Accurate determination of the magnetic hyperfine anomaly in atomic caesium from muonic-atom experiments

G. Sanamyan, B. Roberts, and J. Ginges

School of Mathematics and Physics, The University of Queensland, Brisbane QLD 4072, Australia

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Outline

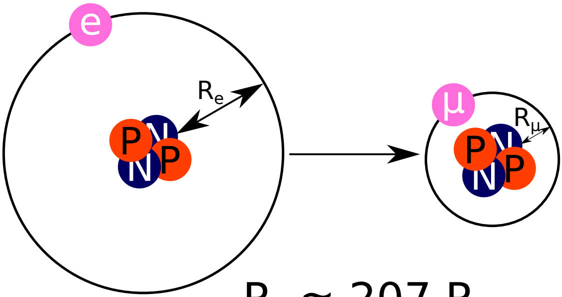
Motivation

To increase the precision of atomic calculations

- Push atomic calculations to 0.1% precision,
  - Improved benchmarking of atomic theory,
- Remove nuclear structure uncertainties (most limiting),
  - Predictions of different nuclear models deviate by 0.5%,
- Muonic atoms are highly sensitive to nuclear structure,
  - Have been used historically to study nuclear structure,
  - New muonic atom experiment underway at Paul Scherrer Institute.

Muonic atoms

Muonic atoms?



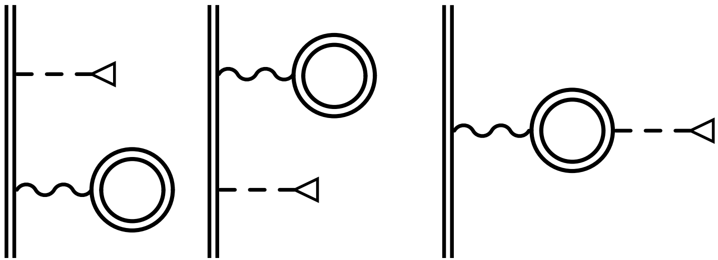
$$R_e \approx 207\ R_\mu$$

$$\left(c\boldsymbol{\alpha}\cdot\boldsymbol{p} + (\beta - 1)\, m_\mu c^2 + V_{\text{nuc}}(r)\right)\phi_{n\kappa m} = \varepsilon\phi_{n\kappa m}$$

Muonic atoms

QED effects

QED effects are dominated by vacuum polarisation consisting of electric and magnetic loop contributions:



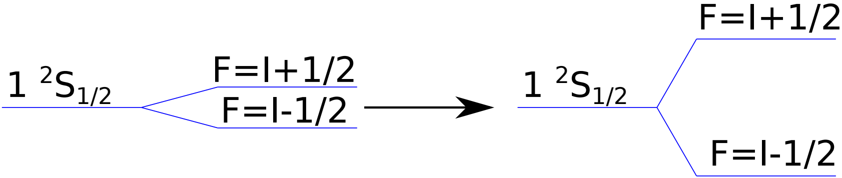
$$V_{\text{nuc}}(r) \to V_{\text{nuc}}(r) + V^{\text{EL}}_{\text{Ueh}}(r) \qquad h_{\text{hfs}}(r) \to h_{\text{hfs}}(r) + h^{\text{ML}}_{\text{Ueh}}(r)$$

Muonic atoms

Hyperfine structure in muonic atoms

Interaction between the spin of a muon (or electron) and nuclear magnetic moments → hyperfine splitting

Closer proximity to the nucleus → larger hyperfine splitting:



- Bohr-Weiskopf (BW) effect (on the order of 1% of the hyperfine constant in electronic system compared to the order of 100% in muonic systems).

Hyperfine structure

Calculations of hyperfine structure

Relativistic expression for the hyperfine interaction is

$$h_{\text{hfs}} = \alpha \frac{\boldsymbol{\mu} \cdot (\boldsymbol{r} \times \boldsymbol{\alpha})}{r^3} F(r),$$

$\boldsymbol{\mu}$ - nuclear magnetic moment
$F(r)$ - describes the nuclear magnetisation distribution:

$$F(r) = \left(\frac{r}{R_m}\right)^3 \Theta(R_m - r) + \Theta(r - R_m) \text{ - uniform spherical model}$$

$$F(r) = \left(\frac{r}{R_m}\right)^3 \left[1 + (\alpha_L - \alpha_S \xi) \ln \left(\frac{r}{R_m}\right)^3\right] \Theta(R_m - r) + \Theta(r - R_m)$$

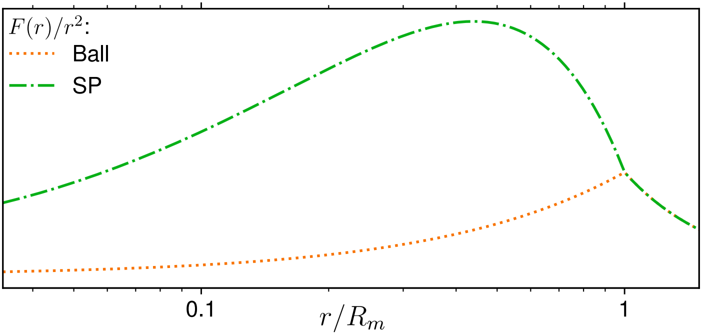
- nuclear single particle model

$\Theta(r)$ - step function
R_m - effective magnetic nuclear radius
α_L, α_S, and ξ - orbital and spin fractions of magnetic moment and spin asymmetry term

Hyperfine structure

Sensitivity to $F(r)$ for muonic and electronic systems

$$\phi(\mathbf{r}) = \frac{1}{r}\begin{pmatrix} f(r)\Omega_{\kappa m}(\boldsymbol{n}) \\ ig(r)\Omega_{-\kappa m}(\boldsymbol{n}) \end{pmatrix} \rightarrow \mathcal{A}_{1s} = \frac{4}{3}\frac{\alpha}{I}\frac{1}{m_p}\frac{\mu}{\mu_N}\int_0^\infty dr\frac{f(r)g(r)}{r^2}F(r).$$



uniform spherical model - "Ball", nuclear single particle model - "SP"

Hyperfine structure

Sensitivity to $F(r)$ for muonic and electronic systems

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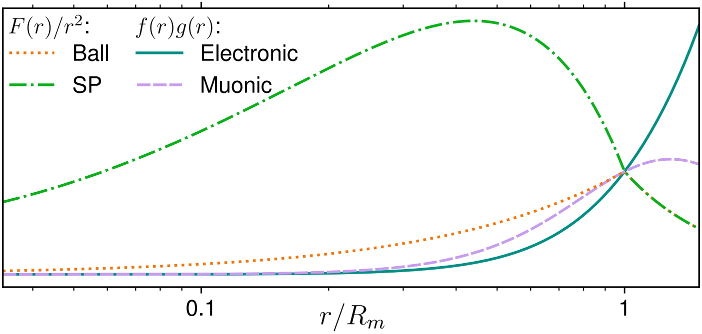

uniform spherical model - "Ball", nuclear single particle model - "SP"

Hyperfine structure

Extraction of $F(r)$ from muonic measurements

$$\left(c\boldsymbol{\alpha}\cdot\boldsymbol{p} + (\beta - 1)\, m_\mu c^2 + V_{\rm nuc}(r)\right)\phi_{n\kappa m} = \varepsilon\phi_{n\kappa m}$$

$\downarrow$ $f(r)g(r)$

$$\mathcal{A}_{1s} = \frac{4}{3}\frac{\alpha}{I}\frac{1}{m_p}\frac{\mu}{\mu_N}\int_0^\infty dr\,\frac{f(r)g(r)}{r^2}F(r)$$

$\downarrow$ $\mathcal{A}_{1s}$

Fit to muonic experimental measurements

Adjust R_m

Hyperfine structure

Relative BW effect in caesium

F(r) - Ball fitted to muonic expt. → Hyperfine constants in caesium atom ← F(r) - SP fitted to muonic expt.

A	I^π	ε (%)
131	$\frac{5}{2}^+$	0.69(19)
133	$\frac{7}{2}^+$	0.24(18)
134	4^+	0.41(18)
135	$\frac{7}{2}^+$	0.27(18)

Hyperfine structure of ^{133}Cs is crucial for APV studies. Our result reduces the nuclear structure uncertainty in the hyperfine structure down to 0.2%!

Final value of 0.24(18)% can be compared to predictions of different nuclear models: 0.67% (Ball), 0.21% (SP), 0.19(14)% (SP-WS), 0.22% (CM).

Summary

- Nuclear structure uncertainty in caesium hyperfine structure is reduced down to 0.2%,
  - The result for ^{133}Cs has implications for the error analysis of theoretical atomic calculations,
  - Further implications on previous ^{133}Cs hyperfine structure calculations,
  - Confirms the validity of the nuclear single-particle model (we advocate for the future use of single particle model in atomic calculations instead of the commonly used uniform spherical model),
  - Emphasises the importance of future muonic (and *H*-like) atom experiments for further reducing uncertainties in BW effect.