Dynamics of quasi-one-dimensional dipolar condensate droplets

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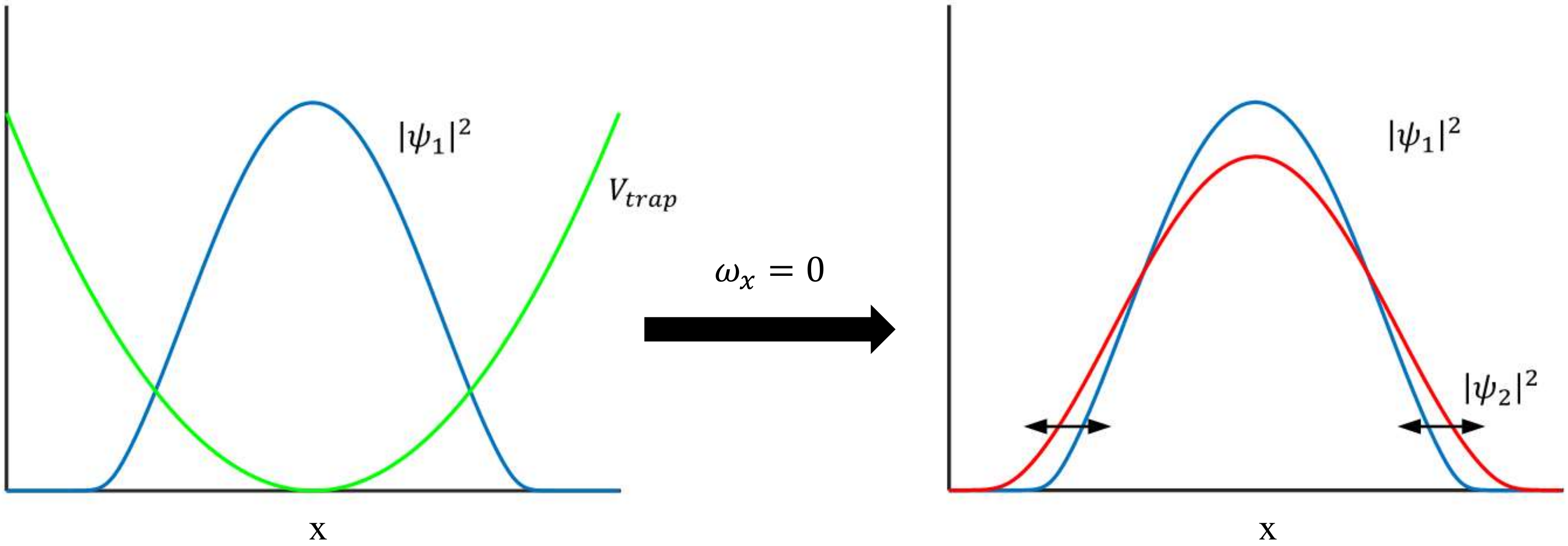


Dipole-Dipole interactions

The dipole-dipole interaction V indicates the interaction between two magnetic or electric dipoles as shown in Figure 1

$$V(\boldsymbol{r}) = \frac{C_{dd}}{4\pi}\frac{(1-3\cos(\theta_{rd})^2)}{|\boldsymbol{r}|^3}$$

$C_{dd} = 3g\varepsilon_{dd}$ ε_{dd}: The dipole strength

$g = \frac{4\pi\hbar^2 a}{m}$ a :the s-wave scattering length

In this work, these dipoles are aligned in x-z plane by an external field, enclosing an angle α with the x-axis.

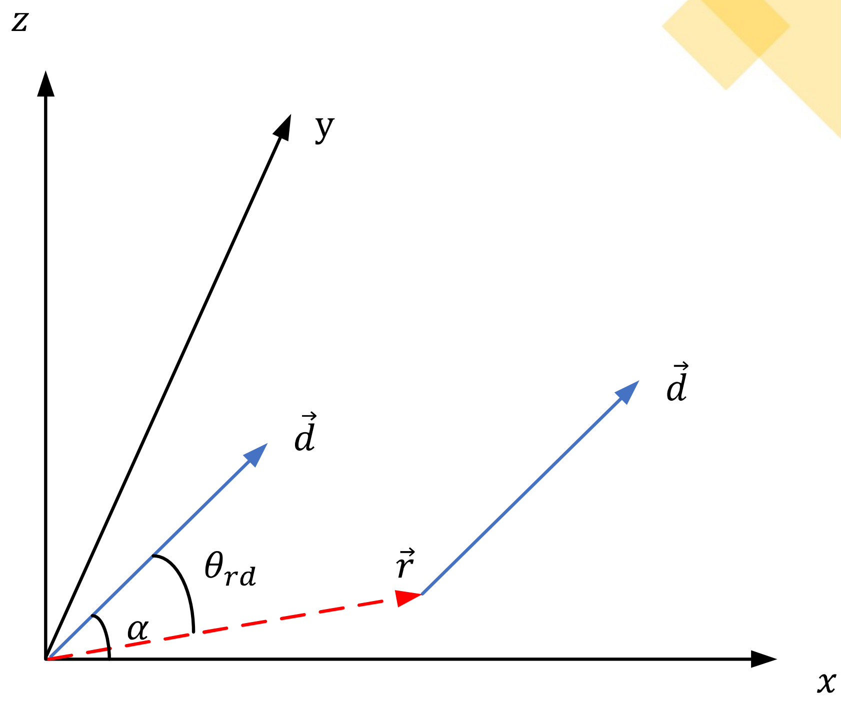


Figure.1 dipole-dipole interaction

Gross-Pitaevskii equation

The time-dependent Gross-Pitaevskii equation (GPE) for atoms with long-range interactions and for a harmonic trap is given by:

$$i\hbar\frac{\partial\phi}{\partial t} = \left[-\frac{\hbar^2}{2m}\nabla^2 + \frac{1}{2}m\omega_\rho^2(y^2+z^2) + \frac{1}{2}m\omega_x^2x^2 + g|\phi|^2 + \int V(\boldsymbol{r}-\boldsymbol{r}')|\phi(\boldsymbol{r}')|^2 d^3\boldsymbol{r}'\right]\phi$$

$g = \frac{4\pi\hbar^2 a}{m}$ The contact interaction

$$a_\rho = \sqrt{\frac{\hbar}{m\omega_\rho}}$$

Cigar-shape trap potential i.e. $\omega_\rho \gg \omega_x$

Why do we want to focus on this frequency?

- If we put the system in a cigar shape potential, according to the simulation, these dipoles tend to orient along the weak confinement axis of the trap so that it is unstable when ε_{dd} is large[1].
- However, experiments show that this form can be preserved even in the absence of external confinement. It is called self-bound droplet.

Figure.2 Intuitive picture of trapped a dipolar BEC in a cigar shape potential[1].

[1]T Lahaye, C Menotti, L Santos, M Lewenstein, and T Pfau. The physics of dipolar bosonic quantum gases. Reports on Progress in Physics, 72(12):126401, nov 2009

Why do we want to focus on this frequency?

- To fix this problem, theorists propose the extended Gross-Pitaevskii equation[2].

$$i\hbar\frac{\partial\phi}{\partial t} = \left[-\frac{\hbar^2}{2m}\nabla^2 + \frac{1}{2}m\omega_\rho^2(y^2+z^2) + \frac{1}{2}m\omega_x^2x^2 + g|\phi|^2 + \int V(\boldsymbol{r}-\boldsymbol{r}')|\phi(\boldsymbol{r}')|^2 d^3\boldsymbol{r}' + \mu_{QF}|\phi|^3\right]\phi$$

$\mu_{QF} = Q_5(\varepsilon_{dd})$ The quantum fluctuations.

$$Q_5(\varepsilon_{dd}) = \int_0^1 (1-\varepsilon_{dd}+3x^2\varepsilon_{dd})^{\frac{5}{2}}dx$$

[2]Aristeu R. P. Lima and Axel Pelster. Quantum fluctuations in dipolar bose gases. Physical Review A, 84(4), oct 2011.

Why do we want to focus on this frequency?

$\mu_{QF} = Q_5(\varepsilon_{dd})$. It is a complex number($\varepsilon_{dd} > 1$).

Rewrite $\mu_{QF} = a + ib$ and neglect all real terms

$i\hbar\frac{\partial\phi}{\partial t} = ib\phi$ leads to $\phi = e^{\frac{b}{\hbar}t}$ the change of number of atoms in the system.

To avoid this problem, people use its quadratic expansion of ε_{dd}, which is

$$Q_5(\varepsilon_{dd}) \approx \frac{32g}{3}\sqrt{\frac{a^3}{\pi}}\left(1 + \frac{3}{2}\varepsilon_{dd}^2\right) = \gamma_{QF}$$

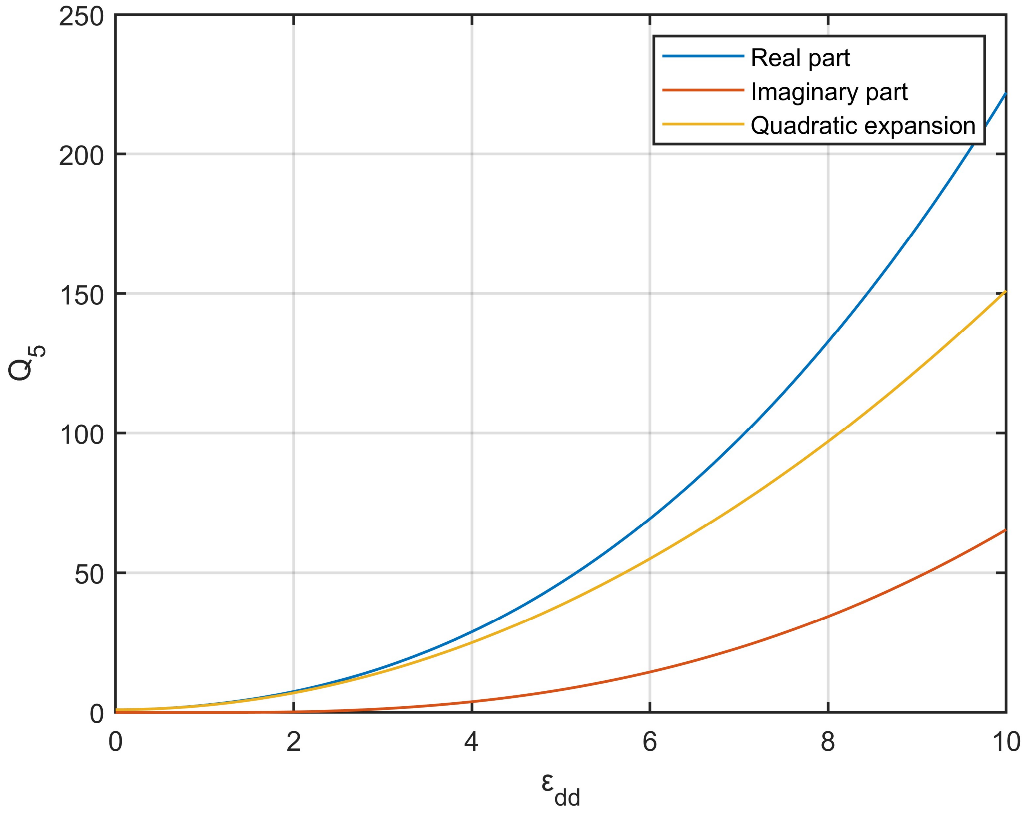
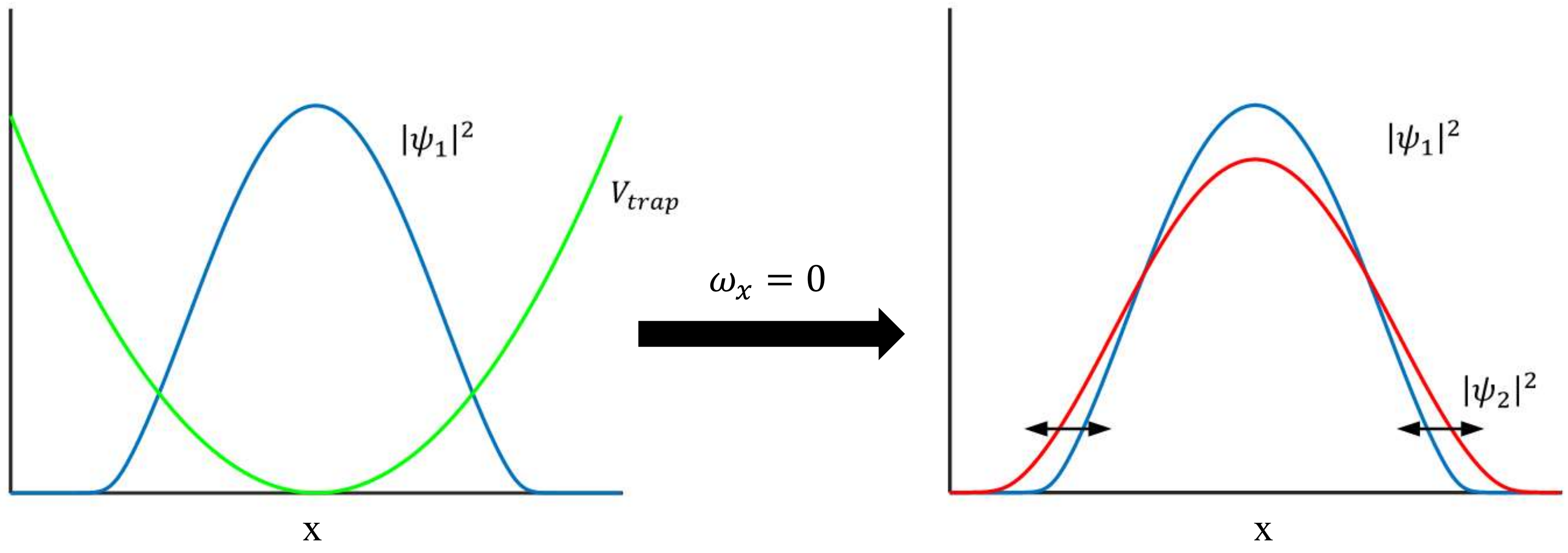


Figure.3 Real part, imaginary part and quadratic expansion of $Q_5(\varepsilon_{dd})$

Why do we want to focus on this frequency?

This oscillation frequency can be compared with experimental results to check whether quantum fluctuations stabilize the droplet system, given some theoretical studies propose that it is the short-range three-body repulsion that stabilizes the droplet system[3].



[3] Yuqi Wang, Longfei Guo, Su Yi, and Tao Shi. Theory for self-bound states of dipolar bose-einstein condensates. Phys. Rev. Research, 2:043074, Oct 2020

Quasi-One-Dimensional Gross-Pitaevskii equation

Following the cigar-shape trap condition, theorists can use the dimensionless quasi-one-dimensional Gross-Pitaevskii equation to deal with the problem.

$$i\frac{\partial\psi}{\partial\tilde{t}} = \left[-\frac{1}{2}\frac{d^2}{d\tilde{x}^2} + \frac{1}{2}\widetilde{\omega}_x^2\tilde{x}^2 + 2\tilde{a}N|\psi|^2 + \widetilde{\Phi}_{dd}^{1D} + \frac{256}{15\pi}N^{3/2}\tilde{a}^{5/2}\left(1+\frac{3}{2}\varepsilon_{dd}^2\right)|\psi|^3\right]\psi$$

- $\tilde{t} = \omega_\rho t$ $\qquad \tilde{x} = \frac{x}{a_\rho}$
- $\int|\psi(\tilde{x},\tilde{t})|^2 d\tilde{x} = 1$ $\qquad \widetilde{\omega}_x = \frac{\omega_x}{\omega_\rho}$
- $\tilde{a} = \frac{a}{a_\rho} = 0.0015$
- $N = 50000$ number of atoms
- $\alpha = 0$

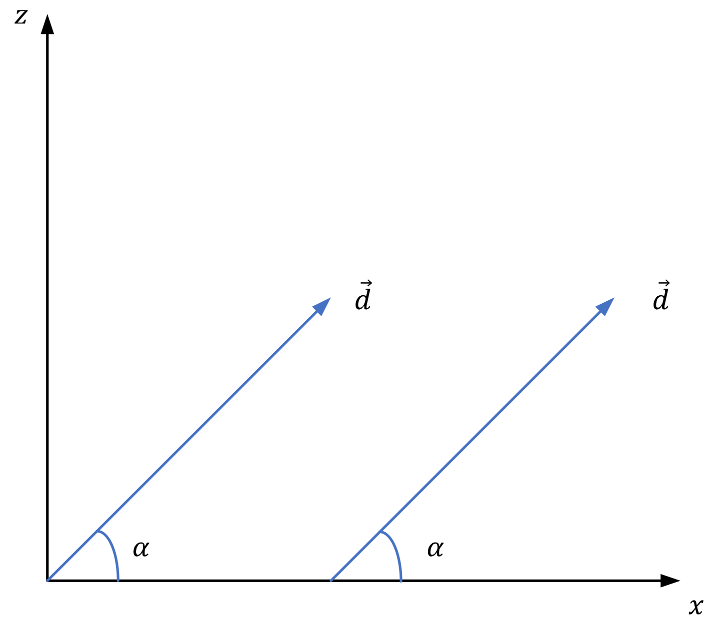


Figure.4 The reduced dipole-dipole interaction

Quasi-one-dimensional Self-Bound Droplet

$\varepsilon_{dd} > \varepsilon_{dd}^{c} \approx 1.1$

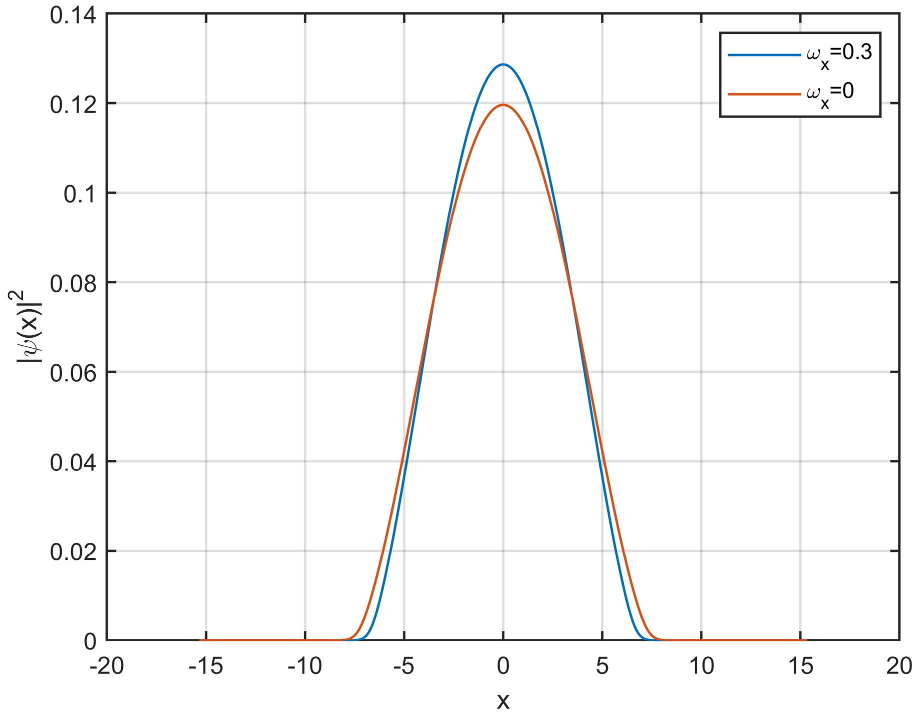


Figure.5(a) Ground states for $\varepsilon_{dd} = 2$

$\varepsilon_{dd} < \varepsilon_{dd}^{c} \approx 1.1$

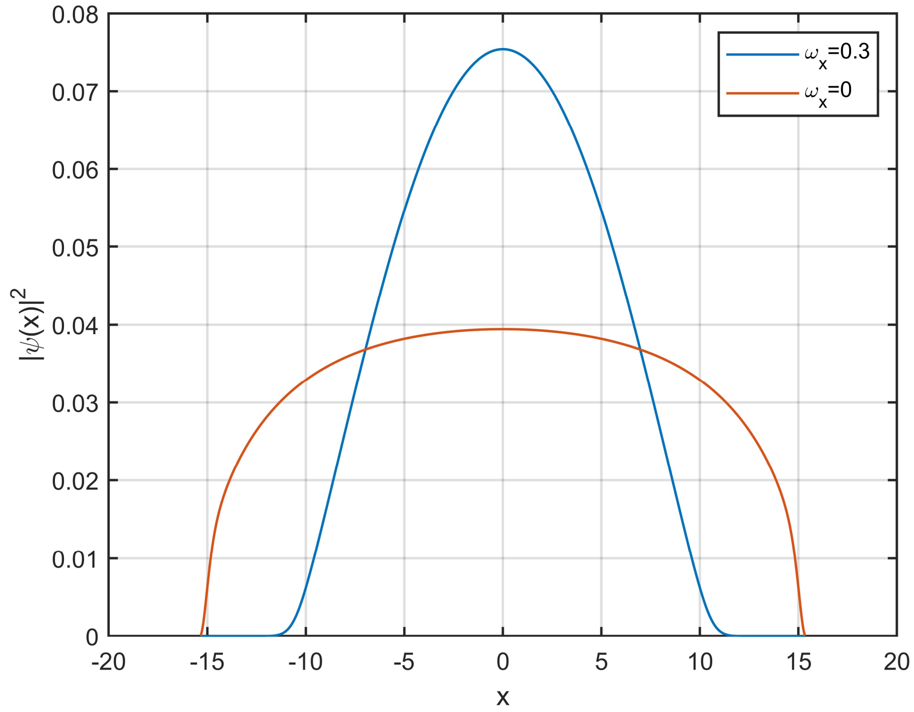


Figure.5(b) Ground states for $\varepsilon_{dd} = 0.8$

Oscillation behaviors

When $t < t_0$ system is in the ground state, after t_0 , what will happen if we switch off the potential?

- For $\varepsilon_{dd} > \varepsilon_{dd}^{c}$, the condensate will oscillate. Take $\varepsilon_{dd} = 2$ as an example.
- For $\varepsilon_{dd} < \varepsilon_{dd}^{c}$, the condensate will continue expanding. Take $\varepsilon_{dd} = 0.8$ as an example.

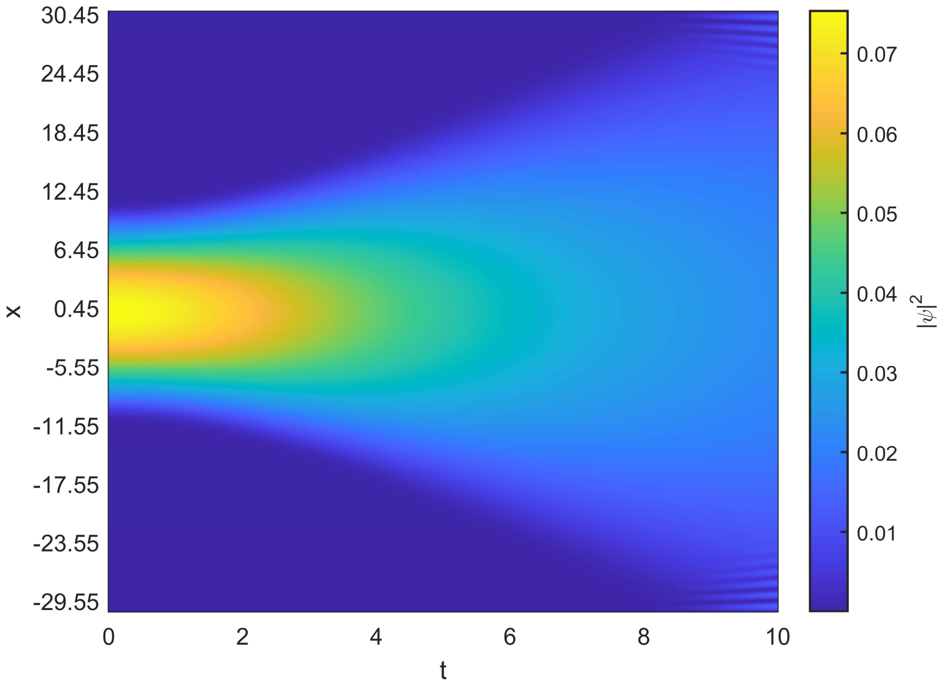


Figure.6(a) $|\psi(x,t)|^2$ for $\varepsilon_{dd} = 0.8$

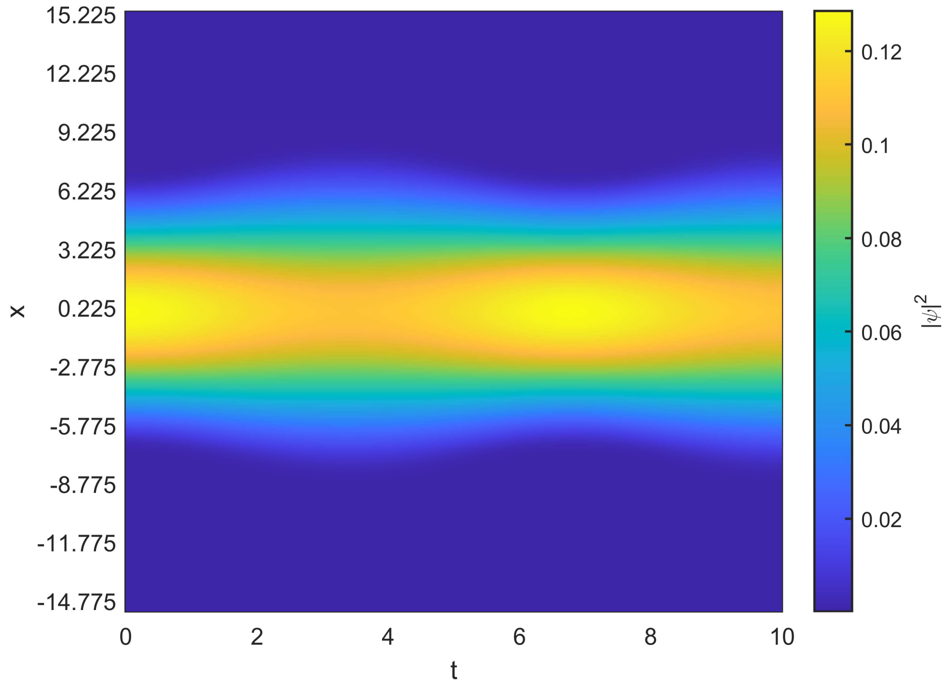


Figure.6(b) $|\psi(x,t)|^2$ for $\varepsilon_{dd} = 2$

The numerical value of the frequency

$$< x^2 > (t) = \int x^2 |\psi(x,t)|^2 dx$$

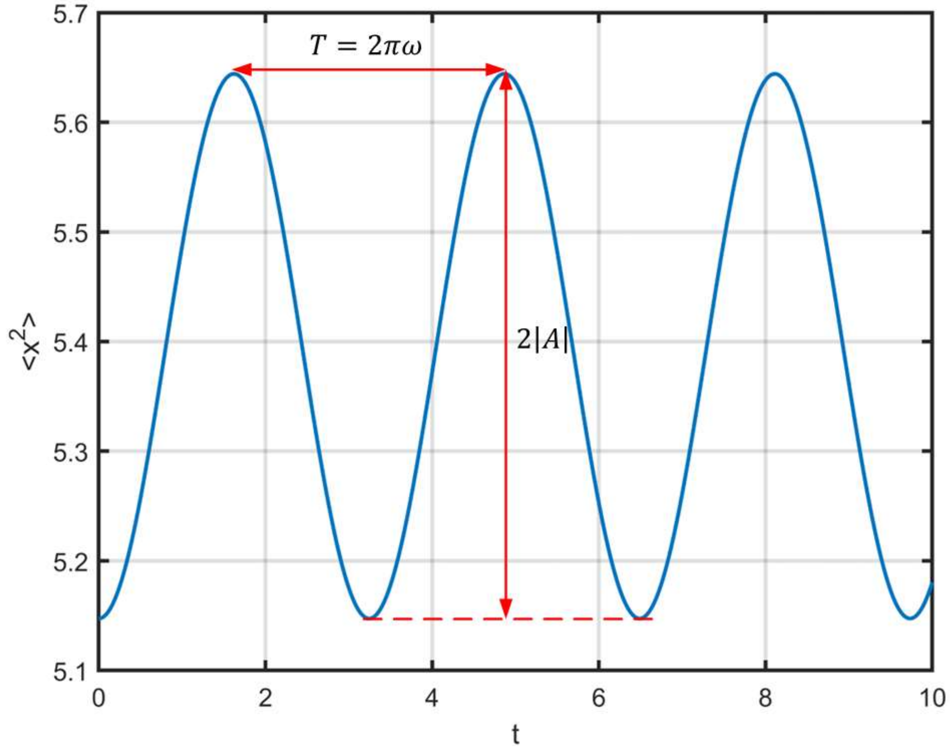


Figure.7(a) $< x^2 > (t)$ for $\varepsilon_{dd} = 3.2$

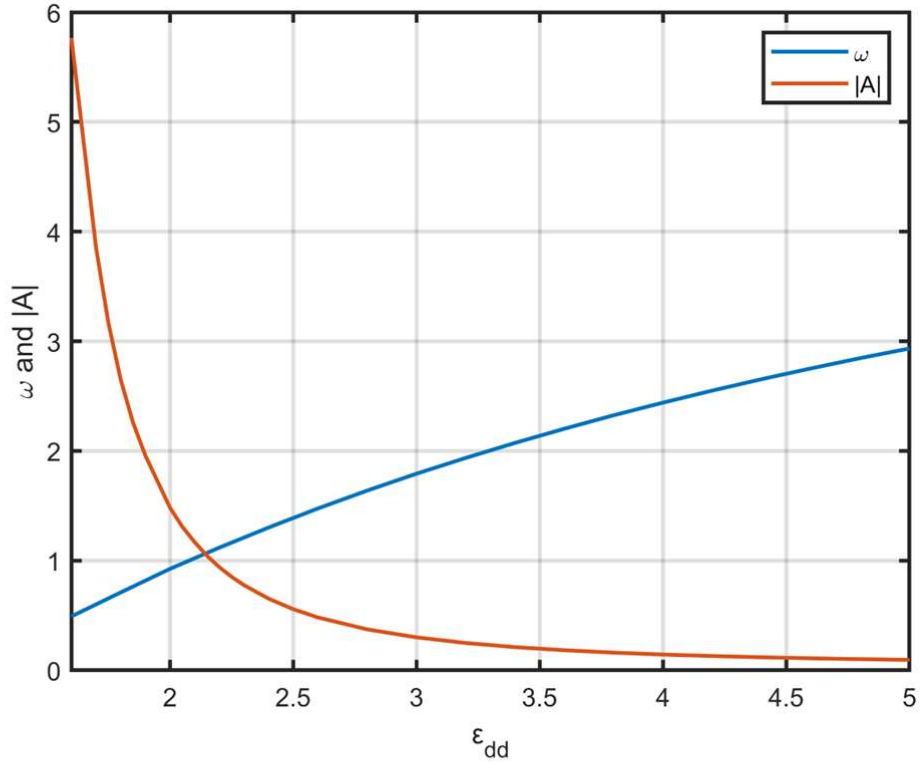


Figure.7(b) The relation between the frequency, the amplitude and ε_{dd}

Analytical Approach

Analytical expression for the frequency

$$\omega = \omega_x \sqrt{\frac{2}{\frac{< x_{droplet}^2 >}{< x_{potential}^2 >} - 1}}$$

Variational wave function of the ground state.

The Gaussian state:

$$\psi(x) = \frac{1}{\sqrt{\pi^{\frac{1}{2}}}} \sqrt{k} \exp(-\frac{x^2}{2} k^2)$$

The new trial wave function:

$$\psi(x) = \frac{1}{\sqrt{\Gamma(\frac{5}{4})}} \sqrt{\frac{k}{2}} \exp(-\frac{x^4}{2} k^4)$$

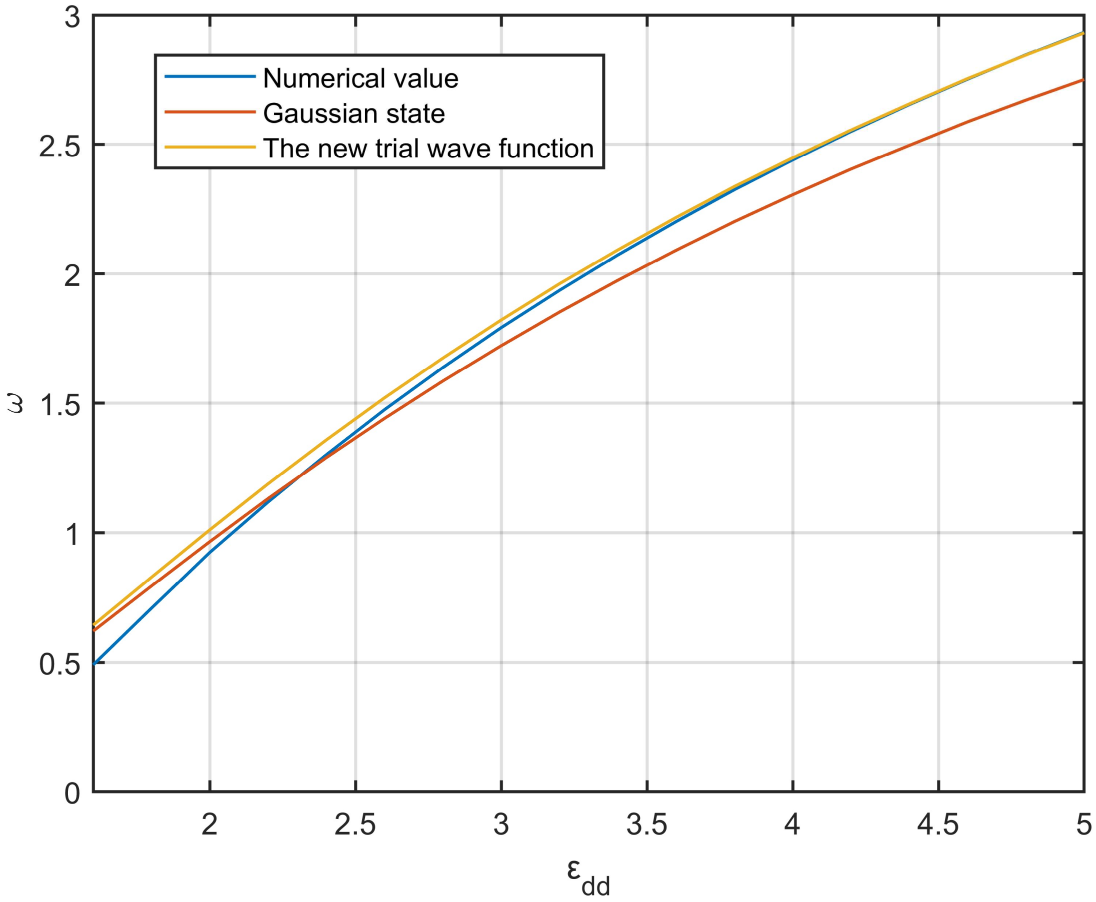


Figure.8 The relation between ω and ε_{dd} by using the variational method

The effect of quantum fluctuations

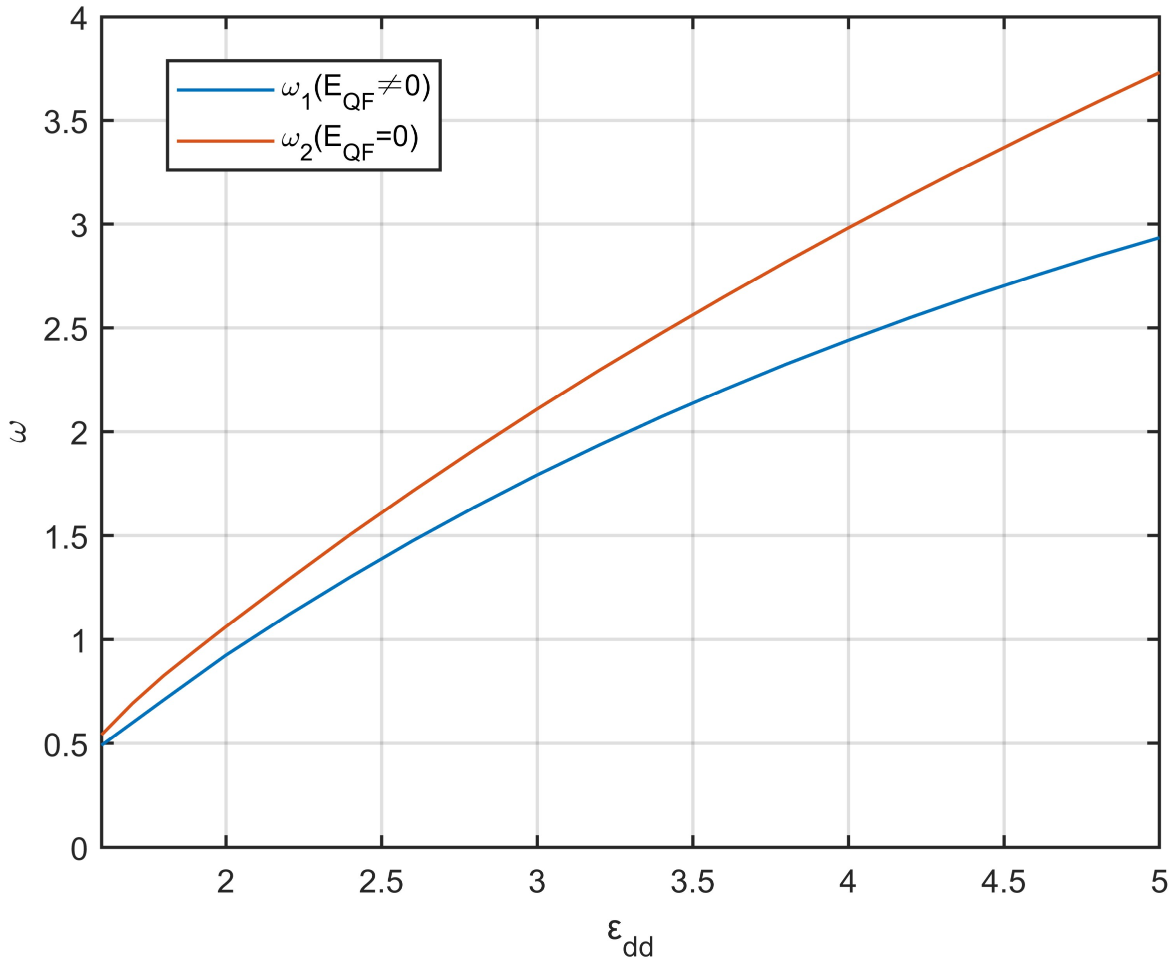


Figure.9 The relation between ω and ε_{dd} with(without) the quantum fluctuations

The quantum fluctuation term is propotional to $\left(1+\frac{3}{2}\varepsilon_{dd}^2\right)$ and the dipolar interaction term is propotional to ε_{dd}. So, this difference is small when ε_{dd} is low and it is large when ε_{dd} is high.

What's more important is that mechanisms other than the quantum fluctuations will cause a change in the frequency.

Conclusion

We give a physical quantity: ω. This frequency can be measured in experiments. By the comparison between them, we can testify whether quantum fluctuations stabilize the droplet system. In addition, we use the variational wave functions to calculate the frequency in a purely analytical way, which behaves well in predicting the oscillation frequency.

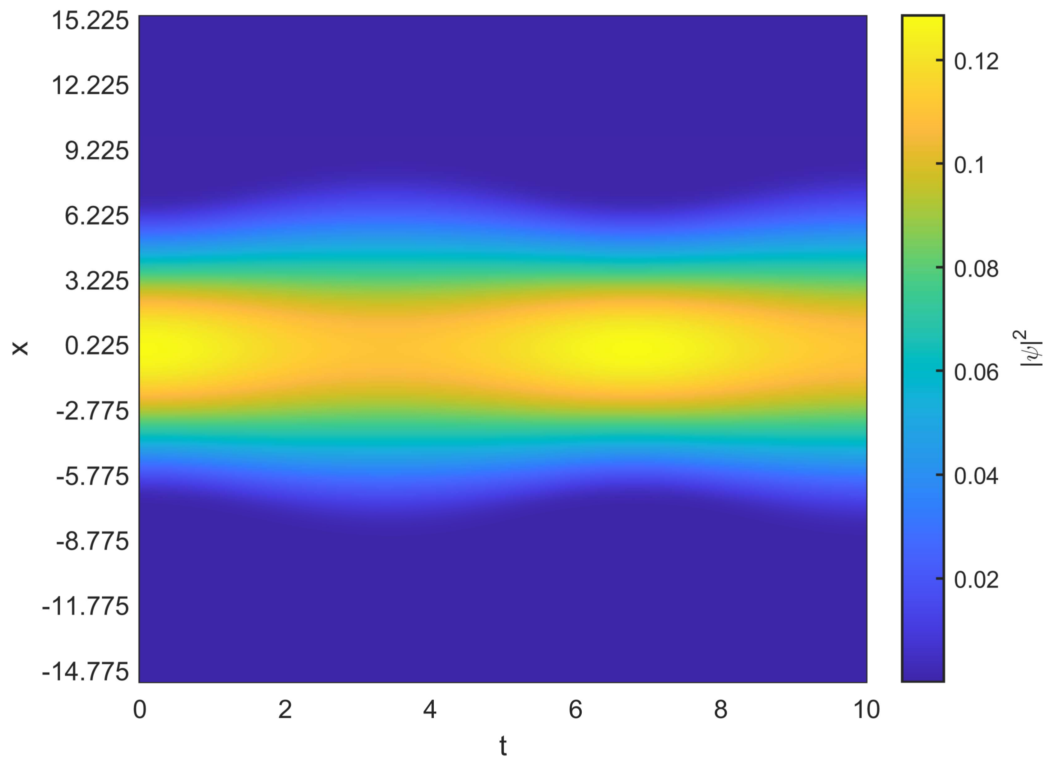


Figure.10 $|\psi(x,t)|^2$ for $\varepsilon_{dd} = 2$