Phase-space stochastic quantum hydrodynamics for interacting Bose gases

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In this work we derive, within the positive-P phase-space formalism, a new stochastic hydrodynamic method for the description of interacting Bose gases. It goes beyond existing hydrodynamic approaches, such as superfluid hydrodynamics or generalised hydrodynamics, in its capacity to simulate the full quantum dynamics of these systems: it possesses the ability to compute non-equilibrium quantum correlations, even for short-wavelength phenomena. Using this description, we derive a linearised stochastic quantum hydrodynamic (LSQHD) scheme which is able to simulate such non-equilibrium situations for longer times than the full positive-P approach (at the expense of approximating the treatment of quantum fluctuations). We additionally elucidate how our formalism can be reduced to, and connected with, other known theoretical approaches. Furthermore, we demonstrate the usefulness and advantages of this formalism by exploring the correlations that arise in the quantum shock wave scenario of Ref. [1] (which themselves have not previously been calculated) and comparing its predictions to other numerical approaches for quantum dynamics. Figure 1 (a) shows the initial (coherent state) and time evolved density profiles of the system: an initial density bump produces a quantum shock wave (density ripples) as it expands into the non-zero background density; (b) shows the correlations at dimensionless time τ which develop across the shock front; (c) shows the results of the full $+P$ approach, LSQHD, truncated Wigner approach (TWA), and an exact matrix product state (MPS) calculation, all separated vertically by 0.15 for clarity (the shaded regions denote one standard error of uncertainty). Here the LSQHD result possesses extremely little noise and lies very close to the exact MPS calculation.

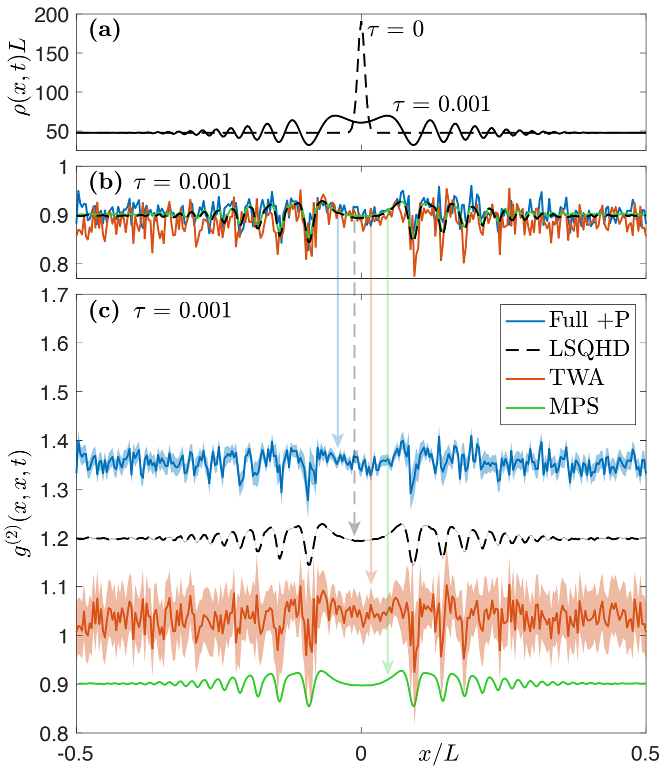

Figure 1: The density profile and normalised local correlation function $g^{(2)}(x,x,t)$ for a weakly interacting one-dimensional Bose gas (with the Lieb-Liniger interaction parameter $\gamma_{\rm bg} = 0.1$), on a ring of circumference L, at zero-temperature.

[1] S. A. Simmons *et al.*, *Phys. Rev. Lett.* **125**, 180401 (2020).