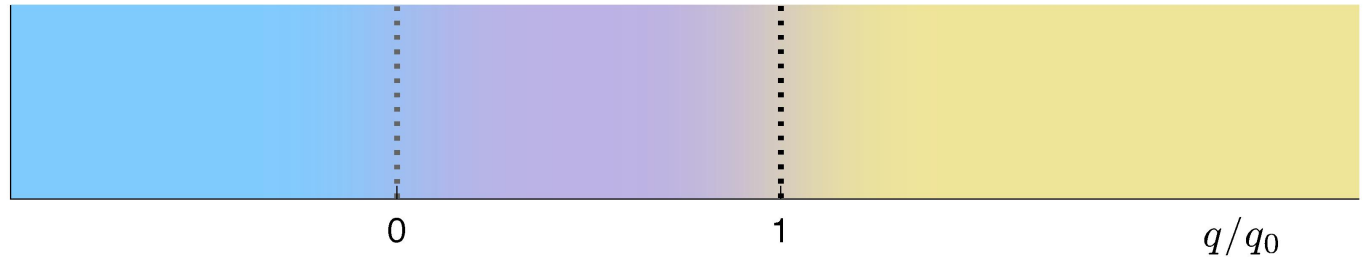
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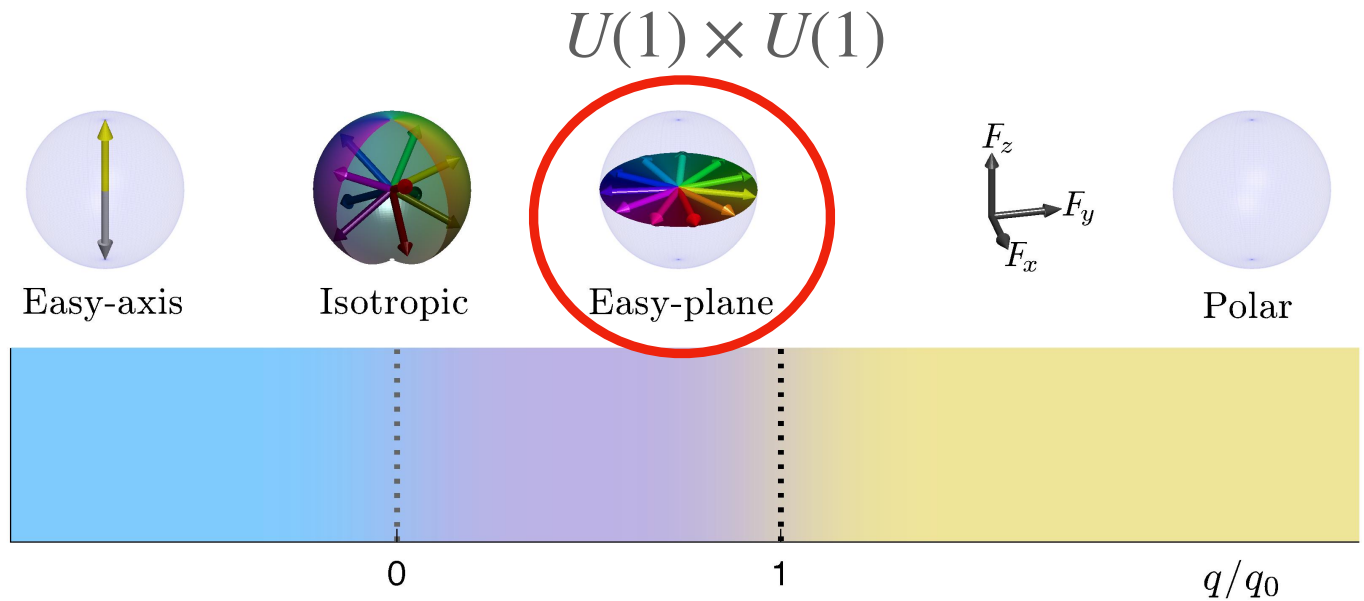
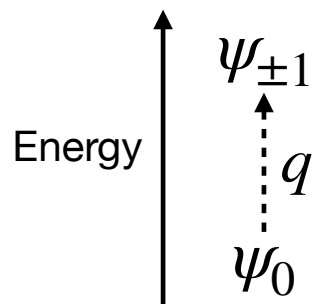
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Y. Kawaguchi and M. Ueda Phys. Rep. **520** 253-381 (2012)

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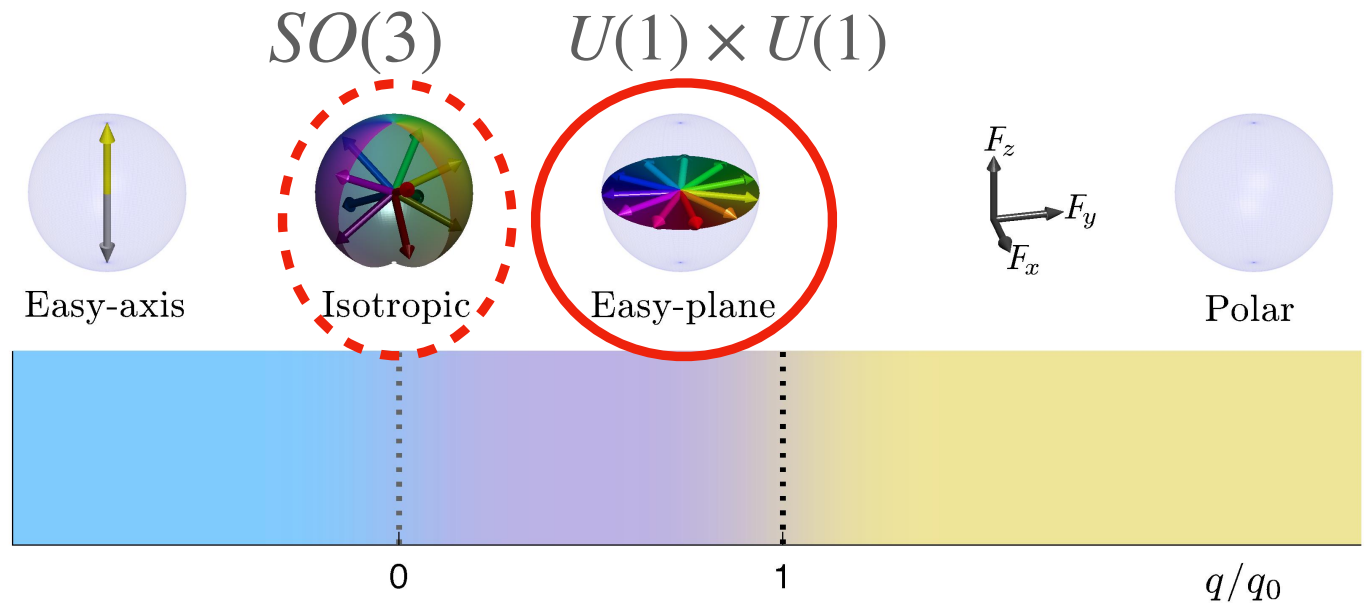
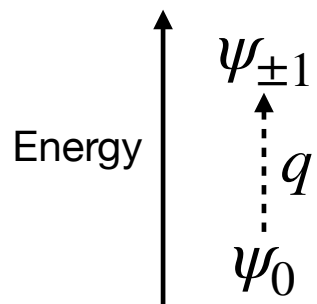
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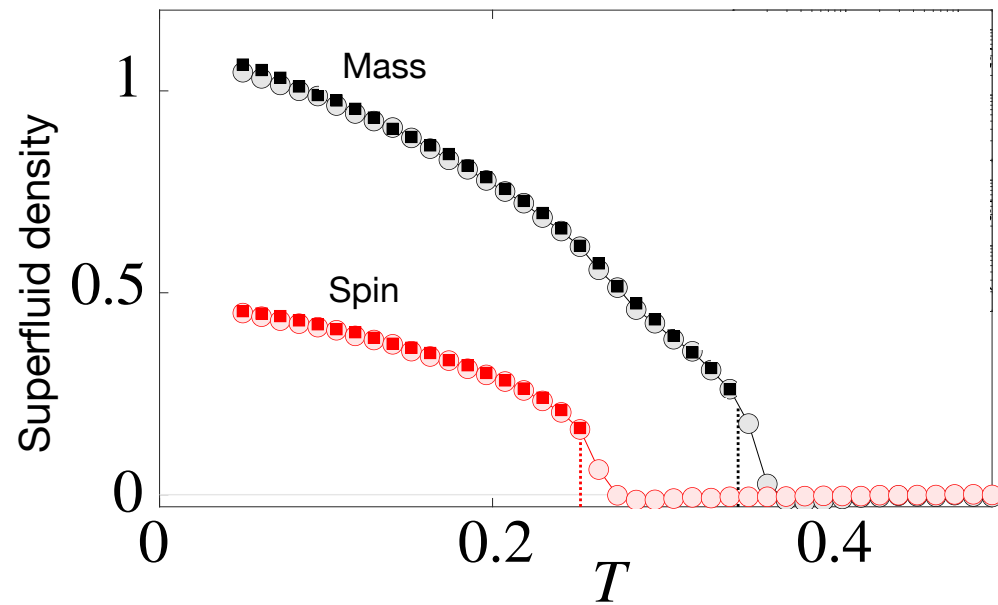
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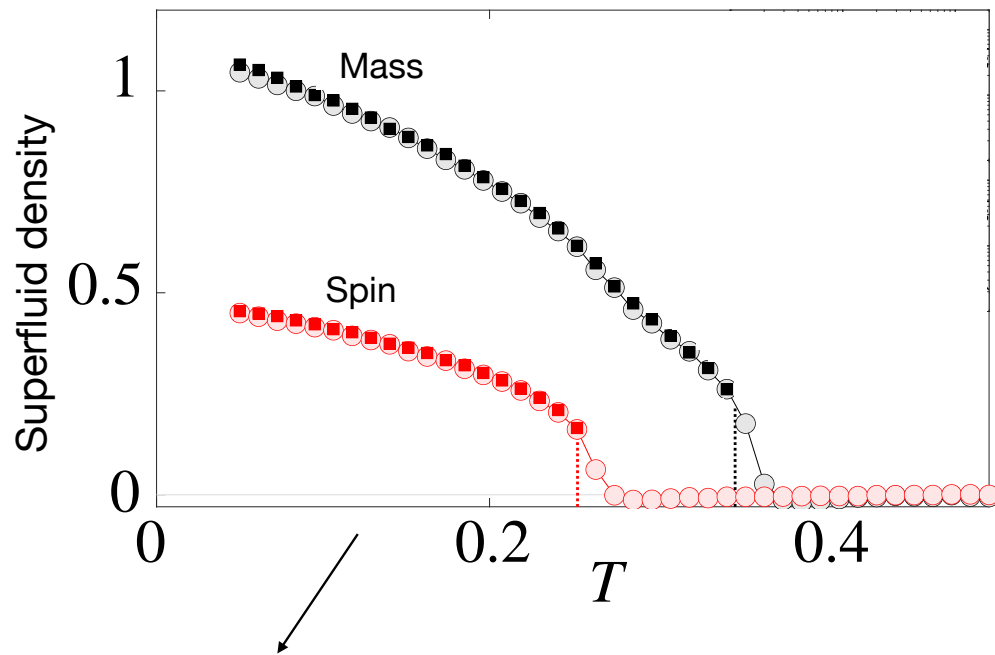
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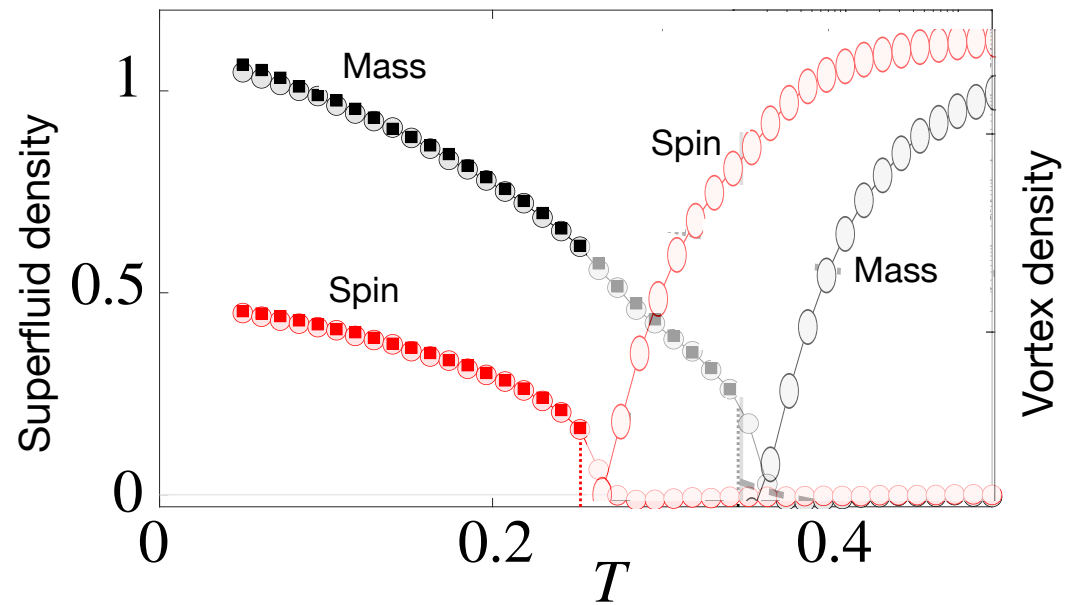
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Also in: M. Kobayashi J. Phys. Soc. Jpn. **88** 094001 (2019)

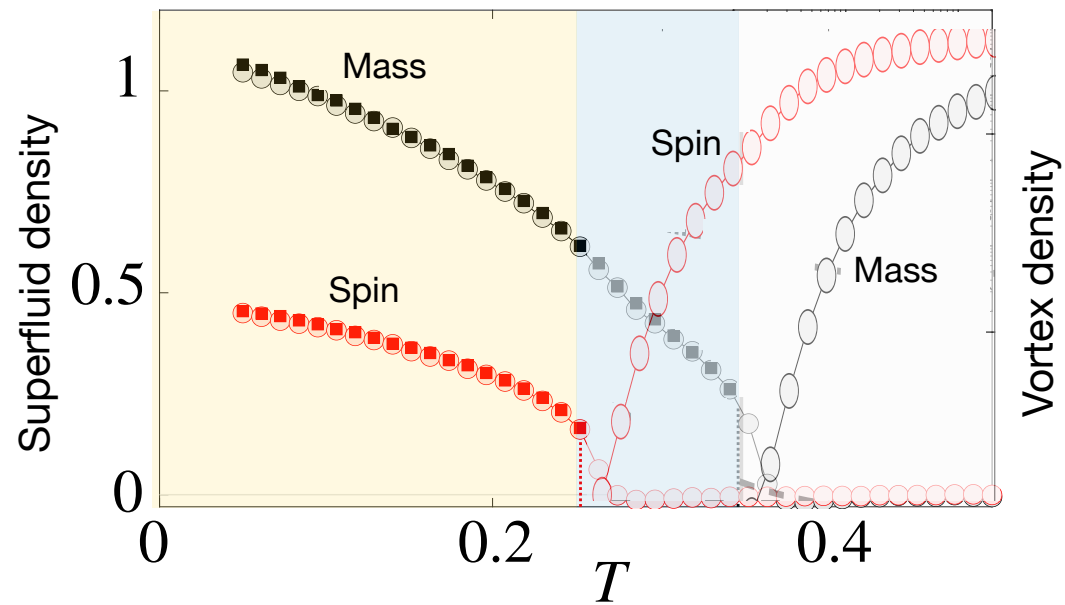
A. P. C. Underwood et al. arXiv:2207.14497

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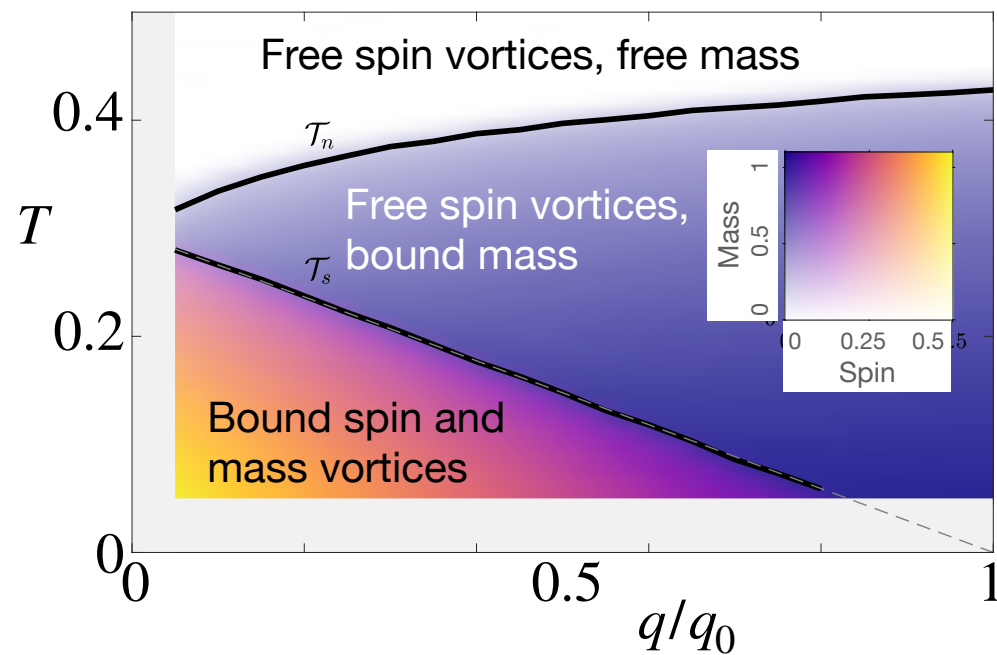
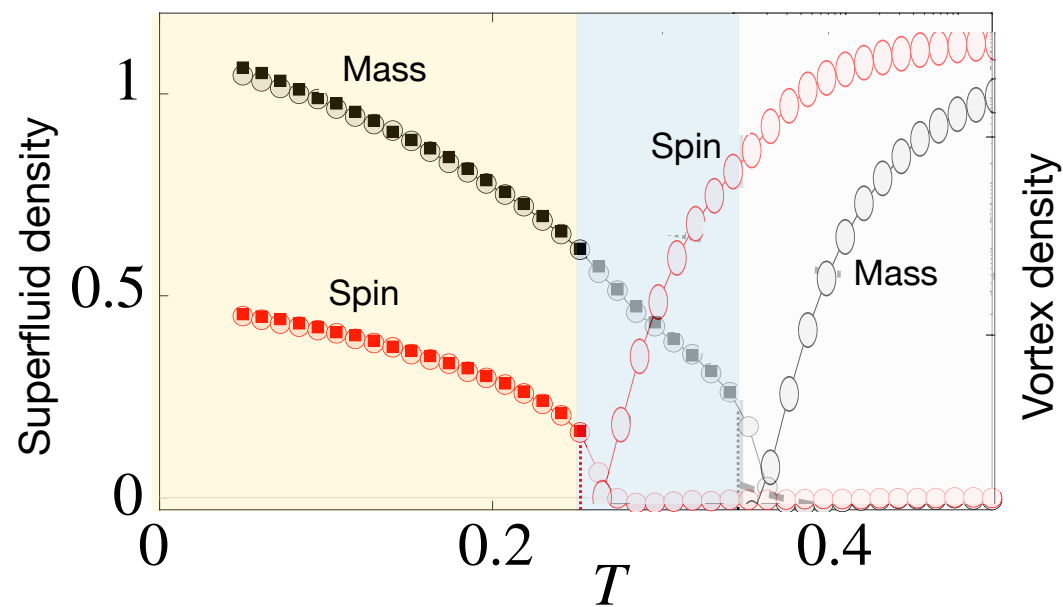




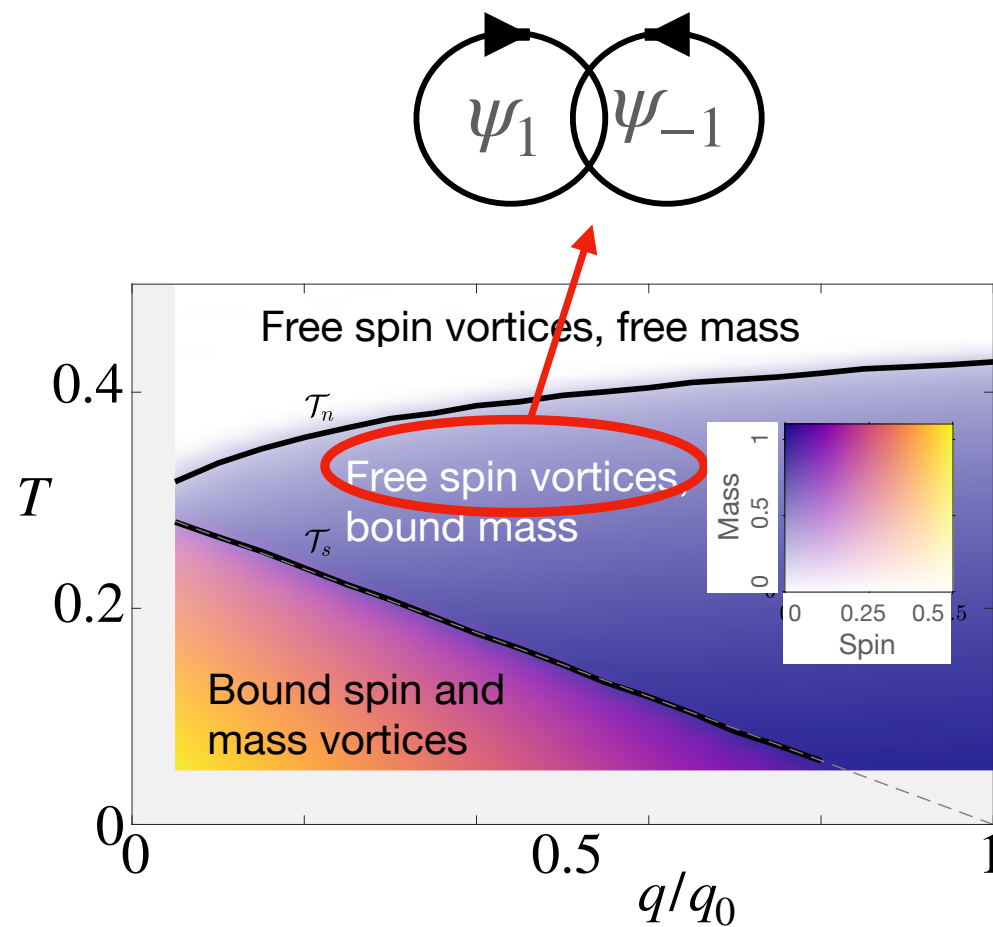
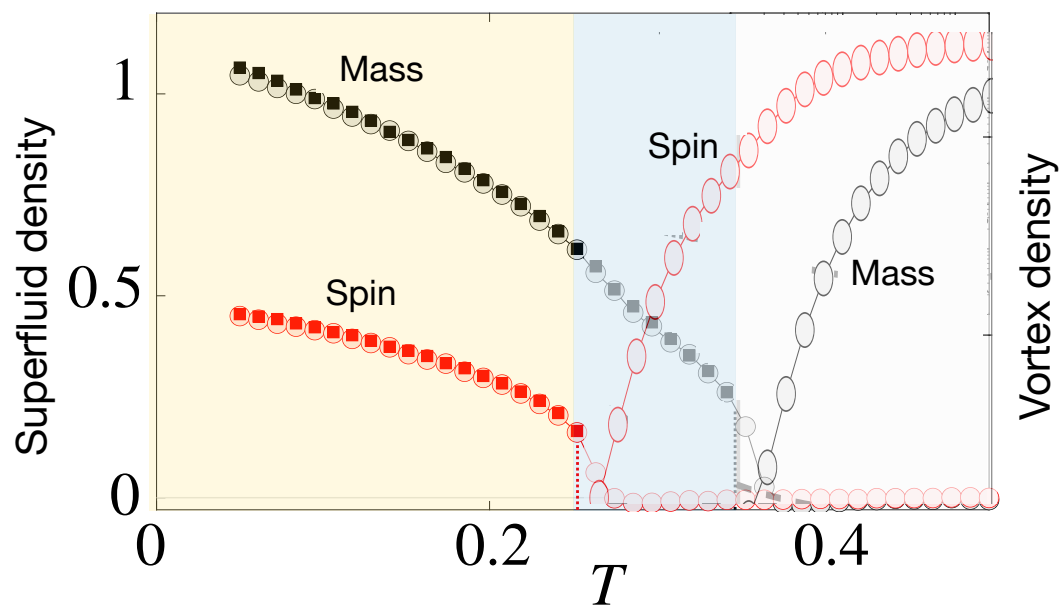
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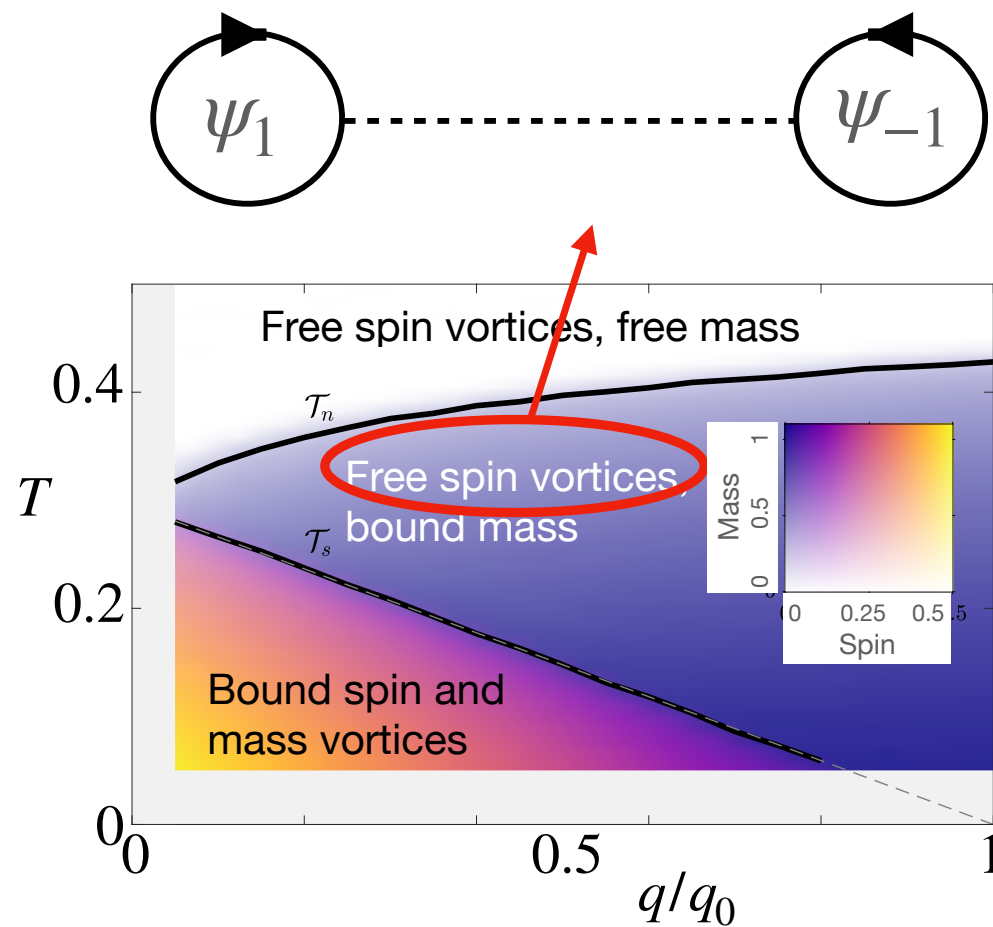
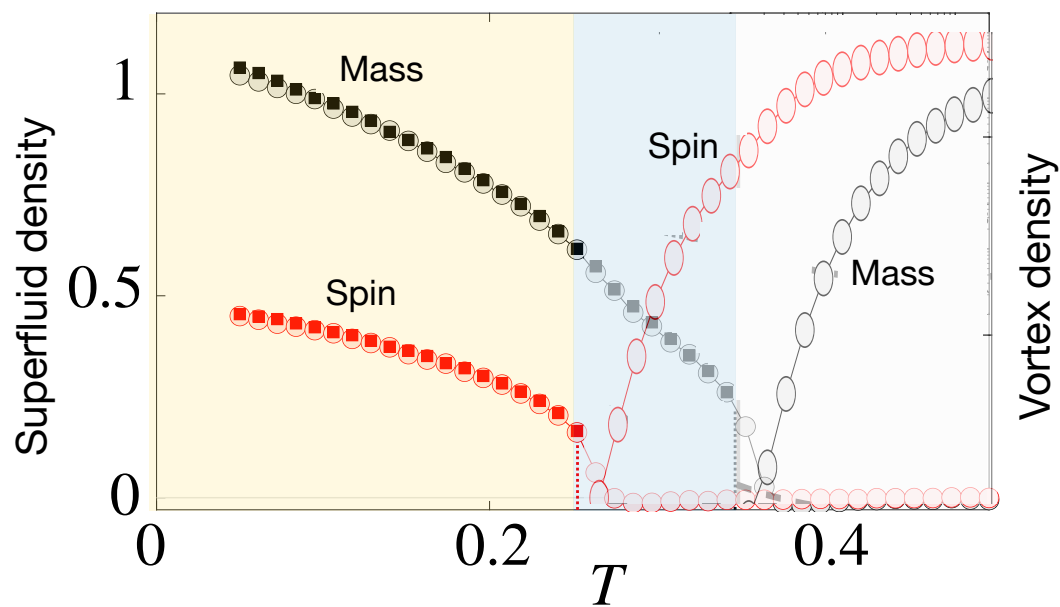
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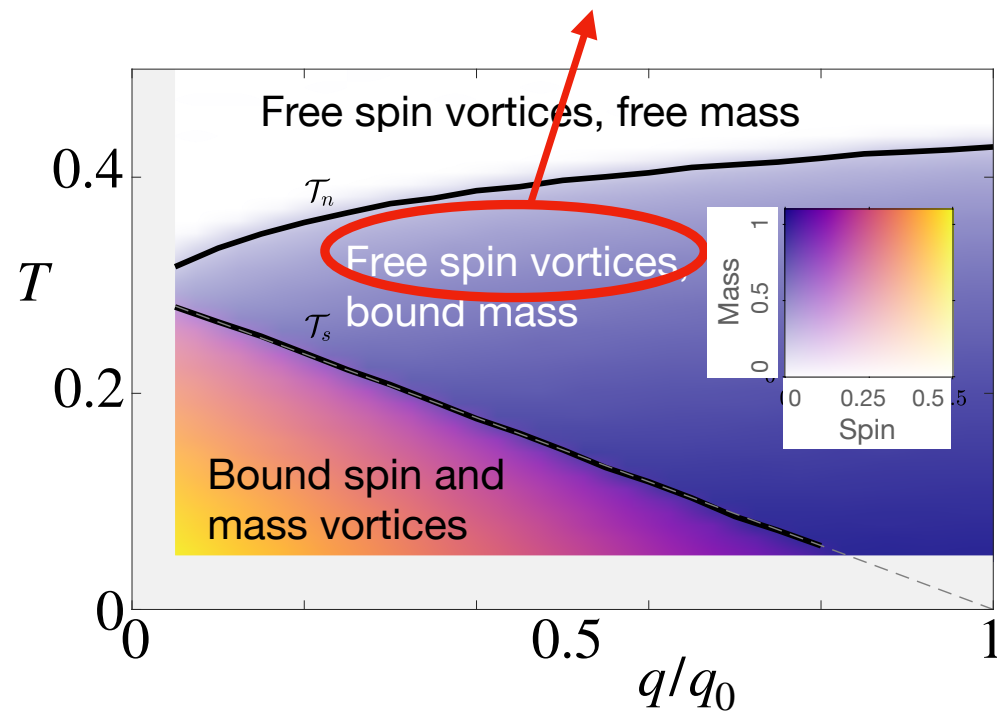
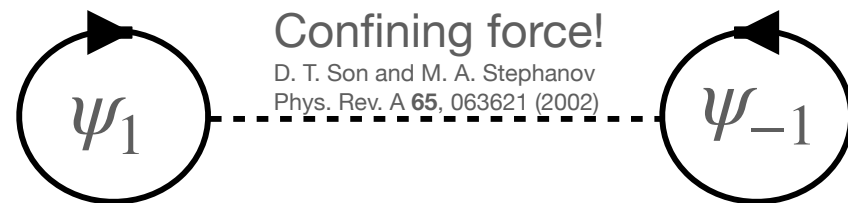
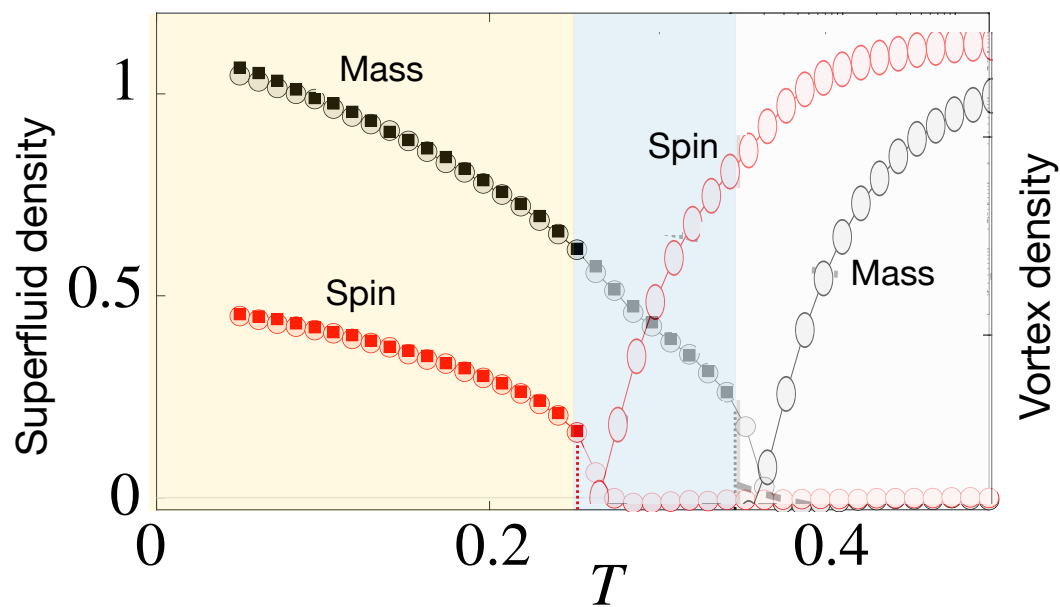
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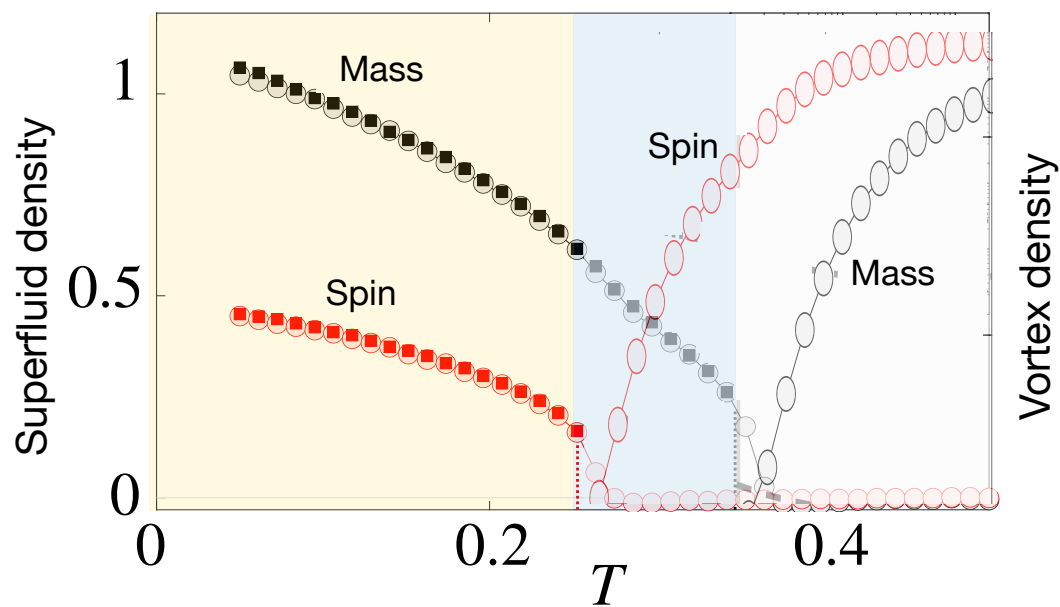
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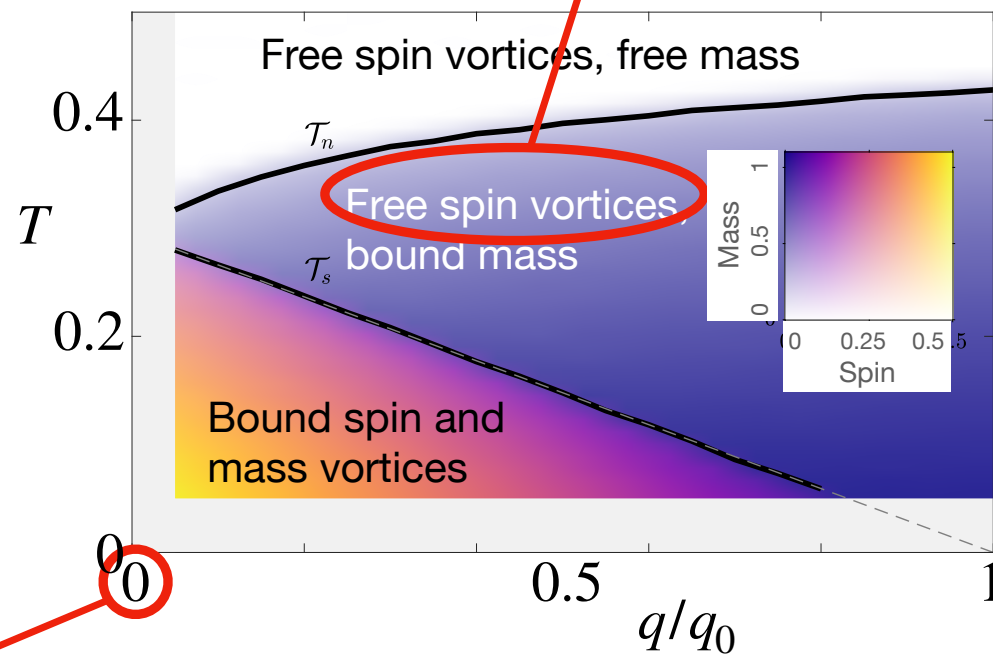
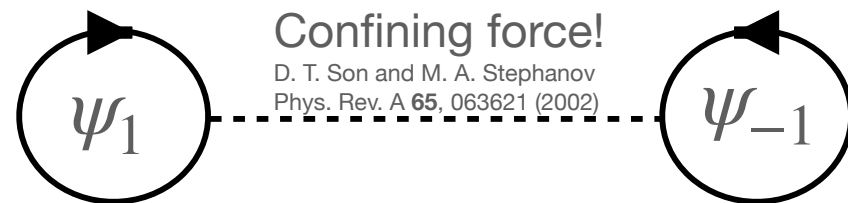
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Fluctuations sensitive to system size:  
no BKT transition in thermodynamic limit.  
But in any finite system...



0

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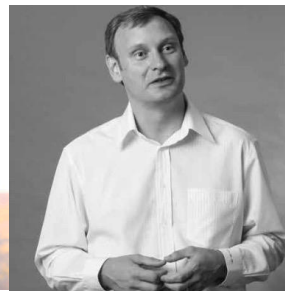
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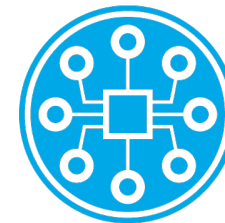
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Xiaoquan Yu



Blair Blake



**EQUUS**  
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Engineered Quantum Systems

Thanks for listening!