The He-McKellar-Wilkins phase shift, atom interferometry tests — recent work related to Aharonov Casher and Klein, Opat et al.

In the AIP session in honour of Tony Klein

24th Australian Institute of Physics National Congress

Bruce H J McKellar 13th December 2022

What is a topological phase?

One of the controversial features of measurements of the "quantum phases", the Aharonov-Bohm, the Aharonov-Casher, and the He-McKellar-Wilkens phase, has always been the question "Is the phase which been measured topological?". That a focus of this talk.

To be able to answer that question we need to understand

1. What is meant by topological phase, or geometric phase?
2. What are the conditions under which the phase in question is topological?
3. Does the experimental set up satisfy these conditions?

Topological phase

The first question

What is meant by *topological phase*, or *geometric phase*?

is already somewhat controversial. I give an answer in three parts:

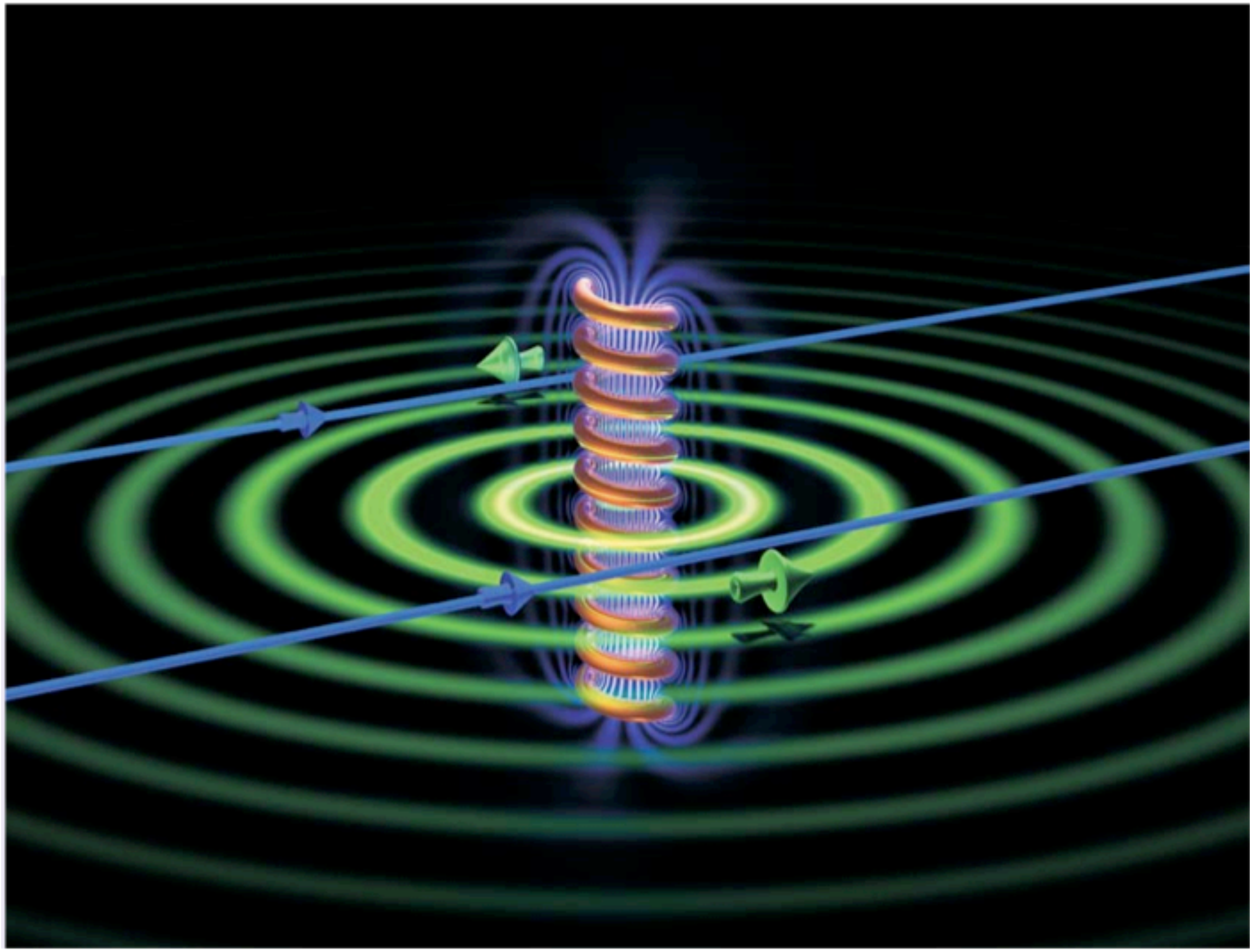
1. the phase is constructed from a position dependent phase $\phi(\mathbf{x})$ such that $\psi'(\mathbf{x}) = \exp\{i\phi(\mathbf{x})\}\,\psi(\mathbf{x})$ satisfies the free wave equation.
2. the phase is independent of the path, except for the number of times it circles some interior excluded region, and
3. the phase is determined by properties of the fields in the excluded region

The answers to the remaining questions are specific to each phase, and will be considered for each of the phases

1. Aharonov-Bohm phase,
2. Aharonov-Casher phase,
3. He-McKellar-Wilkens phase,

in turn.

Aharonov Bohm

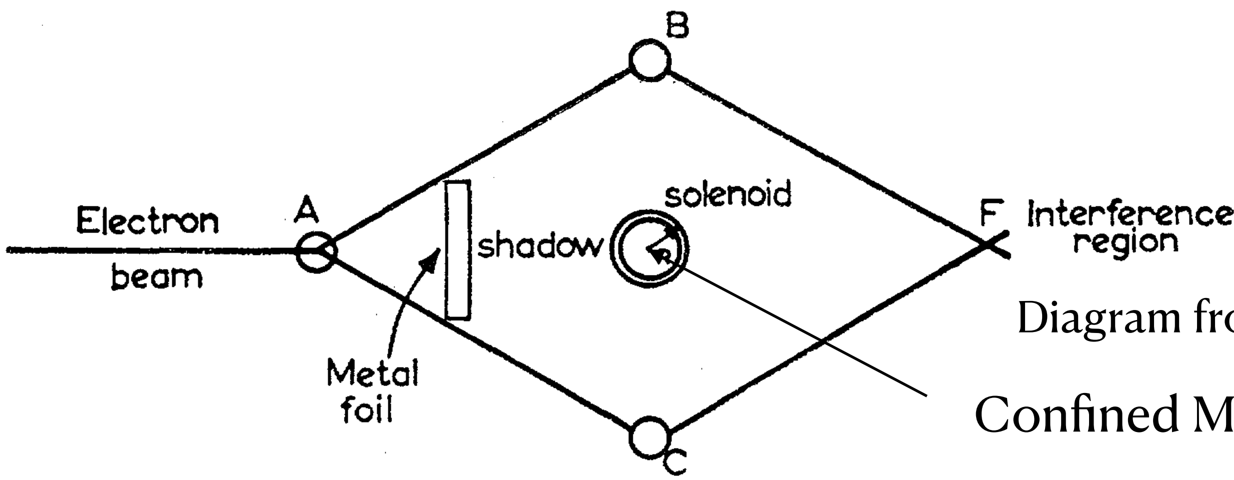


From
Herman Batelaan and
Akira Tonomura

Phys. Today 62(9), 38
(2009)

Aharonov Bohm Effect

The original topological phase



No magnetic field at the particle path

Yet the Magnetic field creates
phase observable by interference

Diagram from the paper Y. Aharonov and D. Bohm, *Phys. Rev* **115** (1959) 485-491.

Confined Magnetic field

Topological means that the phase is independent under deformations of the path
Dependent on the topology, not the details of the path

There is no magnetic field at the path of the charge,
But there is a vector potential **A**, Such that $\mathbf{B} = \nabla \times \mathbf{A}$
The phase shift is proportional to the line integral of the vector potential around the path ***C***,
Which is the flux Φ_C of **B** across the surface ***S*** bounded by the path ***C***

$$e(\chi_1 - \chi_2) = 2\int_C \mathbf{A}\cdot\mathbf{ds} = e\int_S \mathbf{B}\cdot\mathbf{dS} = e\Phi_C$$

which is independent of distortions of the path,
which need not be confined to a plane,
and is thus **topological**

First noted by Eherenberg and Siday (1949), rediscovered by Aharonov and Bohm (1959)
First Measurement Chalmers (1960), definitive measurement Tomomura (1982)

Aharonov Bohm

A derivation

Free Dirac Equation

$$\left(i\gamma^{\mu}\partial_{\mu} - m\right)\psi = 0$$

Maths is easier for the Dirac equation, but Schrödinger, Bargman Wigner or any other wave equation, is just as good

Dirac Equation in an em field

$$\left(i\gamma^{\mu}\left[\partial_{\mu} + ieA_{\mu}\right] - m\right)\psi = 0$$

Phase change $\psi \rightarrow \psi' = e^{i\chi}\psi$

Gives a new field satisfying the free field

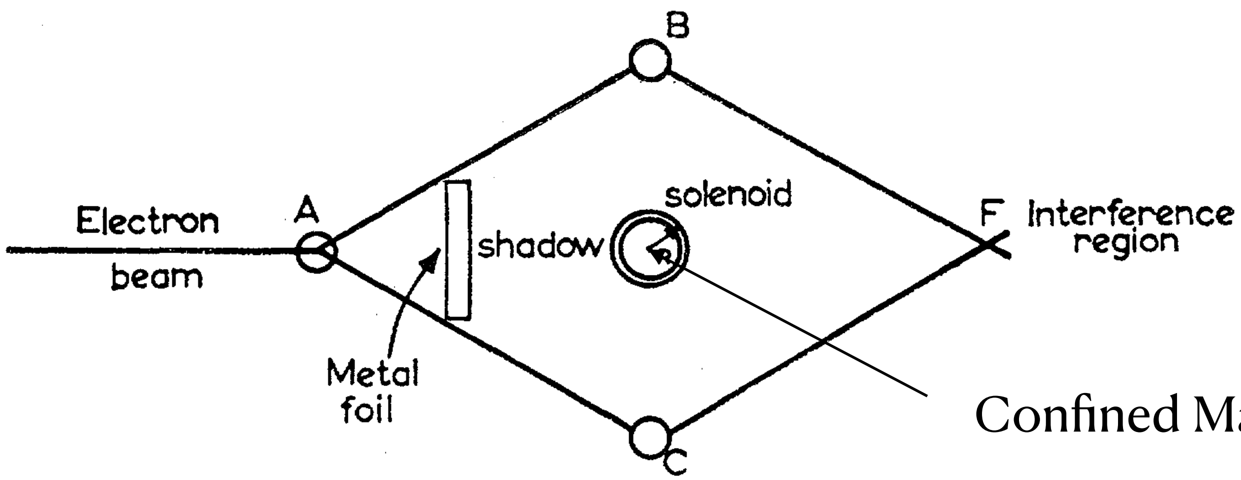
$$\left(i\gamma^{\mu}\partial_{\mu} - m\right)\psi' = 0$$

when the phase is such that

$A_{\mu} = e\partial_{\mu}\chi$ where $\chi(\mathbf{x}) = \int_{\mathbf{P}}^{\mathbf{x}} \mathbf{A}(\mathbf{x})' \cdot \mathbf{ds}'$

Aharonov Bohm Effect

To summarise the original topological phase



No magnetic field at the particle path

Yet the Magnetic field creates
phase observable by interference

Confined Magnetic field

Topological means that the phase is independent under deformations of the path
Depending on the topology, not the details of the path

There is no magnetic field at the path of the charge,
But there is a vector potential **A,** Such that $\mathbf{B} = \nabla \times \mathbf{A}$
The phase shift is proportional to the line integral of the vector potential around the path ***C***,
Which is the flux Φ_C of **B** across the surface ***S*** bounded by the path ***C***

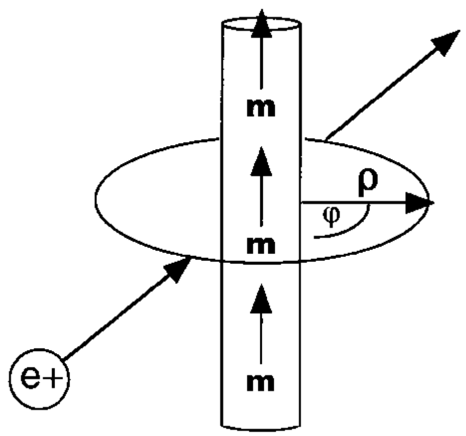
$$e(\chi_1 - \chi_2) = 2\int_C \mathbf{A} \cdot \mathbf{ds} = e\int_S \mathbf{B} \cdot \mathbf{dS} = e\Phi_C$$

which is independent of distortions of the path,
which need not be confined to a plane,
and is thus **topological**

First noted by Eherenberg and Siday (1949), rediscovered by Aharonov and Bohm (1959)
First Measurement Chalmers (1960), definitive measurement Tomomura (1982)

Duality of Quantum mechanical phases

Aharonov-Bohm

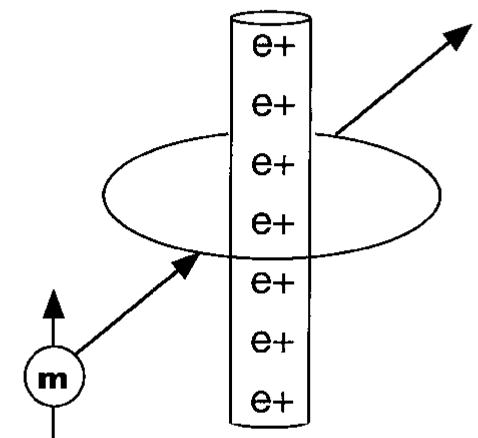


(a)

Aharonov-Casher

electric charge e ↔ magnetic dipole $\mathbf{m}$

AC duality



(b)

EM duality

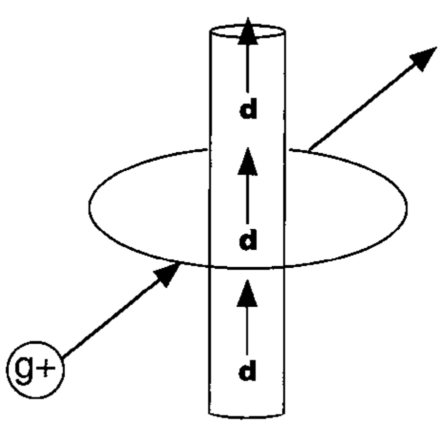
electric charge e ↔ magnetic charge g,

magnetic dipole $\mathbf{m}$ ↔ electric dipole $\mathbf{d}$

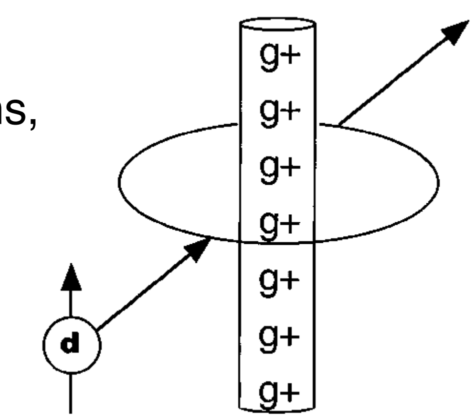
EM duality

See Jackson for an extensive discussion of EM duality

J.~D. Jackson. *Electrodynamics*, John Wiley and Sons, New York, 1998, pp 273-275



(c)



(d)

Dual Aharonov-Bohm

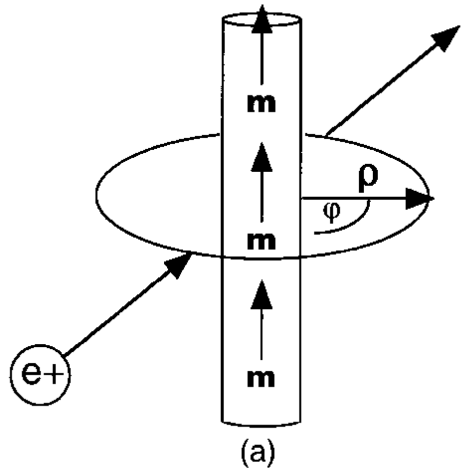
He-McKellar-Wilkens

Magnetic flux in solenoid is equivalent to a line of magnetic dipoles

after Dowling et al, PRL **83** (1999) 2486

Quantum mechanical phases

Aharonov-Bohm

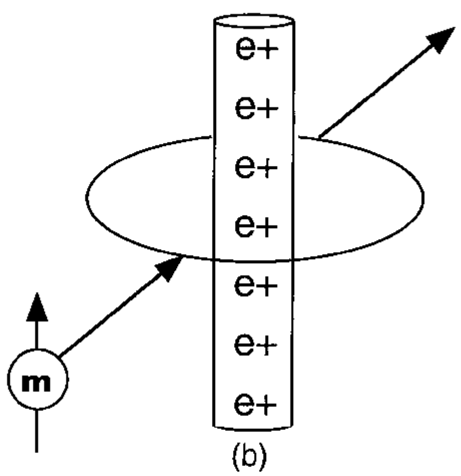


Predicted Aharanov & Bohm, 1959, presaged Eherenberg & Siday, 1949, early measurement Chambers 1960, definitive measurement Tonomura et al 1982

One year before first measurement

$$\phi_{AB} = \frac{q}{\hbar}\int_C \vec{A}\cdot d\vec{l} = \frac{q}{\hbar}\Phi_B^{(C)}$$

Aharonov-Casher



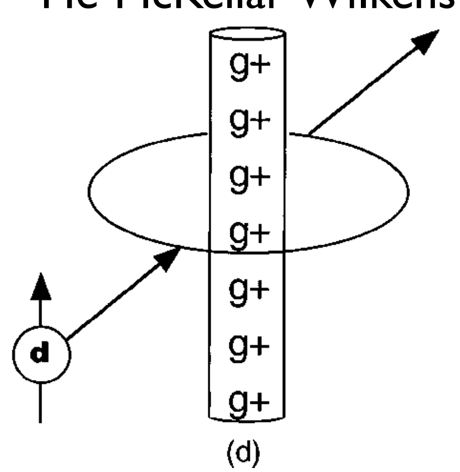
Predicted Aharonov & Casher, 1984, observed Cimmino, Opat, Klein et al, 1989

Five years before first measurement

$$\phi_{AC} = \frac{1}{\hbar c^2}\int_C \vec{\mu}\times\vec{E}\cdot d\vec{l} = \frac{\mu_0}{\hbar}\mu\lambda_e$$

Sorry, **m** and μ are the same magnetic moment

He-McKellar-Wilkens



Predicted He & McKellar, 1993, & Wilkens 1994 observed Lepoutre et al 2012

Nineteen years before first measurement

$$\phi_{HMW} = \frac{-1}{\hbar}\int_C \vec{\mathcal{D}}\times\vec{B}\cdot d\vec{l} = \frac{-1}{\hbar}\mathcal{D}\lambda_m$$

AC and HMW topological?

- The topological nature of the AC or HMW phases occurs under restricted conditions, as in fact does their existence.
- Existence requires
  - In non-relativistic limit, neglecting quadratic terms in the fields

1. a neutral spin half particle with a nonzero magnetic dipole moment is moving in a plane, say the x- y plane, in an external electric field $\vec{E}$;
2. On this plane, $E_3 = 0, \partial_3 E_3 = 0$;and $\partial_3 \psi = 0$,

Topological?

- For the AC effect

It is possible for this particle to travel a closed path, along which the component $\vec{\mu}$ along $\vec{E} \times \vec{dl}$ is constant along the path, which is in the $\nabla \cdot \vec{E} = 0$ region and is such that a surface whose boundary is the path intersects regions where $\nabla \cdot \vec{E} \neq 0$, the accumulated phase ϕ_{AC} independent of the path and thus topological.

- For the HMW effect, you see why He and I proposed monopoles, it seemed that we needed a confined source for the **B** field, $\nabla \cdot \mathbf{B} \neq 0$, in order to get a topological phase.

Klein *et al* measurement of AC

VOLUME 63, NUMBER 4 PHYSICAL REVIEW LETTERS 24 JULY 1989

Observation of the Topological Aharonov-Casher Phase Shift by Neutron Interferometry

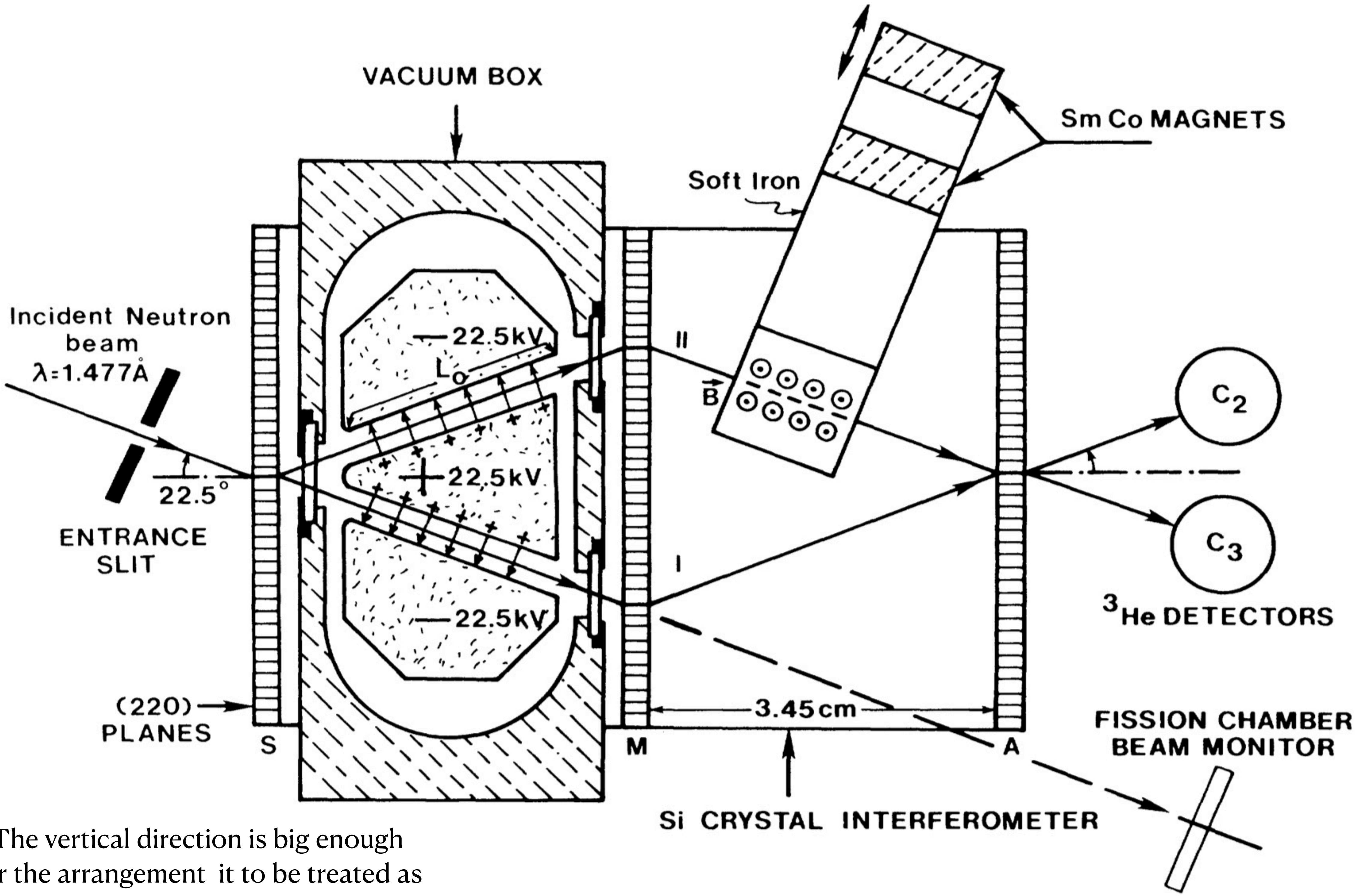
A. Cimmino, G. I. Opat, and A. G. Klein
School of Physics, The University of Melbourne, Parkville, Victoria 3052, Australia

H. Kaiser, S. A. Werner, M. Arif,[a] and R. Clothier
Department of Physics and Research Reactor, The University of Missouri–Columbia, Columbia, Missouri 65211
(Received 1 May 1989)

The phase shift predicted by Aharonov and Casher for a magnetic dipole diffracting around a charged electrode has been observed for the case of thermal neutrons, using a neutron interferometer containing a 30-kV/mm vacuum electrode system. The judicious use of the Earth's gravitational field introduces a spin-independent phase shift which enables unpolarized neutrons to be used. A supplementary magnetic bias field of the correct magnitude allows first-order sensitivity to be achieved; even so, the theoretically predicted phase shift is only 1.50 mrad for the geometry and conditions of the experiment. We observe a phase shift of 2.19 ± 0.52 mrad.

Klein *et al* measurement of AC

The apparatus

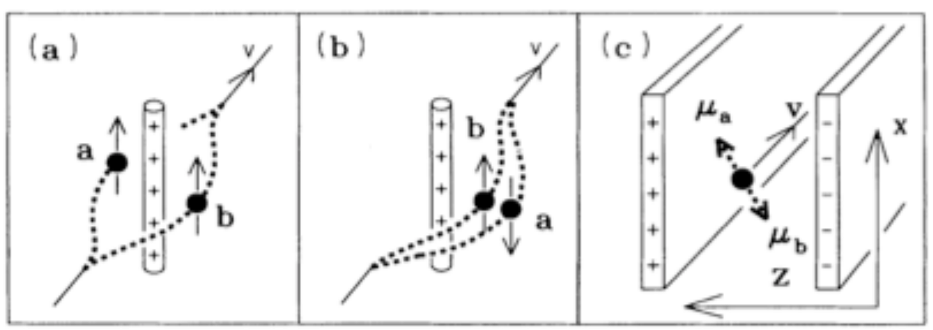


The vertical direction is big enough
for the arrangement it to be treated as
two dimensional
So they measured a ***Topological phase***

Alternative geometries for AC

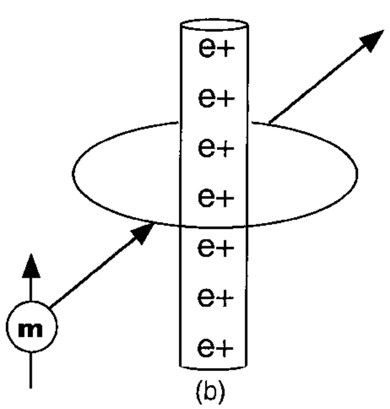
K. Sangster, E.~A. Hinds, S.~M. Barnett, E. Rijs and A.~G. Sinclair, \emph{Phys. Rev. A} \textbf{51}, 1776-1786 (1995).

Sangster et al, attempt to improve the precision, but use different geometry: opposite spins and same side of the wire so replace wire by capacitor



Aharonov-Casher

Sangster geometry



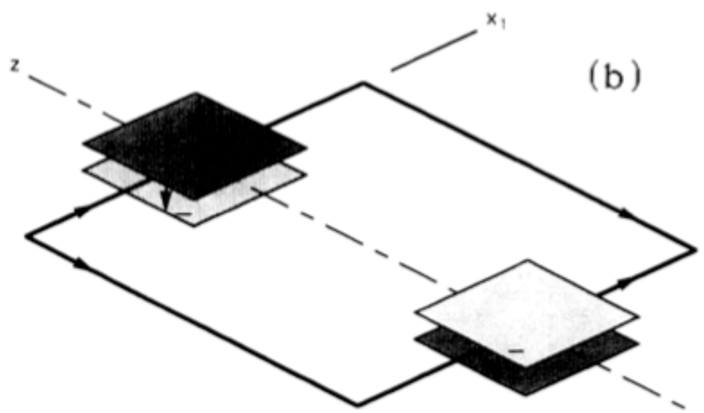
The most precise measurement of the AC phase is the experiment of Sangster *et al*, who used a completely different geometry. They pass coherent beams of magnetic dipoles of different orientation through the same electric field, and observe a phase difference between the beams.

As they point out themselves, the magnetic moment in this case is not constant in magnitude or direction, and so the phase they measure is not topological.

Much more precise results from these methods, but until 2013 the Klein et al result was the most precise for a **topological phase**

Alternative geometries for AC

R.~C. Casella, Phys. Rev. Letters 65, 2217-2220 (1990).



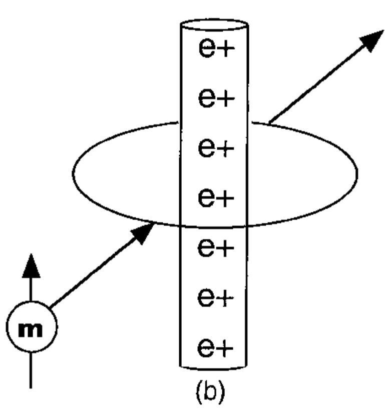
The spin is now in the plane of the path, and the electric field is normal to both, and changes sign on different sections of the path.
The phase is no longer topological.
However, the phase is

$$\chi_c = -\oint_{\mathcal{C}} (\boldsymbol{\mu} \times \boldsymbol{E}) \cdot \boldsymbol{ds},$$

and

1. in the region of the path $\text{curl}(\boldsymbol{\mu} \times \boldsymbol{E}) = \boldsymbol{0}$, so the phase is path independent,
2. in the plane of the circuit, interior to the circuit, there is a region in which $\text{curl}(\boldsymbol{\mu} \times \boldsymbol{E}) \neq \boldsymbol{0}$, not because $\text{div}\boldsymbol{E} \neq 0$, but because $(\boldsymbol{\mu} \cdot \boldsymbol{\nabla})(\boldsymbol{E} \cdot \boldsymbol{d\mathcal{S}}) \neq \boldsymbol{0}$. The phase depends on the fields in the excluded region, even though there are no charges in that region.

Aharonov-Casher



The AC phase for the Casella geometry satisfies the last two of my conditions for a topological phase, but not the first. It is not topological, and Casella (almost) says so

There are no magnetic monopoles, how can we measure HMW?

$$\phi_{HMW} = \frac{-1}{\hbar}\int_C \vec{\mathcal{D}} \times \vec{B} \cdot \vec{dl} = \frac{-1}{\hbar}\mathcal{D}\lambda_m$$

To answer this we note that the important features are, as well as the essential 2+1 dimensional geometry, that curl$\{\mathbf{B} \times \mathcal{D}\}$ should vanish in the region of the path, and be non-vanishing in the excluded region. Note that

$$\begin{aligned}\text{curl}\{\mathbf{B} \times \mathcal{D}\} &= \mathbf{B}(\nabla \cdot \mathcal{D}) - \mathcal{D}(\nabla \cdot \mathbf{B}) \\ &\quad +(\mathcal{D} \cdot \nabla)\mathbf{B} - (\mathbf{B} \cdot \nabla)\mathcal{D}\end{aligned}$$

For curl$\{\mathbf{B} \times \mathcal{D}\}$ to vanish in the region of the path, $\mathbf{B}$ and $\mathcal{D}$ must be constant in that region. If they then vary in an appropriate way in the excluded region, then curl$\{\mathbf{B} \times \mathcal{D}\}$ can be non-vanishing in that region.

In practice the electric dipole must be induced by an electric field $D = \alpha\mathbf{E}$, so

$$\nabla \times (\mathbf{B} \times [\alpha\mathbf{E}]) =$$

$$\alpha\left(\mathbf{B}\,\text{div}\,\mathbf{E} - \mathbf{E}\,\text{div}\,\mathbf{B} + (\mathbf{B} \cdot \nabla)\mathbf{E} - (\mathbf{E} \cdot \nabla)\mathbf{B}\right).$$

To have a topological phase this must vanish in the interference region and not vanish in the excluded region

Now the electric field can generate the HMW phase, but we must have the additional constraint $\mathbf{v} \cdot \mathbf{B} = 0$

This was noted to be possible by H.~Wei, R.~Han and X.~Wei, Phys. Rev. Letters **75**, 2071 (1995)

Leading to the measurement of HMW in 2012

The Toulouse Experiment

PRL **109,** 120404 (2012) PHYSICAL REVIEW LETTERS week ending 21 SEPTEMBER 2012

He-McKellar-Wilkens Topological Phase in Atom Interferometry

S. Lepoutre, A. Gauguet, G. Trénec, M. Büchner, and J. Vigué*
Laboratoire Collisions Agrégats Réactivité-IRSAMC, Université de Toulouse-UPS and CNRS UMR 5589, Toulouse, France
(Received 20 January 2012; published 20 September 2012)

We report an experimental test of the topological phase predicted by He and McKellar in 1993 and by Wilkens in 1994: this phase, which appears when an electric dipole propagates in a magnetic field, is connected to the Aharonov-Casher effect by electric-magnetic duality. The He-McKellar-Wilkens phase is quite small, at most 27 mrad in our experiment, and this experiment requires the high phase sensitivity of our atom interferometer with spatially separated arms as well as symmetry reversals such as the direction of the electric and magnetic fields. The measured value of the He-McKellar-Wilkens phase differs by 31% from its theoretical value, a difference possibly due to some as yet uncontrolled systematic errors.

DOI: 10.1103/PhysRevLett.109.120404 PACS numbers: 03.65.Vf, 03.75.Dg, 37.25.+k

Used electric field to polarise ^{7}Li atoms,

$\mathbf{D} = \alpha\, \mathbf{E}$, *in plane of path, with opposite sign on different sections*

B is normal to the plane of the path

Modified Casella geometry

The Toulouse Experiment

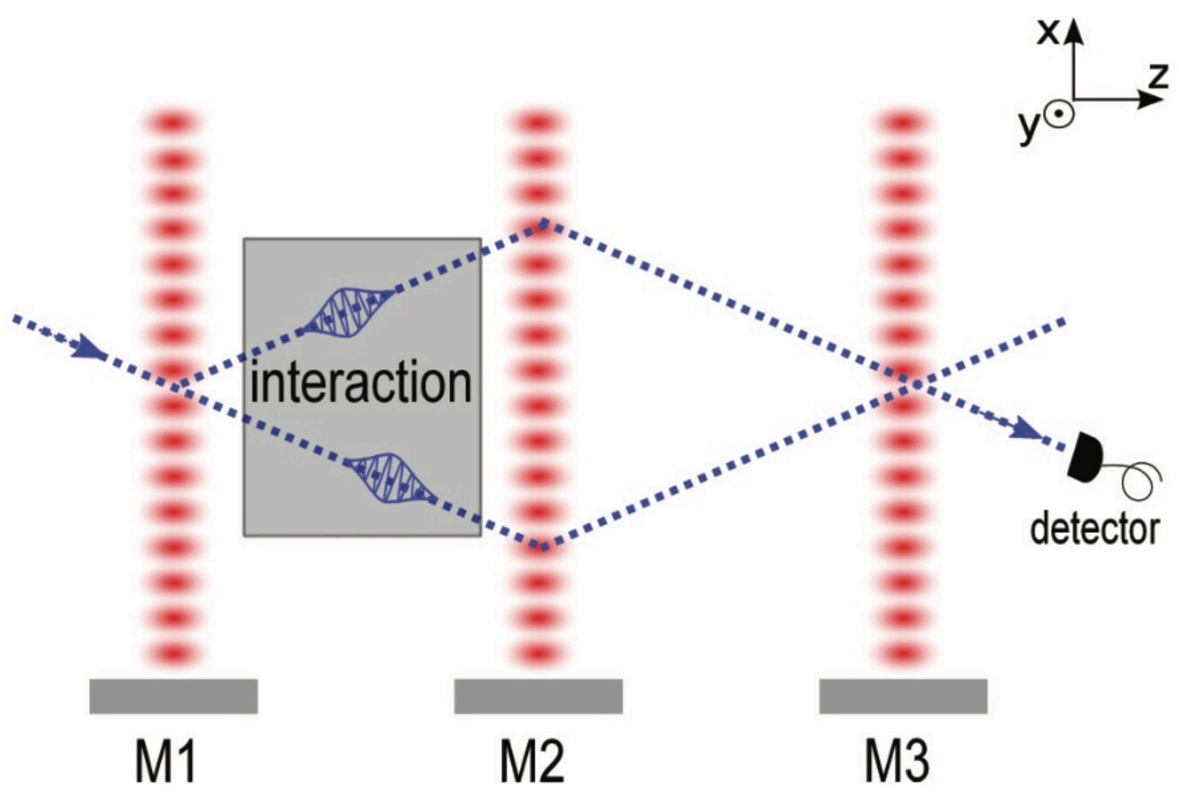


FIG. 4. (Color online) Schematic top view of our atom interferometer: the HMW interaction region is placed just before the second laser standing wave, at the place where the arm separation is largest. It is thus possible to introduce a septum between the two interferometer arms without any perturbation of the interferometer signal.

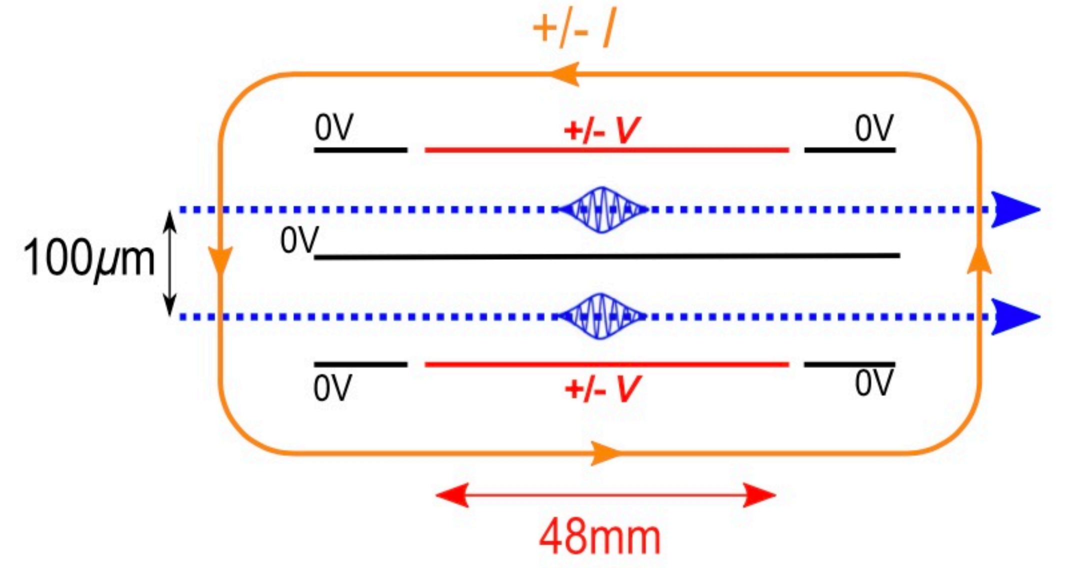


FIG. 1 (color online). Schematic top view of the interaction region (not to scale). The interferometer arms (dotted blue lines) are separated by a septum; the electric fields are produced by two plane capacitors sharing the septum as a common electrode grounded electrode. The high voltage electrodes (in red) are surrounded by grounded guard electrodes (in black). The brown rectangle represents the coils which produce the magnetic field.

The Toulouse group give a very careful analysis of various Stark and Zeeman contributions which can introduce systematic effects. And the effects of electric fields not quite normal to the central electrode giving systematic errors in
S. Lepoutre, J. Gillot, A. Gauguet, M. Büchner, and J. Vigué, *Phys. Rev. A* **88**, 043628 (2013) and
S. Lepoutre, A. Gauguet, M. Büchner, and J. Vigué. *Phys Rev A* **88**, 043627 (2013)

My one paper with Tony

Topological phases reviewed: the Aharonov Bohm, Aharonov Casher, and He McKellar Wilkens phases.

B. H. J. McKellar*, X-G. He† and A. G. Klein**

**ARC Centre of Excellence for Particle Physics at the Terrascale, School of Physics*
University of Melbourne, Australia 3010
†Department of Physics, National Taiwan University, Taipei Taiwan
***School of Physics, University of Melbourne, Australia 3010*

Abstract. There are three topological phases related to electromagnetic interactions in quantum mechanics:

1. The Aharonov Bohm phase acquired when a charged particle encircles a magnetic field but travels through a field free region
2. The Aharonov Casher phase acquired when a magnetic dipole encircles electric charges but travels through a charge free region
3. The He McKellar Wilkens phase acquired when an electric dipole encircles magnetic charges but travels through a charge free region.

We review the conditions under which these phases are indeed topological and their experimental realisation.

Because the He McKellar Wilkens phase has been recently observed we pay particular attention to how the basic concept of "an electric dipole encircles magnetic charges" was realised experimentally, and discuss possible future experimental realisations

AIP Conference Proceedings **1588**, 59 (2014);
https://doi.org/10.1063/1.4866924

We pointed on systematic errors, or breakdown of topological phase from deviations from normality of electric fields
And that using permanent dipoles could give much larger effects,
as later discussed by R. P. Rai and D. Rai, Annals of Physics **383** (2017) 196-206 https://doi.org/10.1016/j.aop.2017.05.007

A measurement of the *topological* AC effect

At last the topological phase is measured more precisely than Klein et al

Eur. Phys. J. D (2014) 68: 168
DOI: 10.1140/epjd/e2014-50277-1

THE EUROPEAN PHYSICAL JOURNAL D

Regular Article

Measurement of the Aharonov-Casher geometric phase with a separated-arm atom interferometer

Jonathan Gillot, Steven Lepoutre, Alexandre Gauguet, Jacques Vigué, and Matthias Büchner[a]

Laboratoire Collisions Agrégats Réactivité-IRSAMC, Université de Toulouse-UPS and CNRS UMR 5589, 31062 Toulouse, France

Table 4. Our measurements of the AC phase slope $\partial\varphi_{AC}/\partial V$ in 10^{-5} rad/V are compared to the theoretical values $\partial\varphi^{c}_{AC}/\partial V$ corrected for imperfect optical pumping (see Eq. (10)). These predicted values show a velocity dependence, which stems from the velocity dependence of the optical pumping efficiency.

v_m (m/s)	$\partial\varphi_{AC}/\partial V$ experiment	$\partial\varphi^{c}_{AC}/\partial V$ theory
1520 ± 18	-8.05 ± 0.20	-7.86 ± 0.12
1062 ± 20	-8.51 ± 0.18	-8.51 ± 0.88
744 ± 18	-8.44 ± 0.41	-8.40 ± 0.48

The first topological phase measurement of AC after Klein *et al* in 1989

HMW and AC influence all of these recent papers

And many more

Tan, S.G., Chen, S.H., Ho, C.S., Huang, C.C., Jalil, M.B., Chang, C.R. and Murakami, S.,.
Yang–Mills physics in spintronics. *Physics Reports*, ***882*** (2020) pp.1-36.

Hassanabadi, H., M. de Montigny, and M. Hosseinpour.
Interaction of the magnetic quadrupole moment of a non-relativistic particle with an electric field in a rotating frame.
Annals of Physics **412** (2020) 168040.

Kholmetskii, A.L., Yarman, T. and Missevitch, O.V.
Quantum phase effects for electrically charged particles: Updated analysis.
Europhysics Letters, ***140*** (2022) p.20001.

Wu, Yijia, Hua Jiang, Hua Chen, Haiwen Liu, Jie Liu, and X. C. Xie.
Non-Abelian braiding in spin superconductors utilizing the Aharonov-Casher effect.
Physical Review Letters, **128** (2022) 106804.

Wood, A., McKellar, B. H. J., and Martin, A. M., ---
Persistent Superfluid Flow Arising from the He-McKellar-Wilkens Effect in Molecular Dipolar Condensates
Physical Review Letters, **116** (2016) 250403

Thank you to

- **He Xiao-Gang** for working with me and educating me about the AC and HMW phases
- **Tony Klein, Jacques Vigré, Lloyd Hollenberg, Ray Volkas, Peter Hannaford, Ed Hinds** for many discussions
- **Andy Martin** and **Alex Wood** for their work on our paper
- The McKellar Superannuation Fund for supporting my work since my retirement
- **You**, my audience for your attention