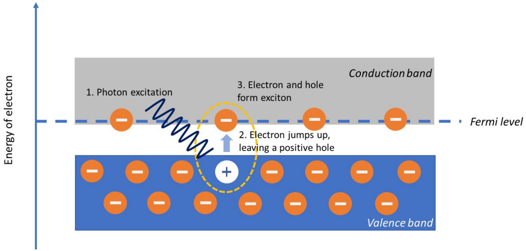
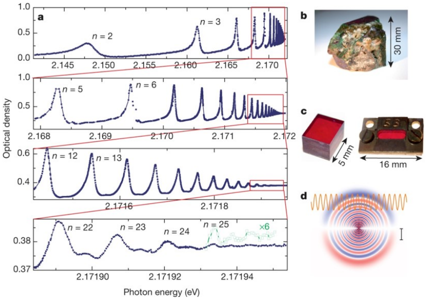
Rydberg Exciton-Polaritons in a Magnetic Field

Emma Laird
Previously: PhD student at Monash University
Now: postdoctoral fellow at the University of Queensland

December 14, 2022

Introduction





Experimental observation of photon absorption by Rydberg excitons in copper oxide.

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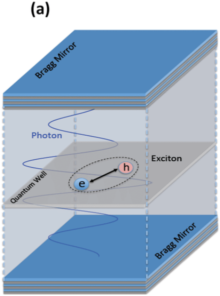

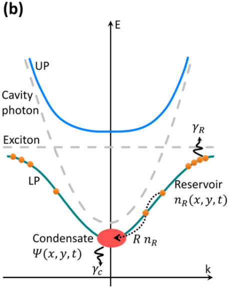

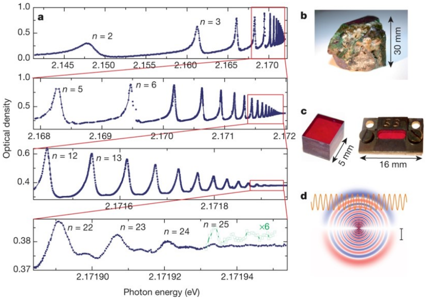

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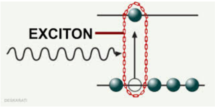
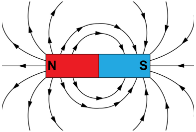
- until our work, there was no theory that could describe the interplay of these effects

Exciton Problem

- the properties of the Rydberg series in a static perpendicular magnetic field





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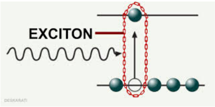

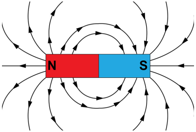

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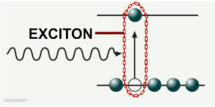

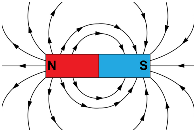

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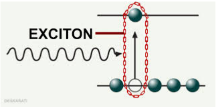

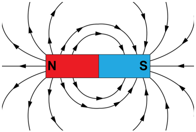

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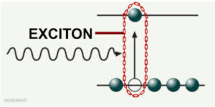

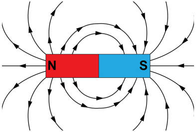

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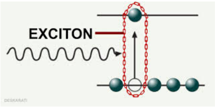

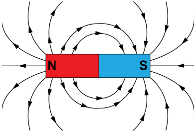

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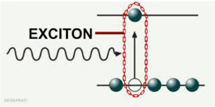

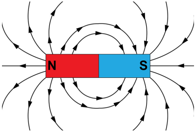

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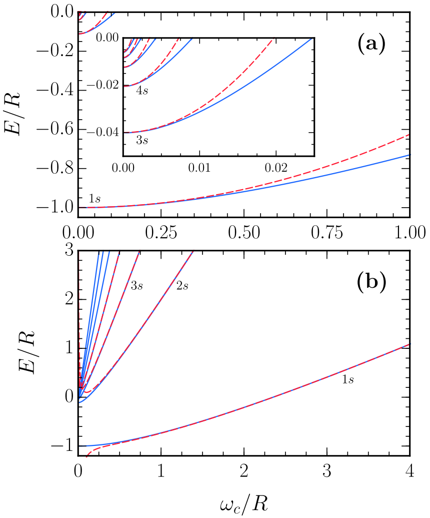

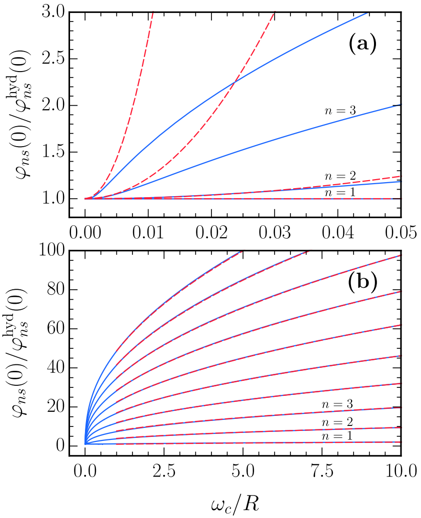

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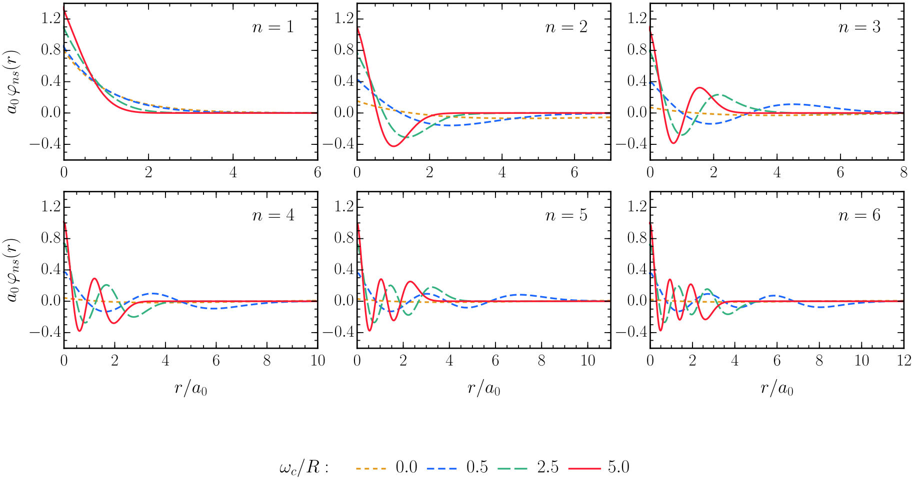

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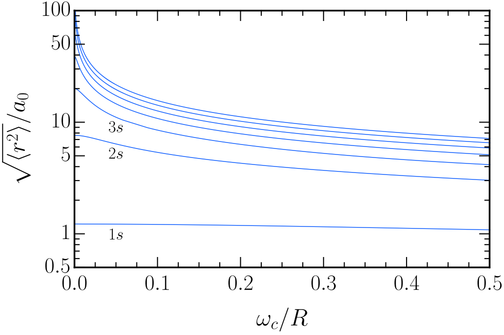

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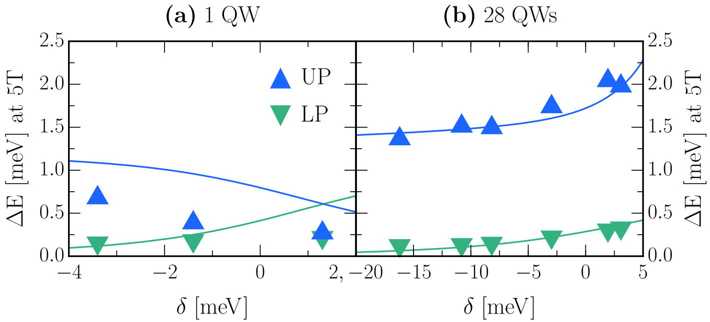
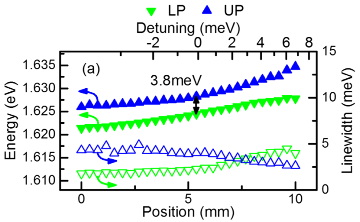
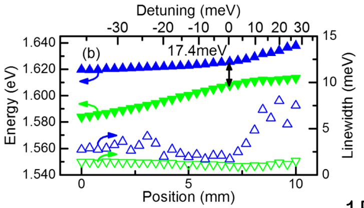
Comparison to Experiment — PRL 119, 027401 (2017)







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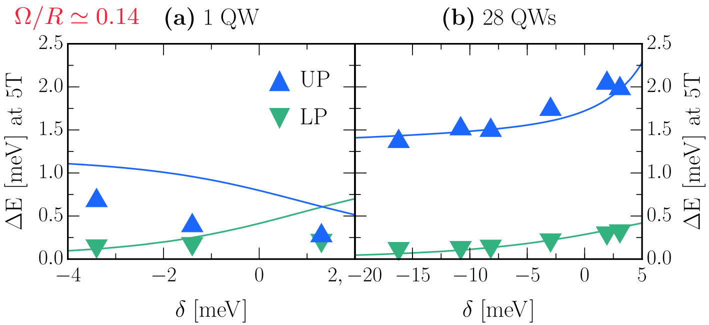

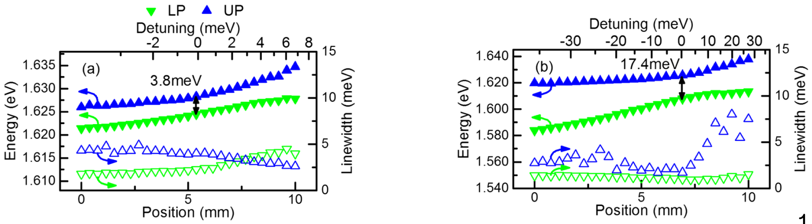

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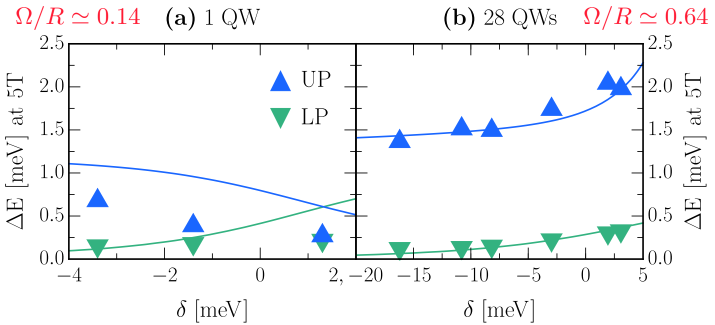

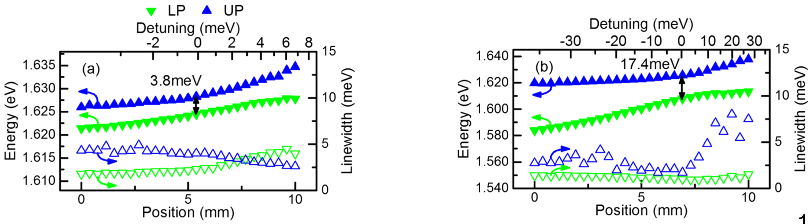

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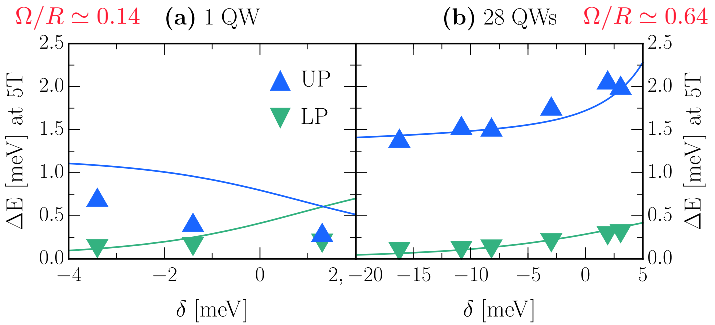

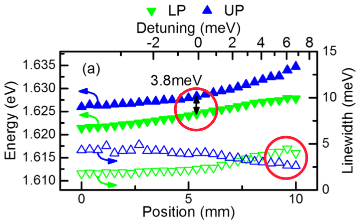

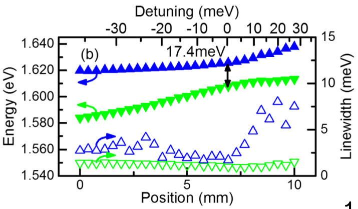

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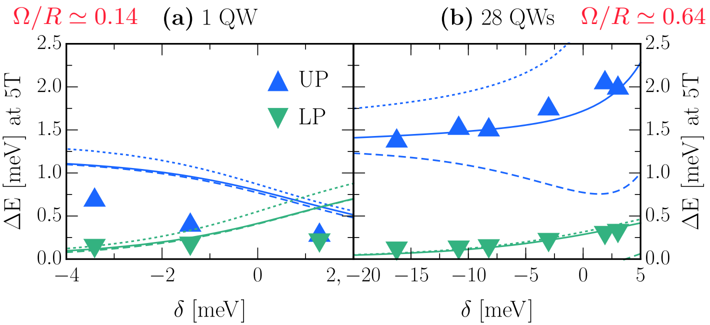

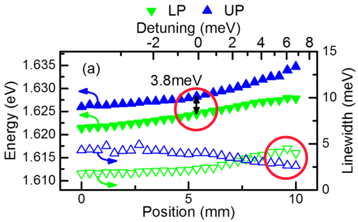

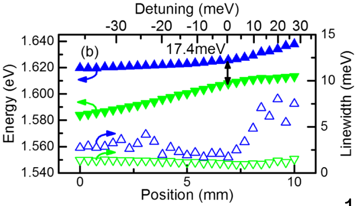

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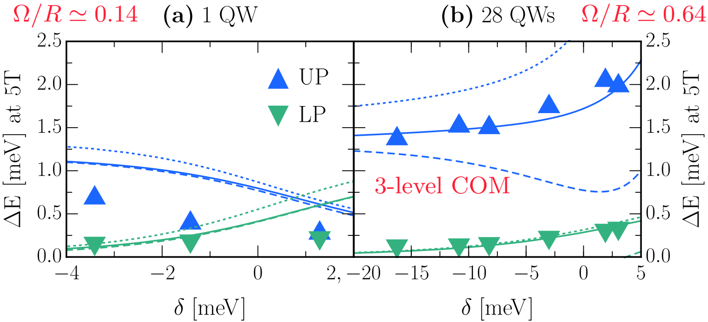

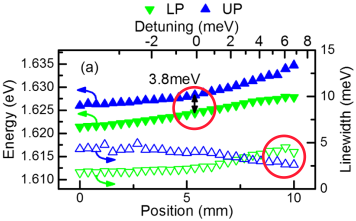

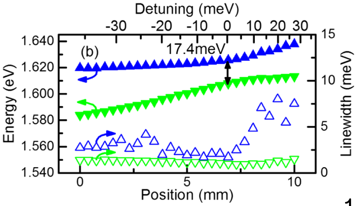

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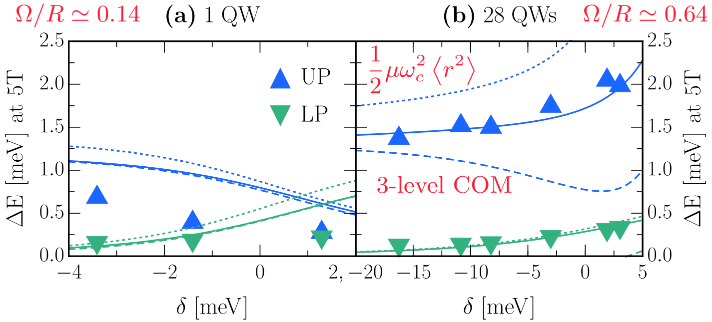

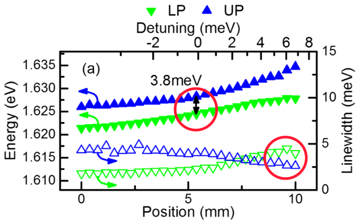

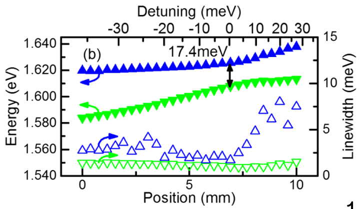

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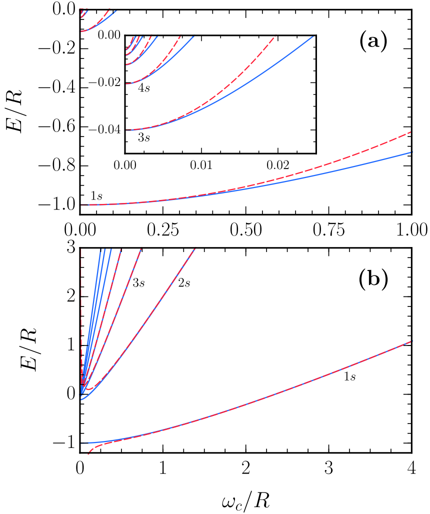
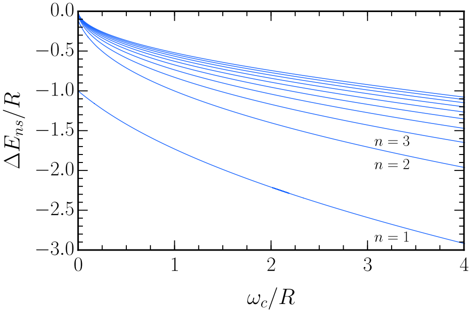
- see our paper: Phys. Rev. B 106, 125407 (2022)
- "Rydberg Exciton-Polaritons in a Magnetic Field"
- by E. Laird, F. M. Marchetti, D. K. Efimkin, M. M. Parish, and J. Levinsen

Extra Slides

Exciton Problem





$$\Delta E_{ns} \equiv E_{ns} - (2n-1)\omega_c$$

Polariton Problem

- Rydberg polaritons in a static perpendicular magnetic field (s excitons and s-wave light-matter coupling)
- Levinsen et al., Phys. Rev. Research 1, 033120 (2019)
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$$\hat{H} = \hat{H}_{\text{mat}} + \hat{H}_{\text{ph}} + \hat{H}_{\text{ph-mat}}$$

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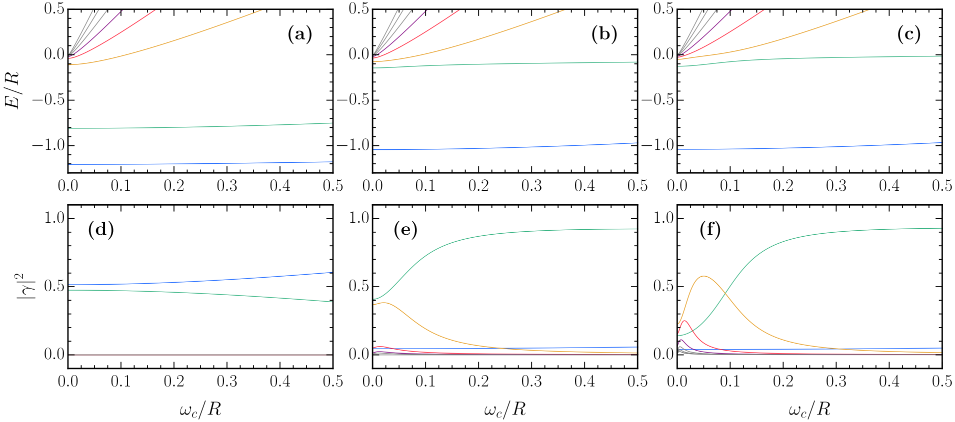
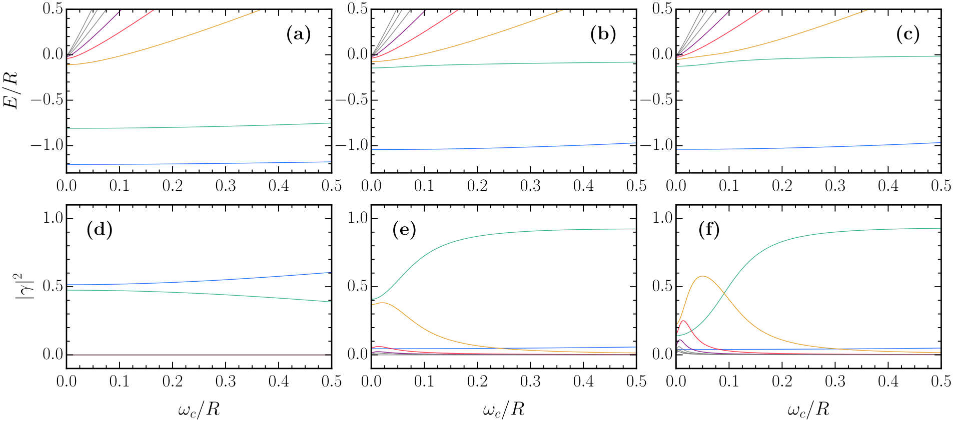
Numerically Exact Polariton Spectra



Numerically Exact Polariton Spectra

$\Omega/R = 0.2$



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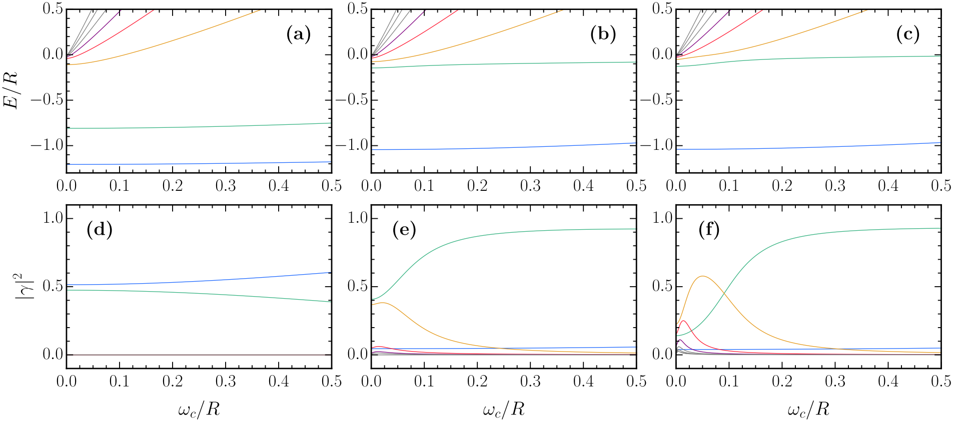

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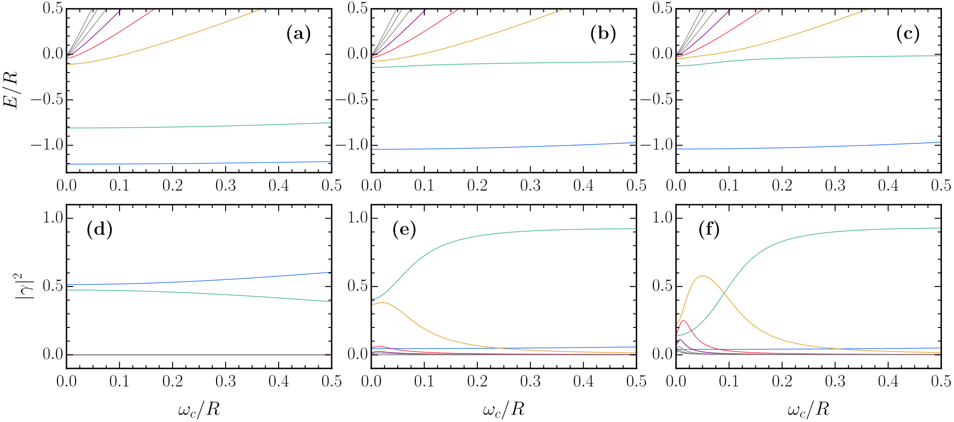

$\delta/R = 0$

$\delta/R = 8/9$

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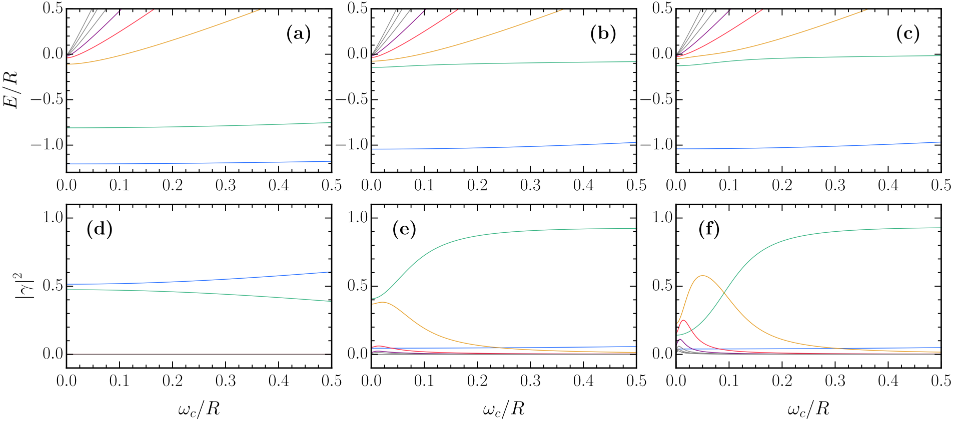
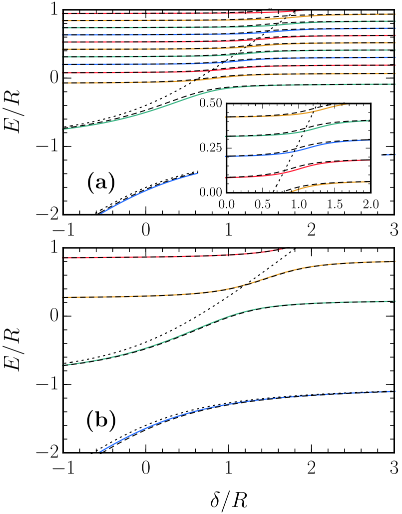


$\delta/R = 0$ $\delta/R = 8/9$ $\delta/R = 24/25$

Comparison to a Coupled Oscillator Model (COM)



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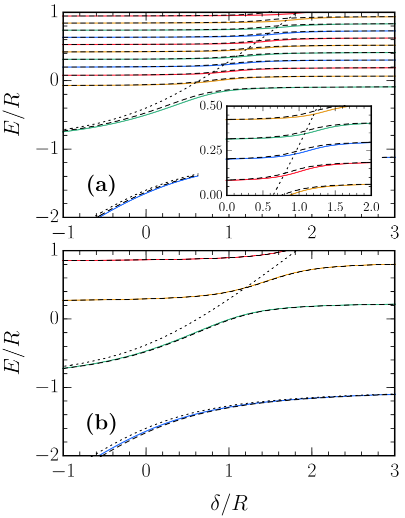

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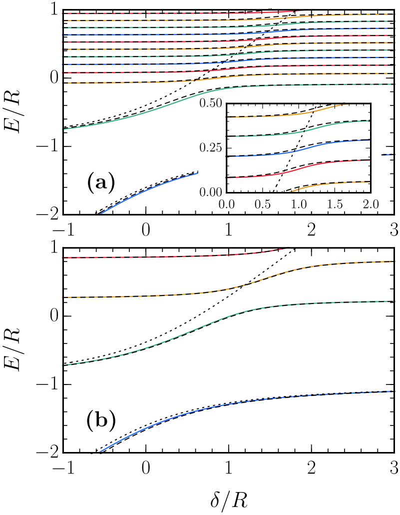

$\frac{\omega_c}{R} = 0.05$

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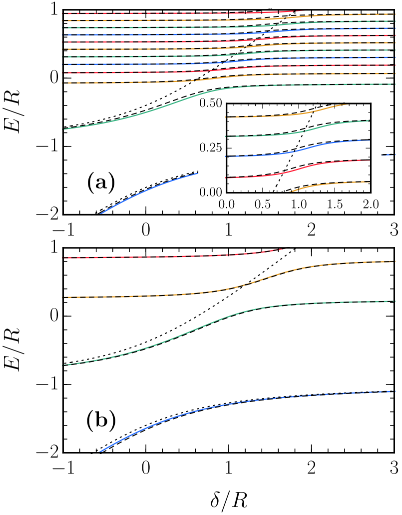

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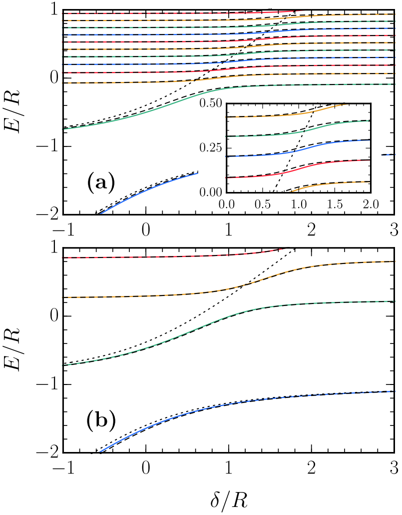

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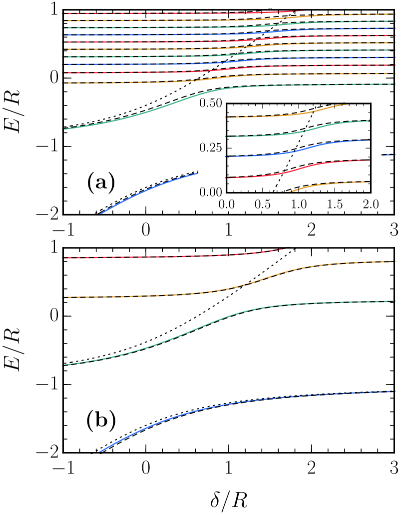

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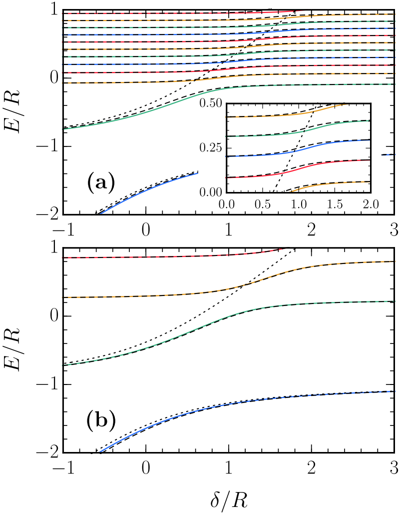

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