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Second sound with ultracold atoms

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Theory: PRA **98**, 011602(R) (2018) & *Experiment*: Science **375**, 528 (2022)

A short review: AAPPS Bulletin **32**, 26 (2022)

A$: Australian Government
Australian Research Council

Acknowledgements

Prof. Xia-Ji Liu
(Swinburne University)

A/Prof. Peng Zou
(Qingdao University)

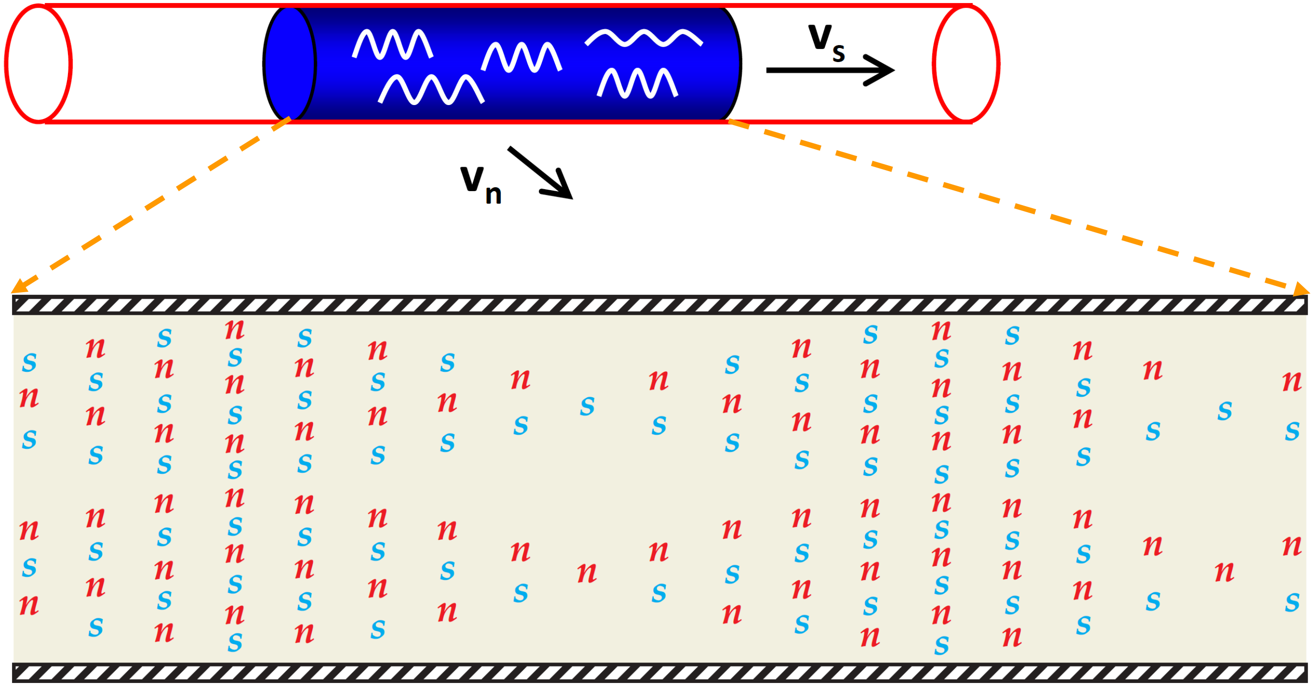
Prof. XingCan Yao
and his group
(USTC, Shanghai)

Outline of the talk

- **First sound and second sound (^{4}He)**
- **New opportunity: the novel unitary Fermi superfluid**
- **Second sound propagation**
- **Second sound attenuation & quantum transport near the quantum critical regime**
- **Conclusions**

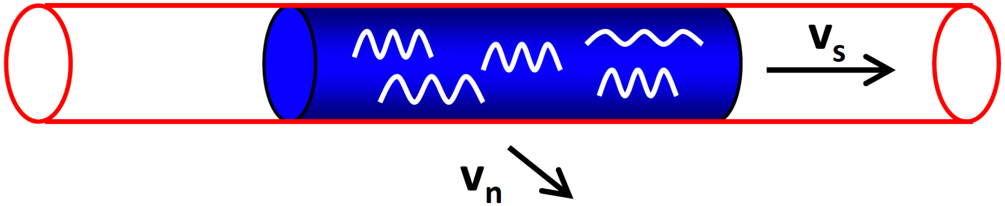
Two-fluid model of superfluid helium (4He)

Superfluid helium moving in a tube at nonzero temperature (= ***s*** + ***n***)



Laszlo Tisza

Two-fluid model of superfluid helium (4He)



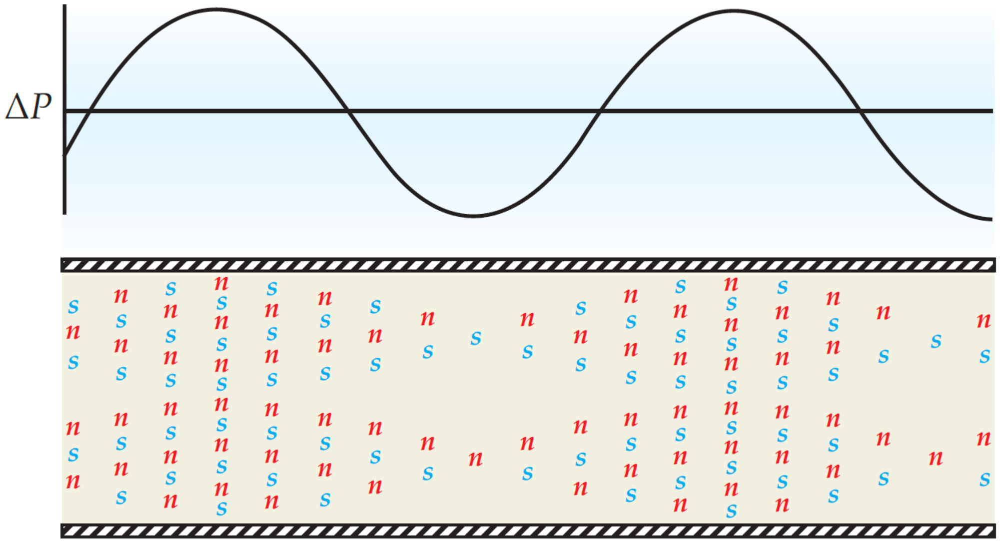
$$n = n_n + n_s$$

normal part superfluid part

Laszlo Tisza

- Superfluid component (s) moving with velocity $\mathbf{v}_s$ - zero viscosity, zero entropy
- Normal component (n) moving with velocity $\mathbf{v}_n$ - finite viscosity, entropy carrier

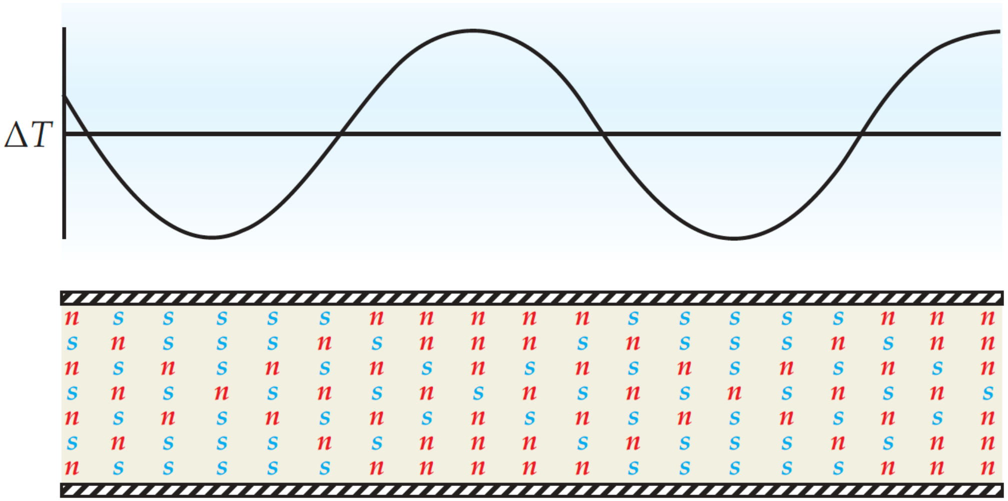
First sound and second sound (4He)



First ordinary sound: **in-phase motion**

density wave

$$c_1 = \sqrt{\frac{1}{m}\left(\frac{\partial P}{\partial n}\right)_{\boxed{\bar{s}}}}$$



Second sound: **out-of-phase motion**

entropy or temperature wave

$$c_2 = \sqrt{\frac{1}{m}\frac{T\bar{s}^2}{\boxed{\bar{c}_p}}\boxed{\frac{n_s}{n_n}}}$$

Picture credit: Russell Donnelly, Phys. Today **62**, 34 (2009).

Significance of second sound

Why second sound is so important?

- **A critical mode describes the dynamics of the order parameter**: it is a temperature **wave**, instead of a thermal diffusion

- **A hallmark of superfluidity** – the second sound velocity directly measures the **superfluid density** n_s

$$c_2 = \sqrt{\frac{1}{m}\frac{T\bar{s}^2}{\boxed{\bar{c}_p}}\boxed{\frac{n_s}{n_n}}}$$

- Second sound attenuation (i.e., sound diffusivity D_2) determines the crucial **quantum transport coefficients**

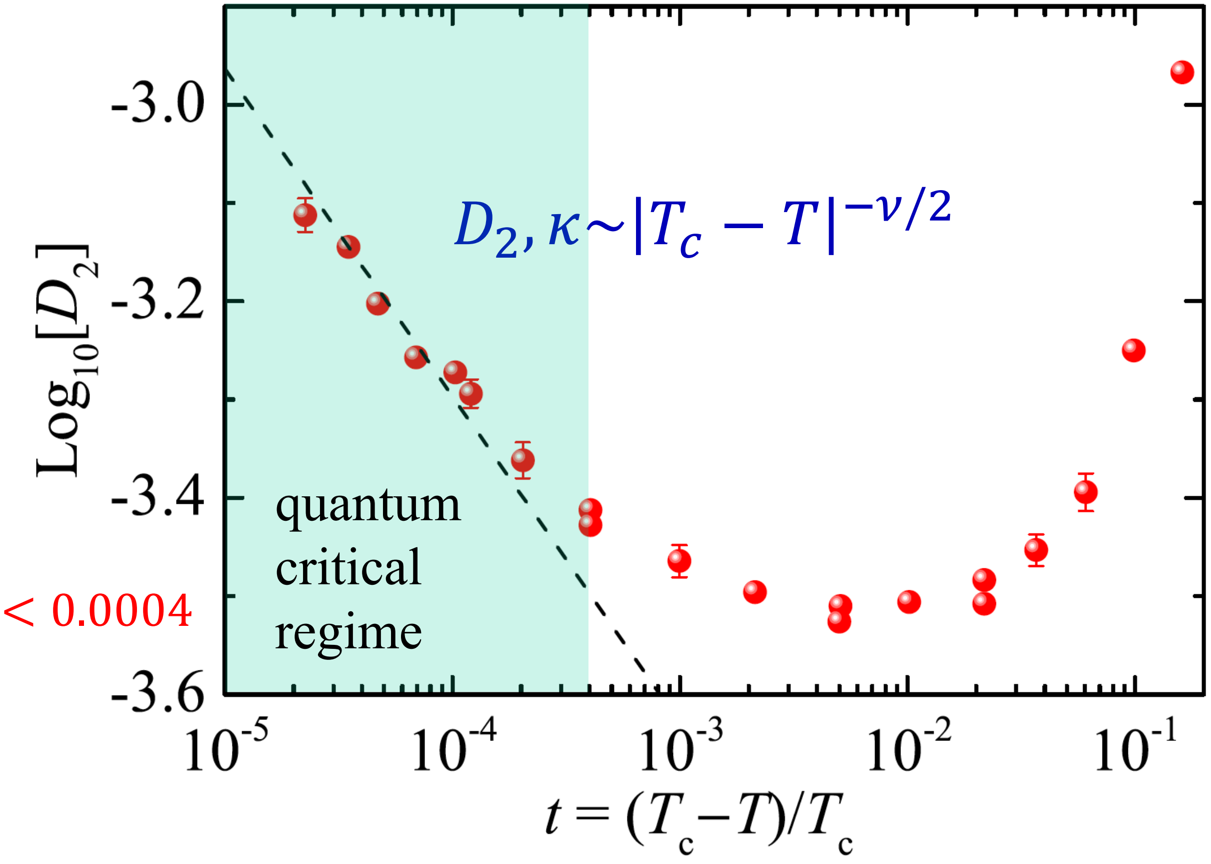
$$D_2 \simeq \frac{n_s}{n_n}\left[\frac{4\boxed{\eta}}{3\rho} + \left(\frac{\zeta_2}{\rho} + \rho\zeta_3 - 2\zeta_1\right)\right] + \frac{\boxed{\kappa}}{\rho c_P}$$

thermal conductivity

shear viscosity

Significance of second sound

- Historically, the measurement of second sound attenuation in superfluid helium plays a crucial role to establish the **dynamic scaling theory of superfluid phase transition** (the critical exponent $\nu \approx 2/3$ for superfluid helium)



Two-fluid hydrodynamic theory

Landau two-fluid hydrodynamic theory

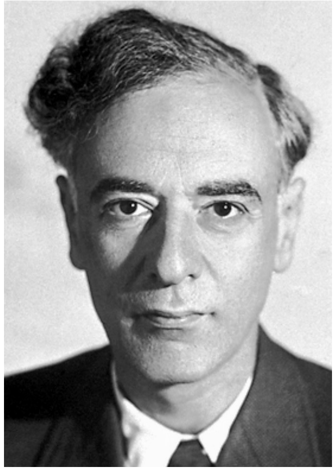
In the collisional regime, the **conservation laws** at **local equilibrium** leads to the celebrated **Landau two-fluid hydrodynamic equations**:

$$m\partial_t n + \nabla \cdot \mathbf{j} = 0,$$

$$m\partial_t \mathbf{v}_s + \nabla\left(\mu + V_{\mathrm{ext}}\right) = 0,$$

$$\partial_t j_i + \partial_i P + n\partial_i V_{\mathrm{ext}} = \partial_t\left(\eta \Gamma_{ik}\right),$$

$$\partial_t s + \nabla \cdot (s\mathbf{v}_n) = \nabla \cdot (\kappa \nabla T / T),$$



Lev Landau

where $\mathbf{j} = m(n_s \mathbf{v}_s + n_n \mathbf{v}_n)$ is the current density, n_s and n_n are the superfluid and normal density, $\mathbf{v}_s$ and $\mathbf{v}_n$ are the corresponding velocity fields, $\Gamma_{ik} \equiv (\partial_k v_{ni} + \partial_i v_{nk} - 2\delta_{ik}\partial_j v_{nj}/3)$, and finally η and κ are the shear viscosity and thermal conductivity, respectively. In the above equations, we have kept only linear terms in the velocity, as we are interested in small-amplitude dynamics in the linear response regime. Moreover, we have omitted bulk viscosity terms which give smaller contributions.

Hydrodynamic density response function

Hydrodynamic **density response function** (Hohenberg & Martin 1965):

$$\boxed{\chi_{nn} = \frac{\left(nk^2/m\right)\left(\omega^2 - v^2k^2 + iD_sk^2\omega\right)}{\left(\omega^2 - c_1^2k^2 + iD_1k^2\omega\right)\left(\omega^2 - c_2^2k^2 + iD_2k^2\omega\right)}}$$

involving **velocities**: and **diffusion coefficients**:

$$v^2 \equiv T\frac{s^2}{c_v}\frac{\rho_s}{\rho_n}$$

$$c_1^2 + c_2^2 = v^2 + v_s^2,$$

$$c_1^2c_2^2 = v_T^2v^2 = \frac{v^2v_s^2}{\gamma},$$

$$D_1 + D_2 = \frac{4}{3}\frac{\eta}{\rho} + \frac{\kappa}{\rho c_v},$$

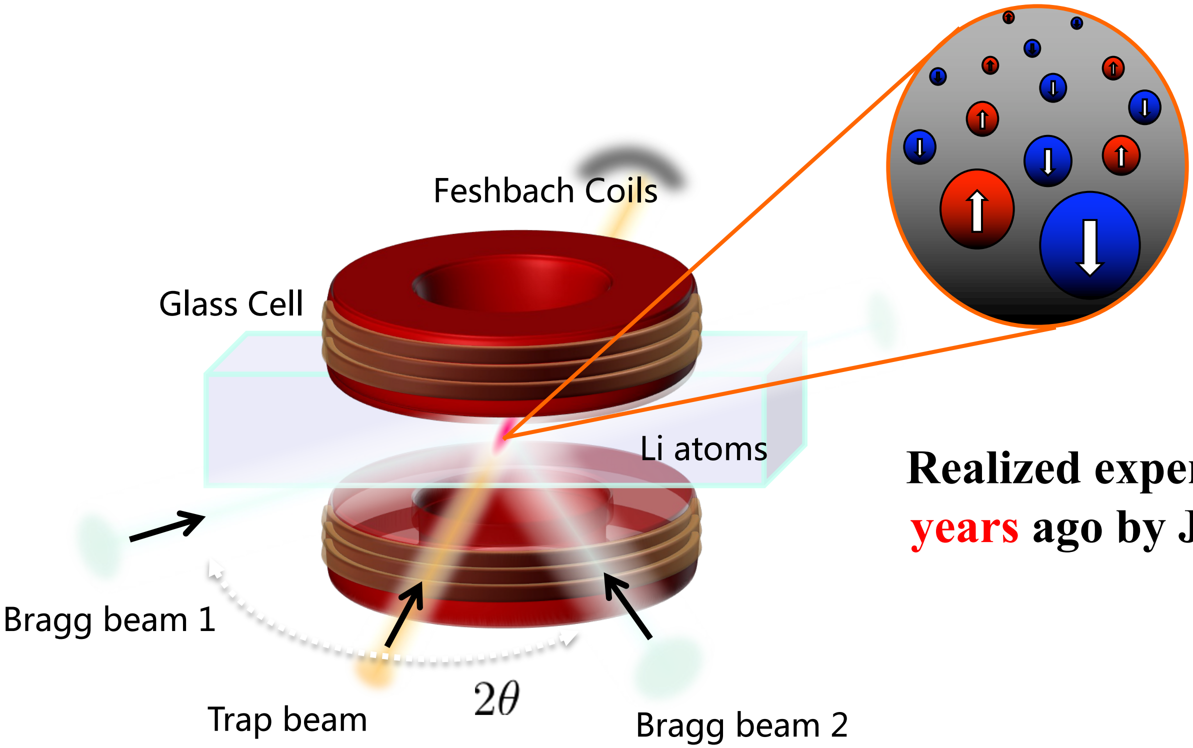
$$\frac{c_1^2D_2 + c_2^2D_1}{v_s^2} = \frac{4}{3}\frac{\eta}{\rho}\left[\frac{v^2}{v_s^2} - \frac{2v^2}{\rho s}\frac{\left(\frac{\partial P}{\partial T}\right)_\rho}{v_s^2} + \frac{\rho_s}{\rho_n}\right] + \frac{\kappa}{\rho c_p},$$

$$D_s = \frac{4}{3}\frac{\eta}{\rho}\frac{\rho_s}{\rho_n} + \frac{\kappa}{\rho c_v},$$

Note : various **second viscosities** are negligible at unitarity.

P. C. Hohenberg and P. C. Martin, *Ann. Phys.* (N. Y.) **34**, 291 (1965).

Unitary Fermi superfluid: New opportunity from 2002?



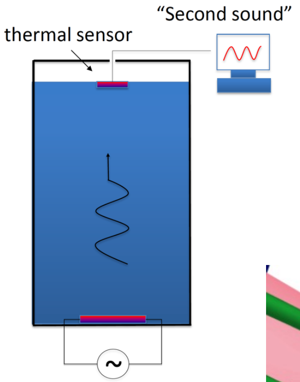
Realized experimentally 20 years ago by John Thomas

Ultracold atoms is an ideal platform to emulate many-body physics

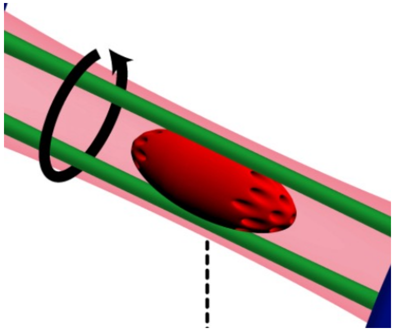
<u>Toolbox</u>: magnetic Feshbach resonance (MFR) + optical lattice + disorder + spin-orbit coupling (SOC) + optical control of MFR

Questions for a unitary Fermi gas

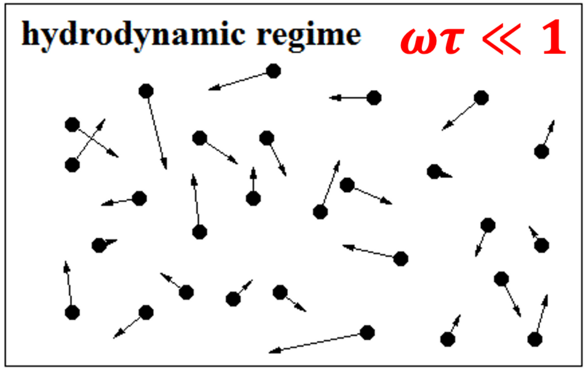
Difficulties for observing second sound in a strongly interacting Fermi superfluid:



- Lack of accurate **thermometry at *nK***
- **Inhomogeneity** due to external harmonic traps



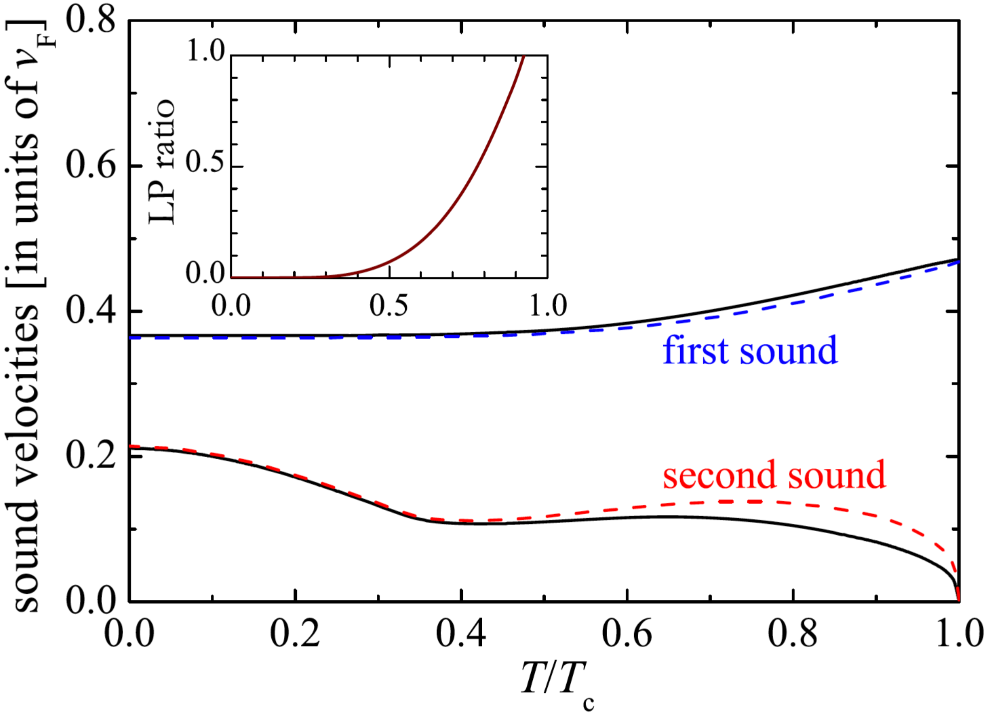
- **Low-energy** (ω) and **long-wavelength excitations** are required

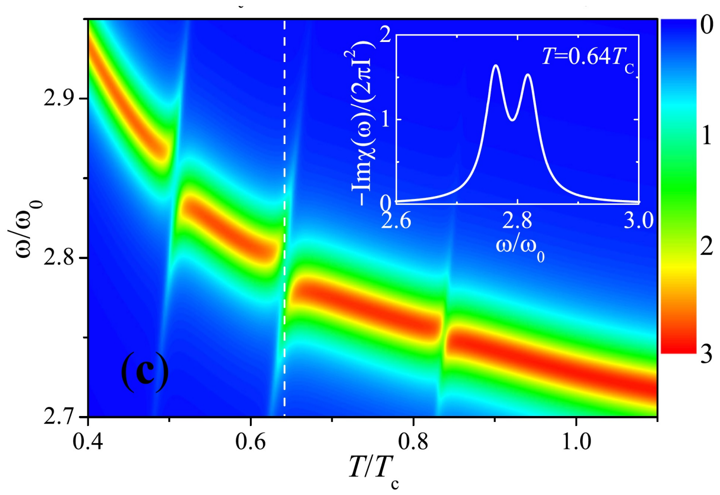


Questions for a unitary Fermi gas

The problem of thermometry can be solved by measuring the density wave (due to its coupling to the temperature wave)

Large Landau-Placzek (LP) ratio: second sound appears in the **density response**



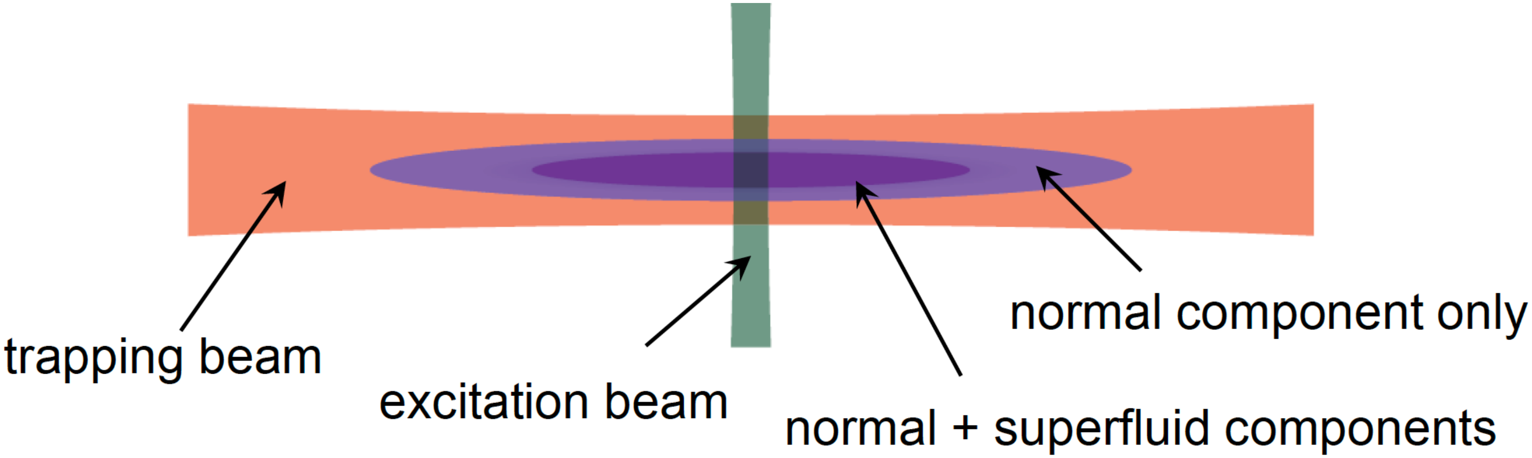


In traps: **avoided crossing** between 1st and 2nd sounds

E. Taylor, H. Hu, X.-J. Liu, L. Pitaevskii, A. Griffin, and S. Stringari, *PRA* **80**, 053601 (2009).

Questions for a unitary Fermi gas

The problem of inhomogeneity can be partly solved by considering a highly elongated harmonic trap (i.e., the quasi-one-dimensional configuration)

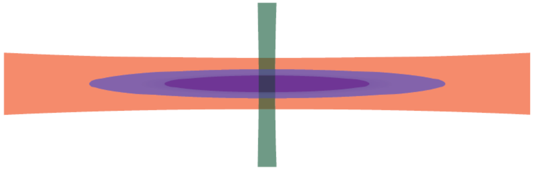


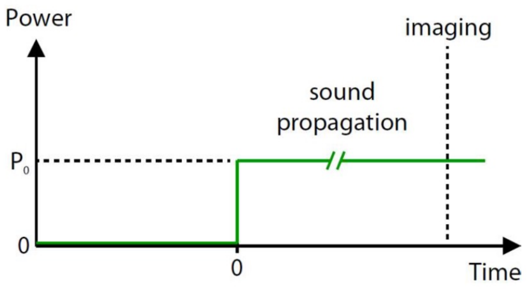
- **Pros:** Useful effective **dissipationless** quasi-1D hydrodynamic description by assuming relatively large shear viscosity and thermal conductivity

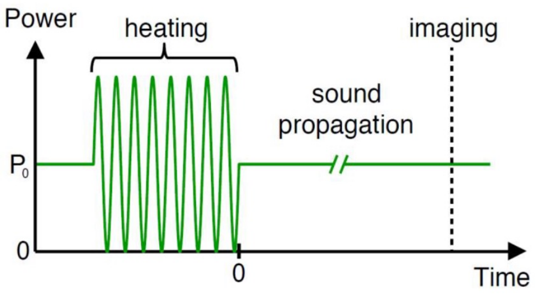
- Cons: (i) **Quasi-1D to 3D conversion** is needed for extracting the superfluid density; (ii) Quantum **transport coefficients cannot be extracted**, in principle.

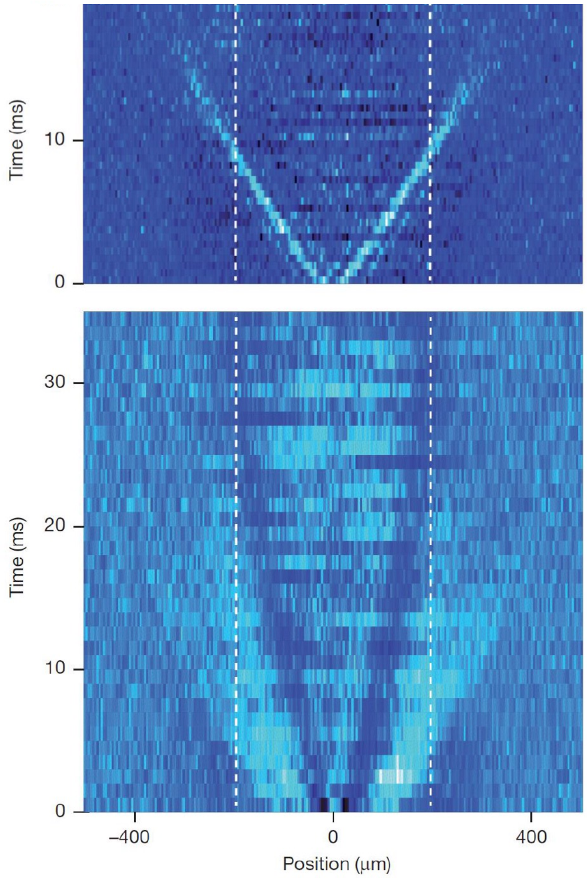
Questions for a unitary Fermi gas

The problem of inhomogeneity can be partly solved by considering a highly elongated harmonic trap (i.e., the quasi-one-dimensional configuration)



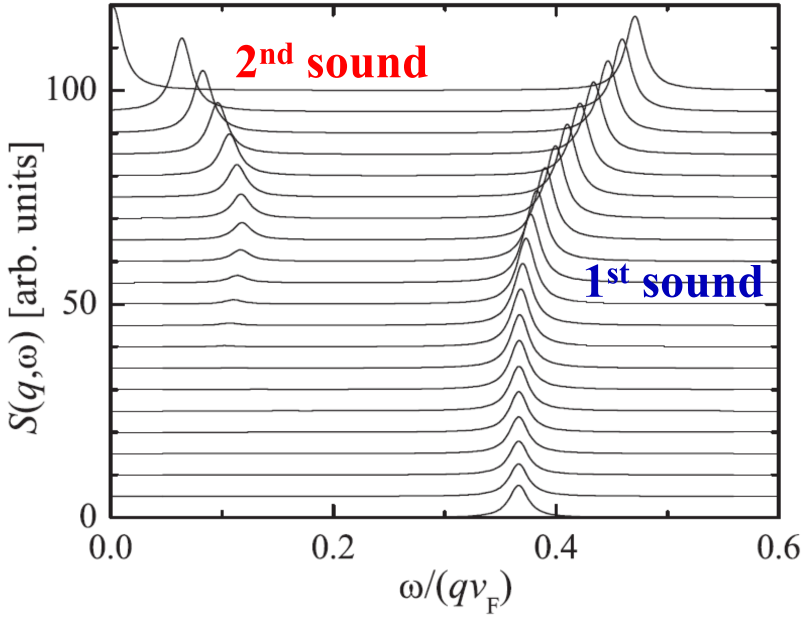






Sidorenkov et al., Nature 498, 78 (2013)

Hydrodynamic description of a unitary Fermi gas



$$\xcancel{\begin{aligned}
D_1 + D_2 &= \frac{4}{3}\frac{\eta}{\rho} + \frac{\kappa}{\rho c_v}, \\
\frac{c_1^2 D_2 + c_2^2 D_1}{v_s^2} &= \frac{4}{3}\frac{\eta}{\rho}\left[\frac{v^2}{v_s^2} - \frac{2v^2}{\rho s}\frac{\left(\frac{\partial P}{\partial T}\right)_\rho}{v_s^2} + \frac{\rho_s}{\rho_n}\right] + \frac{\kappa}{\rho c_p}, \\
D_s &= \frac{4}{3}\frac{\eta}{\rho}\frac{\rho_s}{\rho_n} + \frac{\kappa}{\rho c_v},
\end{aligned}}$$

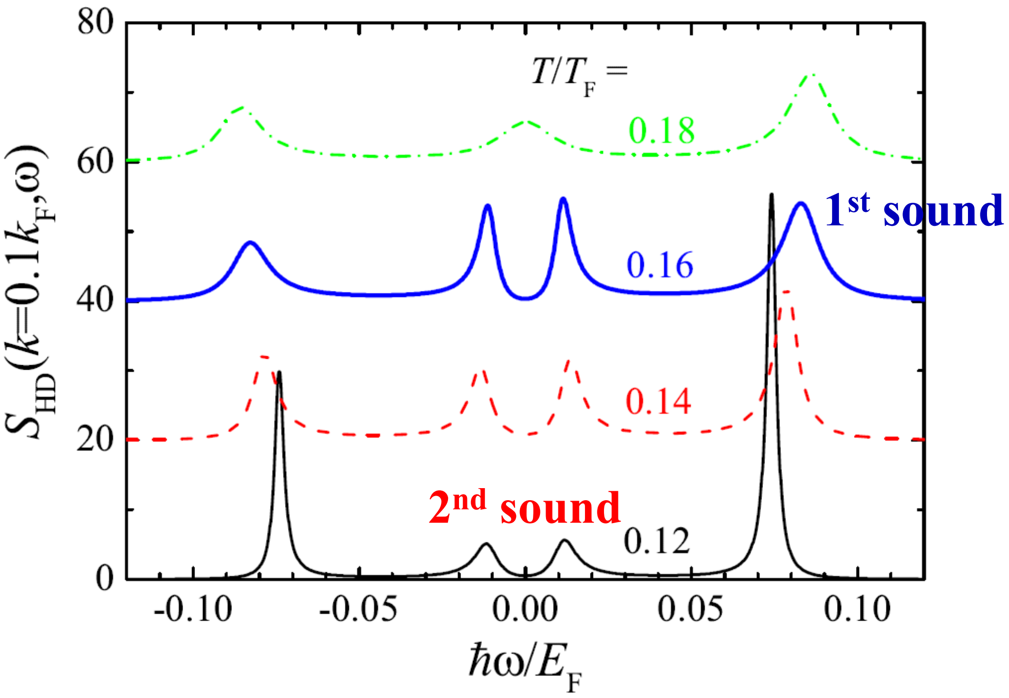
New Journal of Physics
The open-access journal for physics

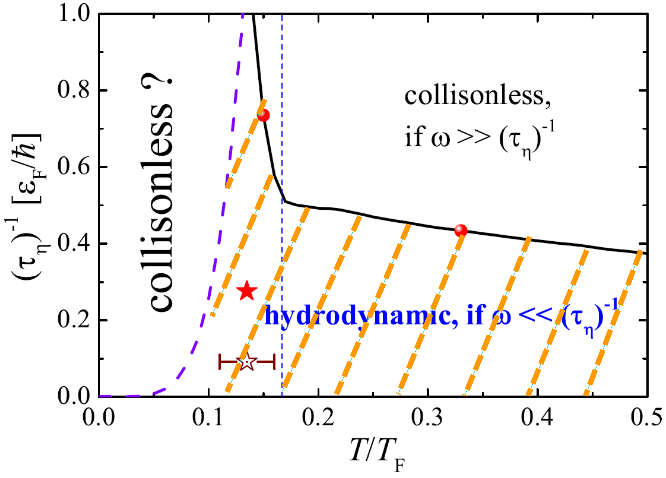
Second sound and the density response function in uniform superfluid atomic gases without dissipation terms

H Hu[1,2,6], E Taylor[3], X-J Liu[1], S Stringari[4] and A Griffin[5]

New Journal of Physics **12** (2010) 043040 (24pp)
Received 7 January 2010
Published 22 April 2010
Online at http://www.njp.org/
doi:10.1088/1367-2630/12/4/043040

Hydrodynamic density response at small momentum





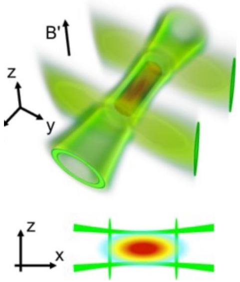
We definitively can observe the 2nd sound in the density response at small momentum (i.e., $k < 0.1k_F$), if the homogeneous unitary Fermi gas is in hydrodynamic regime as predicted in the right figure (shaded area)!

H. Hu, P. Zou, and X.-J. Liu, *Phys. Rev. A* **97**, 023615 (2018).

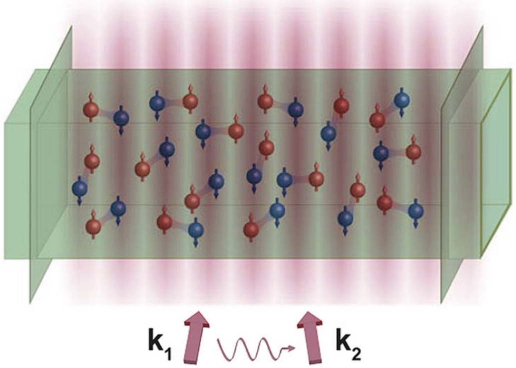
Unitary Fermi superfluid: New opportunity from 2002?

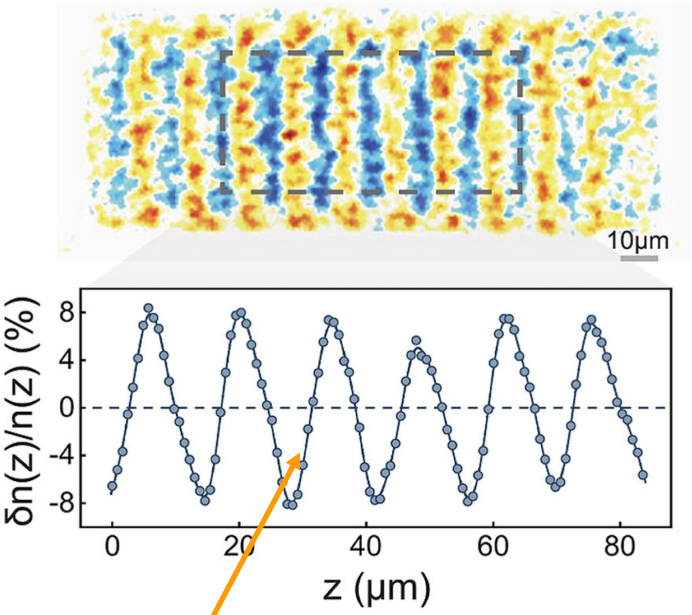
All these technical difficulties (i.e., homogeneity, low-momentum etc) have been solved by XingCan Yao's group at the University of Science and Technology of China (USTC)

- **Uniform** box-potential trap



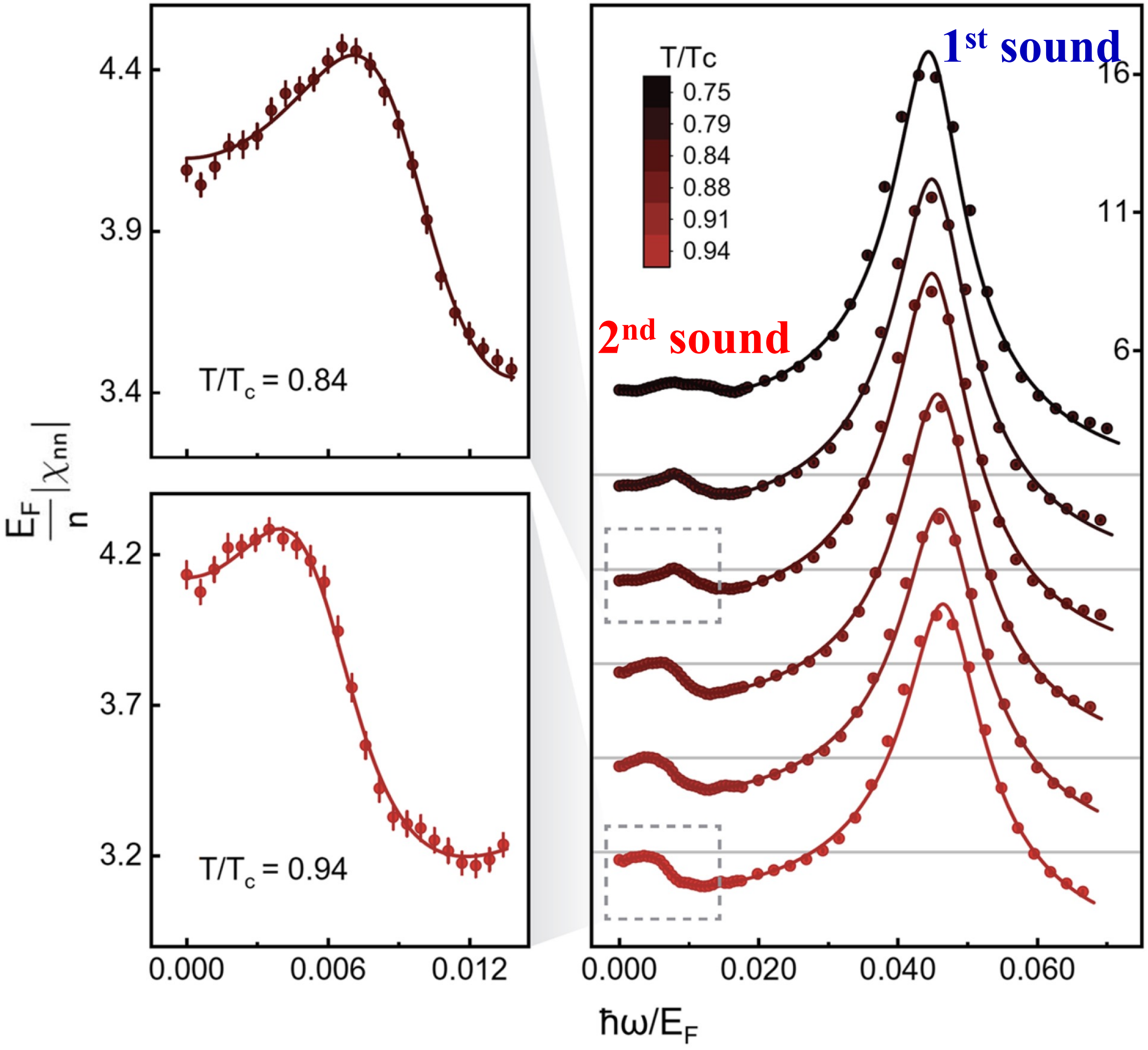
- **Novel Bragg** scattering spectroscopy

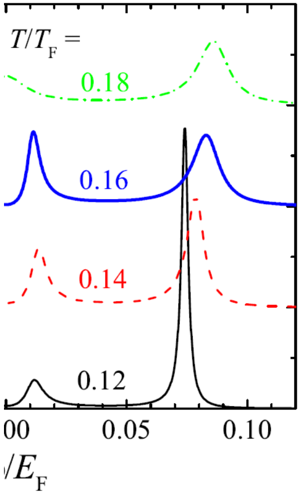




- Extremely large Fermi cloud (density 10× , Fermi energy 5× …) and hence low-enough momentum (**$0.05k_F$**) to reach the **quantum critical regime** near the superfluid phase transition.

Second sound propagation

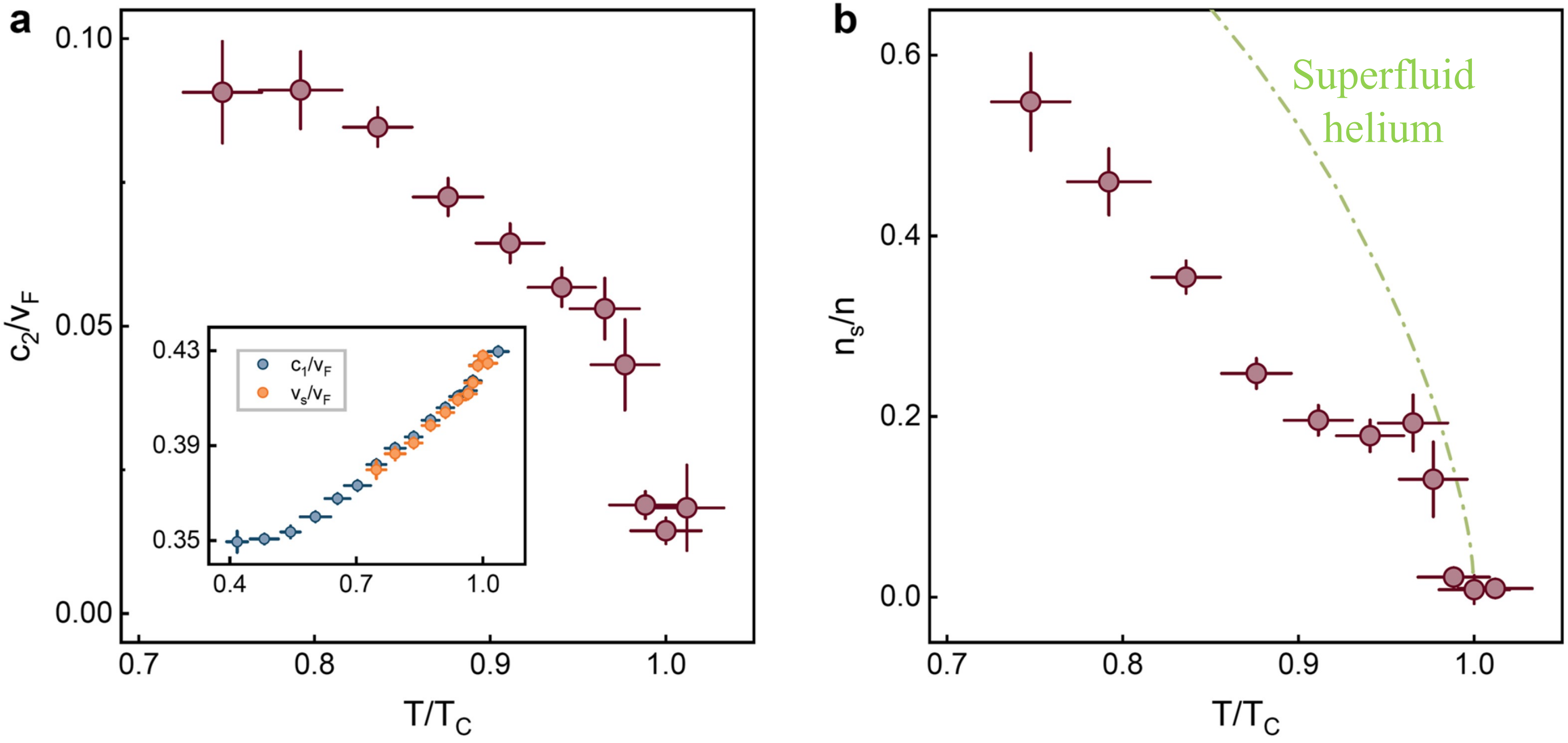




From curve fitting, various **sound velocities** and **sound diffusion coefficients** are determined.

$$\boxed{\chi_{nn} = \frac{\left(nk^2/m\right)\left(\omega^2 - v^2k^2 + iD_sk^2\omega\right)}{\left(\omega^2 - c_1^2k^2 + iD_1k^2\omega\right)\left(\omega^2 - c_2^2k^2 + iD_2k^2\omega\right)}}$$

Second sound propagation

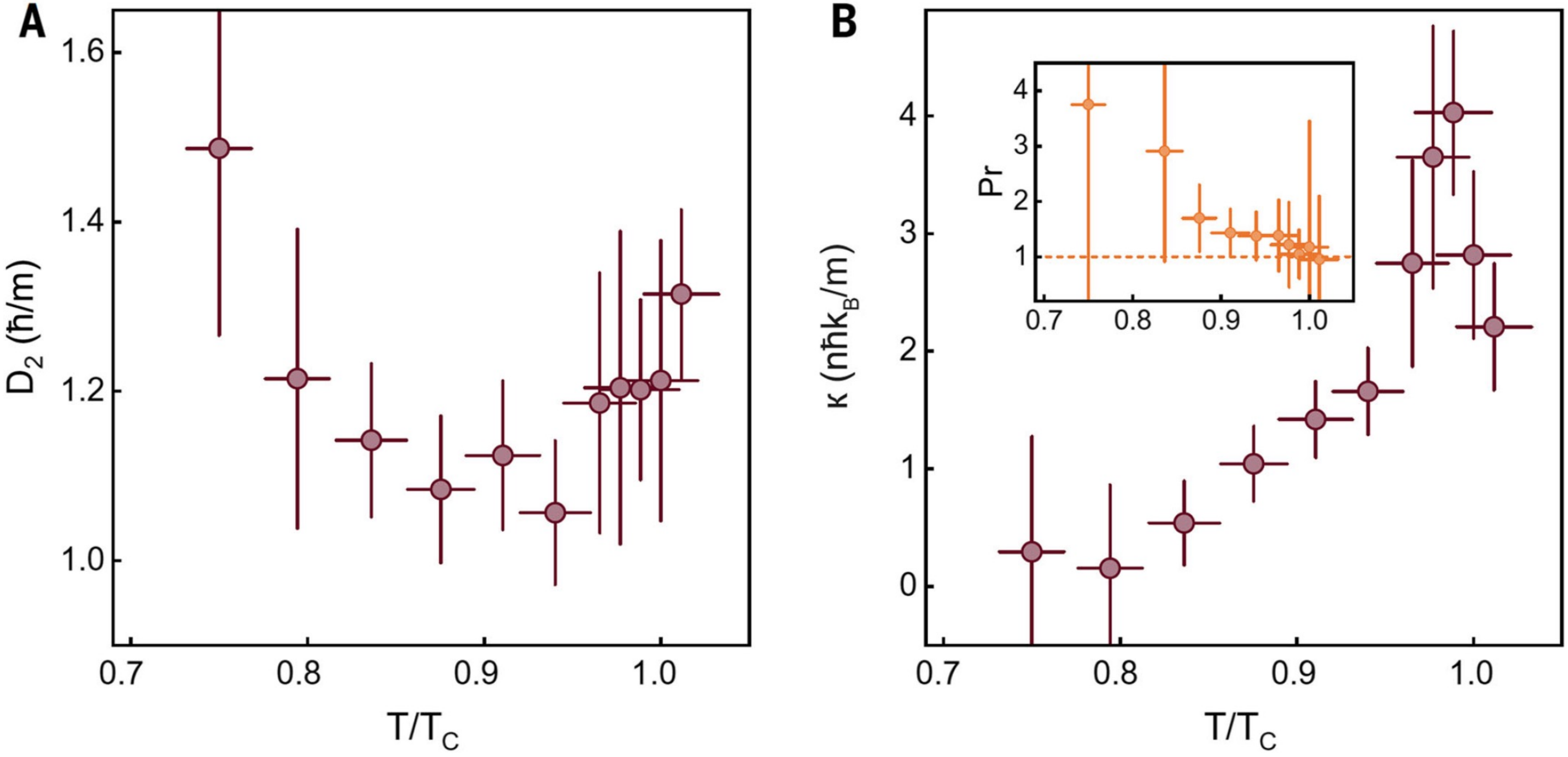


$$c_2 = \sqrt{\frac{1}{m}\frac{T\bar{s}^2}{\boxed{\bar{c}_p}}\boxed{\frac{n_s}{n_n}}} \longrightarrow$$

Superfluid fraction $\frac{n_s}{n}$

Second sound attenuation

$D_2 \sim |T_c - T|^{-\nu/2}$ $\kappa \sim |T_c - T|^{-\nu/2}$



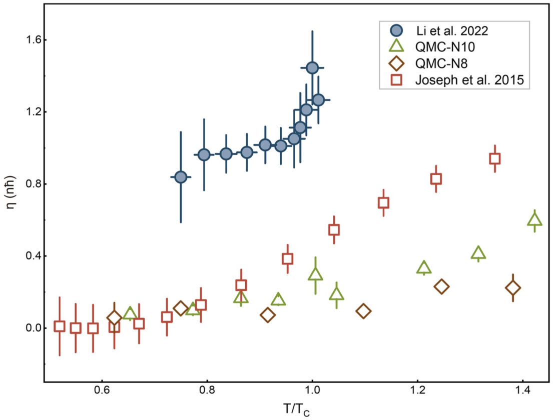
First measurement of the second sound diffusivity and thermal conductivity indicates a **critical divergence**, as observed in superfluid helium. However, a much large **quantum critical regime** is found, i.e., $|T_c - T|/T_c < 0.02$.

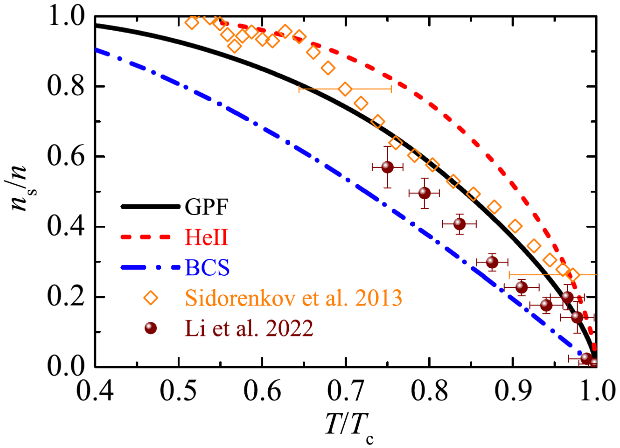
Taking home message

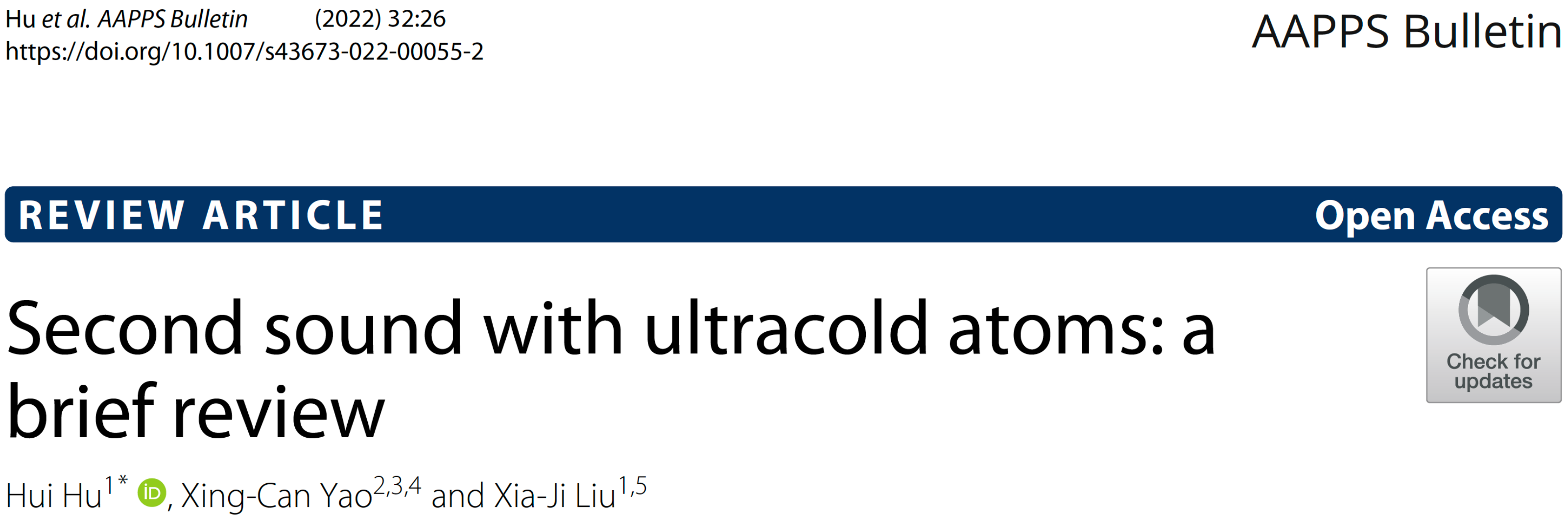
- Full characterization of **second sound** in novel unitary Fermi superfluid (**^{4}He** vs **unitary Fermi gas**)

- An **unexpected large** quantum critical regime (50×) and a pathway to understand **anomalous quantum transport at quantum criticality (cf. high-T_c materials)**

- **New challenges** for many-body physics (i.e., η and n_s)







Hu *et al. AAPPS Bulletin* (2022) 32:26
https://doi.org/10.1007/s43673-022-00055-2

AAPPS Bulletin

REVIEW ARTICLE **Open Access**

Second sound with ultracold atoms: a brief review

Hui Hu[1*], Xing-Can Yao[2,3,4] and Xia-Ji Liu[1,5]

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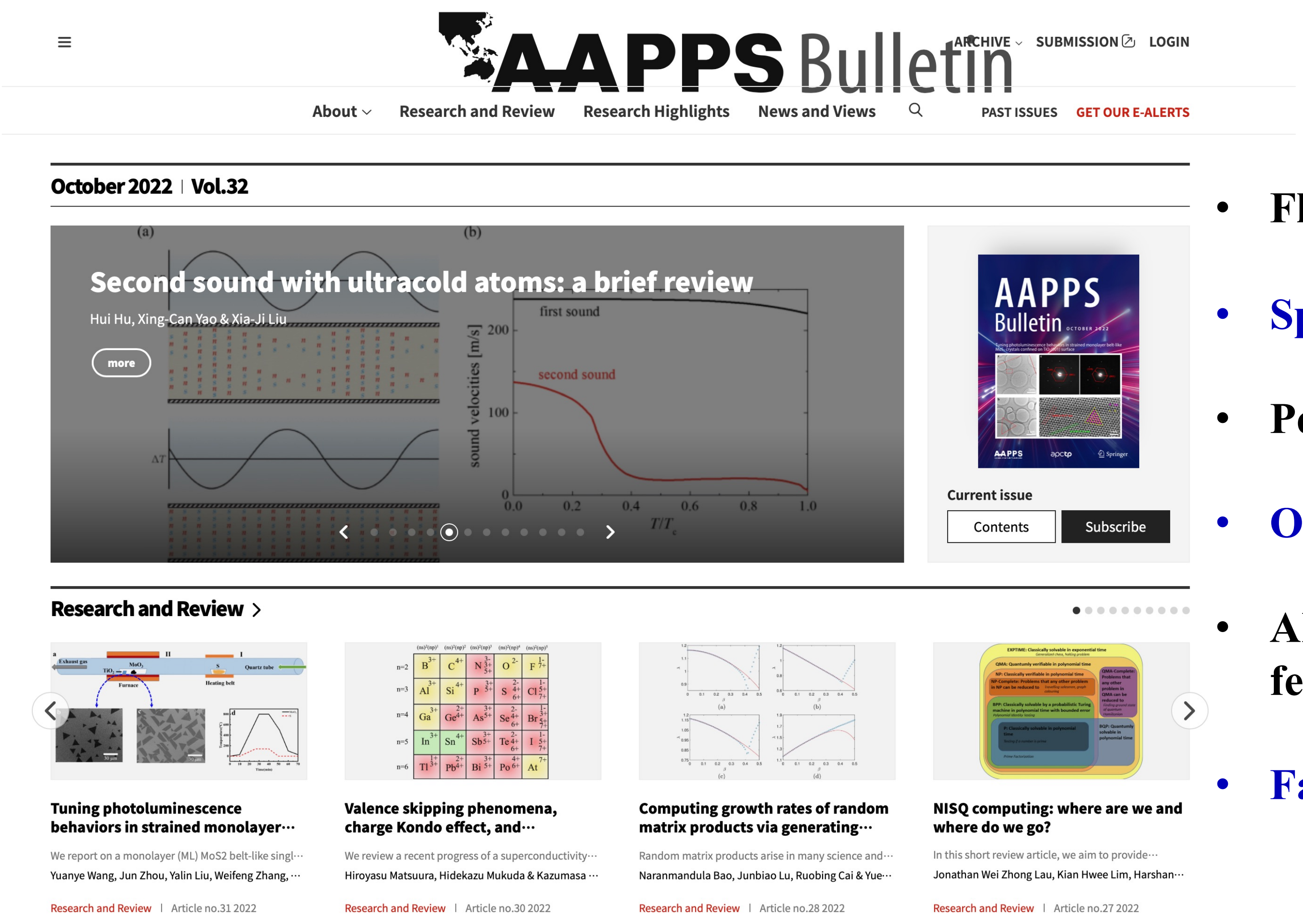

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Second sound with ultracold atoms: a brief review

Hui Hu, Xing-Can Yao & Xia-Ji Liu

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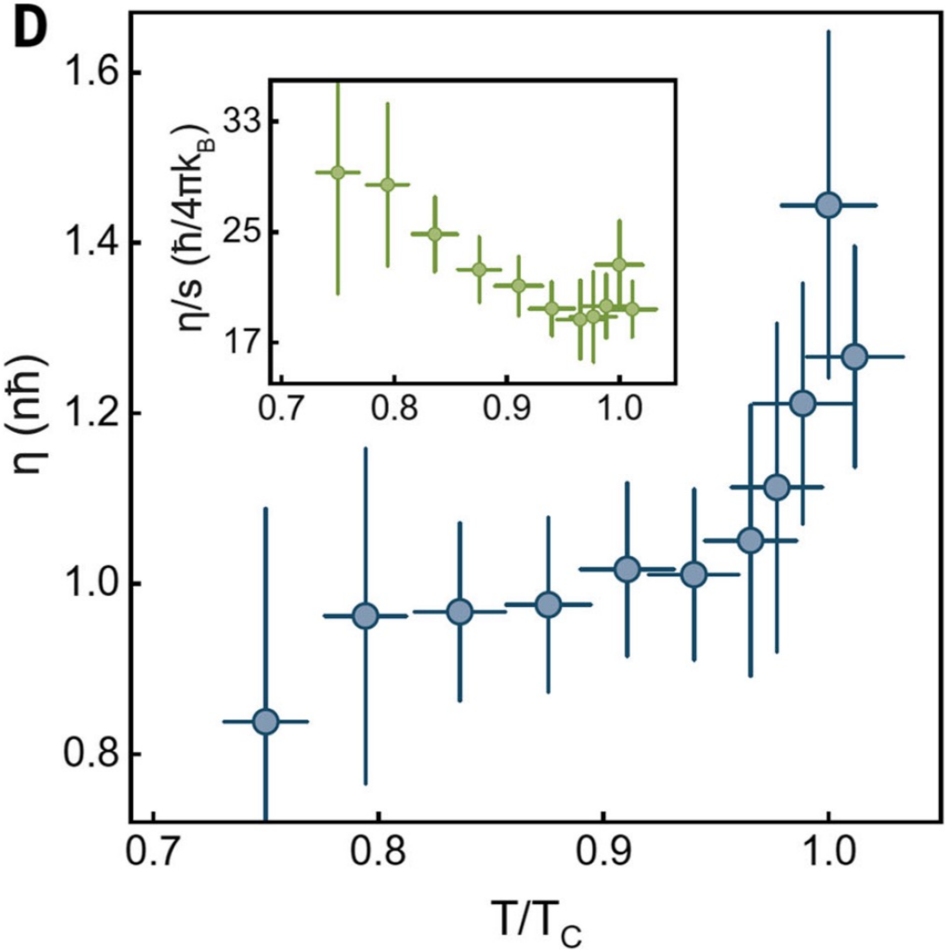
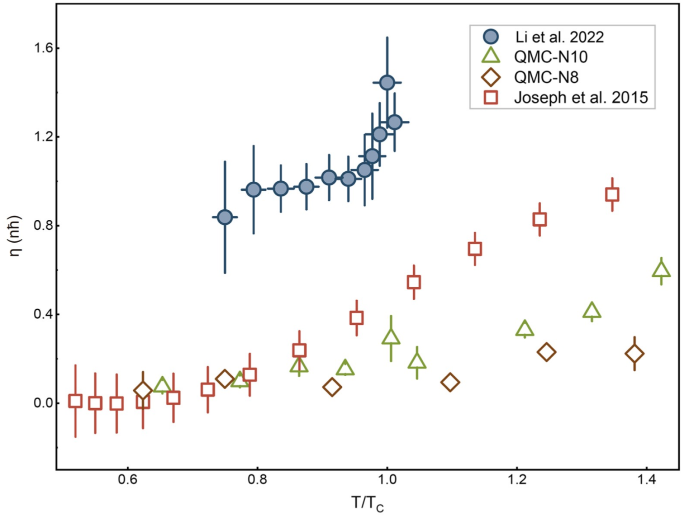
Second sound attenuation





Direct measurement of the shear viscosity of the homogenous unitary Fermi gas greatly improve its accuracy, compared with a previous result extracted from the trapped measurement (John Thomas group, Science 2011). Hence, the **shear viscosity-to-entropy** ration is much larger than the **universal low-bound** for a perfect liquid.