



Authors: Gary J Mooney, Gregory A L White,
John Fidel Kam, Charles D Hill, and
Lloyd C L Hollenberg

Generation of Large-Scale Entanglement on Physical Quantum Devices

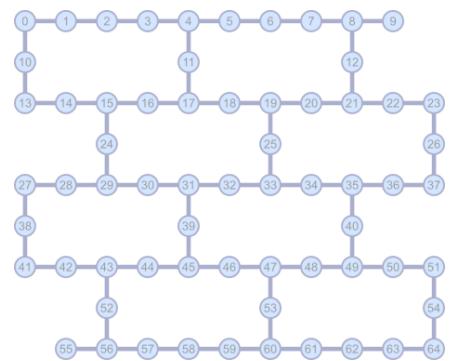
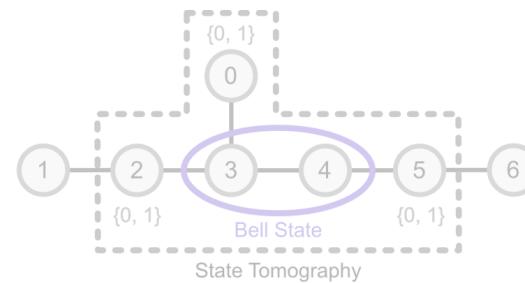
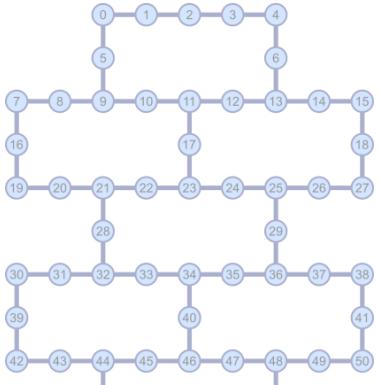
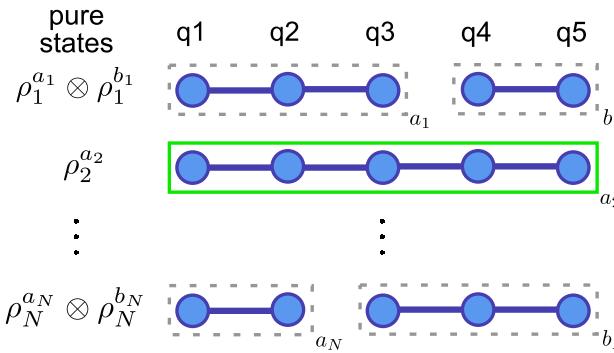


**IBM Quantum Network Hub
at the University of Melbourne**

Overview

Forms of Multipartite Entanglement

- Bipartite and Genuine Multipartite entanglement



Detecting Bipartite entanglement

- By preparing Graph states on *IBM Quantum* devices

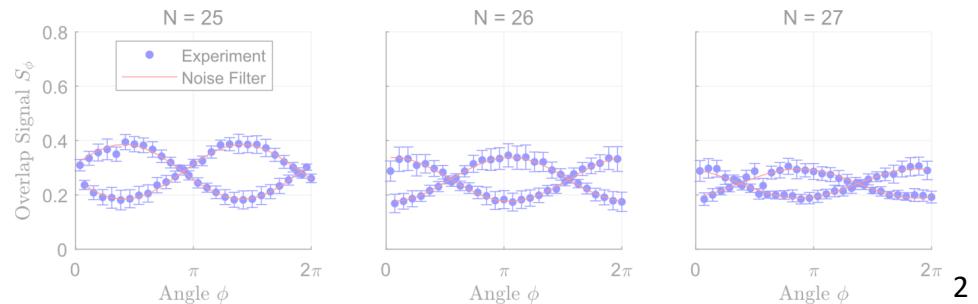
Mooney, Hill and Hollenberg, Sci. Rep. (2019)

Mooney, White, Hill and Hollenberg, Adv. Quantum Technol (2021)

Detecting Genuine multipartite entanglement

- By preparing GHZ states
- Also apply parity checking error detection
 - Effects on fidelity

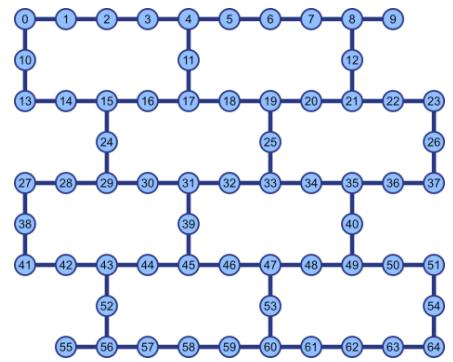
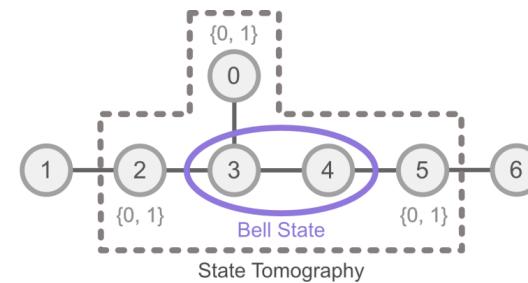
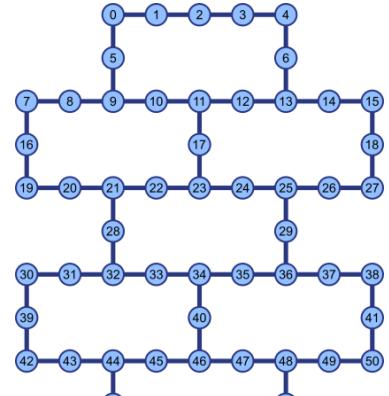
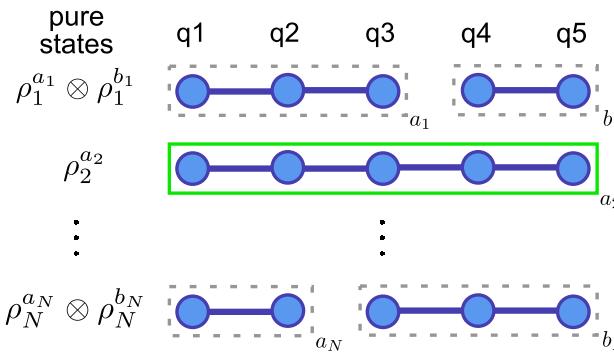
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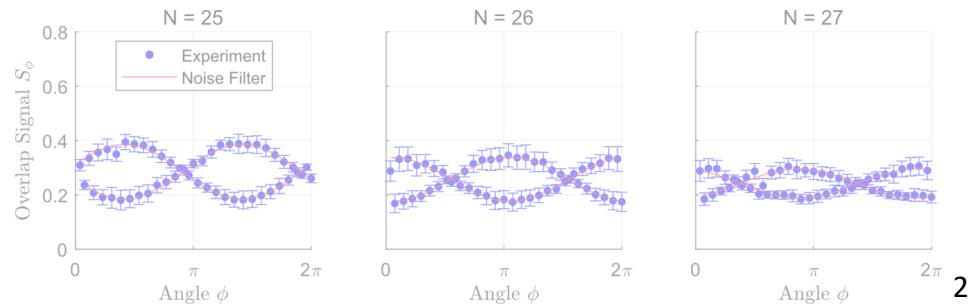
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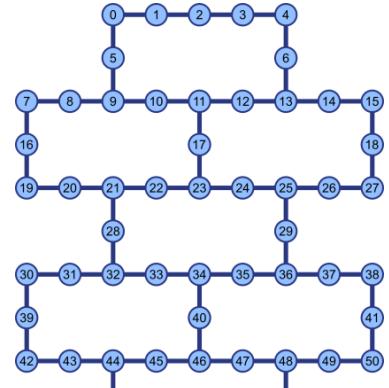
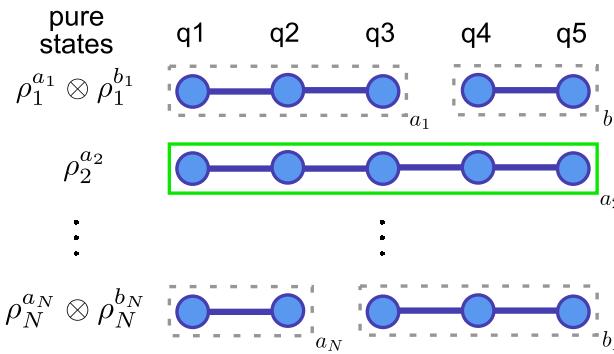
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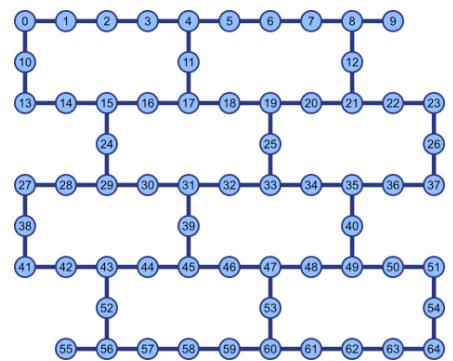
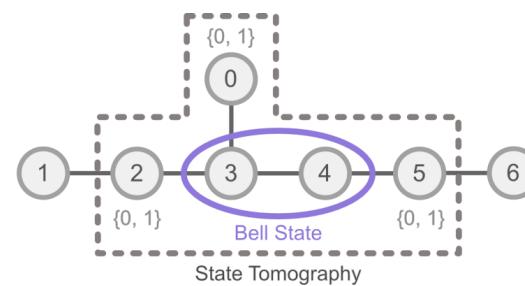


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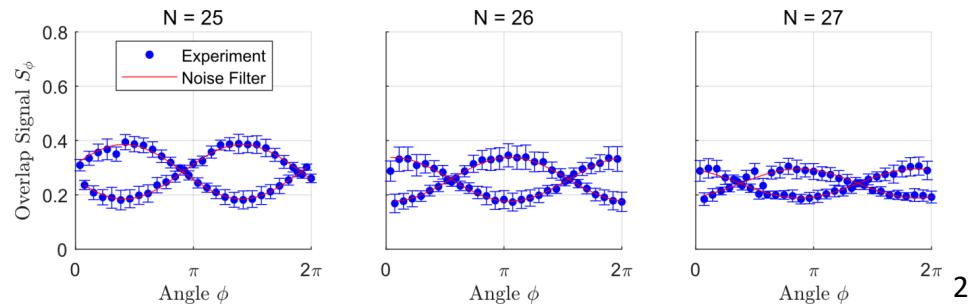
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Forms of Multipartite Entanglement

Real quantum device → Noise

Quantum state is a **Mixed state**: probabilistic mixture of pure states

Mixed State

$$\rho = \sum_{i=1}^N p_i \rho_i$$

Density matrix ρ

Pure states ρ_i

Probabilities p_i

Forms of Multipartite Entanglement

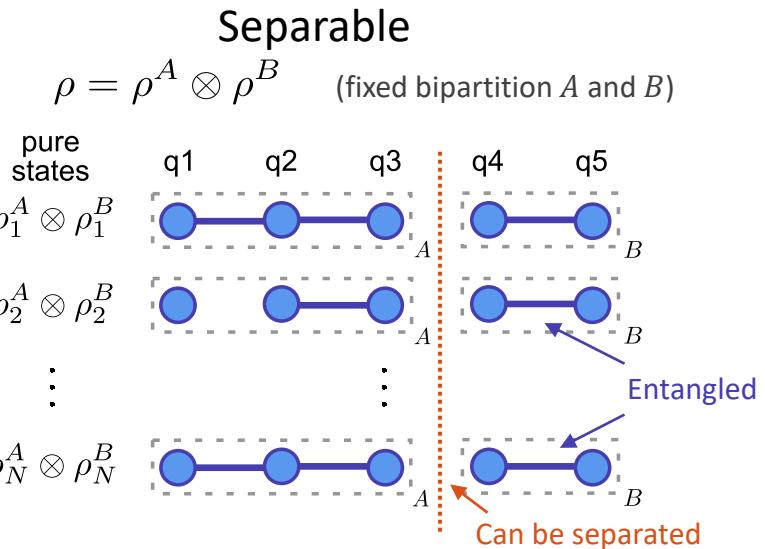
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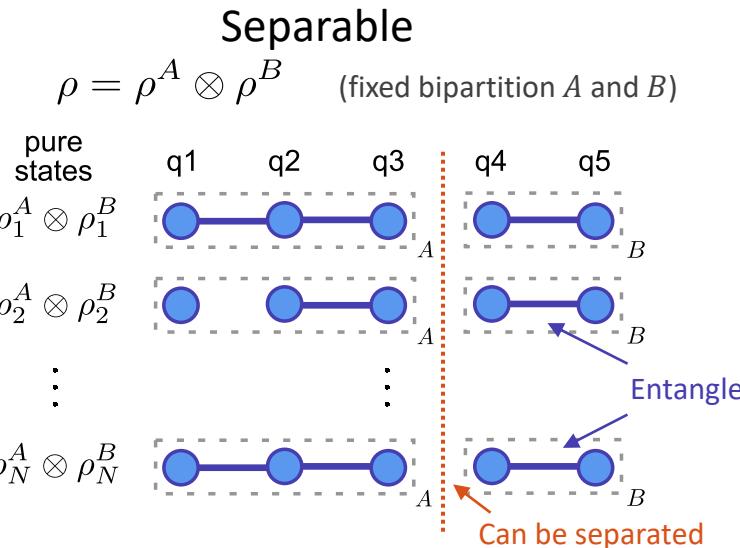
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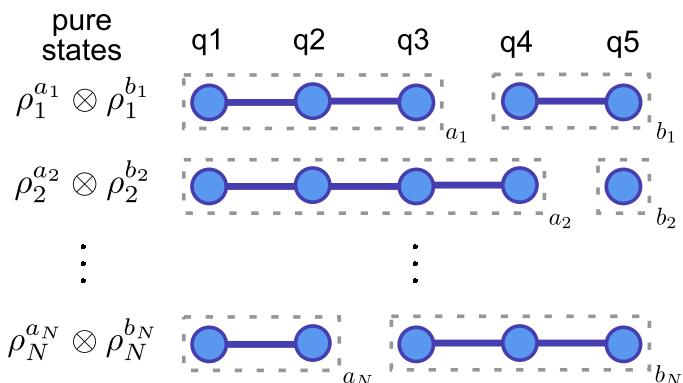
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Bipartite Entanglement

- State is **not** separable
- Can't write the state as a tensor product of any bipartition



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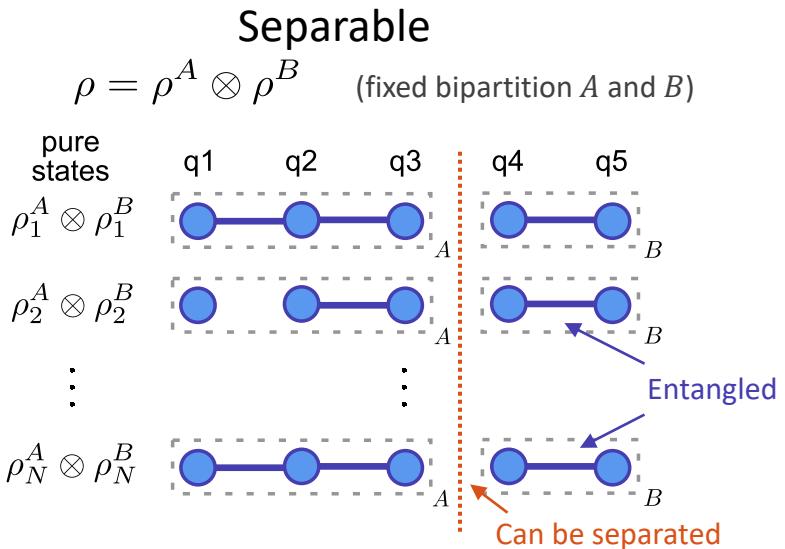
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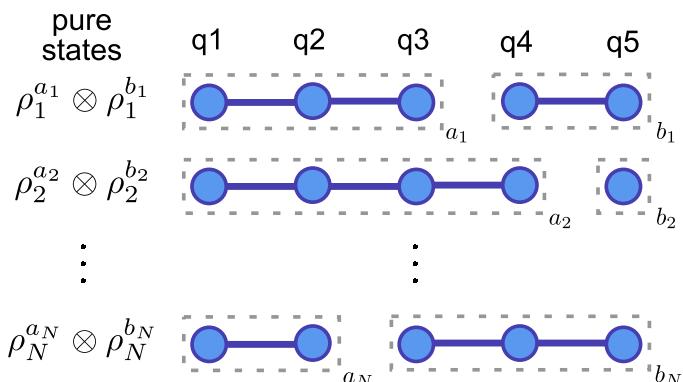
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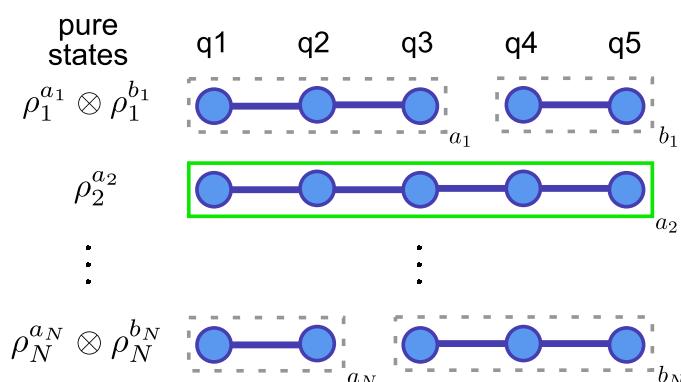
Bipartite Entanglement

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Genuine Multipartite Entanglement (GME)

- Stronger form of entanglement
- Contains pure states → entangled over all qubits





Bipartite Entanglement in Graph States

Graph State (Cluster State) → detect bipartite entanglement

- Robust to noise
 - Requires $n/2$ local measurements to disentangle
- Low circuit depth

Briegel and Raussendorf, Phys. Rev. Lett. (2001)

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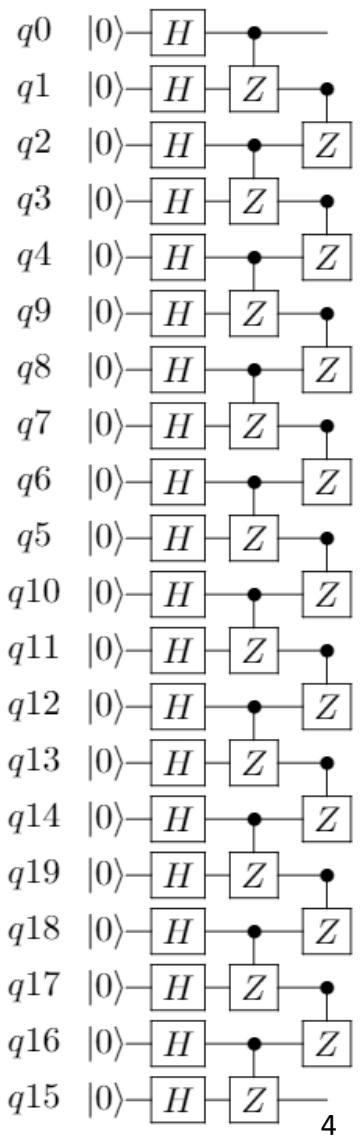
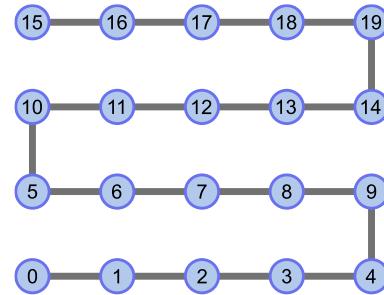
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$$|G_n\rangle = \prod_{\substack{(\alpha, \beta) \in E \\ E := (\text{edge set})}} \text{CZ}_{\beta}^{\alpha} |+\rangle^{\otimes n}$$

↑
Controlled-phase gate

Defined on a graph

20-qubit *ibmq_poughkeepsie*
device



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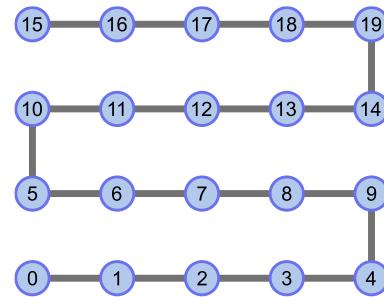
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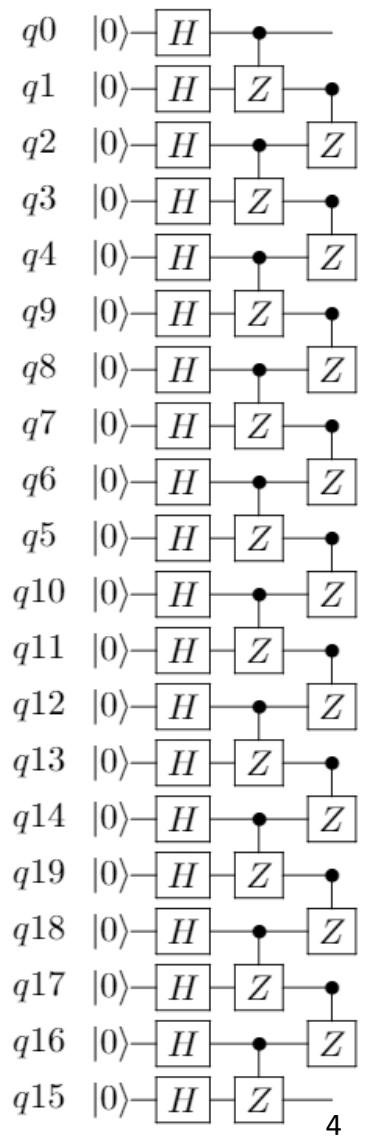
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20-qubit *ibmq_poughkeepsie* device



Preparation



20-qubit line

Detection Strategy

Show state is **not separable**
 ⇒ At least **bipartite entangled**

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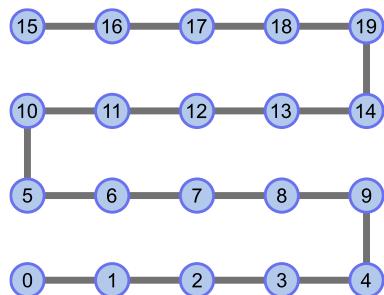
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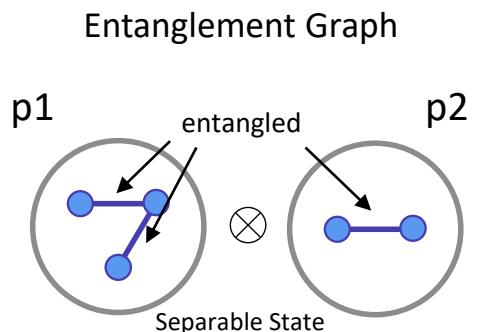
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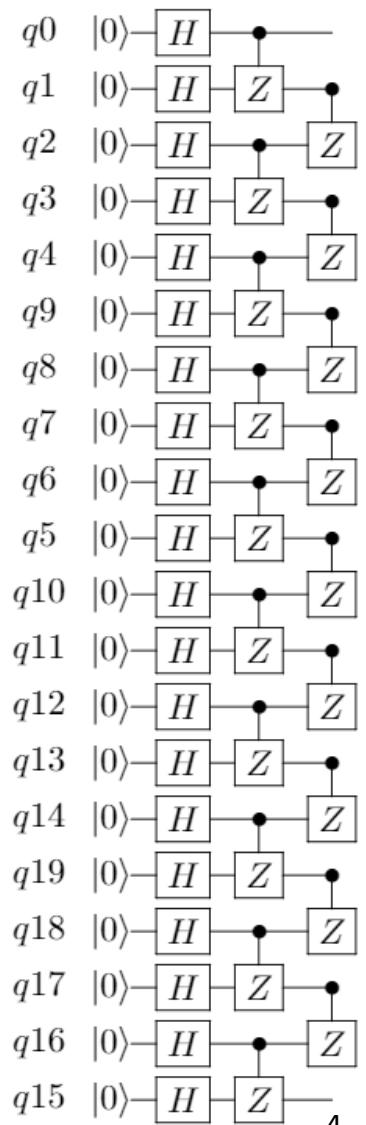
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 ⇒ At least **bipartite entangled**

Generate
Entanglement
Graph



Check each pair of connected qubits
 → Detect entanglement ⇒ Add edge

If entanglement graph is connected ⇒ State is entangled



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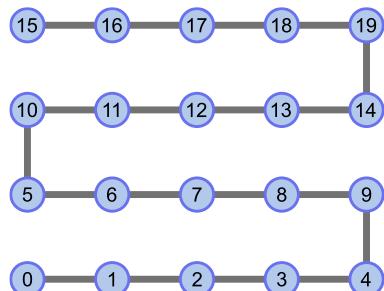
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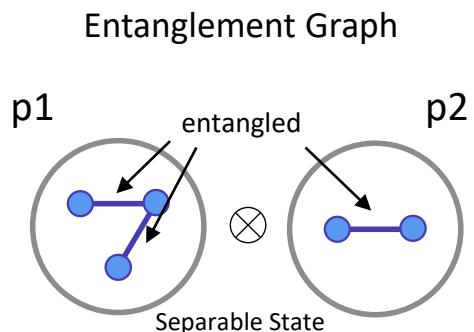
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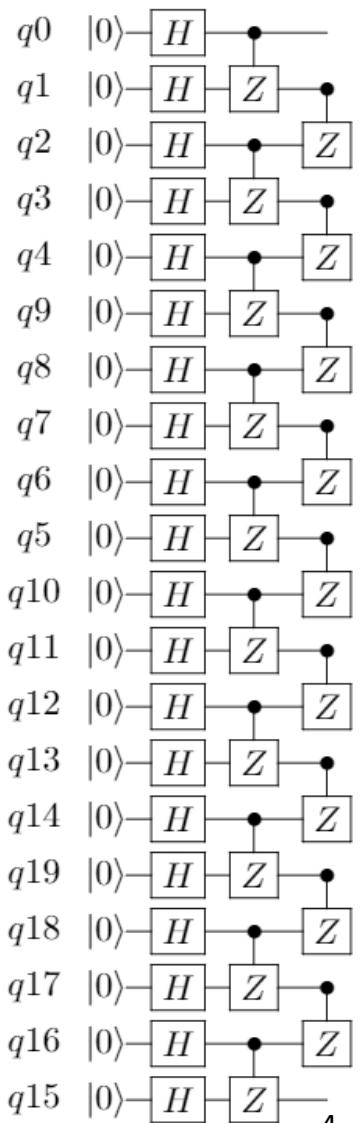
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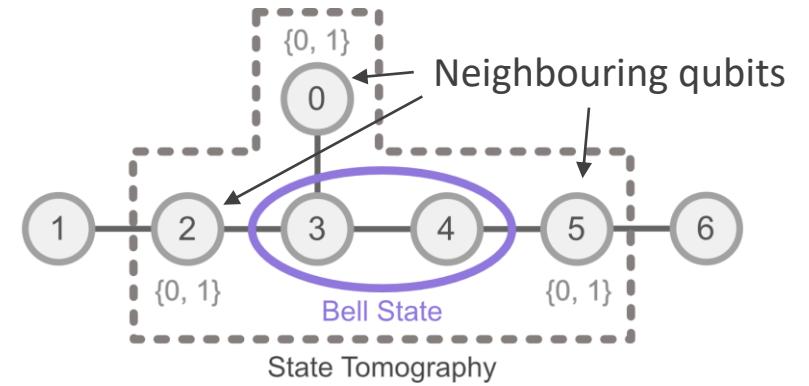


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How to Detect 2-Qubit Entanglement

Full state tomography on target pair of qubits and their neighbours

Example
Entanglement detection between qubits 3 and 4



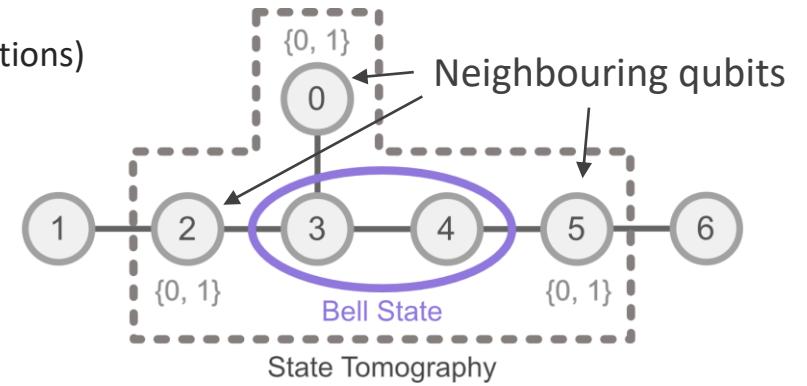
How to Detect 2-Qubit Entanglement

Full state tomography on target pair of qubits and their neighbours

Turns out: project neighbour-qubits into Z-basis → **Bell state** (up to local operations)

- Each combination of projection states: $\{0, 1\}^{\#(\text{neighbours})}$ (produces a Bell state)

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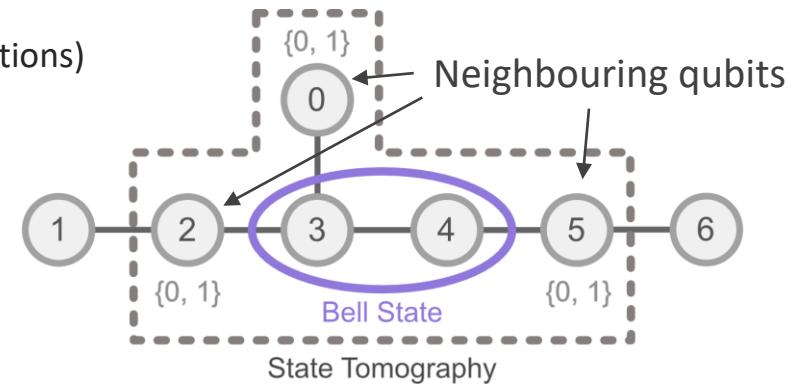
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Entanglement measured as **negativity** of partial transpose

- Calculate → Sum over magnitudes of negative eigenvalues of $\rho_{3,4}^{T_4}$

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Negativity

$$\mathcal{N}(\rho_{3,4}^{T_4}) = \sum_{\lambda_i < 0} |\lambda_i|$$

Negative eigenvalues of $\rho_{3,4}^{T_4}$

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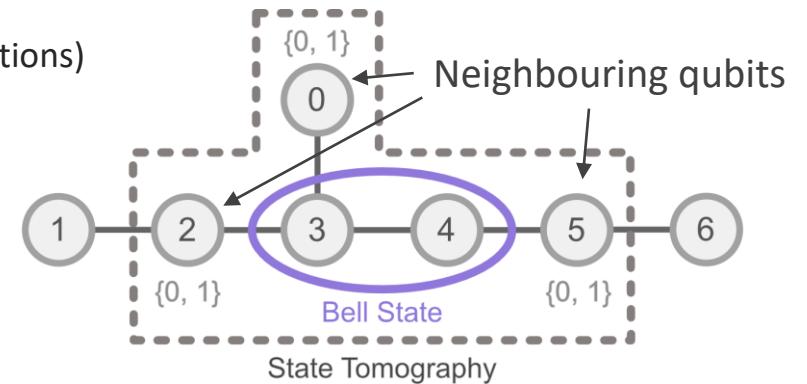
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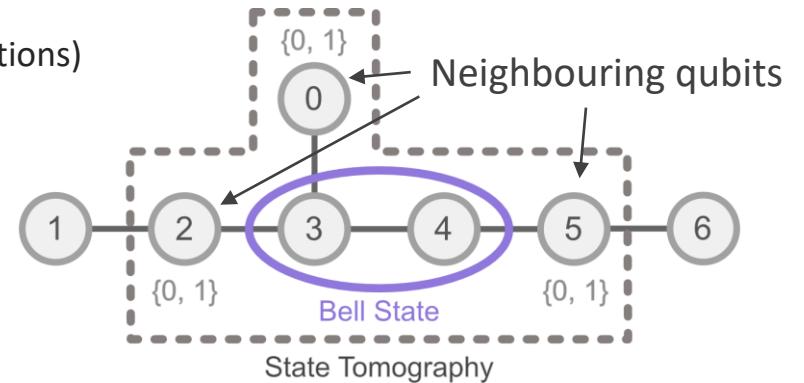
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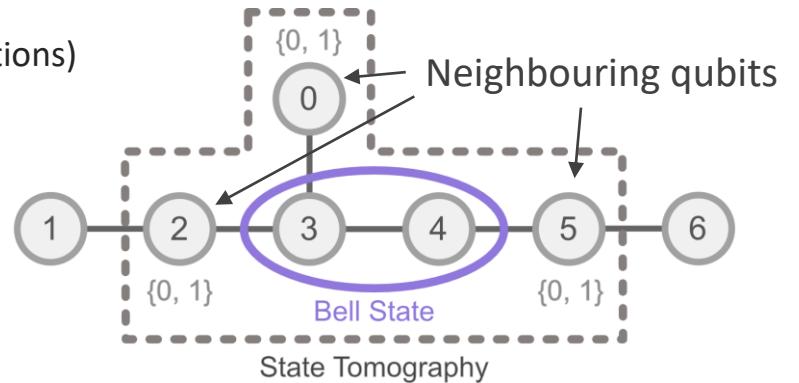
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→ Extent of entanglement is the largest negativity (among the combinations)

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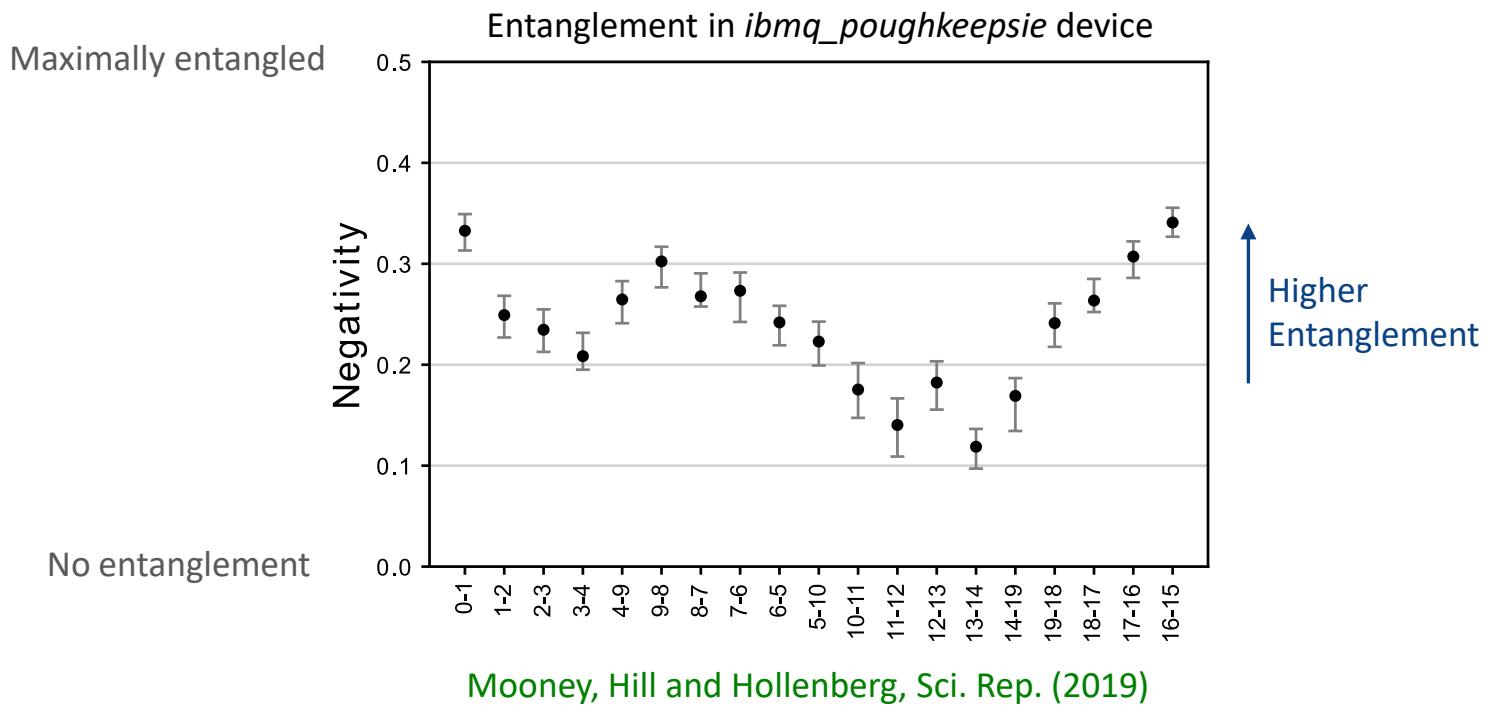
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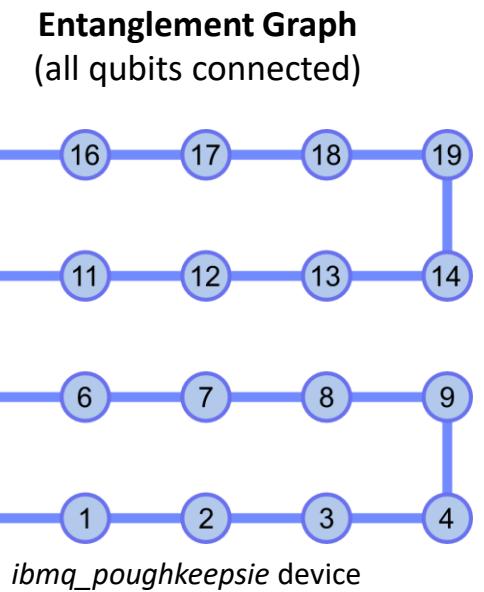
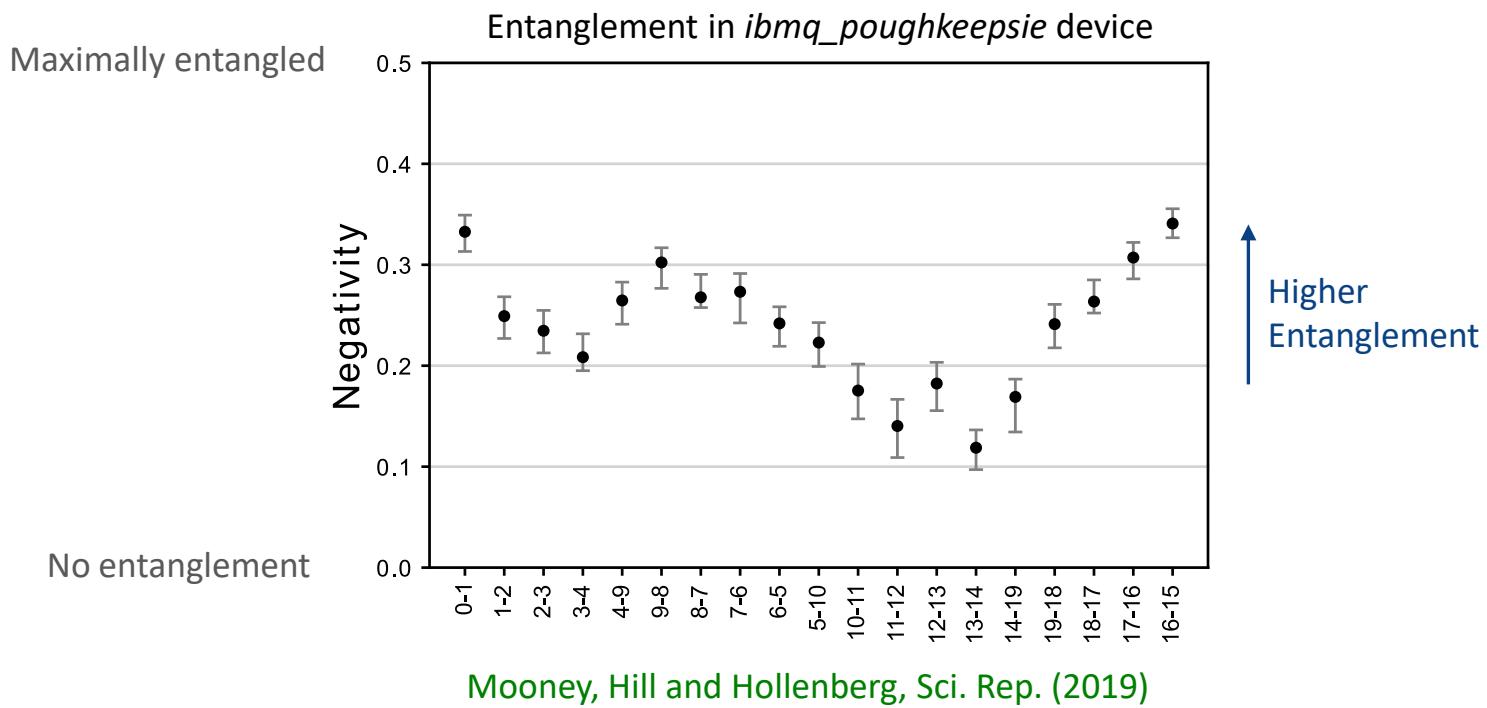
Results: Negativities on an *IBM Quantum* device

Apply these techniques to the **20-qubit *ibmq_poughkeepsie* device**
→ Generate entanglement graph



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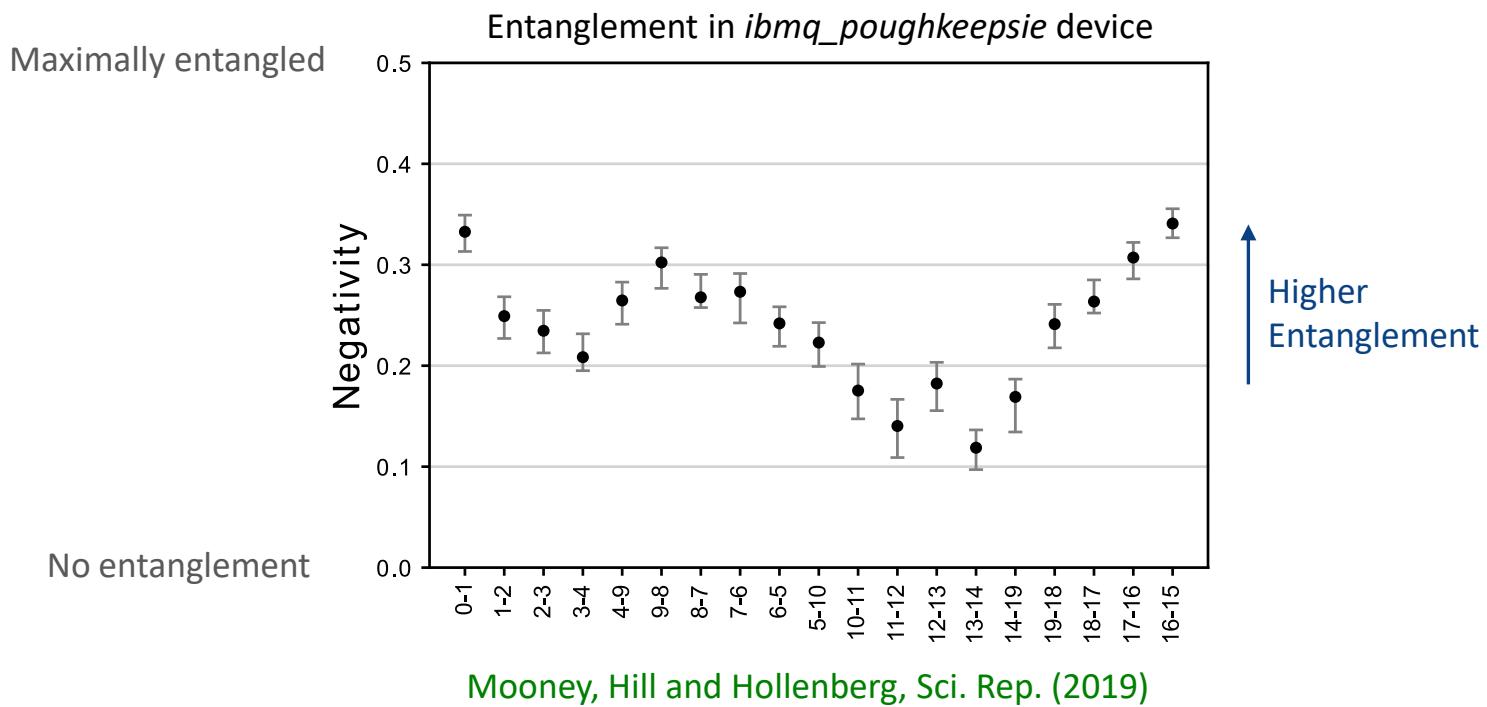
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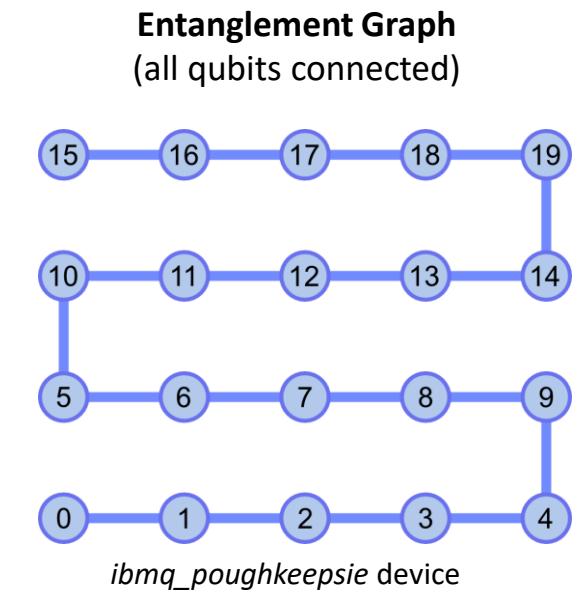
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Higher Entanglement

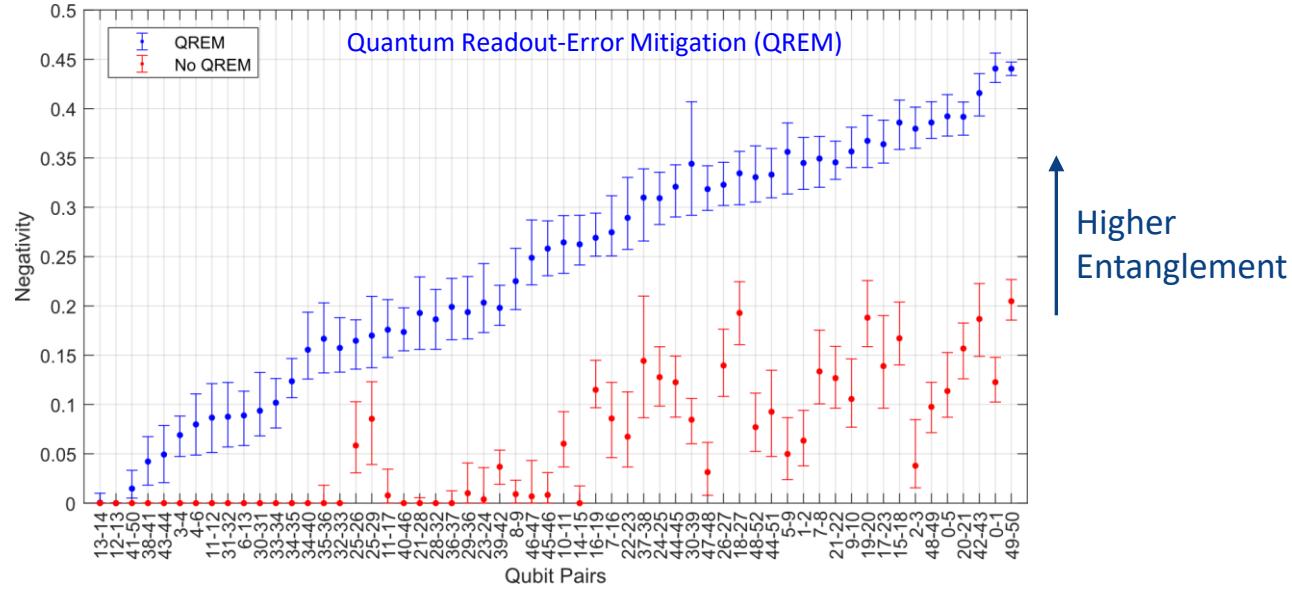
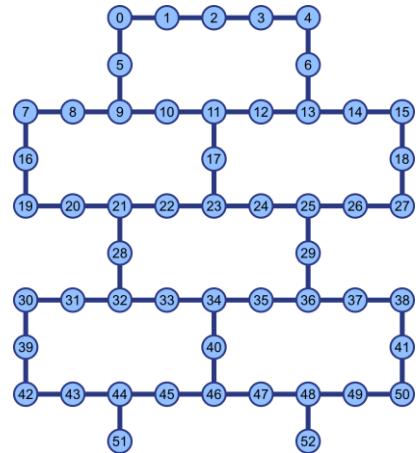


Entanglement graph is connected → Whole device is bipartite entangled

At the time → largest quantum entangled state (on a physical universal quantum computer)

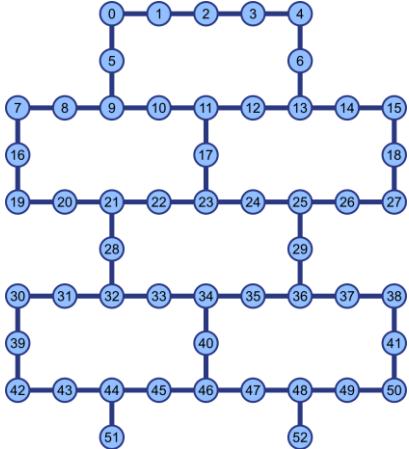
Results: Newer IBM Quantum devices

53-qubit *ibmq_rochester* device

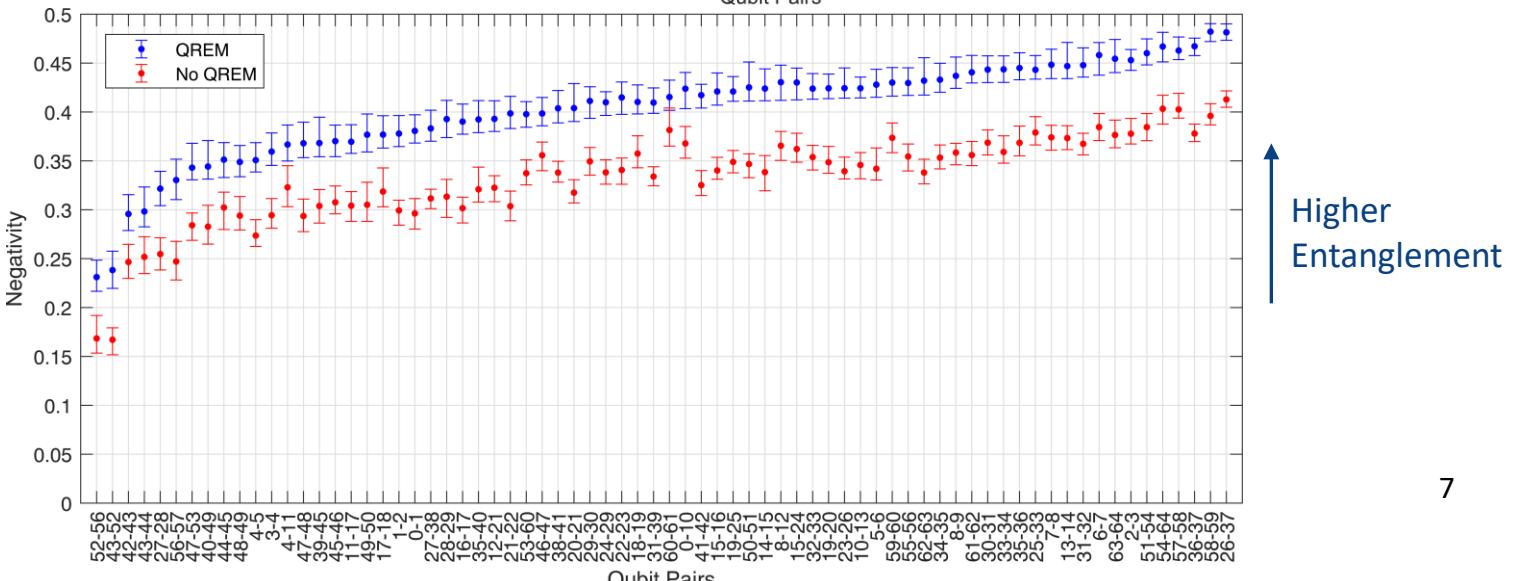
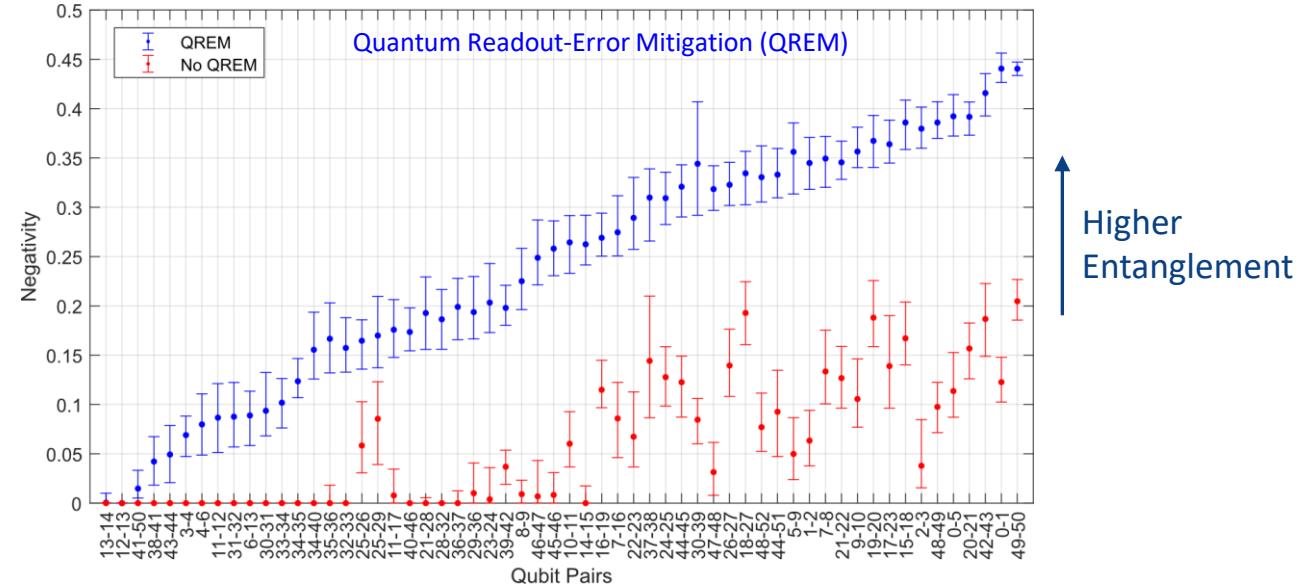
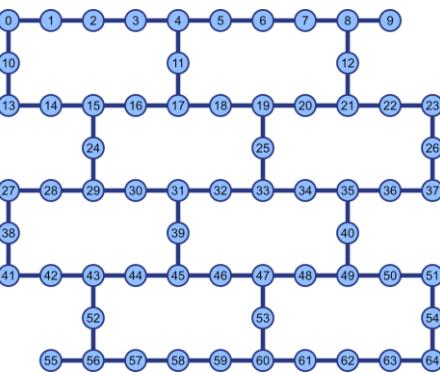


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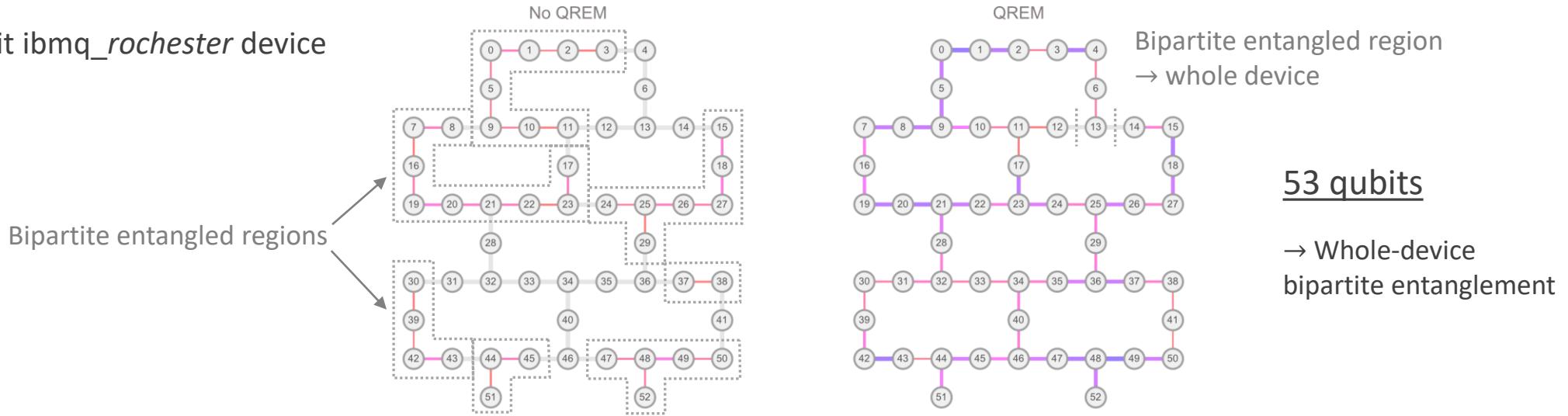


65-qubit *ibmq_manhattan* device

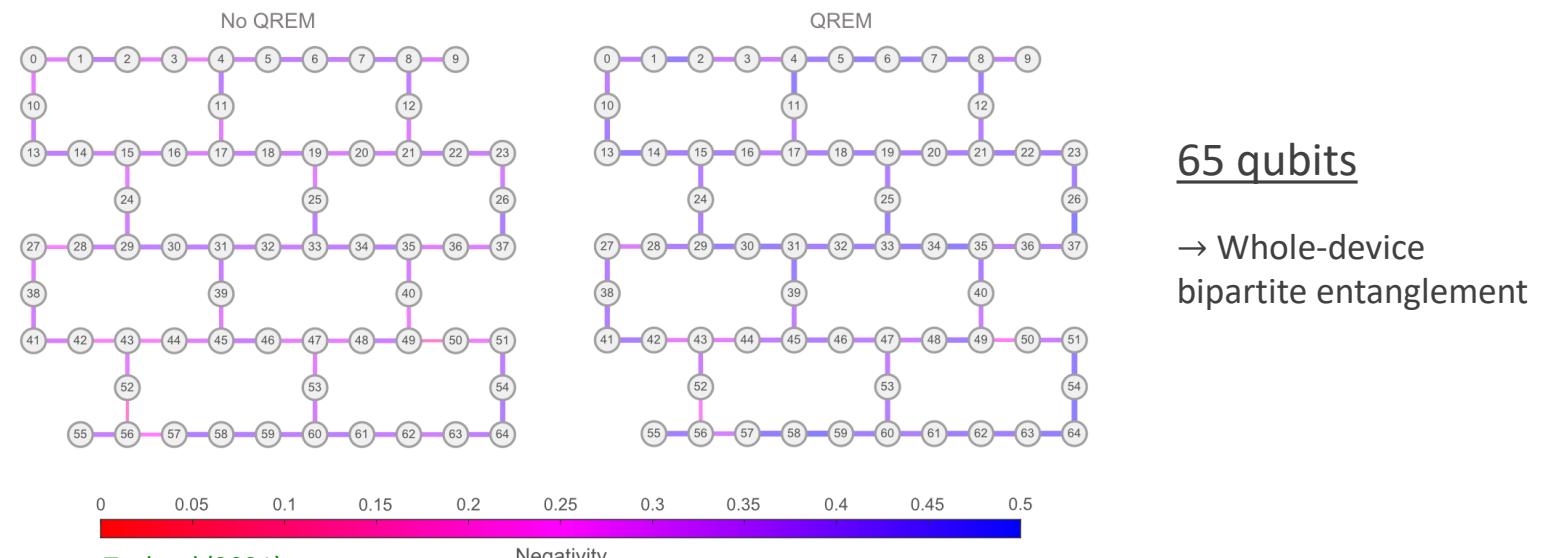


Results: Entanglement Graphs

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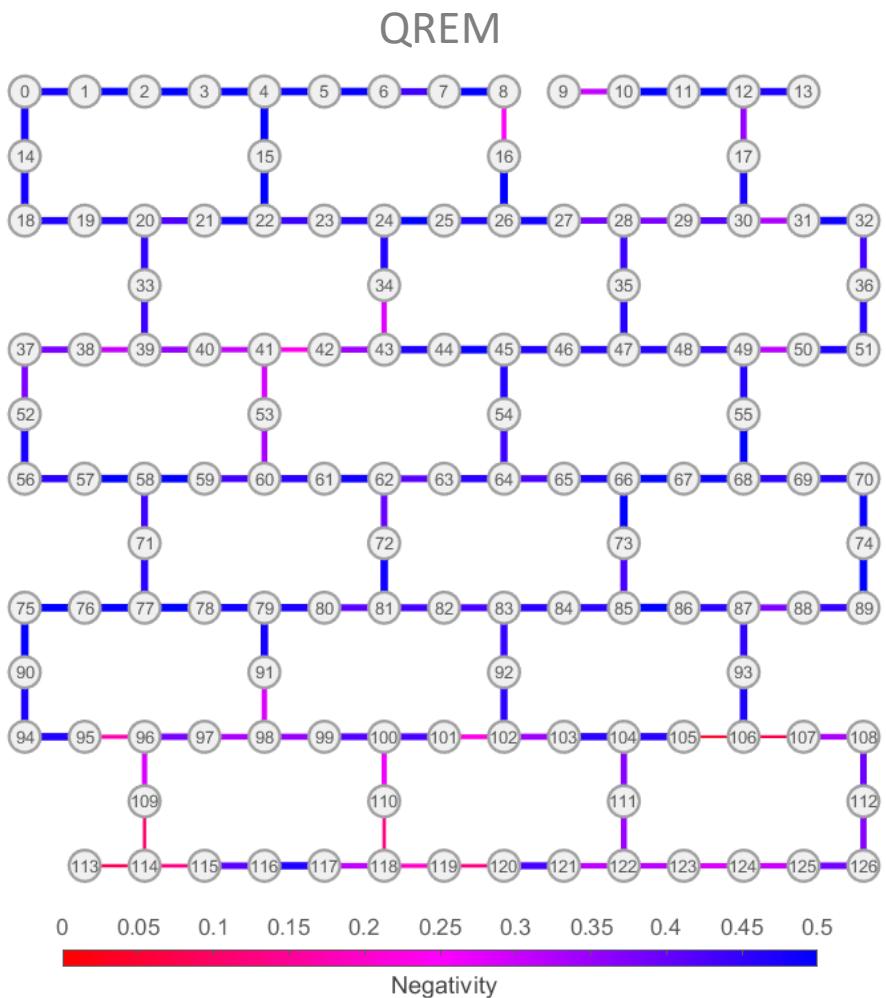
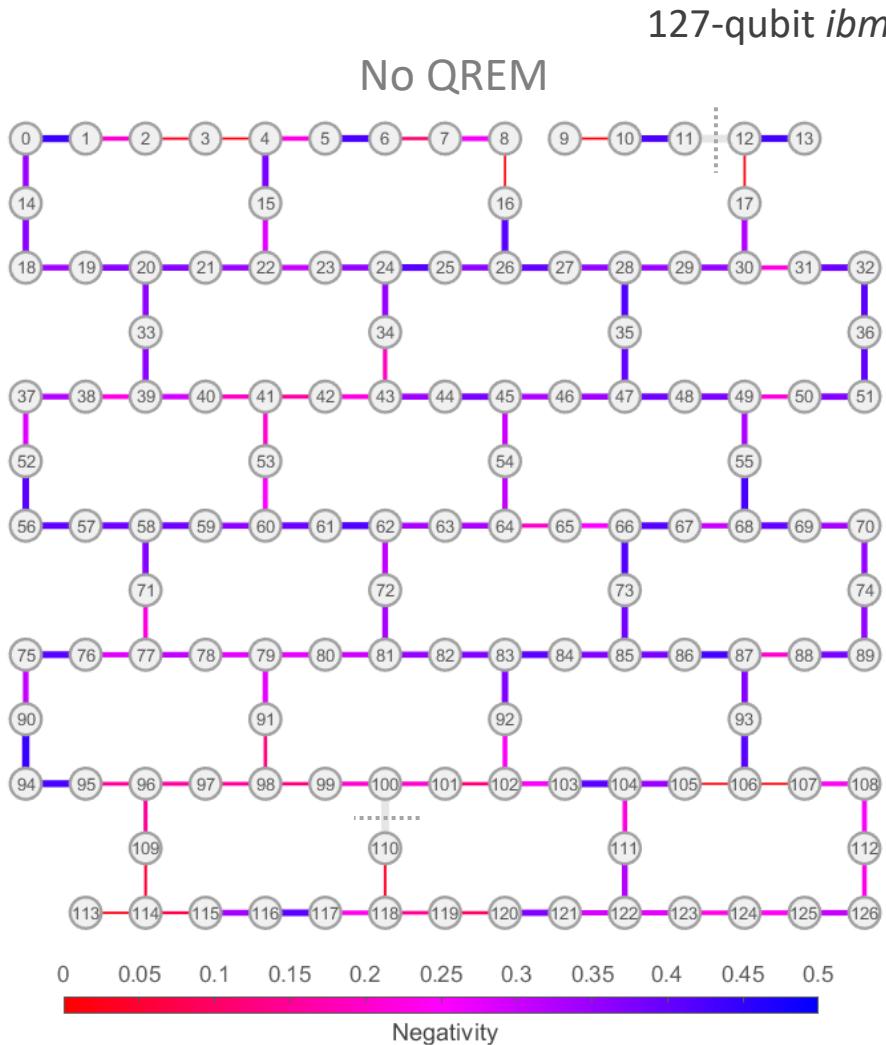
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Latest: 127-qubit *ibm_washington* Device

Latest: 127-qubit *ibm_washington* Device



127 qubits
 → Whole-device
 bipartite entanglement

John Fidel Kam *et al.*,
 (paper in production)

Genuine Multipartite Entanglement (GME) in GHZ states

Ideal GHZ State

State Vector	Density Matrix	Coherence
$\frac{ 00\dots0\rangle + 11\dots1\rangle}{\sqrt{2}}$	$\begin{pmatrix} \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} \end{pmatrix}$	Coherence Population

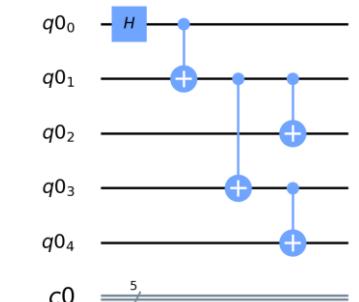
Preparation

Hadamard $\rightarrow \frac{(|0\rangle + |1\rangle)|00\dots0\rangle}{\sqrt{2}}$

Grow the state with CNOTs

CNOT $\rightarrow \frac{(|00\rangle + |11\rangle)|0\dots0\rangle}{\sqrt{2}}$

CNOT $\rightarrow \frac{(|000\rangle + |111\rangle)|\dots0\rangle}{\sqrt{2}}$



```

    graph TD
        H[q0_0] -- "H" --> P0[q0_1]
        P0 -- "+" --> P1[q0_2]
        P1 -- "+" --> P2[q0_3]
        P2 -- "+" --> P3[q0_4]
        P3 -- "+" --> P4[q0_5]
        P4 -- "+" --> c0[5]
    
```

Genuine Multipartite Entanglement (GME) in GHZ states

GHZ Fidelity (> 0.5) \rightarrow GME

Fidelity = (Population)/2 + (Coherence)/2

- Population: Occupancies of $|00 \dots 0\rangle$ and $|11 \dots 1\rangle$
- Coherence: Multiple Quantum Coherences (MQC) Wei *et al.*, Phys. Rev. A (2020)

Ideal GHZ State

State Vector	$\frac{ 00 \dots 0\rangle + 11 \dots 1\rangle}{\sqrt{2}}$
Density Matrix	
$\begin{pmatrix} \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} \end{pmatrix}$	

Coherence

Population

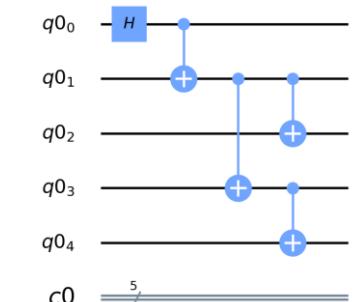
Preparation

Hadamard $\rightarrow \frac{(|0\rangle + |1\rangle)|00\dots 0\rangle}{\sqrt{2}}$

Grow the state with CNOTs

CNOT $\rightarrow \frac{(|00\rangle + |11\rangle)|0\dots 0\rangle}{\sqrt{2}}$

CNOT $\rightarrow \frac{(|000\rangle + |111\rangle)|\dots 0\rangle}{\sqrt{2}}$



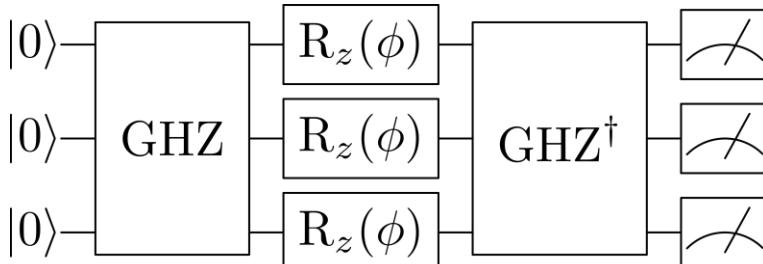
Genuine Multipartite Entanglement (GME) in GHZ states

GHZ Fidelity (> 0.5) \rightarrow GME

$$\text{Fidelity} = (\text{Population})/2 + (\text{Coherence})/2$$

- Population: Occupancies of $|00 \dots 0\rangle$ and $|11 \dots 1\rangle$
- Coherence: Multiple Quantum Coherences (MQC) Wei *et al.*, Phys. Rev. A (2020)

Multiple Quantum Coherences (MQC)



Ideal state: phase of $N\phi$

$$\frac{|00 \dots 0\rangle + e^{iN\phi}|11 \dots 1\rangle}{\sqrt{2}}$$

Ideal GHZ State

State Vector

$$\frac{|00 \dots 0\rangle + |11 \dots 1\rangle}{\sqrt{2}}$$

Density Matrix

$$\begin{pmatrix} \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} \end{pmatrix}$$

Coherence

Population

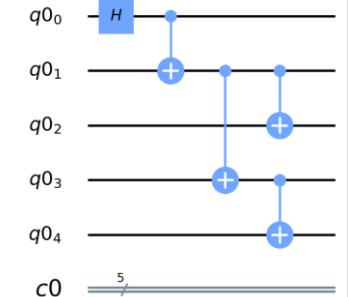
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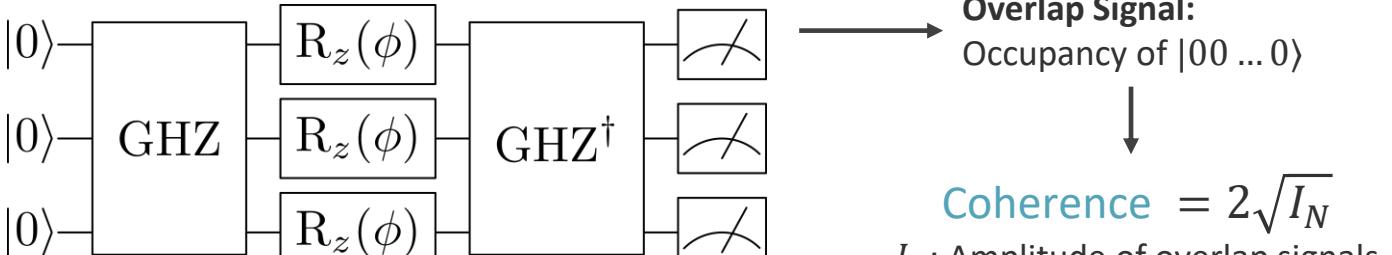
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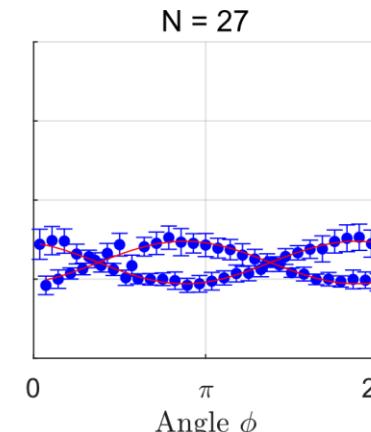
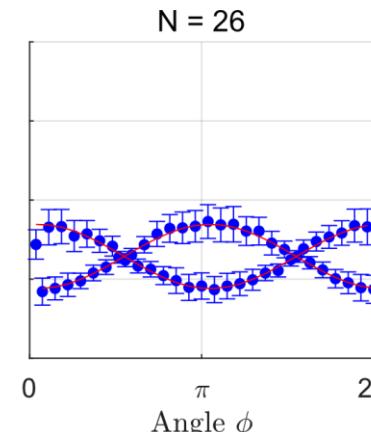
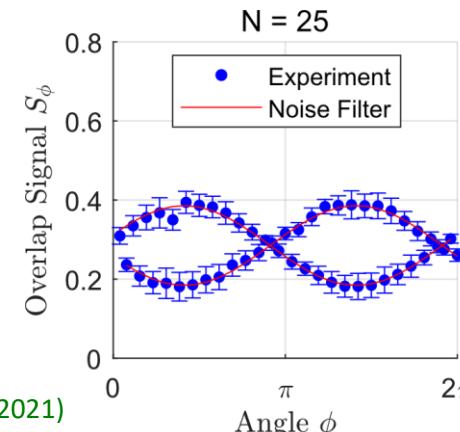
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Ideal GHZ State

State Vector

$$\frac{|00 \dots 0\rangle + |11 \dots 1\rangle}{\sqrt{2}}$$

Density Matrix

$$\begin{pmatrix} \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} \end{pmatrix}$$

Coherence (circled in red)

Population (circled in blue)

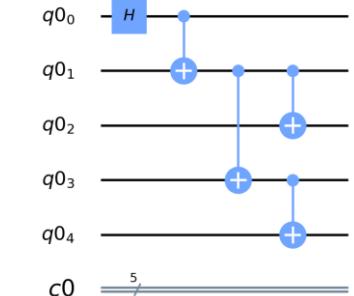
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Grow the state with CNOTs

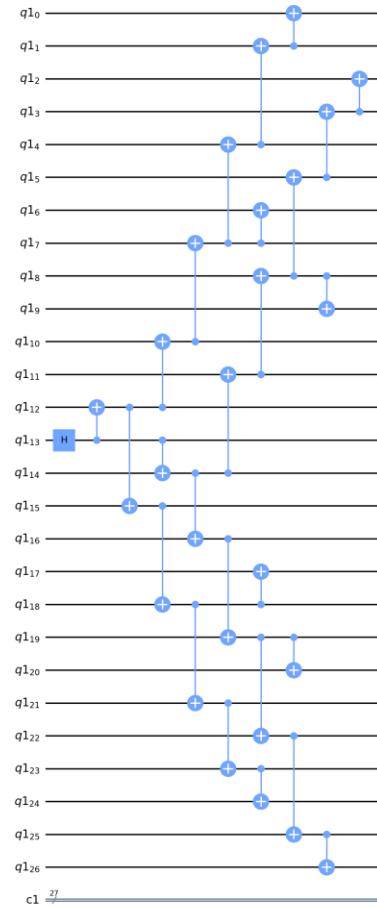
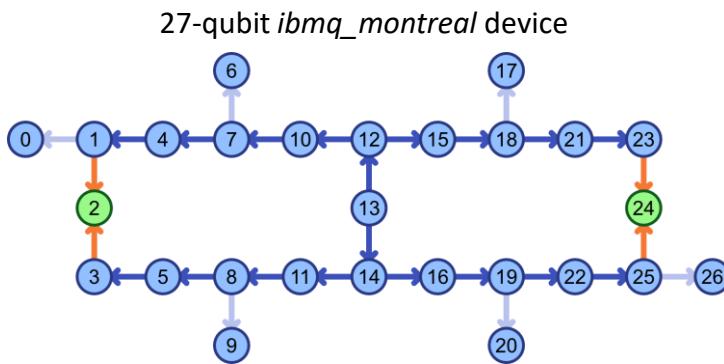
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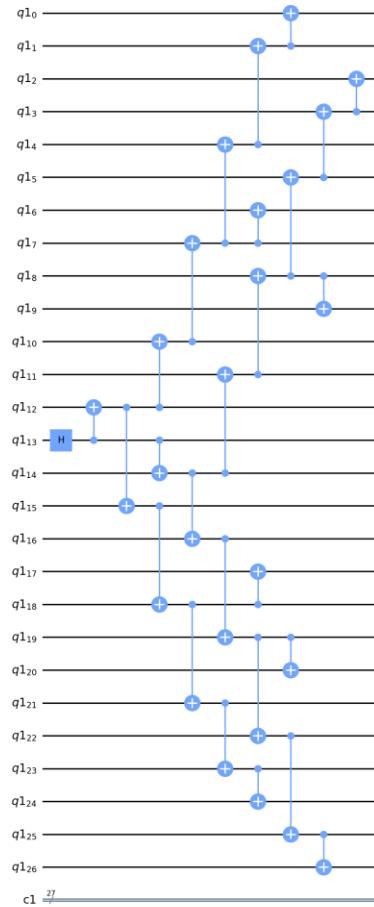
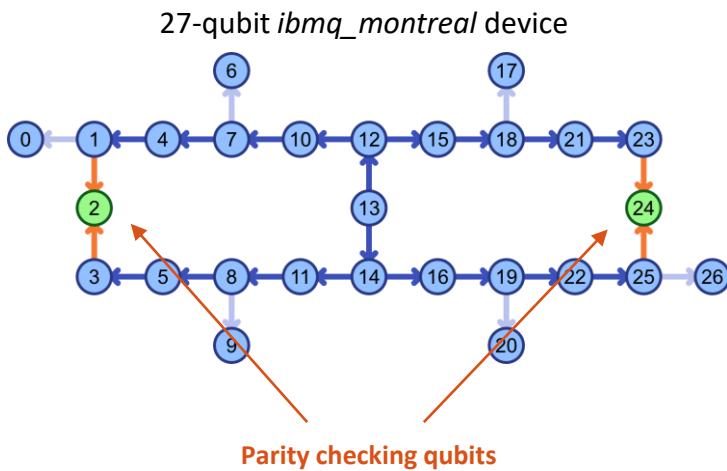
Results: On the 27-qubit *ibmq_montreal* device

- Prepare GHZ States



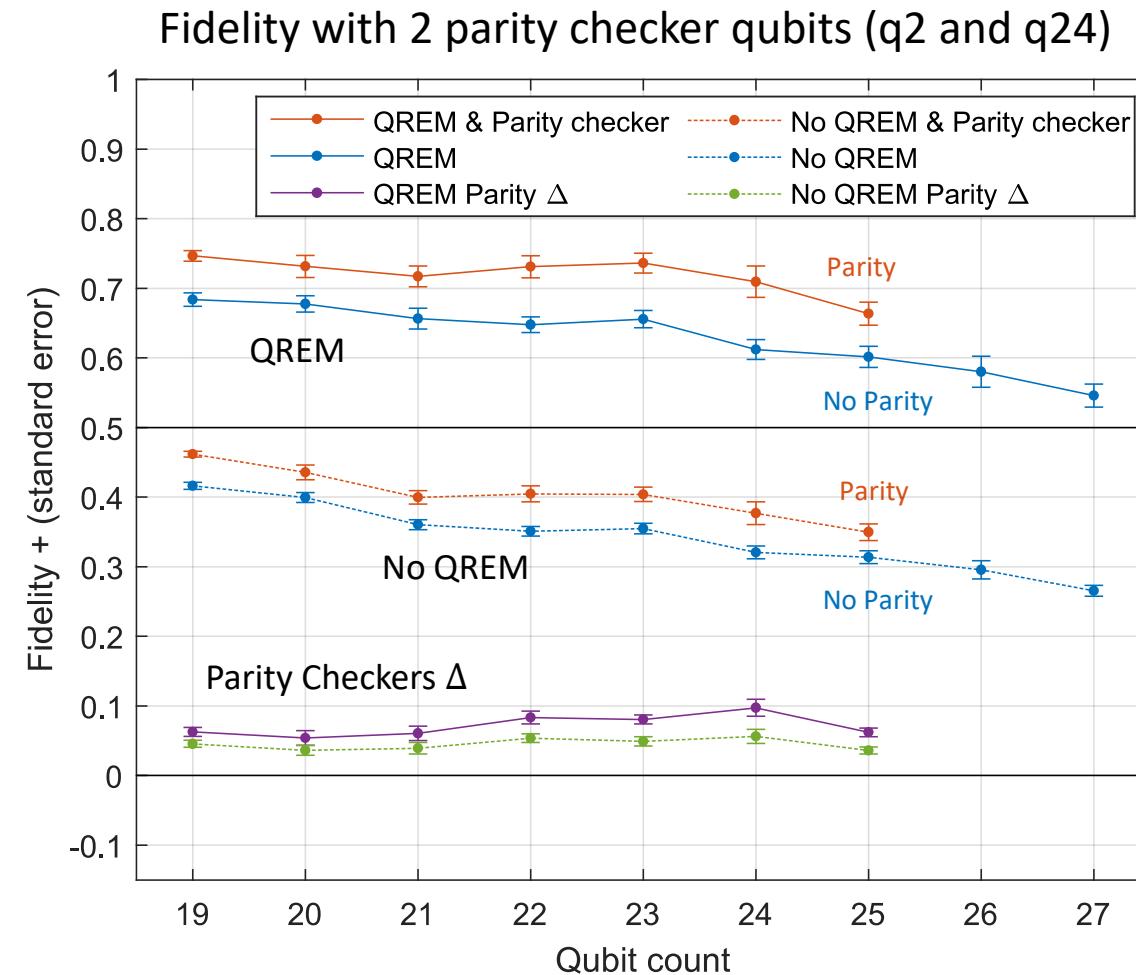
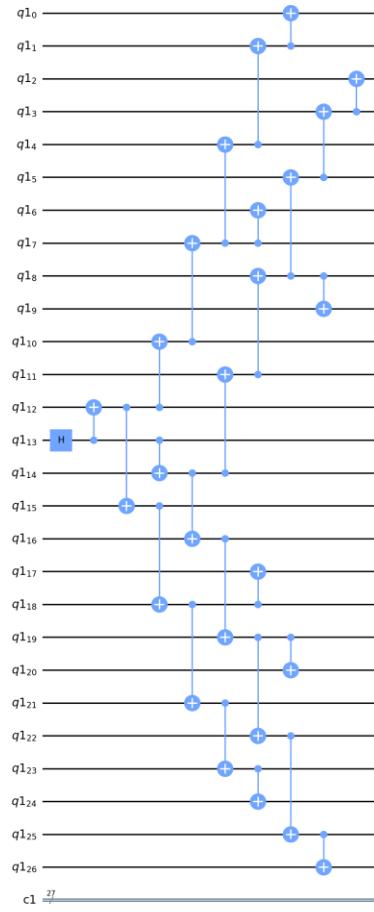
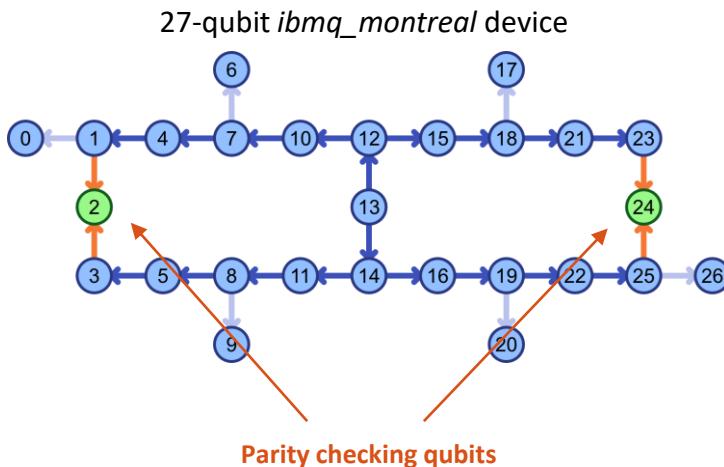
Results: On the 27-qubit *ibmq_montreal* device

- Prepare GHZ States
- Add parity verification
→ effects on fidelity



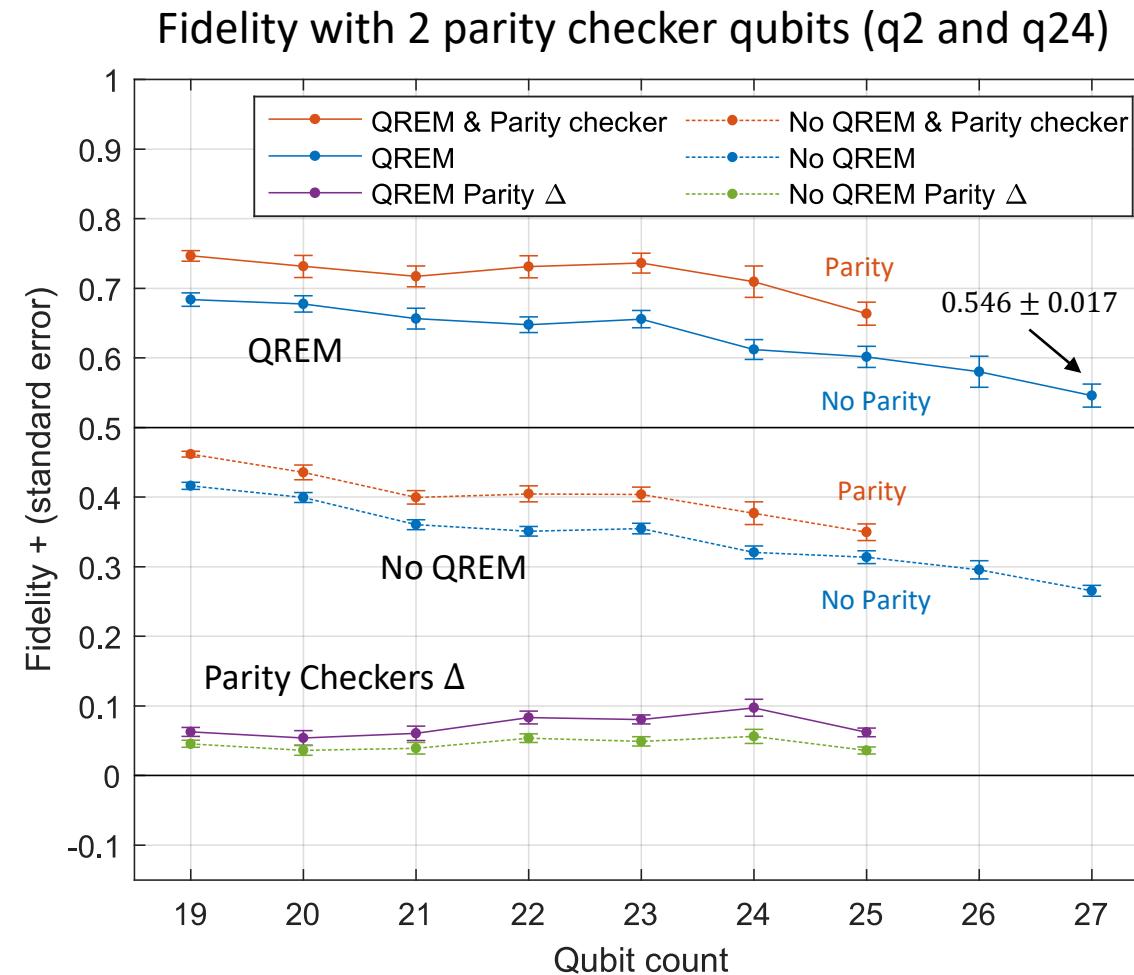
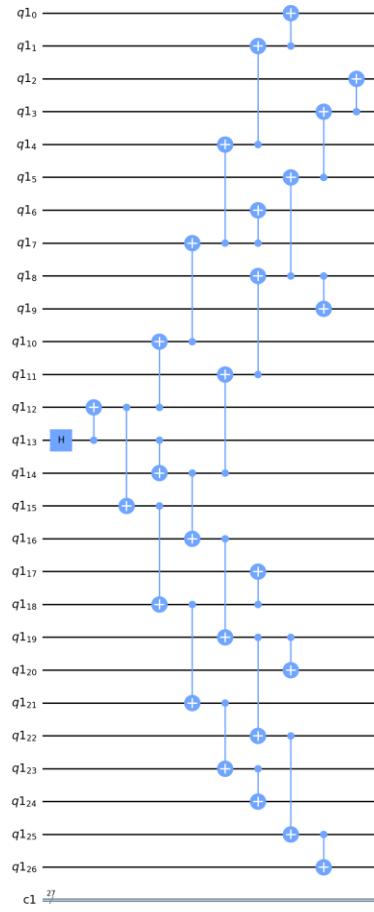
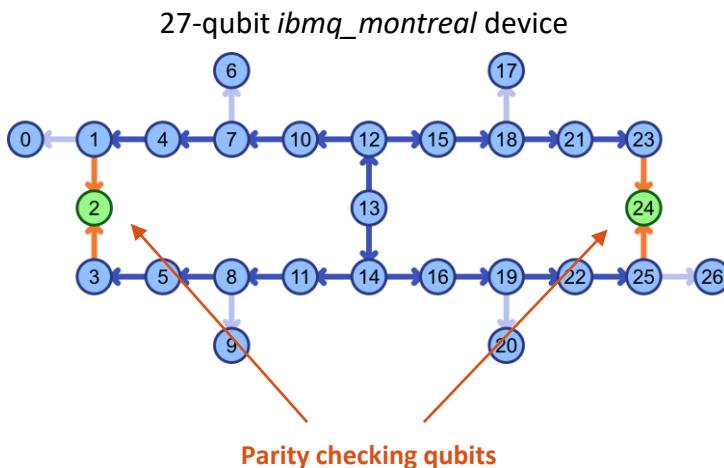
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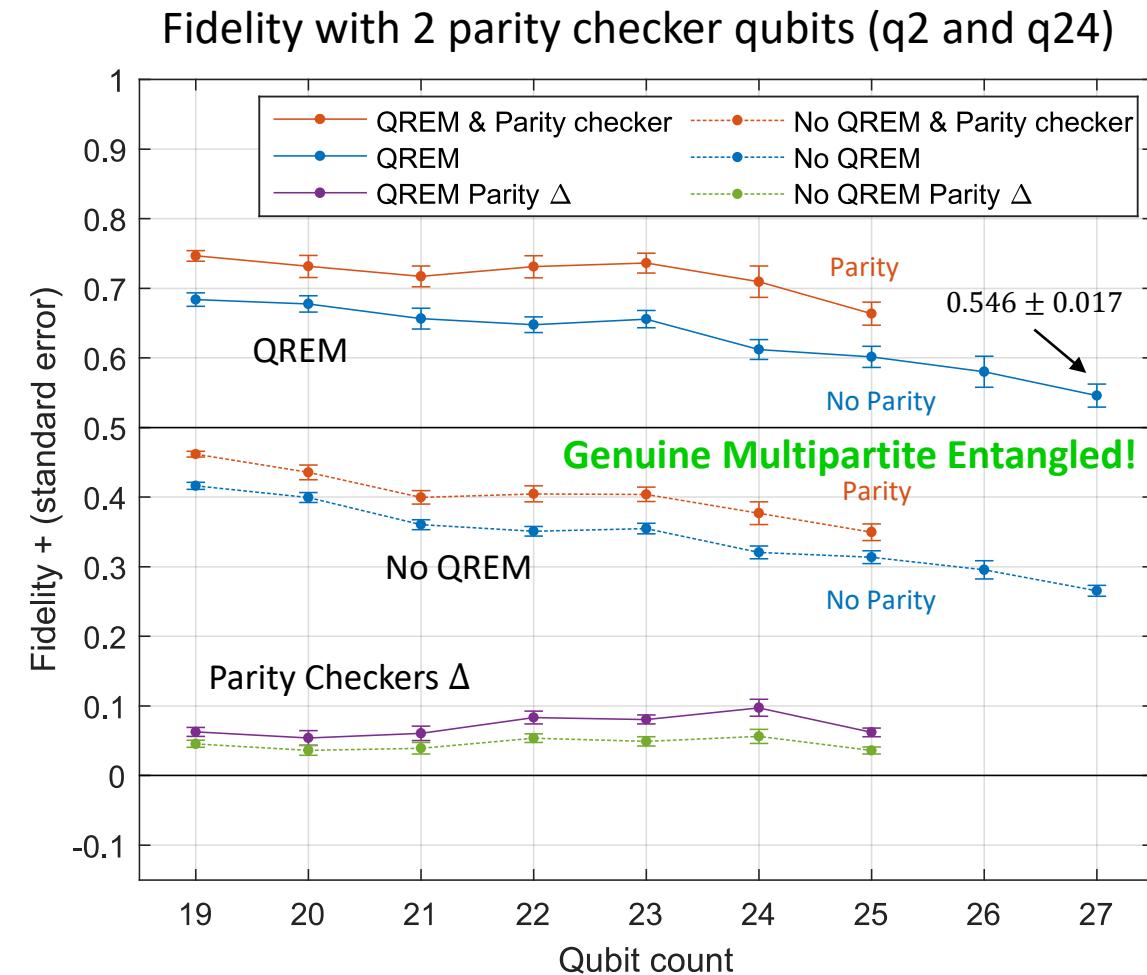
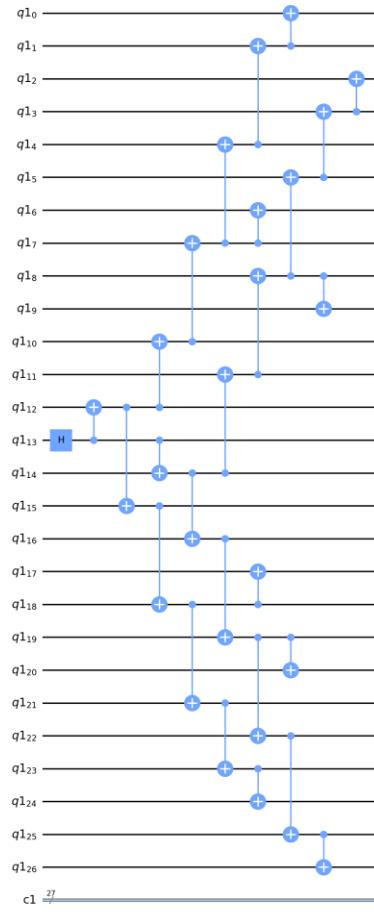
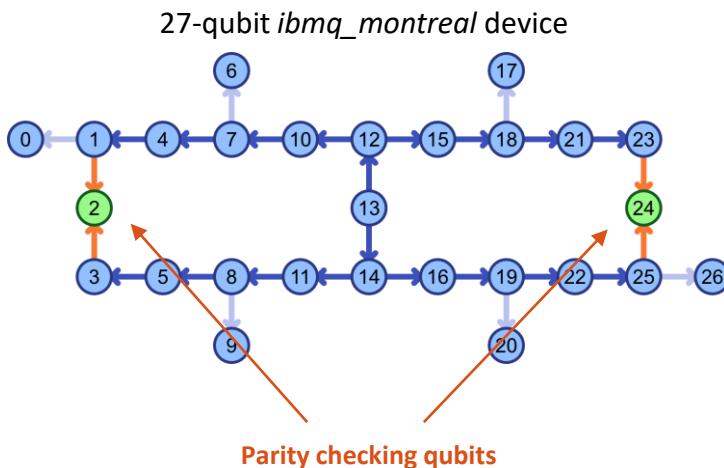
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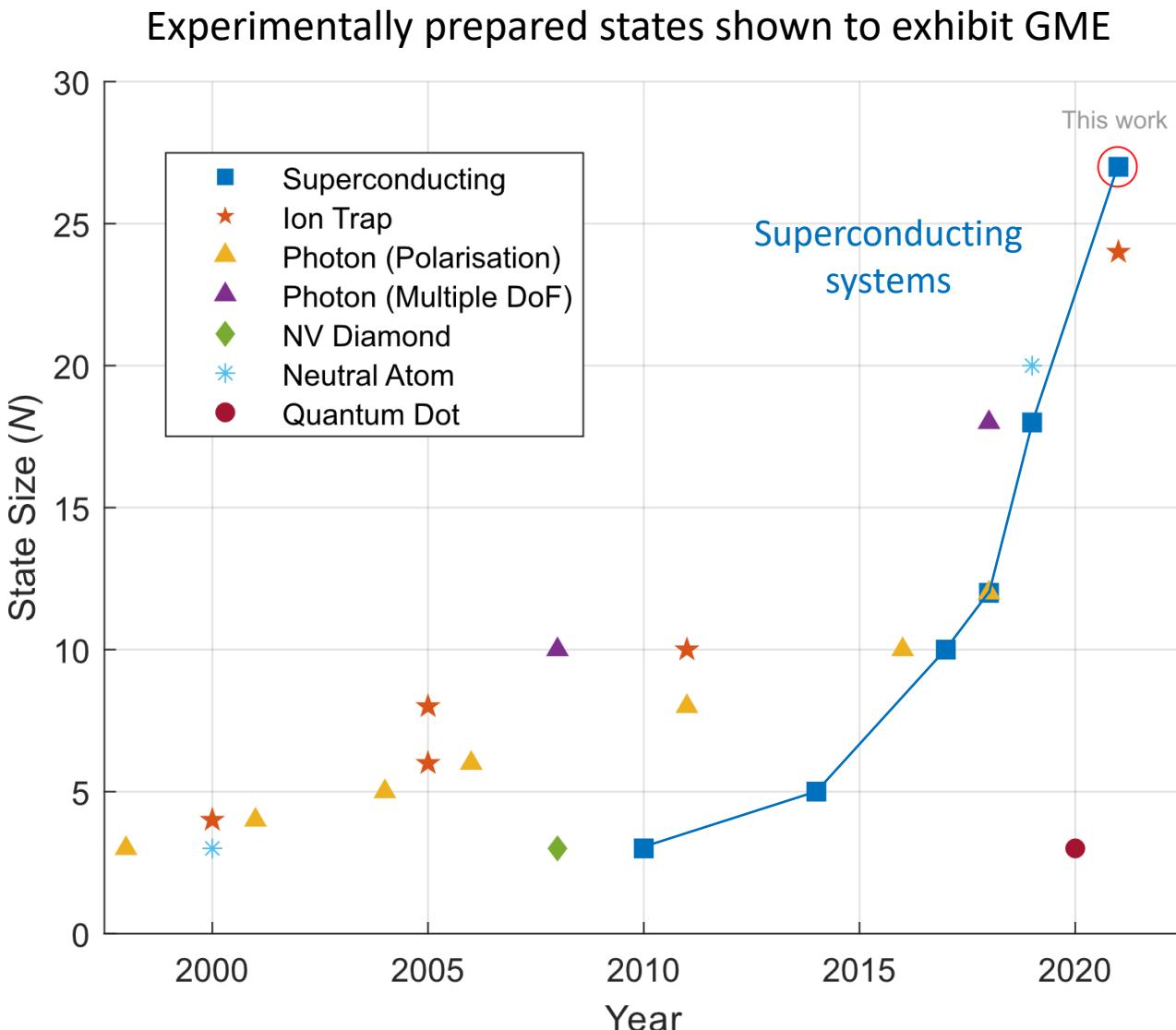


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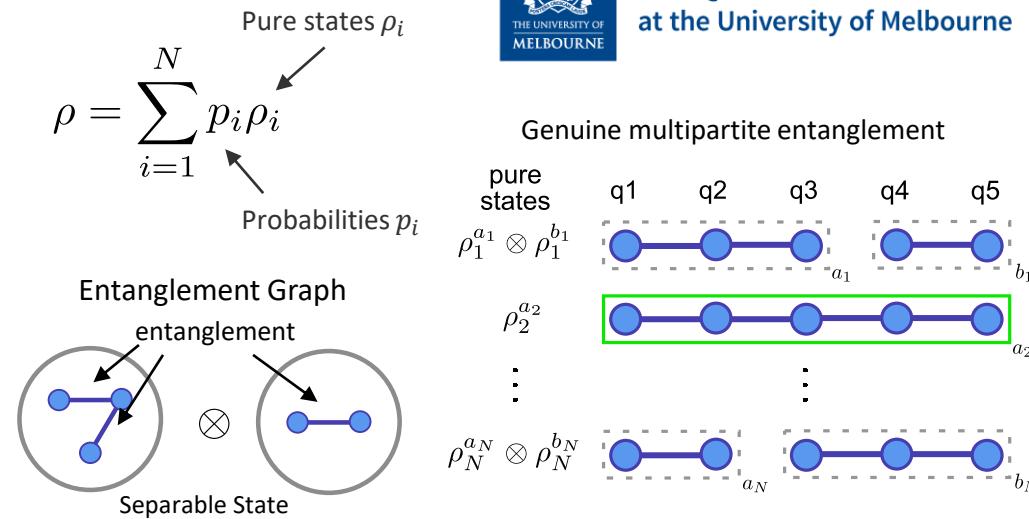
History of Genuine Multipartite Entanglement



Summary

Forms of Multipartite Entanglement

- Bipartite entanglement
- Genuine multipartite entanglement



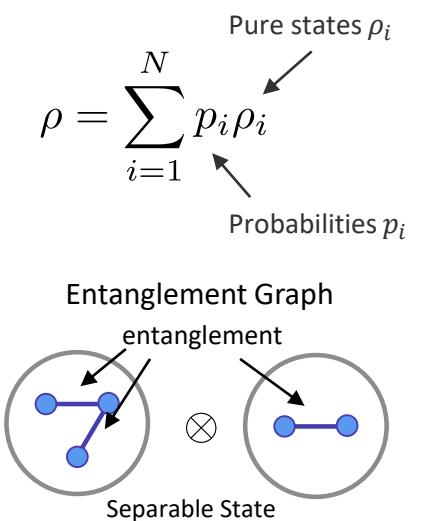
IBM Quantum Network Hub
at the University of Melbourne



Summary

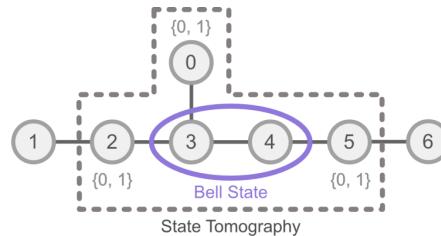
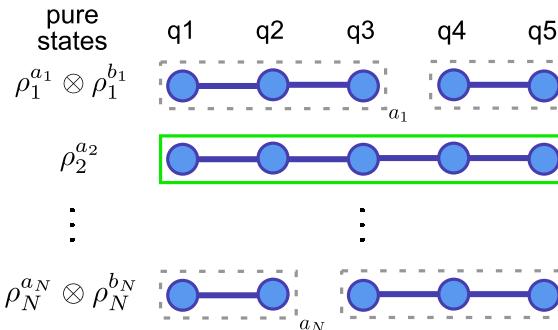
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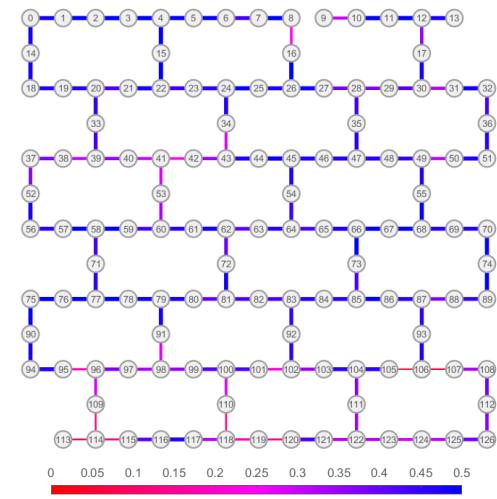


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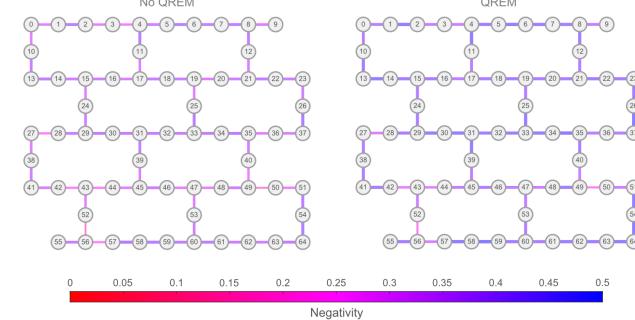
Genuine multipartite entanglement



127-qubit *ibm_washington* device



65-qubit *ibmq_manhattan* device



Bipartite entanglement in graph states

- Found whole-device entanglement on **20, 53, 65**, and now **127-qubit** devices
- All pairs of the 127-qubit *ibmq_manhattan* entangled



Mooney, Hill and Hollenberg, Sci. Rep. (2019)

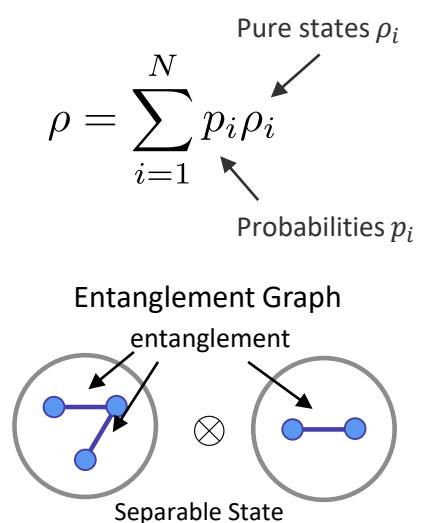
Mooney, White, Hill and Hollenberg, Adv. Quantum Technol (2021)



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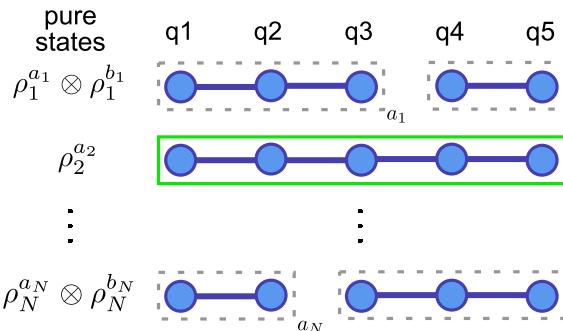
Mooney, Hill and Hollenberg, Sci. Rep. (2019)

Mooney, White, Hill and Hollenberg, Adv. Quantum Technol (2021)



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Genuine multipartite entanglement



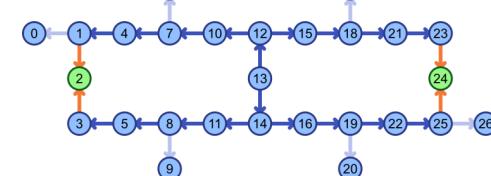
Genuine multipartite entanglement in GHZ state

- Found GME across all **27** qubits of *ibmq_montreal* device
- Applied parity checking error detection
 - Found detectable improvement in fidelity (relatively modest)

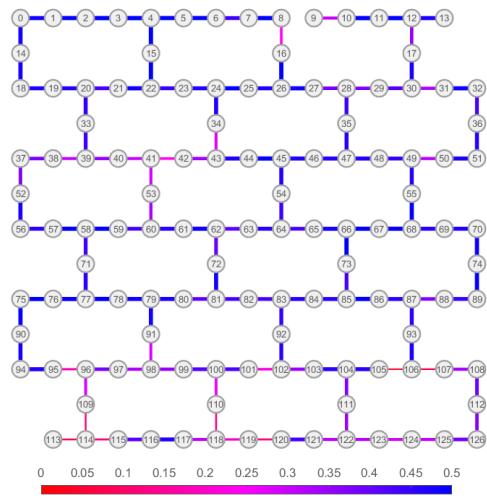
Mooney, White, Hill and Hollenberg, J. Phys. Commun (2021)



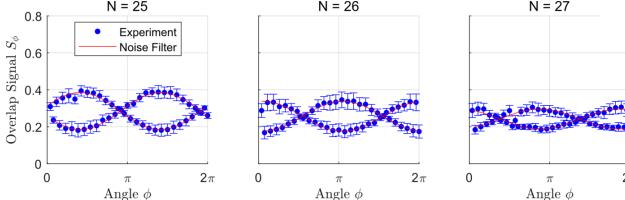
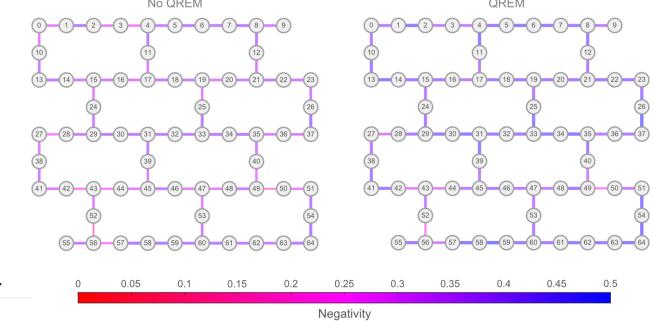
27-qubit *ibmq_montreal* device



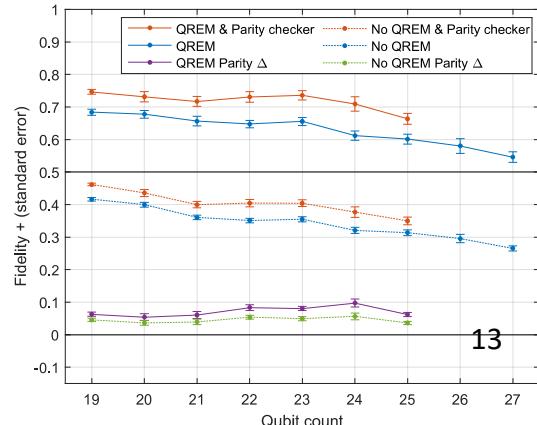
127-qubit *ibm_washington* device



65-qubit *ibmq_manhattan* device



Fidelities of GHZ states

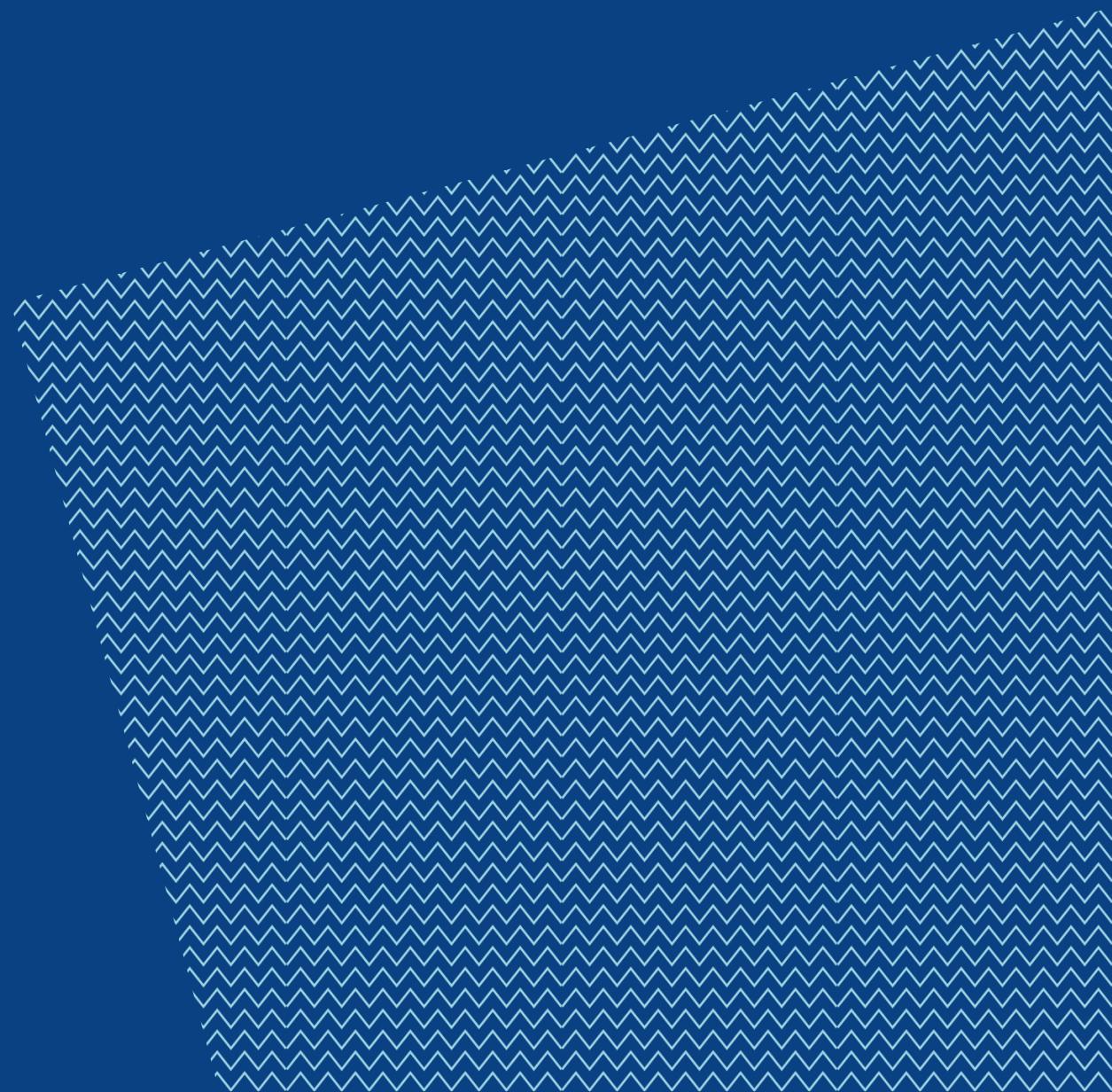




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Thank you

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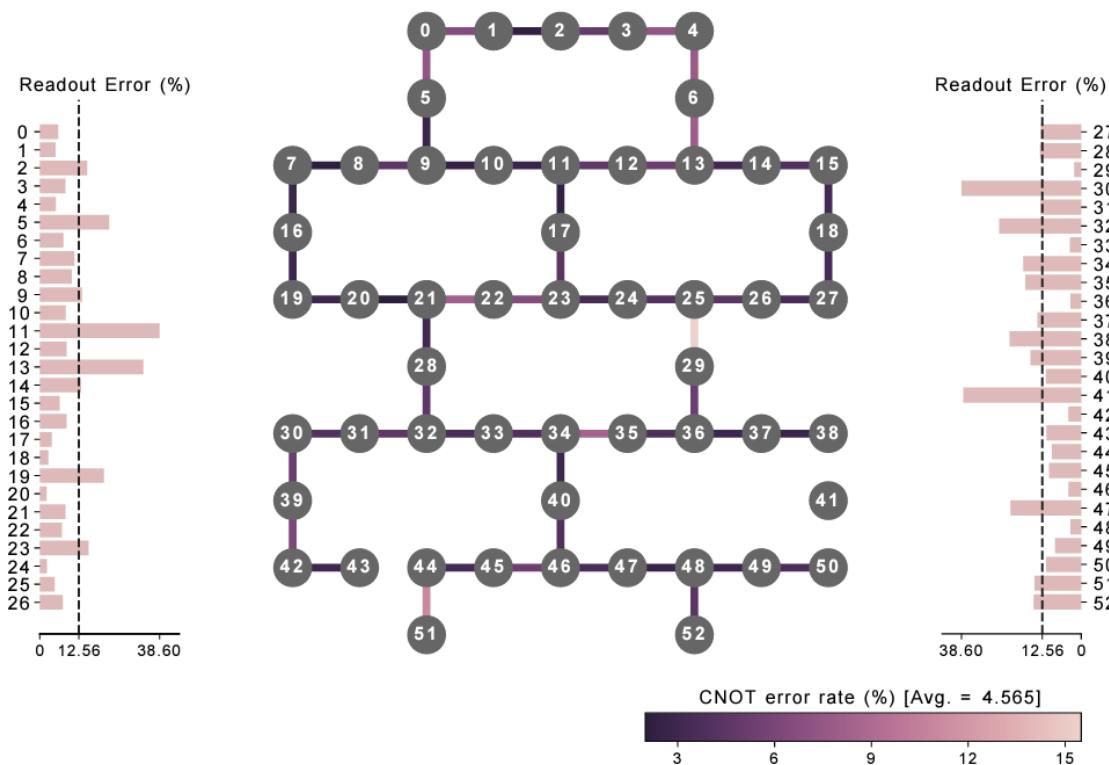
GHZ State Circuit Info

Table 1. CNOT circuit depths and counts required to perform the corresponding experiments.

State size (qubits)	Population		Population (parity)		Coherence		Coherence (parity)	
	Depth	Count	Depth	Count	Depth	Count	Depth	Count
Embedding 1—GHZ state sizes 11 to 19 (one parity-checker qubit)								
11	6	10	7	12	12	20	13	22
12	6	11	7	13	12	22	13	24
13	7	12	8	14	14	24	15	26
14	7	13	8	15	14	26	15	28
15	8	14	8	16	16	28	16	30
16	8	15	8	17	16	30	16	32
17	9	16	9	18	18	32	18	34
18	9	17	9	19	18	34	18	36
19	10	18	10	20	20	36	20	38
Embedding 2—GHZ state sizes 19 to 27 (two parity-checker qubits)								
19	7	18	8	22	14	36	15	40
20	7	19	8	23	14	38	15	42
21	7	20	8	24	14	40	15	44
22	7	21	8	25	14	42	15	46
23	7	22	8	26	14	44	15	48
24	7	23	9	27	14	46	16	50
25	7	24	9	28	14	48	16	52
26	7	25	14	50
27	7	26	14	52

Device calibration data

53-qubit *ibmq_rochester*



65-qubit *ibmq_manhattan*

