Frequency beating and damping of breathing oscillations of a harmonically trapped 1D quasicondensate

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Motivation and Background

- **Mechanisms of relaxation of isolated quantum systems** (integrable, non-integrable, weakly integrable)
  - ***Would a harmonically trapped, pure 1D Bose gas (no transverse excitations) thermalize after a quench?***
    - ✧ A uniform 1D Bose gas is integrable (Lieb-Liniger model) and therefore it shouldn't thermalise
    - ✧ Harmonic confinement breaks the integrability, but only weakly. ***Would it thermalize now? If yes, on what time scale?***
    - ✧ Experiments are quasi-1D (not deep in 1D) ➡ transverse excitations also break the integrability and the system thermalizes – presumably faster than a pure 1D would. ***Can the difference be observed?***
- Revisit and scrutinize characterization of **breathing-mode oscillations** of a harmonically trapped 1D Bose gas in the weakly interacting ***quasi-condensate regime***
  - **Observed beating of two distinct frequencies (unlike all previous studies, which report a single frequency,**
  - In a partially Bose condensed 3D gas, such beating would be common; the two frequencies would be associated with the breathing of the *condensate* and *thermal components*
  - However, a phase fluctuating 1D quasi-condensate is *not* a true condensate (occupation of the lowest energy mode does not dominate all others), ***so why do we still see two frequencies like in 3D?***

Previous work

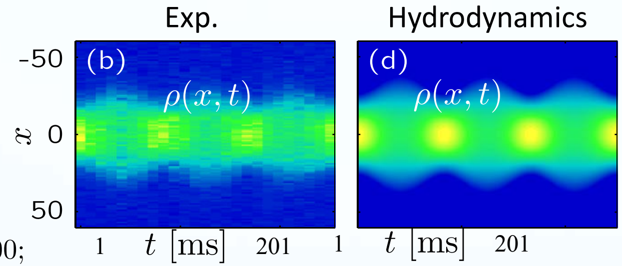
T=0 weakly interacting, TF regime

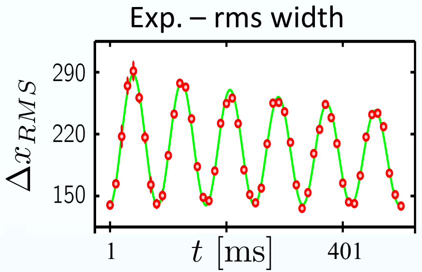
classical gas

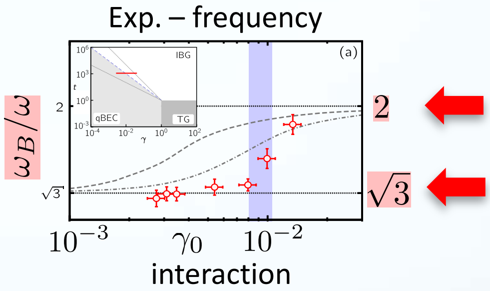
- All predicted a single oscillation frequency crossing over from $\omega_B \simeq \sqrt{3}\,\omega$ to 2ω, depending on the temperature and interactions strength
- Frequency of low-energy excitations is a `fingerprint' of collective effects in interacting many-body systems

Fang *et al.*, PRL **113**, 025301 (2014)

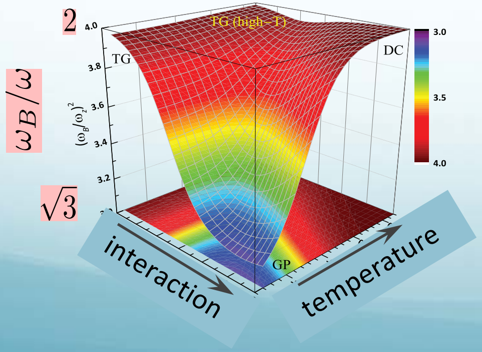
$[N = 800 - 8000;$ $T \sim 100$ nK; ^{87}Rb$]$

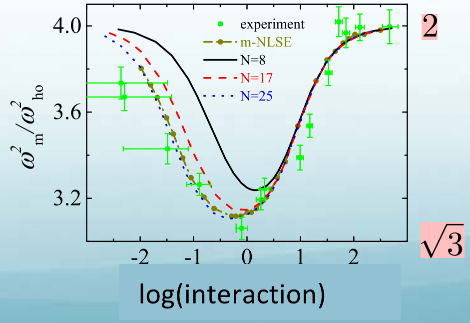






Hui Hu *et al.*, PRA **90**, 013622 (2014)





Moritz et al., PRL **91**, 250402 (2003).

Haller et al., Science **325**, 1224 (2009)

Menotti et al., PRA **66**, 043610 (2002)

Hui Hu et al., PRA **90**, 013622 (2014)

Gudima et al., PRA A **92**, 021601 (2015)

Choi et al., PRL **115**, 115302 (2015)

De Rosi et al., PRA **92**, 053617 (2015)

De Rosi et al., PRA **94**, 063605 (2016)

Bouchoule et al., PRA **94**, 051602 (2016)

1D quasicondensate and c-field simulations

- **Lieb-Liniger model in a harmonic trap**

interaction strength

$$\hat{H} = -\frac{\hbar^2}{2m}\int dx\ \hat{\Psi}^\dagger \frac{\partial^2}{\partial x^2}\hat{\Psi} + \int dx V(x,t)\hat{\Psi}^\dagger\hat{\Psi} + \frac{g}{2}\int dx\ \hat{\Psi}^\dagger\hat{\Psi}^\dagger\hat{\Psi}\hat{\Psi}$$

- **Finite-temperature c-field method (SPGPE - Stochastic Projected Gross-Pitaevskii equation):**

Projector for c-field region

Growth term

'Preparation' of the initial thermal ensemble:

$$\mathrm{d}\Psi_{\mathbf{C}}(x,t) = \mathcal{P}^{(\mathbf{C})}\left\{-\frac{i}{\hbar}\mathcal{L}_0^{(\mathbf{C})}\Psi_{\mathbf{C}}(x,t)\,\mathrm{d}t + \frac{\kappa}{\hbar}(\mu - \mathcal{L}_0^{(\mathbf{C})})\Psi_{\mathbf{C}}(x,t)\,\mathrm{d}t + dW_\Gamma(x,t)\right\}$$

Mean-field GPE operator (for subsequent evolution)

$$\mathcal{L}_0^{(\mathbf{C})} = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x,t) + g|\Psi_{\mathbf{C}}(x,t)|^2$$

$$\langle dW_\Gamma^*(x,t)dW_\Gamma(x',t) = \frac{2\kappa k_B T}{\hbar}\delta(x-x')dt$$

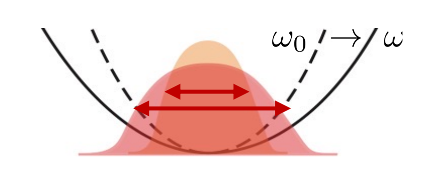
Noise term (seeding of thermal fluctuations)

[Castin et al., J. Mod. Opt. **47**, 2671 (2000); Blakie et al., Adv. Phys. **57**, 363 (2008)]

- Trap quench protocol for exciting the **breathing mode oscillations**:

$$V(x,t) = \begin{cases} \frac{1}{2}m\omega_0^2 x^2, & \text{for } t \le 0, \\ \frac{1}{2}m\omega^2 x^2, & \text{for } t > 0. \end{cases}$$

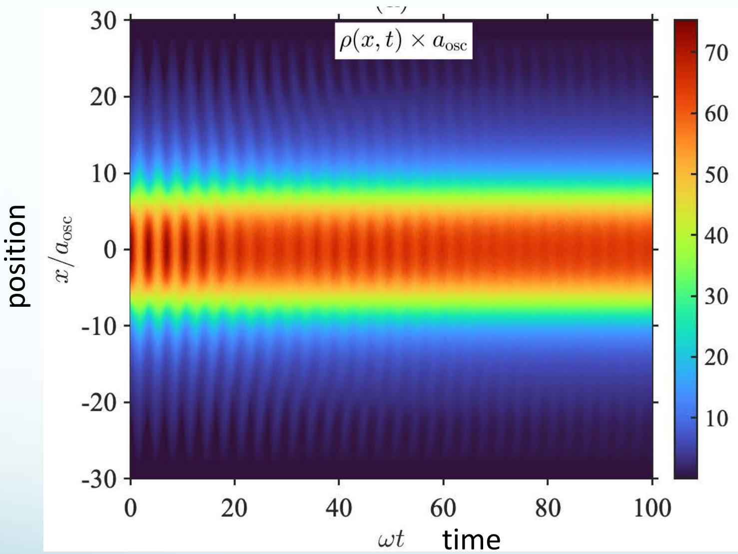
$\omega_0 \to \omega \quad \text{at} \quad t = 0$



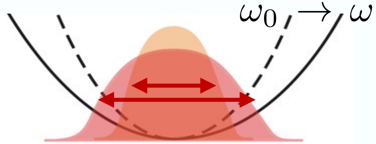
c-field simulation results

- Breathing mode oscillations of the density profile in the quasi-condensate regime:

$[\gamma_0^{3/2}\mathcal{T} \simeq 0.4;\quad N = 1110;\quad \omega/\omega_0 = 0.8;\quad {}^{87}\mathrm{Rb}]$



$\omega_0 \to \omega \quad \text{at} \quad t = 0$



$[\omega_0/2\pi = 10\ \mathrm{Hz}]$

- Dimensionless interactions strength in the trap centre:

$$\gamma_0 = \frac{mg}{\hbar^2\rho(0)} \qquad [\gamma_0 = 8.3 \simeq 10^{-3}]$$

- Dimensionless initial temperature:

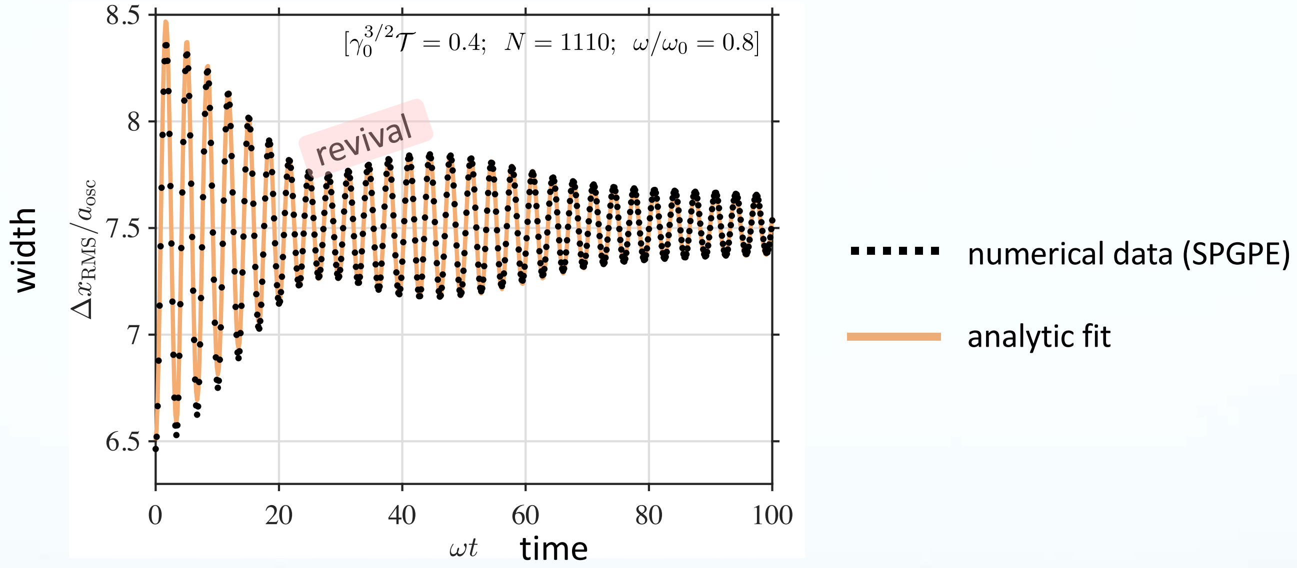
$$\mathcal{T} = \frac{k_{\mathrm{B}}T}{mg^2/(2\hbar^2)} \qquad [\mathcal{T} \simeq 517]$$

- c-field equations in dimensionless form depend on the combination:

$$\gamma_0^{3/2}\mathcal{T} \qquad [\gamma_0^{3/2}\mathcal{T} \simeq 0.4]$$

- Extract the rms width: $\Delta x_{\mathrm{RMS}}(t) = \left[\frac{1}{N}\int \mathrm{d}x\,\rho(x,t)x^2 - \left(\frac{1}{N}\int \mathrm{d}x\,\rho(x,t)x\right)^2\right]^{1/2}$

RMS width of the density profile



- Fit with a superposition of **two** damped cosines ( ):

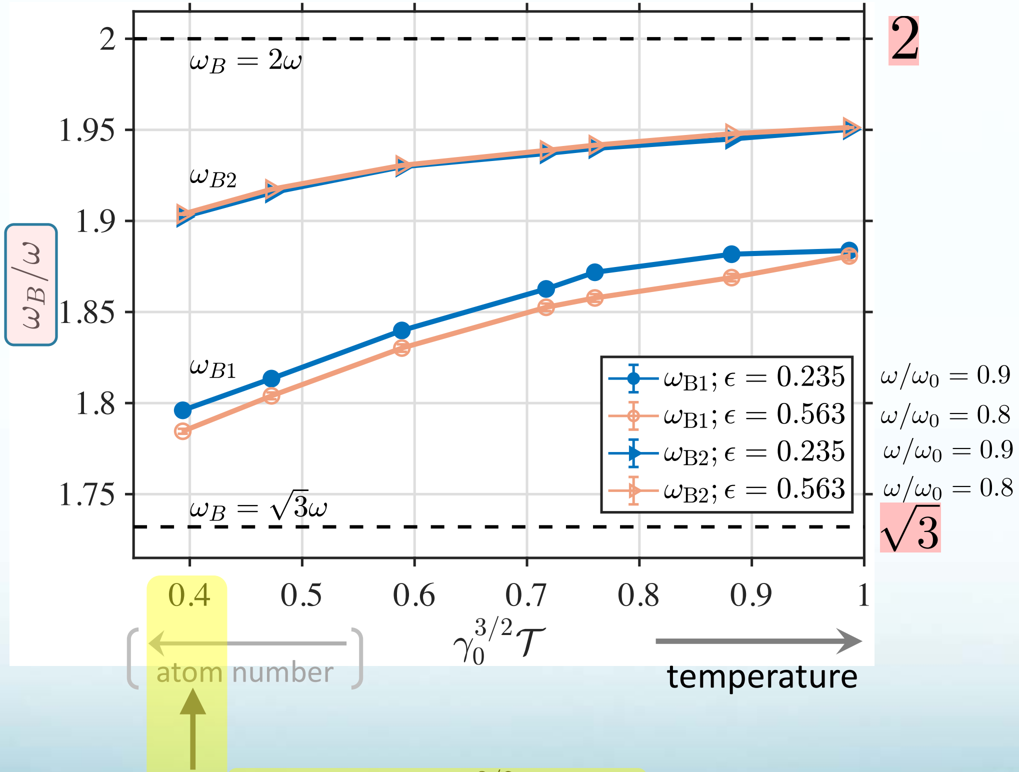
damping rates

$$\Delta x_{\mathrm{RMS}}(t) = A\left[\sqrt{K}\cos(\omega_{\mathrm{B1}}t+\phi_1)e^{-\Gamma_1 t} + \sqrt{1-K}\cos(\omega_{\mathrm{B2}}t+\phi_2)e^{-\Gamma_2 t}\right] + C,$$

relative weights

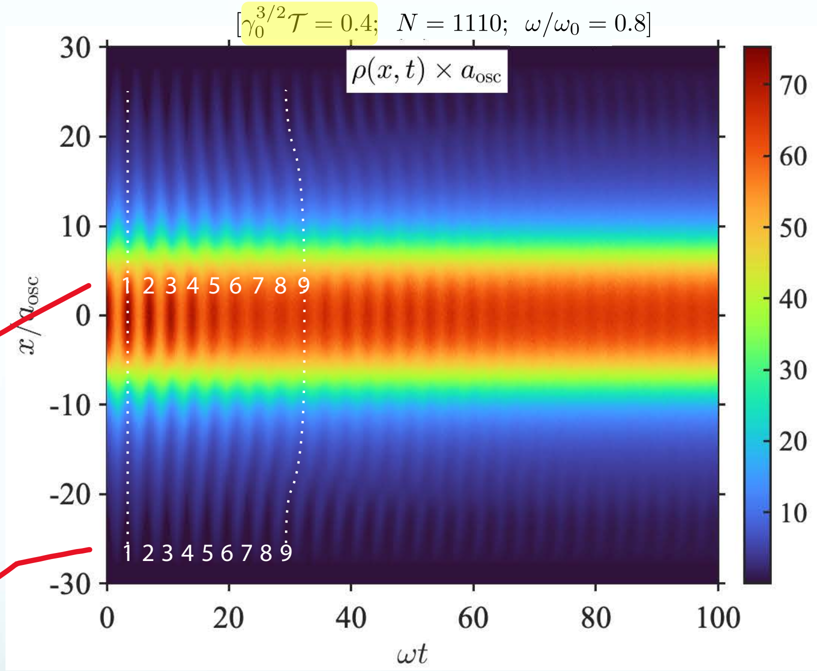
- Beating of two breathing modes with frequencies ω_{B1} and ω_{B2}, each with their own damping rates Γ_1 & Γ_2

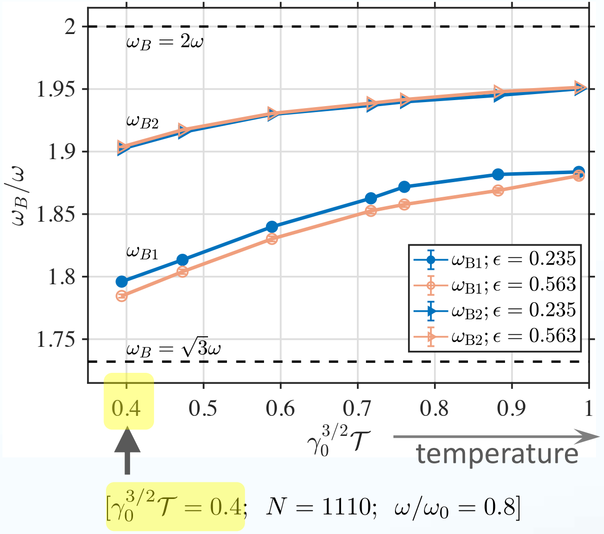
Frequencies of the two breathing modes



Previous example $[\gamma_0^{3/2}\mathcal{T} = 0.4; \;\; N = 1110; \;\; \omega/\omega_0 = 0.8]$

‘Bulk’ and ‘tail’ components



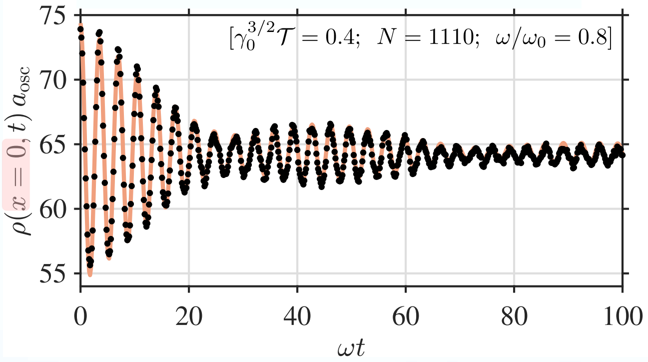


ω_{B1} - frequency of the **bulk** component

ω_{B2} - frequency of the **tail** component

Beating in the bulk and tail components

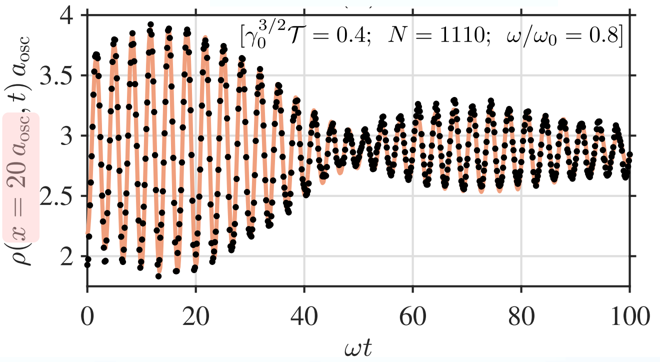
Density In the trap centre (bulk)



Relative weight of $\omega_{B1}: K = 0.96$

$\omega_{B2}: 1 - K = 0.04$

Density In the tails



Relative weight of $\omega_{B1}: K = 0.23$

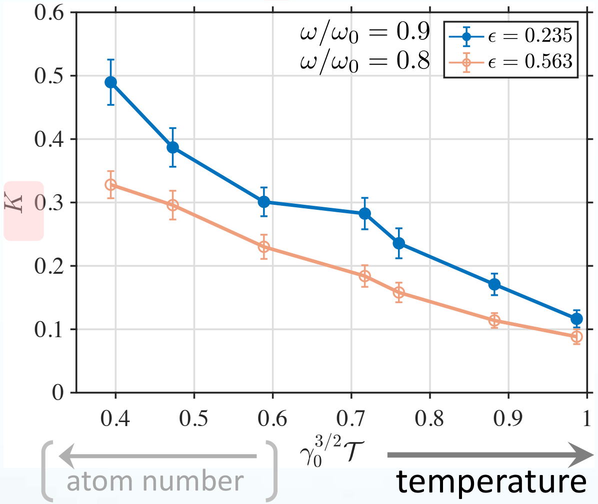
$\omega_{B2}: 1 - K = 0.77$

Fit: $\ldots = A\left[\sqrt{K}\cos(\omega_{\mathrm{B1}}t + \phi_1)e^{-\Gamma_1 t} + \sqrt{1-K}\cos(\omega_{\mathrm{B2}}t + \phi_2)e^{-\Gamma_2 t}\right] + C$

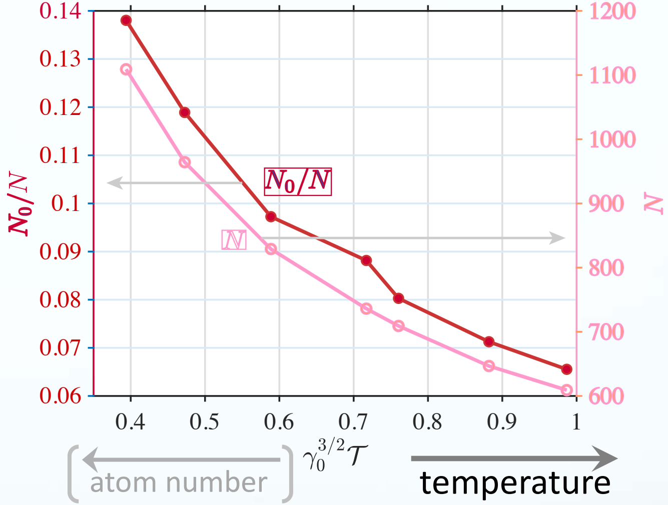
- The lower frequency breathing mode ω_{B1} is associated with the bulk
- The higher frequency breathing mode ω_{B2} is associated with the tails

Relative weight and condensate fraction

Relative power K of the ω_{B1} component



Condensate fraction (Penrose-Onsager)



$$\Delta x_{\rm RMS}(t) = A\left[\sqrt{K}\cos(\omega_{\rm B1}t+\phi_1)e^{-\Gamma_1 t} + \sqrt{1-K}\cos(\omega_{\rm B2}t+\phi_2)e^{-\Gamma_2 t}\right] + C$$

- $N_0/N \ll K$, therefore, **the 'bulk' component (oscillating at ω_{B1}) is not just the condensate mode, but many low-energy, highly occupied modes**
- In 3D the beating is common [see, .e.g, PRA **94**, 043640 (2016)]; the two frequencies are associated with the condensate and thermal atoms; but in 1D, we have a phase fluctuating quasi-condensate (not a true BEC)

Crossover phase diagram of asymptotic regimes in the weakly interacting uniform 1D Bose gas

- Dimensionless parameters for **a uniform** system:

interaction strength: $\gamma = \dfrac{mg}{\hbar^2 \rho}$

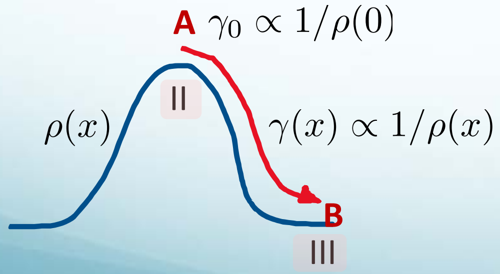
$\gamma \ll 1$ – weakly interacting ($\gamma = 0$ – ideal Bose gas)
$\gamma \gg 1$ – strongly interacting ($\gamma \to \infty$ – Tonks-Girardeau gas)

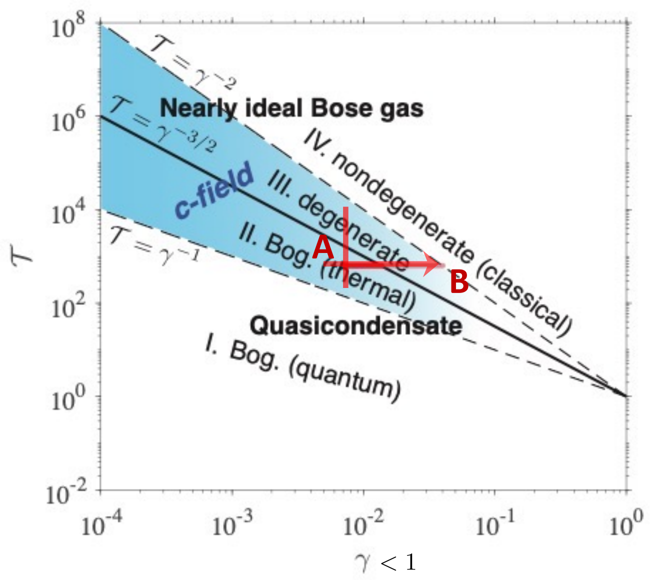
temperature: $\mathcal{T} = \dfrac{k_{\mathrm{B}} T}{mg^2/(2\hbar^2)}$

c-field method: $|\mu| \ll k_B T \implies \gamma_0^{1/2} \ll \gamma_0^{3/2}\mathcal{T} \ll \gamma_0^{-1/2}$

single dimensionless parameter (γ_0 – in the trap center), plus N

Density of a **non-uniform** (trapped) system, in the local density approximation, can explore different regimes in this phase diagram



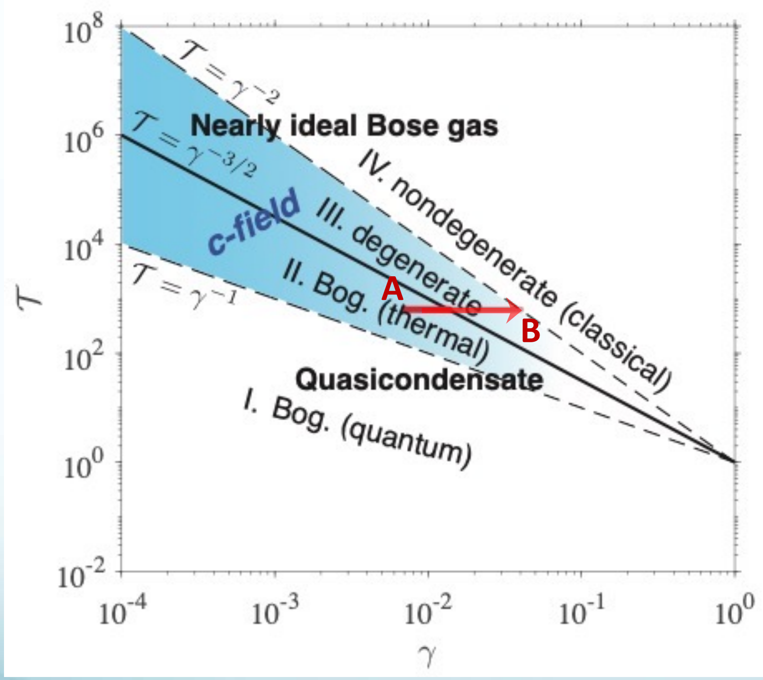


Scan $\gamma_0^{3/2}\mathcal{T} \in [0.4, 1]$

$N \in [1100, 600]$

[$\gamma > 1$ not shown]

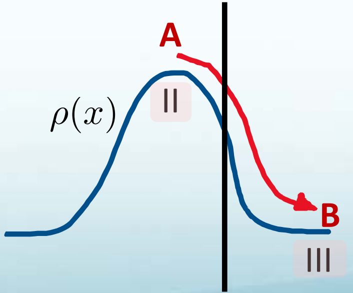
`Bulk' and `tails'



- **'Bulk'** of the gas is locally in **Regime II**. Bogoliubov (thermal) – higly degenerate modes, with suppressed density fluctuations (dominated by thermal fluctuations), but fluctuating phase

  [Regime I. Bolgoliubov (quantum) – highly degenerate, dominated by quantum, rather than thermal, fluctuations]

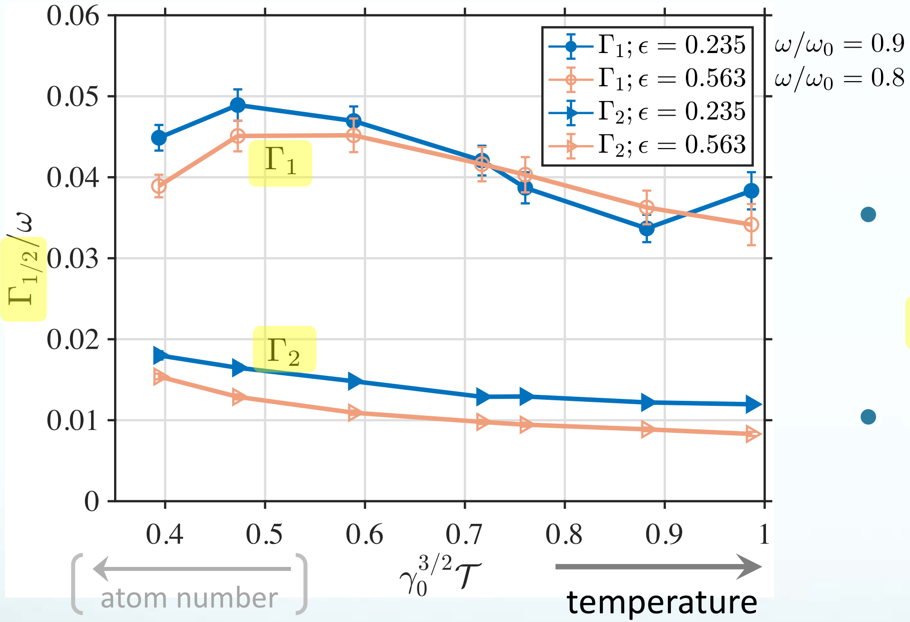
- **`Tails'** are locally in **Regime III.** Degenerate nearly ideal Bose gas, but with both density and phase fluctuations present



$\gamma(x) \propto 1/\rho(x)$

$\gamma_0 \propto 1/\rho(0)$

Damping rates



- For typical experimental parameters, $\Gamma_1/\omega \simeq 0.04$ converts to $\Gamma_1 \simeq 2\ \mathrm{s}^{-1}$ or a damping time constant $\tau_1 = 1/\Gamma_1 \sim 0.5\ \mathrm{s}$
- Γ_2 is smaller ➡ tails thermalise slower ($\tau_2 = 1/\Gamma_2 \sim 2\ \mathrm{s}$)

$$\Delta x_{\mathrm{RMS}}(t) = A\left[\sqrt{K}\cos(\omega_{\mathrm{B1}}t + \phi_1)e^{-\Gamma_1 t} + \sqrt{1-K}\cos(\omega_{\mathrm{B2}}t + \phi_2)e^{-\Gamma_2 t}\right] + C$$

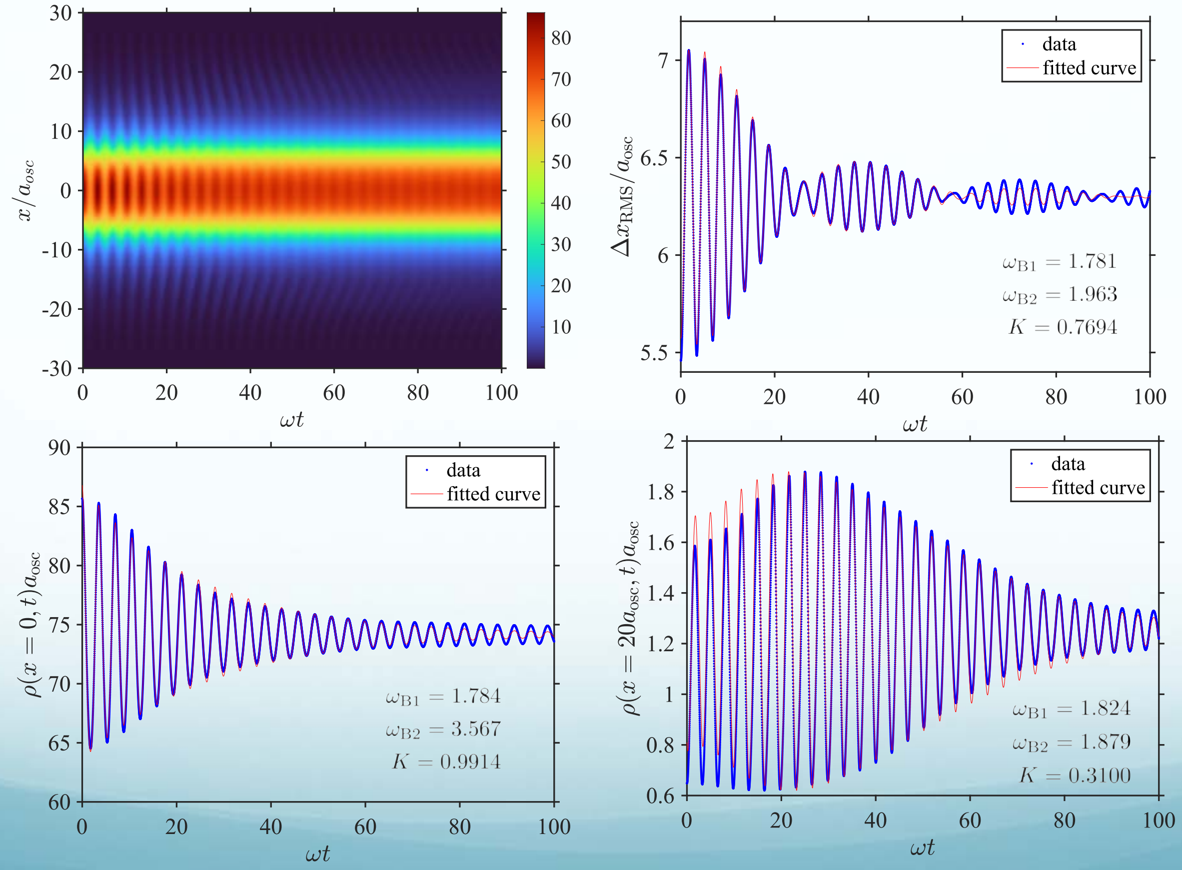
- Damping mechanism? – theory of Landau damping is not straightforward to adopt in 1D (and disagreed with data)

Conclusions and outlook [arXiv:2207.00209]

- Observe ***beating of two breathing modes*** in a harmonically trapped 1D quasi-condensate at nonzero temperature, using c-field simulations
- The frequencies of the breathing modes are intermediate between $\sqrt{3}\omega$ and 2ω
- $\omega_{B1} \simeq \sqrt{3}\omega$ component dominates at lower temperature; associated with the bulk of the quasi-condensate
- $\omega_{B2} \simeq 2\omega$ component dominates at higher temperatures; associated with the tails of the quasi-condensate
- The two breathing modes have ***two distinct damping rates***
- The results:
  - Call for ***a two-fluid model of a weakly interacting 1D Bose gas***, despite the absence of true long-range order or superfluidity at nonzero temperature in 1D
  - Call for revisiting the theory of ***Landau damping in 1D*** and comparing its predictions with the c-field results [See, e.g.: A. Micheli and S. Robertson, arXiv:2205.15826 (2022)]

GHD simulations show beating too!

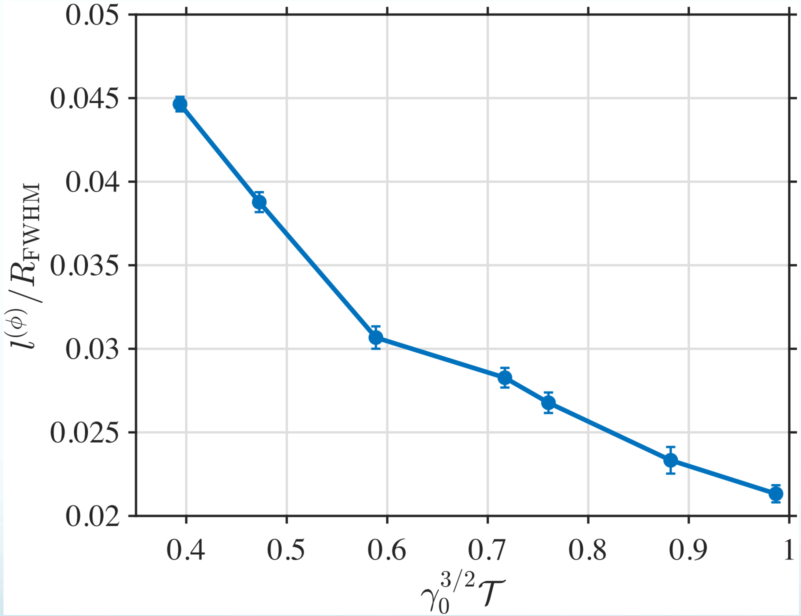
(GHD=Generalised Hydrodynamics; see B. Doyon, SciPost Phys. Lect. Notes 18 (2020) for a review)

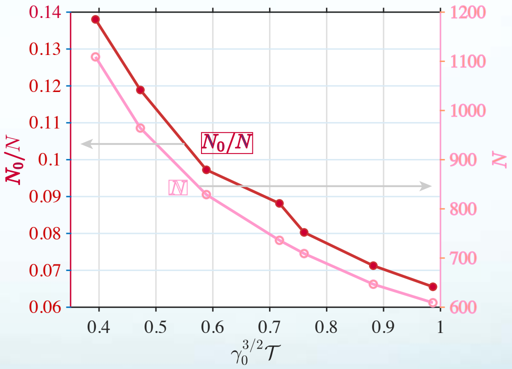


Credit to Raymon Watson

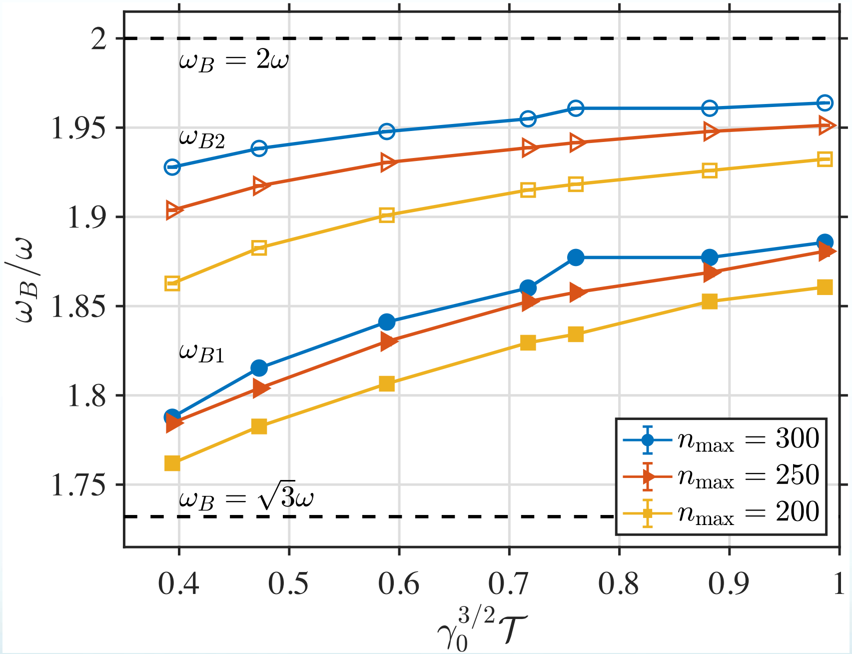
Phase coherence length

- $l_\phi = \frac{2\hbar^2\rho}{mk_BT}$ from $g^{(1)}(x,x';t=0) = \exp(-|x-x'|/2l_\phi)$





Cutoff dependency of SPGPE



Cutoff mode occupancy