Proton and nuclear structure from lattice QCD

Phiala Shanahan, MIT

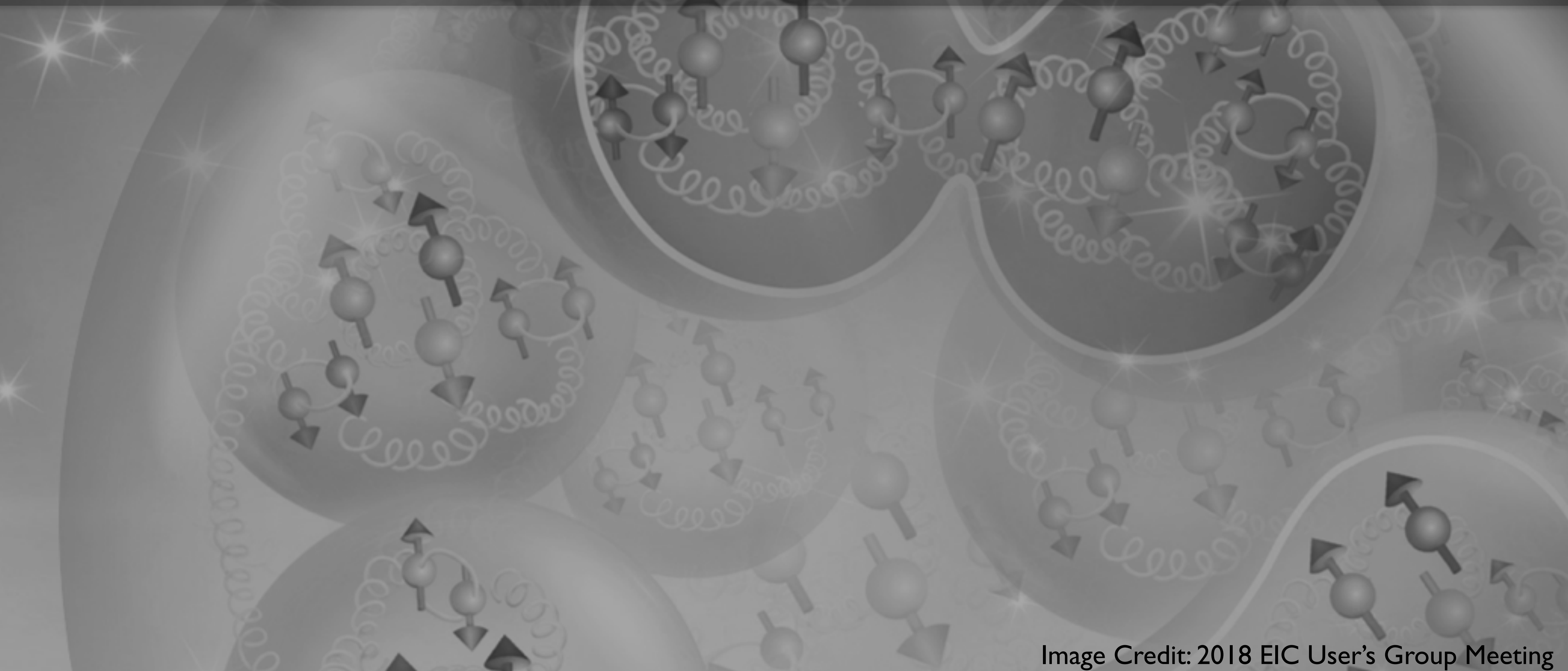


Image Credit: 2018 EIC User's Group Meeting

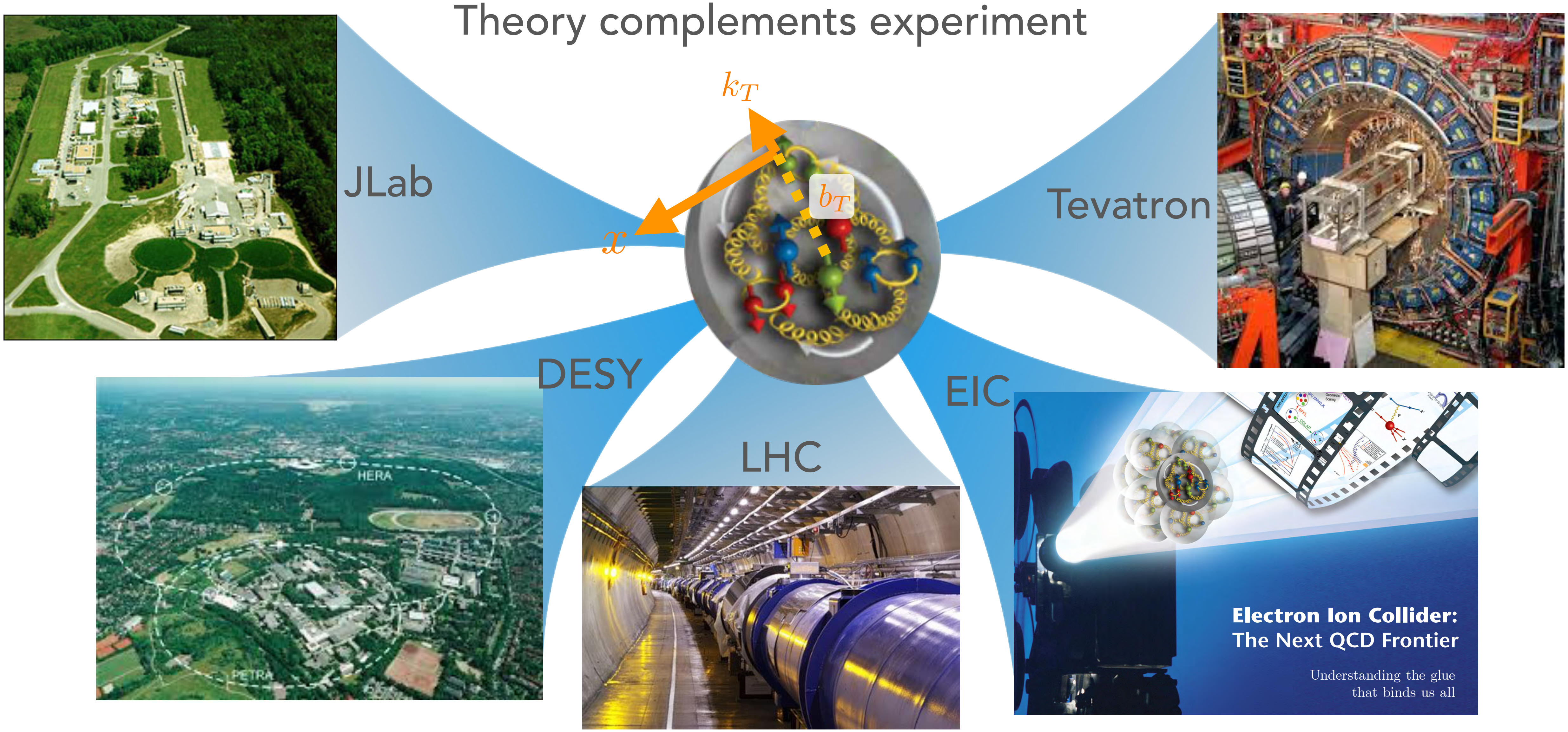
2022 AIP Congress

Massachusetts Institute of Technology

Proton and nuclear structure

Understanding the quark and gluon structure of matter

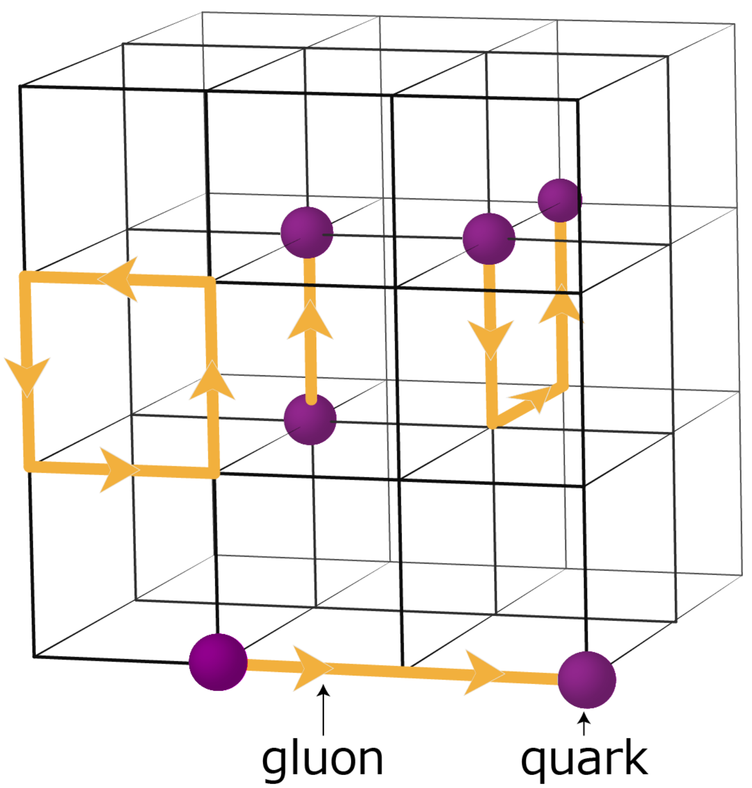
Theory complements experiment



Lattice QCD

Numerical first-principles approach to non-perturbative QCD

- Discretise QCD onto 4D space-time lattice
- Approximate QCD path integral using Monte-Carlo methods and importance sampling
- Run on supercomputers and dedicated clusters
- Take limit of vanishing discretisation, infinite volume, physical quark masses



Lattice QCD

Numerical first-principles approach to non-perturbative QCD

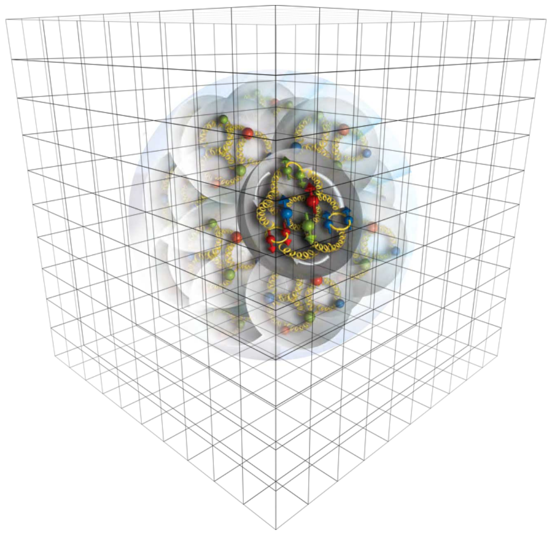
INPUT

Lattice QCD action has same free parameters as QCD: quark masses, α_S

- Fix quark masses by matching to measured hadron masses, e.g., π, K, D_s, B_s for u, d, s, c, b
- One experimental input to fix lattice spacing in GeV (and also α_S), e.g., $2S$-$1S$ splitting in Υ, or f_π or Ω mass

OUTPUT

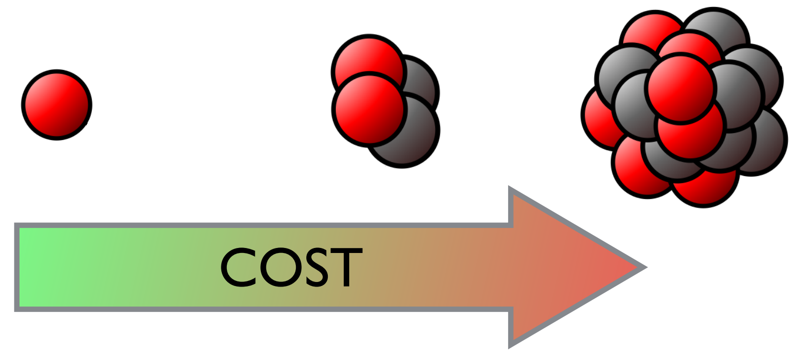
Calculations of all other quantities are QCD predictions



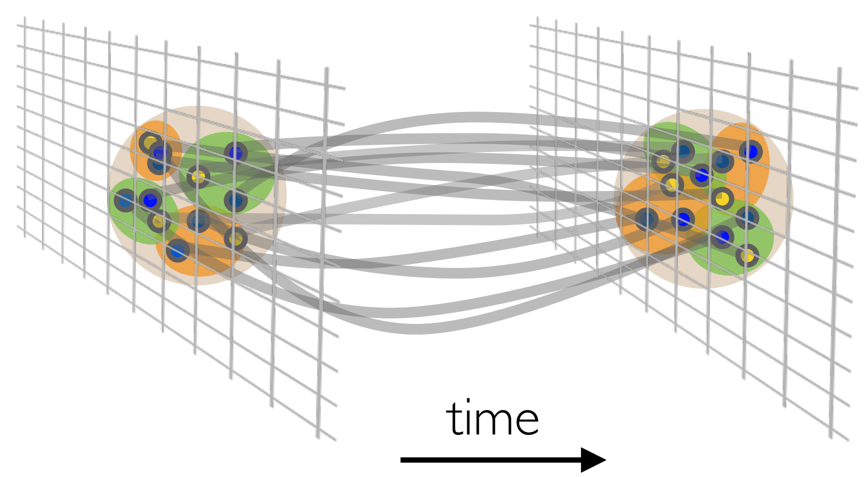
Nuclear physics from lattice QCD

Nuclei on the lattice are HARD

- **Noise:**
  Statistical uncertainty grows exponentially with number of nucleons



- **Complexity:**
  Number of contractions grows factorially

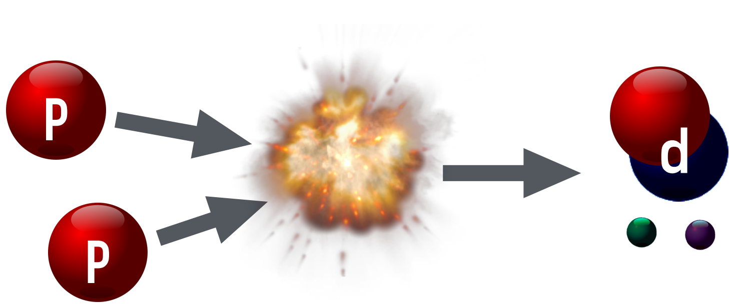


Calculations possible for A<5

Nuclear physics from lattice QCD

"Nuclear physics from LQCD Collaboration"
NPLQCD

- Nuclei with A<5 unphysical quark masses
- Physical-mass calculations begun 2021

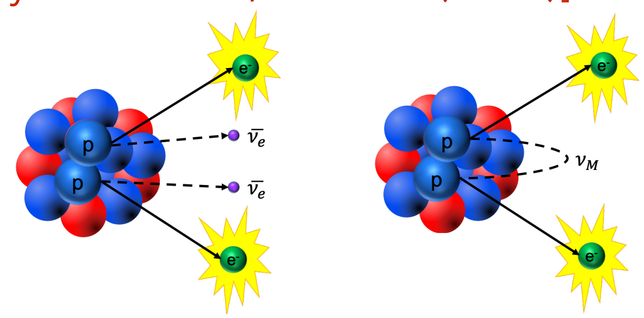


Proton-proton fusion and tritium β-decay
[Phys.Rev.Lett. 119, 062002 (2017)]

Scalar, axial, tensor matrix elements
[Phys.Rev.Lett. 120 (2018), Phys.Rept. 900 (2021), Phys.Rev.D 103, 074511(2021)]

Baryon-baryon interactions, incl. QED
[Phys.Rev.D 96, 114510 (2017), Phys.Rev.D 103, 054504 (2021), Phys.Rev.D 103, 054508 (2021), 2108.10835 (2021)]

Double β-decay
[Phys.Rev.Lett. 119, 062003 (2017), Phys.Rev.D 96, 054505 (2017)]

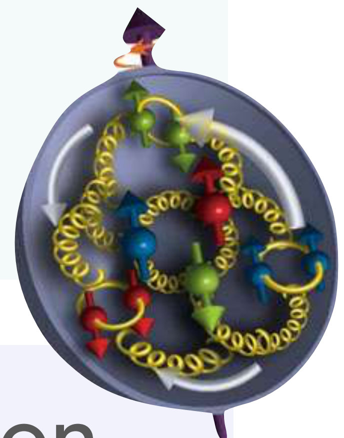


Nuclear parton distribution functions
[Phys.Rev.D 96, 094512 (2017), Phys.Rev.Lett. 126, 202001 (2021)]

Parton physics from Lattice QCD

How much do quarks/gluons contribute to the proton's

- Momentum
- Spin
- Mass

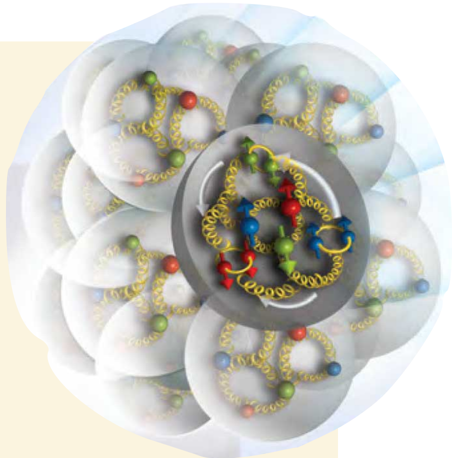


What is the 3D distribution of quarks/gluons in a proton

- Encoded in parton distribution functions and form factors
- Pressure distribution

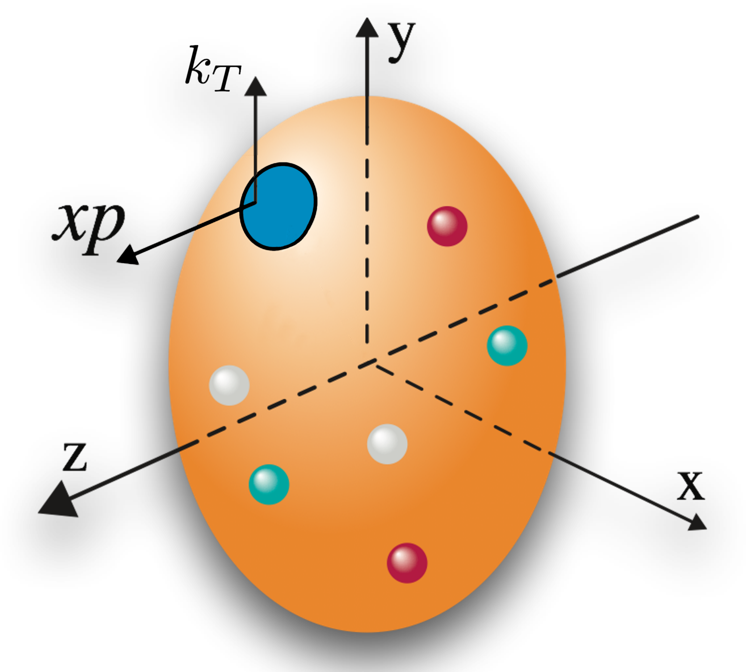
How is the quark/gluon structure of a proton modified in a nucleus

- 'EMC' effects
- Exotic glue



Parton physics from Lattice QCD

Understanding the quark and gluon structure of matter



Three-dimensional partonic structure of the proton

PDF
$$f_{q/H}(x)$$
longitudinal

TMD
$$f_{q/H}(x, k_T)$$
+ transverse

$$f_{q/H}(x, b_T)$$
+ impact parameter

Fourier transform

$\xi = 0$

GPD
$$H_{q/H}(x, \xi, \Delta_T^2)$$

Community overview: Constantinou et al., "2020 PDFLattice Report" arXiv:2006.08636

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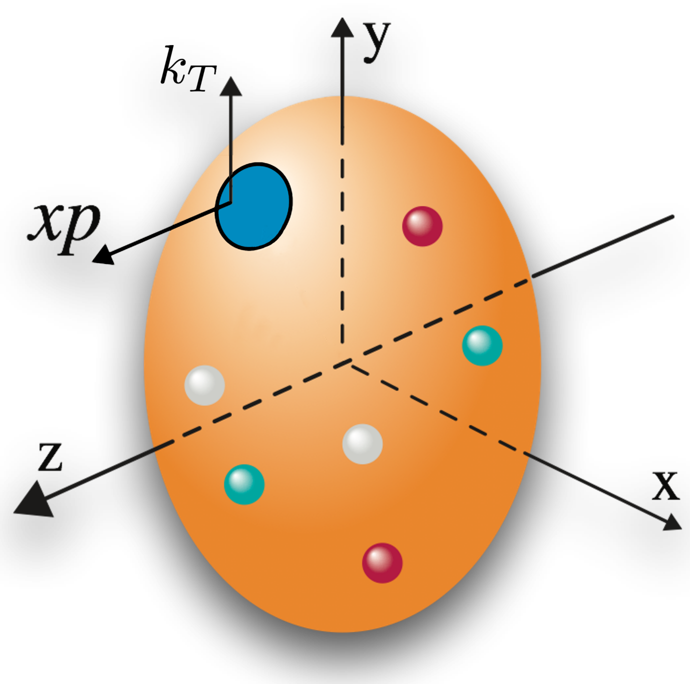

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Community overview:
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Decomposition of proton momentum

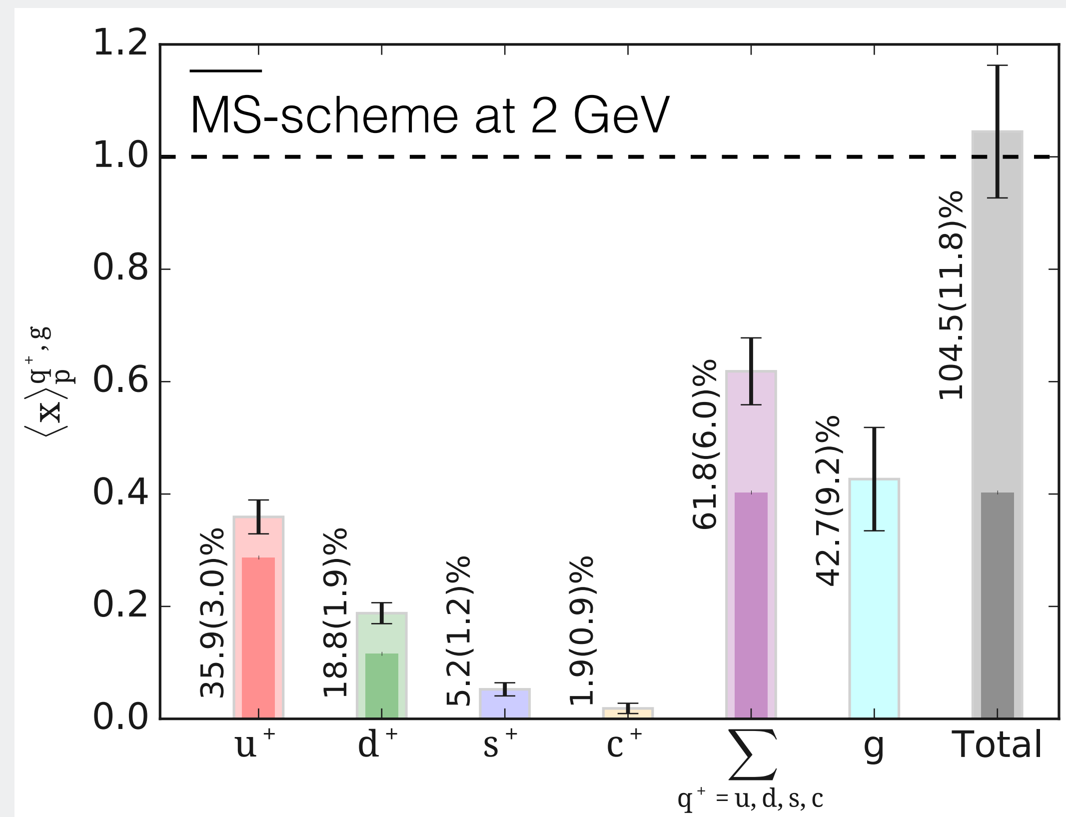
Moments of PDFs encode key aspects of hadron structure

$$\int_0^1 dx\, x^n f(x,\mu^2) = \langle x^n \rangle_f(\mu^2)$$

- Lattice QCD can cleanly access low moments of PDFs (n ≲ 3)
- Massive community efforts to study x-dependence from LQCD + perturbative matching and other approaches
  [Ji, PRL 110 (2013) 262002, Radyushkin, PRD 96 (2017) 034025, Ma & Qiu, PRL 120 (2018) 022003, Braun & Müller, EPJ C55 (2008) 349; Chambers et al., PRL 118 (2017) 242001, Detmold & Lin, PRD 73 (2006) 014501, Liu & Dong, PRL 72 (1994) 1790+many more]

Example: Lowest moment defines contribution of each type of parton to the hadron momentum

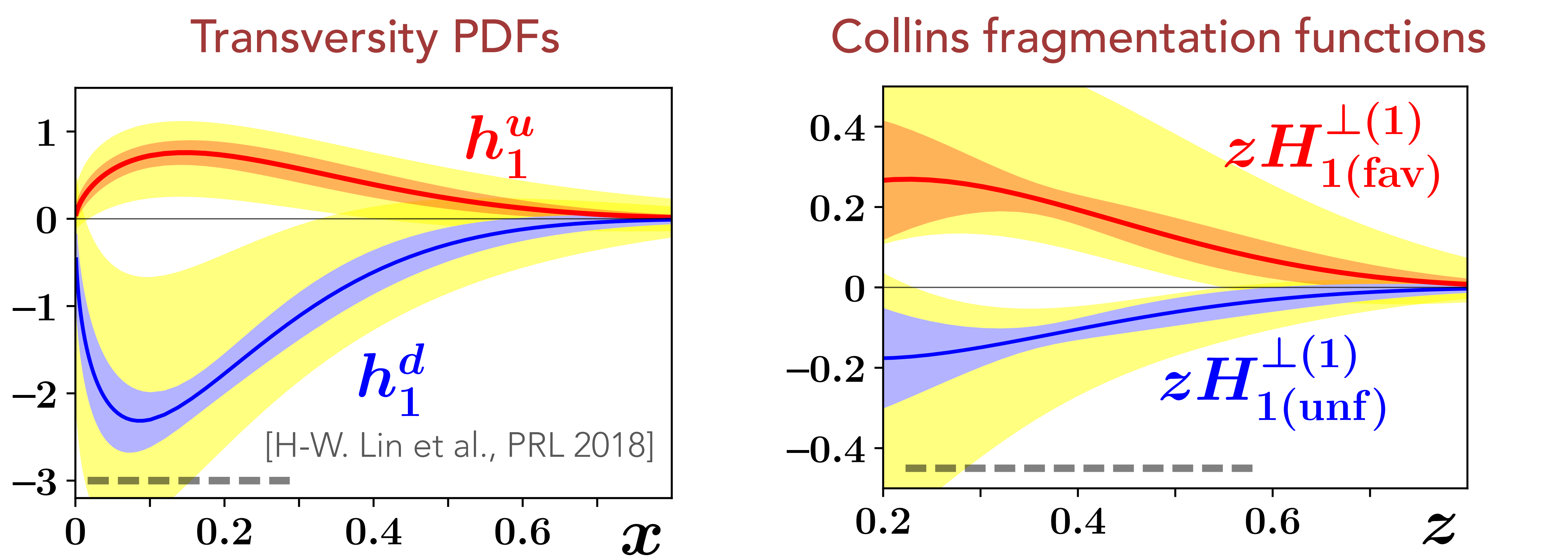
2020 Highlight:
Proton momentum decomposition



[C. Alexandrou et al., PRD 101 (2020)]

Constraints on global PDF fits

- Including lattice QCD results for moments in global PDF fits can yield significant improvements
- Community white paper (LQCD + phenomenologists) assessed potential impacts [Lin et al., Prog. Part. Nucl. Phys 100 (2018), 107]

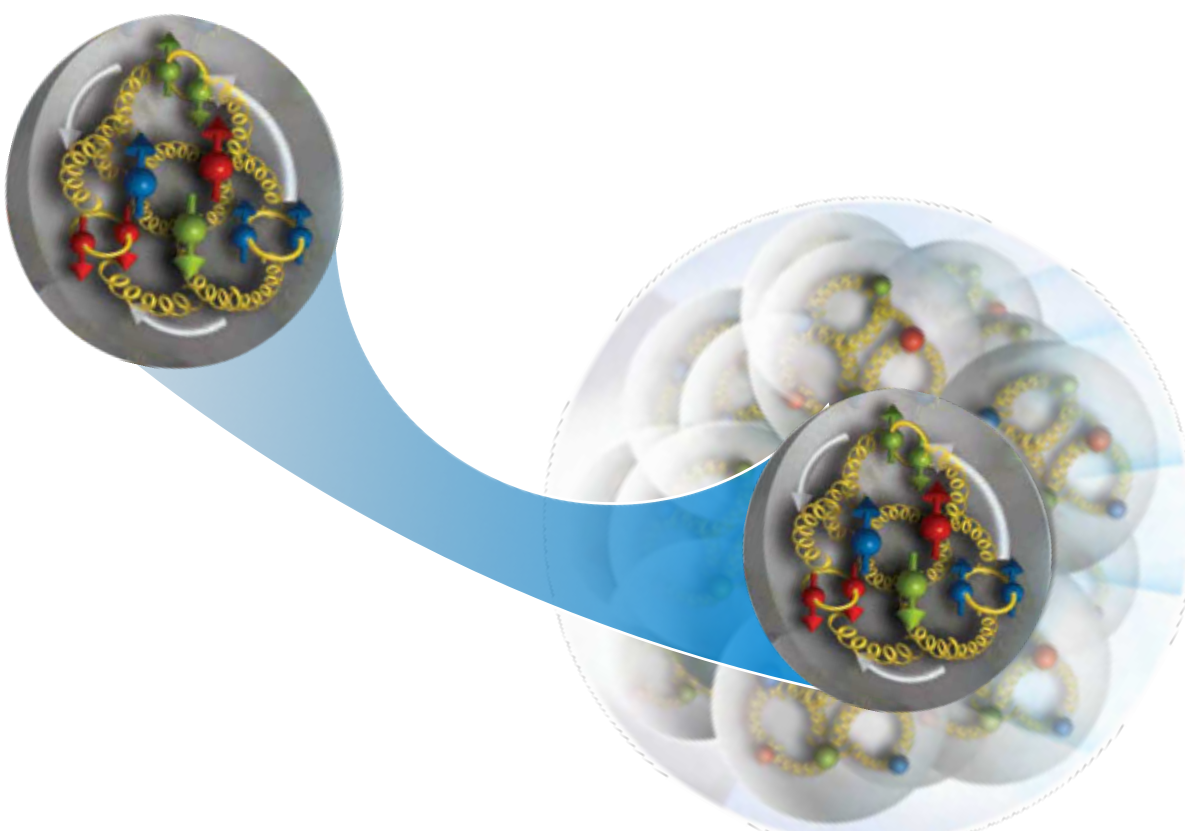


Yellow: SIDIS data only: direct constraints in region indicated by dashes
Blue/Red: SIDIS + lattice QCD for tensor charge (zeroth moment)

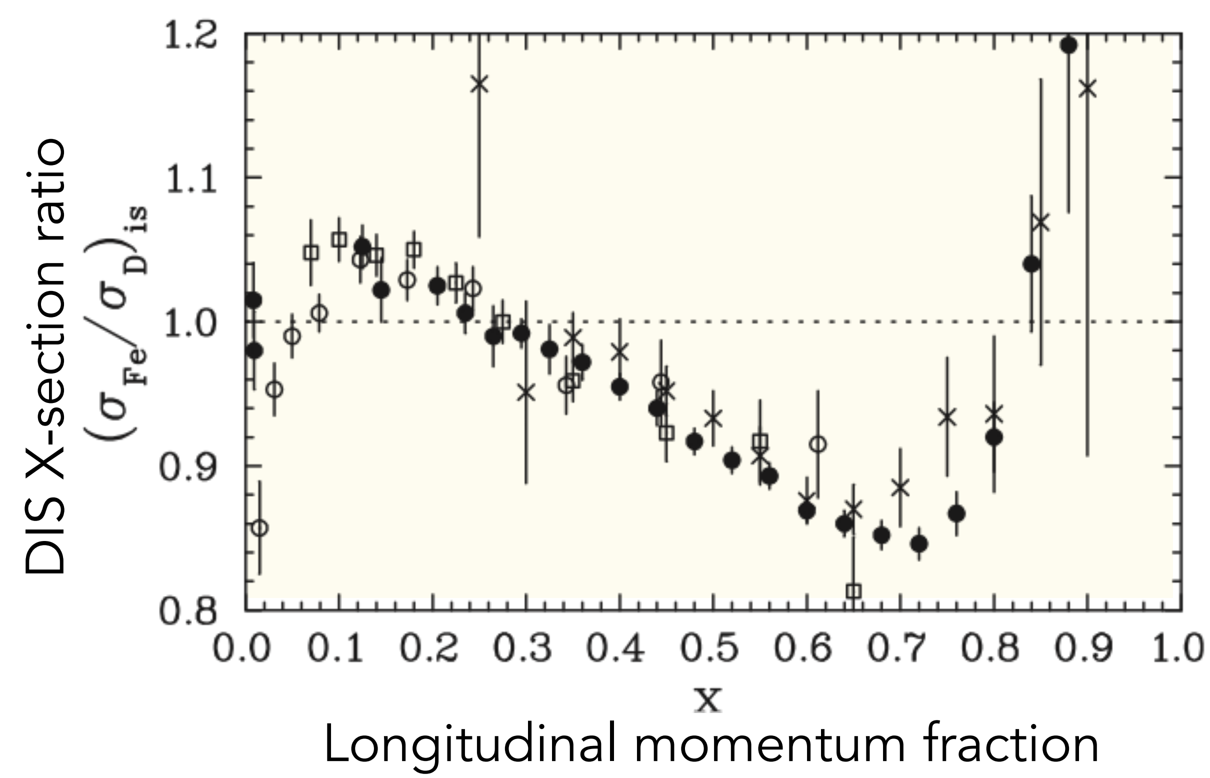
The structure of matter

Understanding the quark and gluon structure of matter

How is the partonic structure of nuclei different from that of nucleons?



Encoded in EMC-type effects



(EMC: Aubert et al., 1983)

EMC-type effects from Lattice QCD

Understanding the quark and gluon structure of matter

Many aspects of EMC effects will be accessible at a future Electron-Ion Collider

Cover image from EIC whitepaper arXiv::1212.1701

- Polarised EMC (polarised light ions)
- Isovector EMC (SIDIS)
- Gluon EMC (quarkonium production)
- LQCD will make predictions!

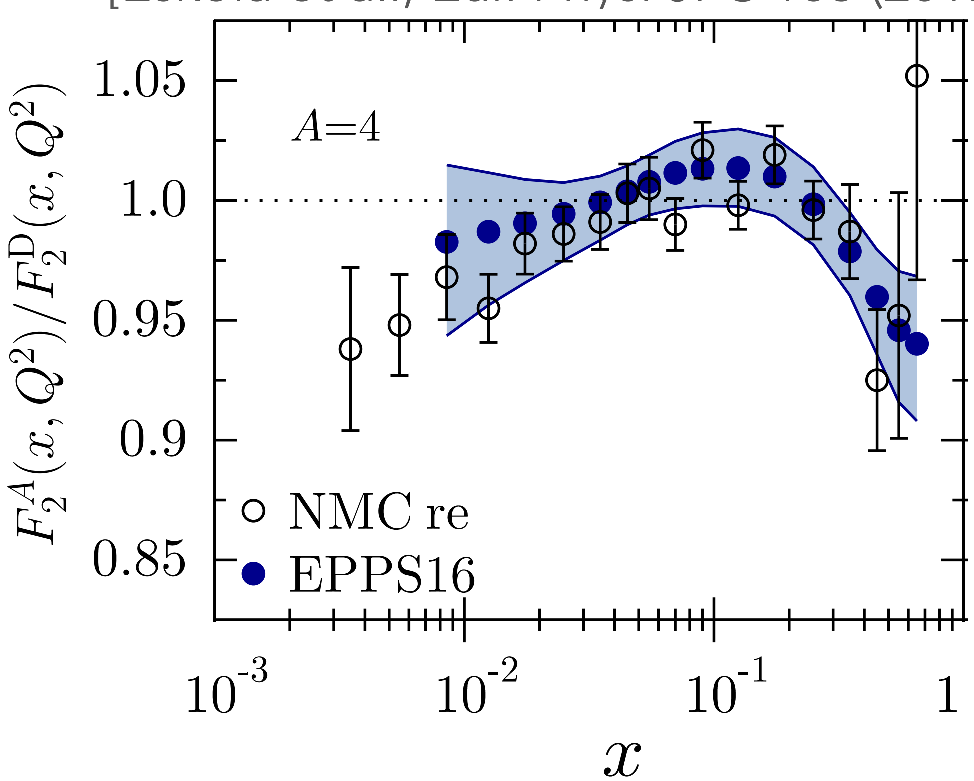
Phiala Shanahan, MIT

EMC effects in Mellin moments

First investigation of EMC-type effects from LQCD:
Nuclear effects in Mellin moments of PDFs

- Calculable from local operators
- BUT EMC effects in moments are very small

[Eskola et al., Eur. Phys. J. C 163 (2017)]



Classic EMC effect is defined in F_2:

$$F_2(x,Q^2) = \sum_{q=u,d,s\ldots} x\, e_q^2 \left[q(x,Q^2) + \overline{q}(x,Q^2)\right]$$

Number density of partons of flavour q

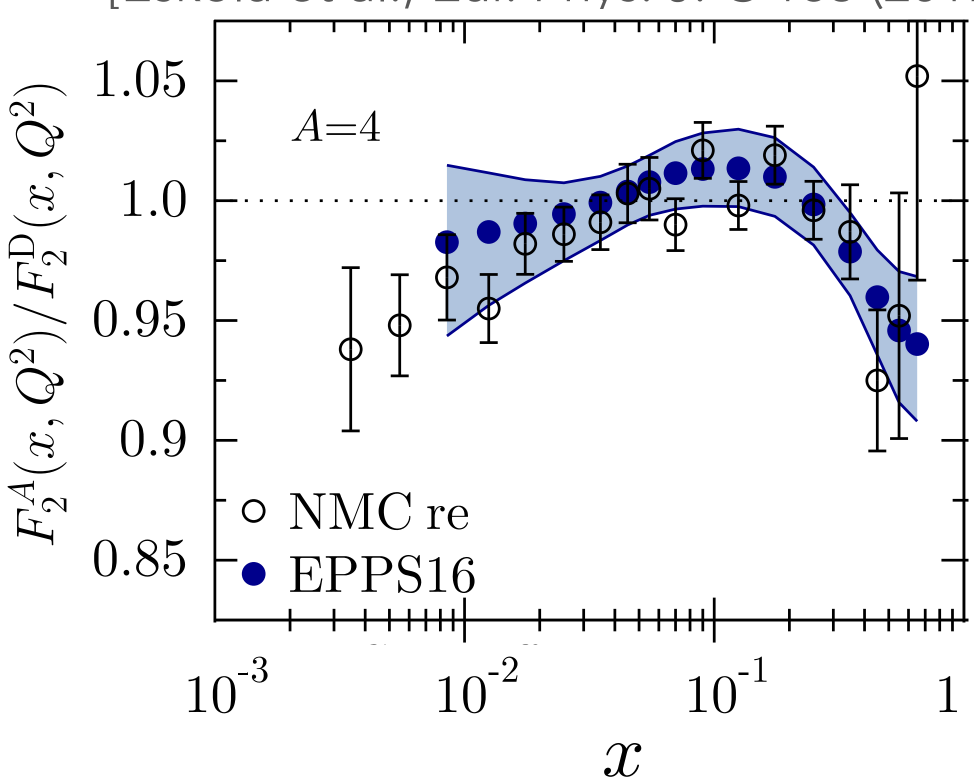
x-integrals of numerator and denominator $\int_0^1 dx\, x^n q(x,Q^2)$

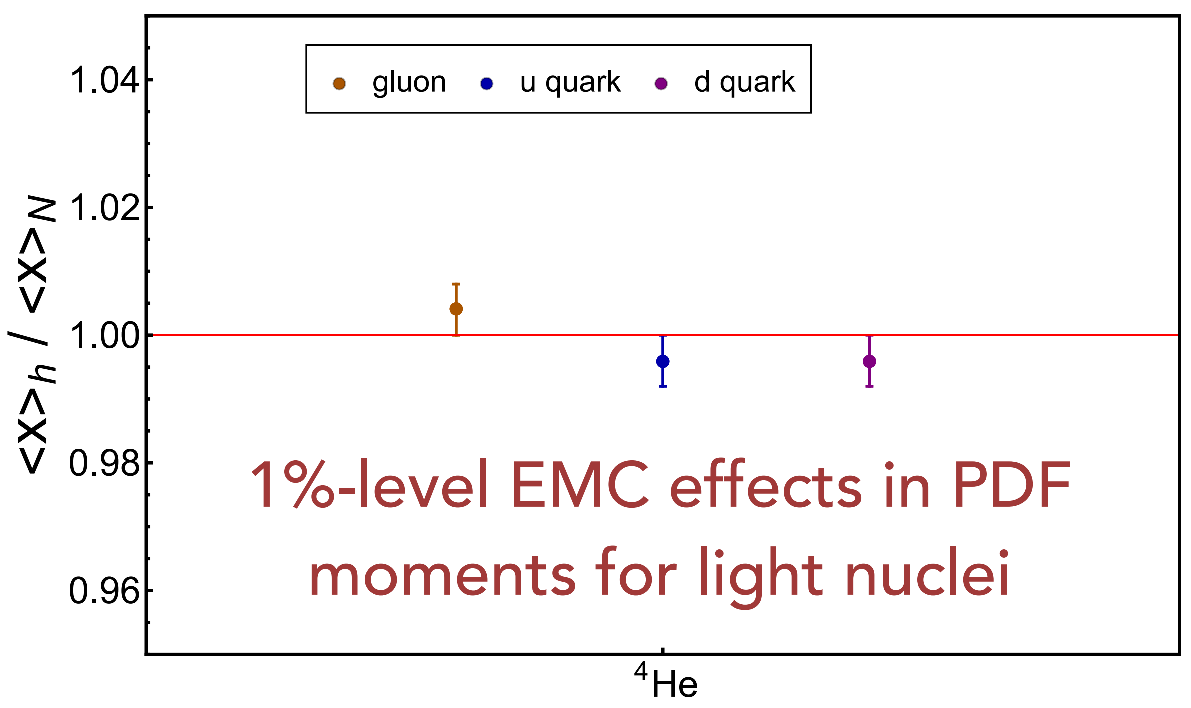
EMC effects in Mellin moments

First investigation of EMC-type effects from LQCD:
Nuclear effects in Mellin moments of PDFs

- Calculable from local operators
- BUT EMC effects in moments are very small

[Eskola et al., Eur. Phys. J. C 163 (2017)]





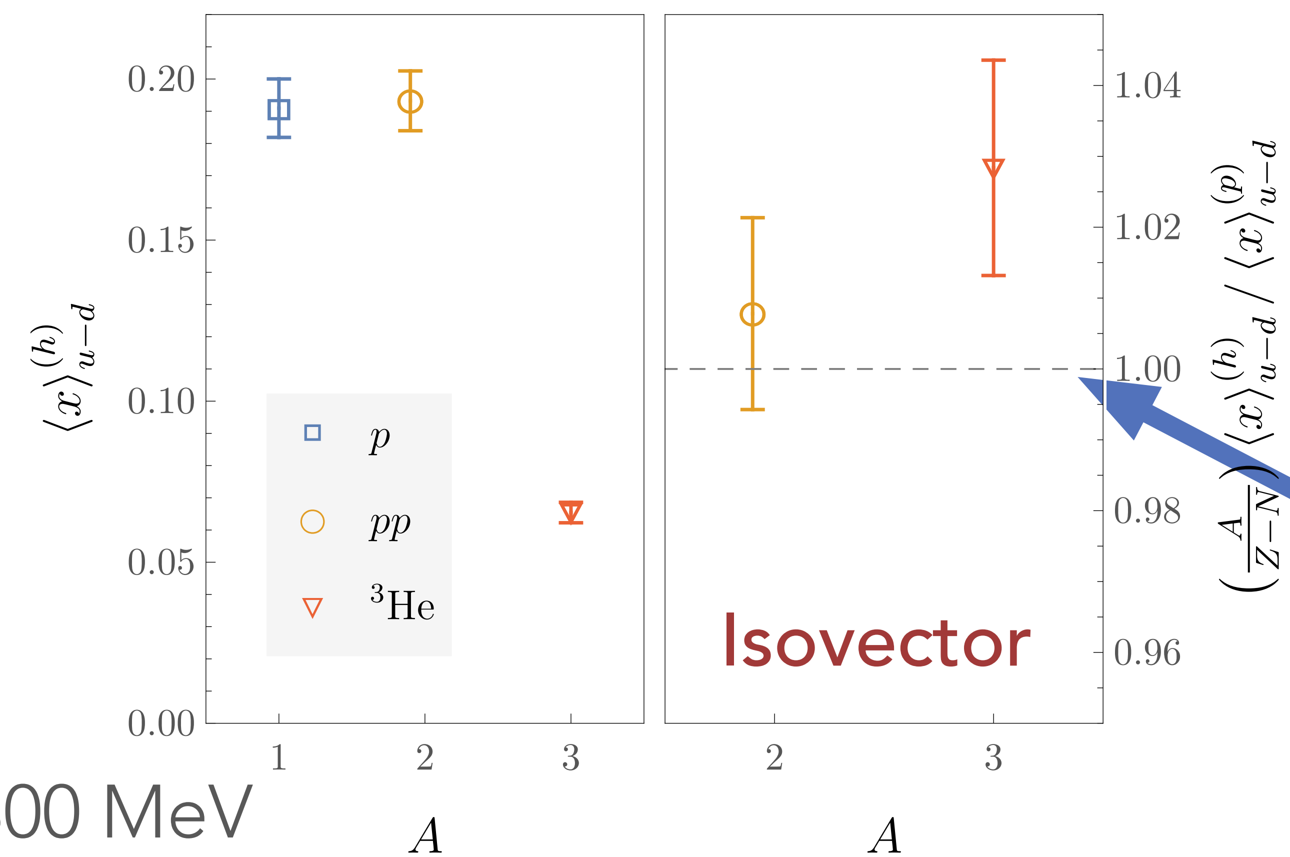
Momentum fraction of nuclei

Matrix elements of the Energy-Momentum Tensor in light nuclei

→ **first QCD determination of momentum fraction of nuclei**

- Bounds on EMC effect in moments at ~few percent level, consistent with phenomenology [2009.05522 [hep-lat] (2021)]

Ratio of quark momentum fraction in nucleus to nucleon



Normalised to proton result

- No mixing
- No sum rule constraint

m_π ~800 MeV

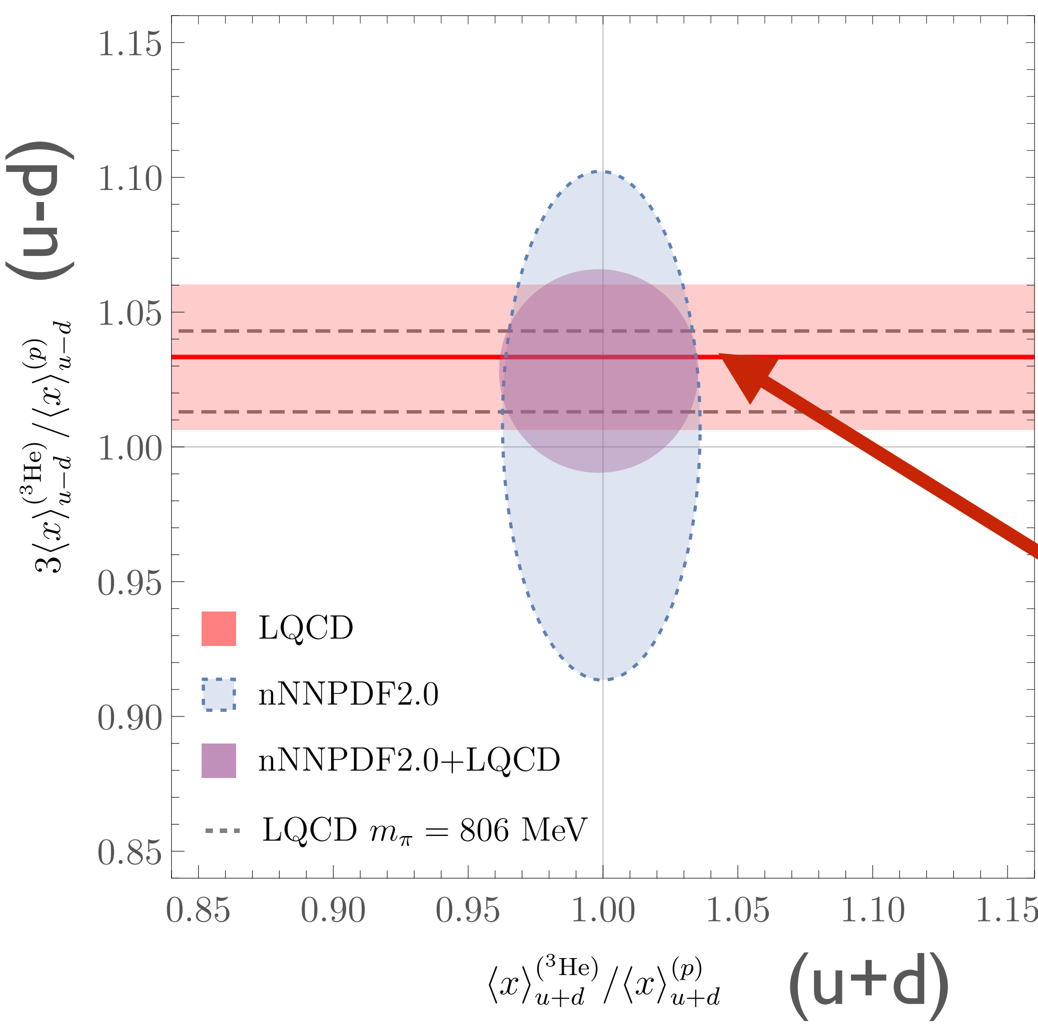
[NPLQCD PRL 126, 202001 (2021) [2009.05522]]

Momentum fraction of ^{3}He

Matrix elements of the Energy-Momentum Tensor in light nuclei

→ **first QCD determination of momentum fraction of nuclei**

Ratio of ^{3}He to proton momentum fractions



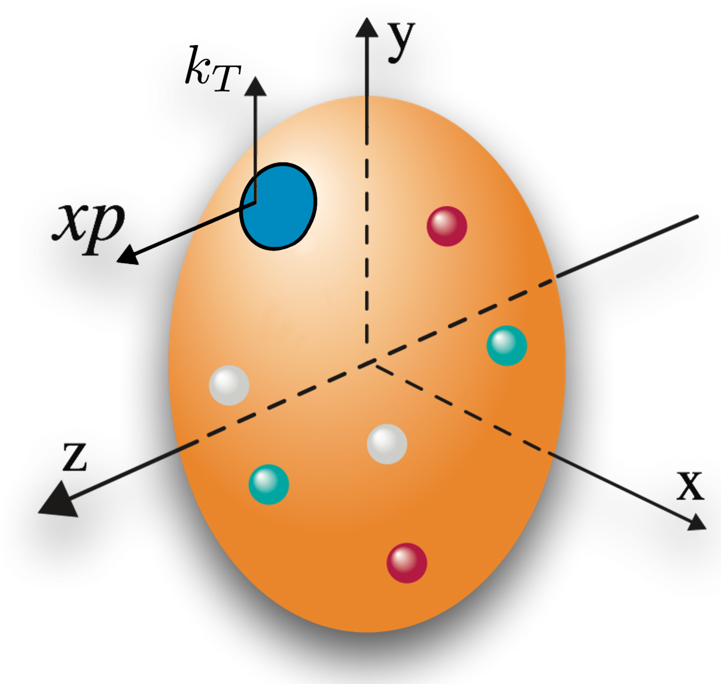
- Match isovector (u-d quark combination) momentum fraction to low-energy constants of effective field theory, extrapolate to physical quark masses
- Include into nNNPDF global fits of experimental lepton-nucleus scattering data

Blue → Purple:
Improvement using theory constraints

[NPLQCD PRL 126, 202001 (2021) [2009.05522]]

Parton physics from Lattice QCD

Understanding the quark and gluon structure of matter



Three-dimensional partonic structure of the proton

PDF
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Fourier transform
$\xi = 0$

GPD
$H_{q/H}(x, \xi, \Delta_T^2)$

Community overview:
Constantinou et al.,
"2020 PDFLattice Report"
arXiv:2006.08636

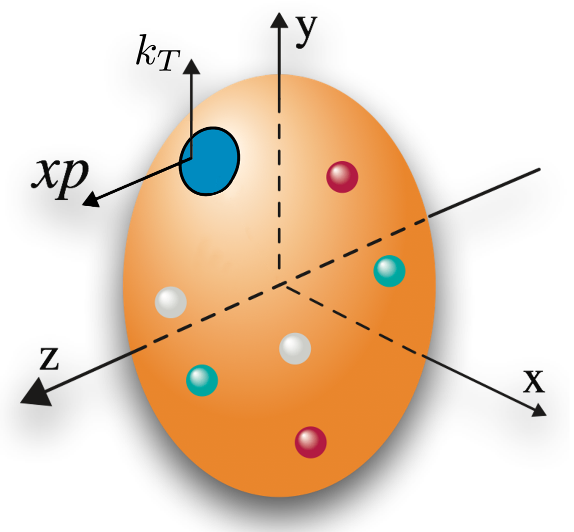
Collins-Soper evolution kernel

- More detailed picture of nucleon including transverse structure

PDF
$f_{q/P}(x)$
longitudinal

→

TMD
$f_{q/P}(x, k_T)$
+ transverse



Collins-Soper Evolution Kernel

$$\gamma_\zeta^q(\mu, b_T) = \zeta \frac{d}{d\zeta} \ln f_q(x, \vec{b}_T, \mu, \zeta)$$

- Governs TMD evolution
- Needed to match quasi-TMD (lattice QCD) to physical TMD

- Perturbative at short distances $\mu, b_T^{-1} \gg \Lambda_{\mathrm{QCD}}$
- Non-perturbative for $b_T^{-1} \lesssim \Lambda_{\mathrm{QCD}}$ → lattice QCD?
- Independent of hadron

Collins-Soper evolution kernel

Estimates of size of nonperturbative contributions to CS kernel

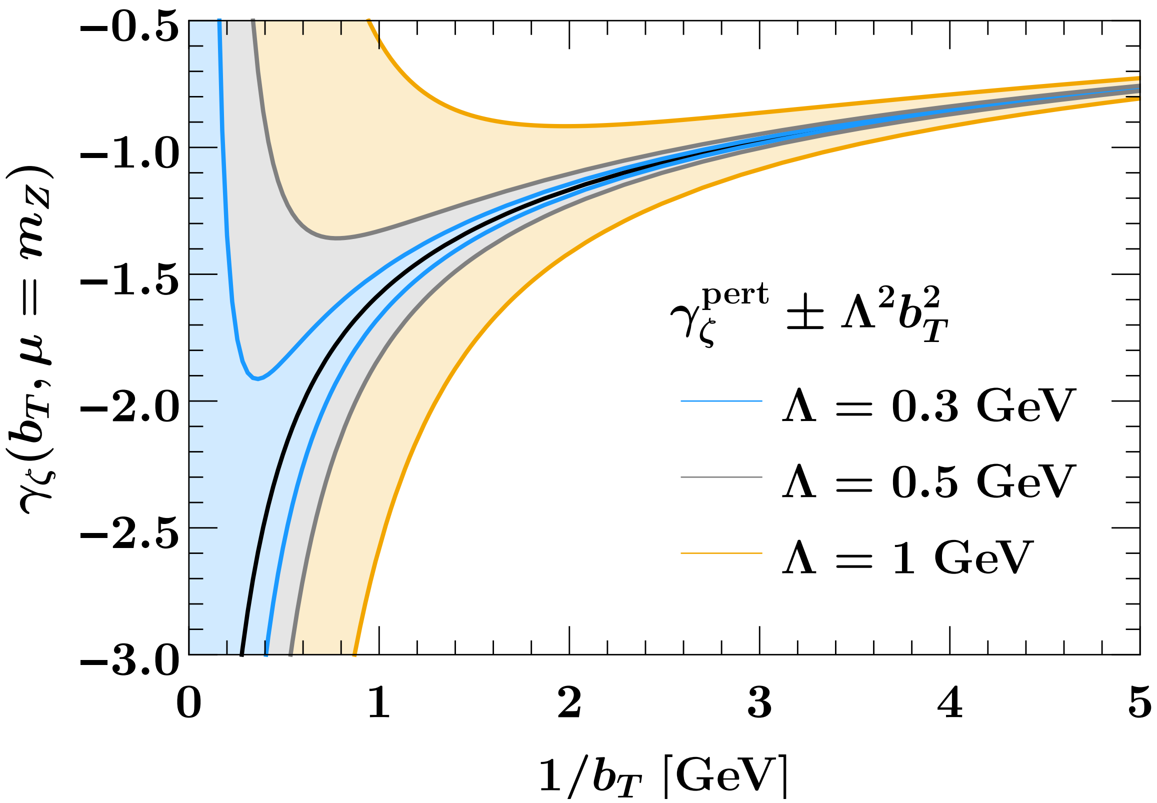


Figure: Iain Stewart

- Large uncertainties in estimates of the size of nonperturbative contributions to the CS kernel

CS kernel from phenomenological fits

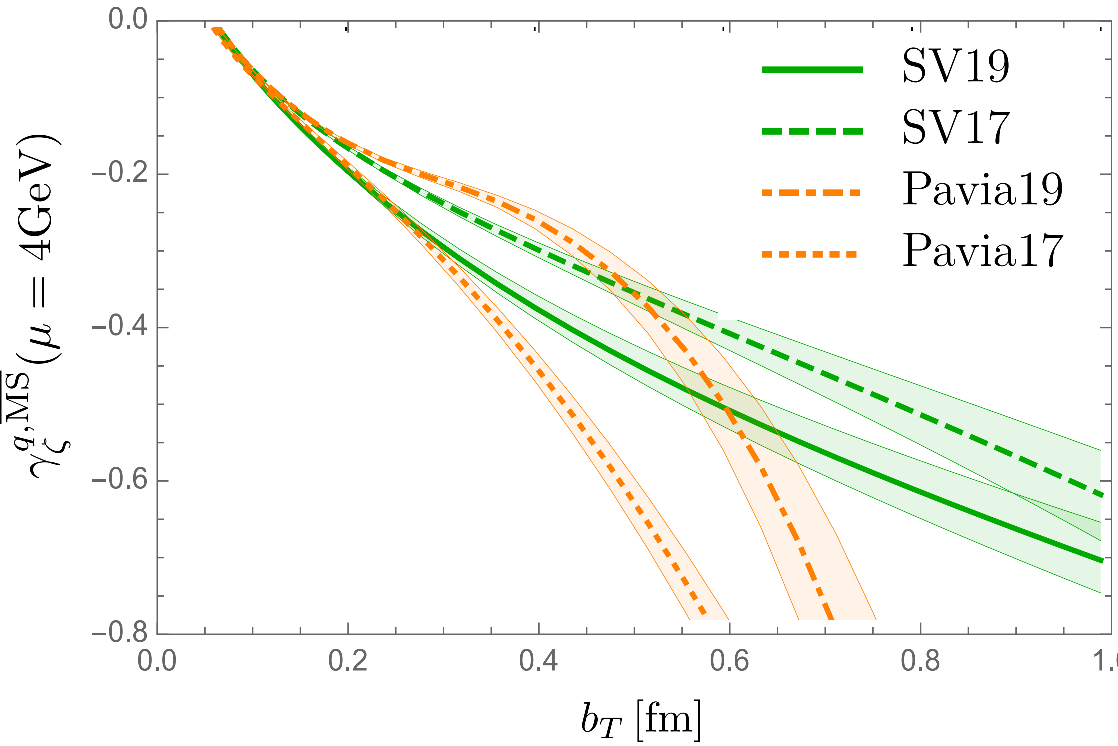
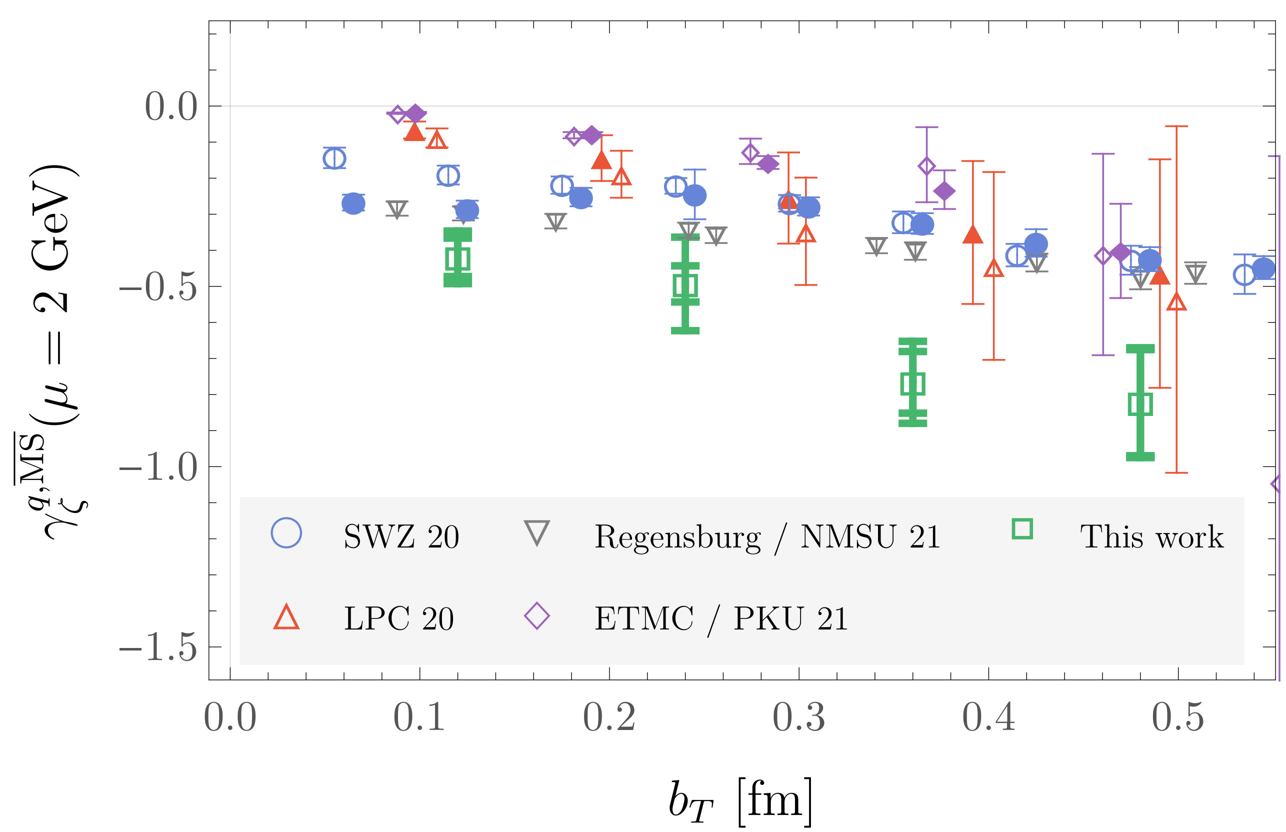


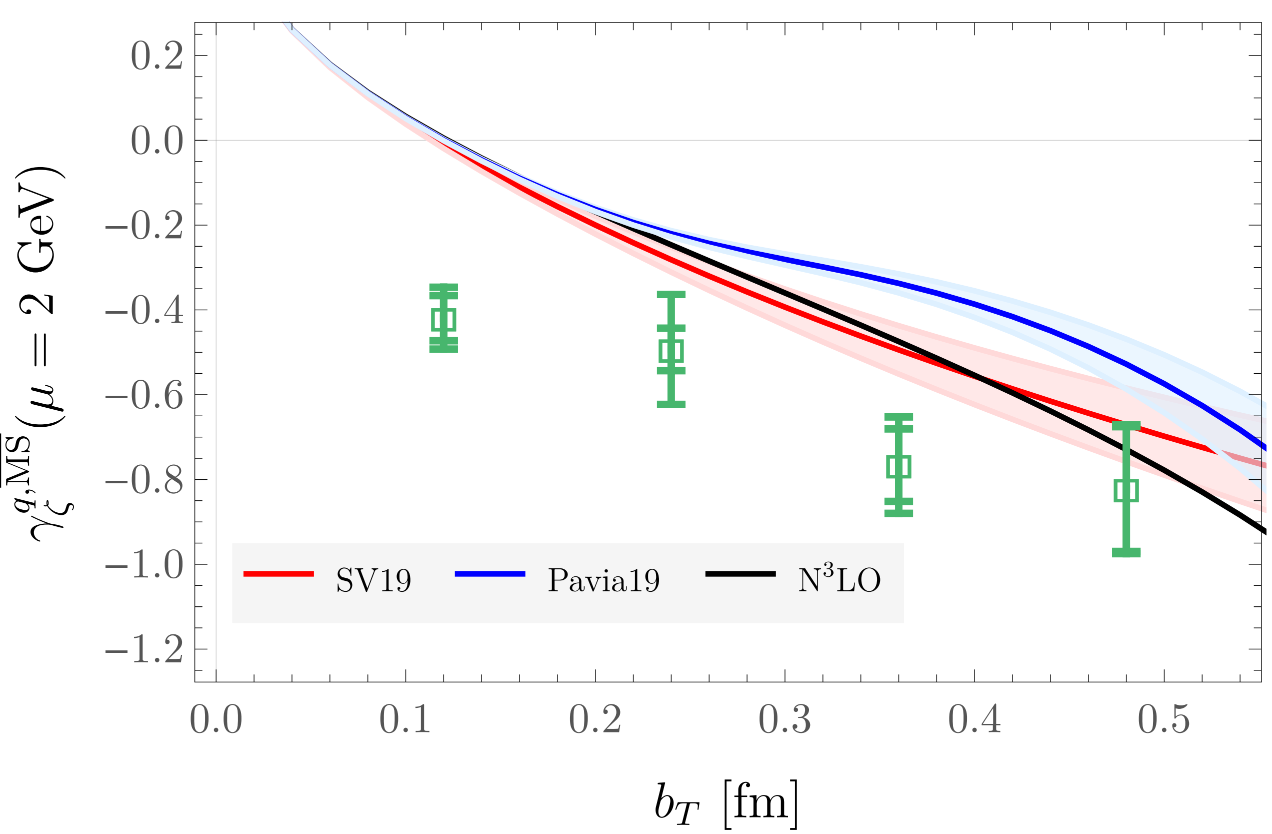
Figure adapted from Vladimirov [2003.02288]
[see also e.g., Kang, Prokudin, Sun,Yuan Phys.Rev.D 93 (2016), Collins, Rogers Phys.Rev.D 91 (2015), Sun, Yuan Phys.Rev.D 88 (2013)]

- LQCD constraints in nonperturbative region with ~10% uncertainties would contribute to model differentiation

Collins Soper kernel from lattice QCD

- First dynamical results 2021
- Currently: Significant spread in results from different LQCD studies from analysis systematics (e.g., NLO matching is important)
- Fully-controlled results at similar precision would already impact phenomenology
- Will require more robust treatment of power corrections





[LPC 2005.14572; Regensburg/NMSU 2103.16991; ETMC/PKU 2106.13027; SWZ 2003.06063, 2107.11930]

Parton physics from Lattice QCD

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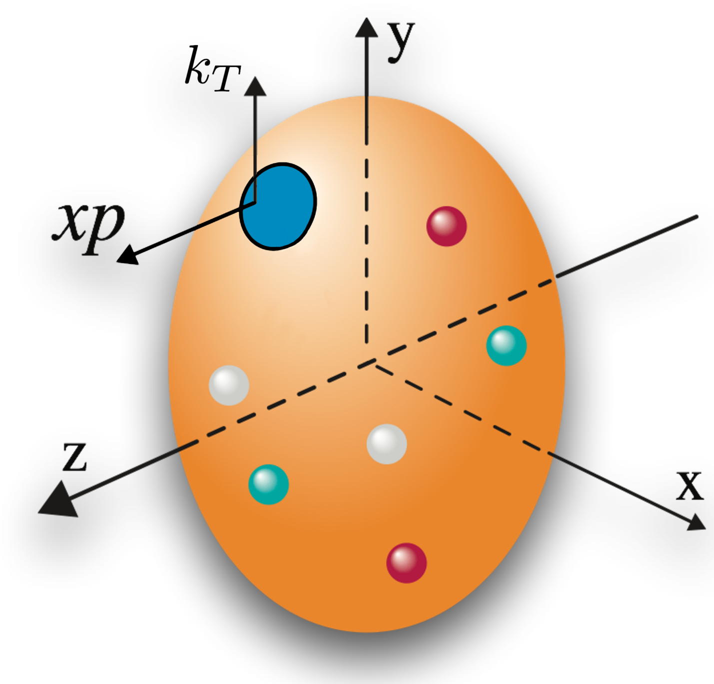

Three-dimensional partonic structure of the proton

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Community overview:
Constantinou et al.,
"2020 PDFLattice Report"
arXiv:2006.08636

Moments of GPDs

- Moments of GPDs define generalised form factors which encode e.g., Energy-Momentum Tensor
- Matrix elements of traceless gluon EMT for spin-half nucleon:

$$\langle p', s' | G^{a}_{\{\mu\alpha} G^{a\alpha}{}_{\nu\}} | p, s \rangle = \bar{U}(p', s') \left(A_g(t)\, \gamma_{\{\mu} P_{\nu\}} + B_g(t) \frac{i\, P_{\{\mu} \sigma_{\nu\}\rho} \Delta^{\rho}}{2M_N} + D_g(t) \frac{\Delta_{\{\mu} \Delta_{\nu\}}}{4M_N} \right) U(p, s)$$

Gluon field-strength tensor

Generalised gluon form factors

$\Delta_\mu = p'_\mu - p_\mu \quad P_\mu = (p_\mu + p'_\mu)/2 \quad t = \Delta^2$

- Sum rules of gluon and quark GFFs in forward limit
  - Momentum fraction $A_a(0) = \langle x \rangle_a$ → $\sum_{a=q,g} A_a(0) = 1$
  - Spin $J_a(t) = \frac{1}{2}(A_a(t) + B_a(t))$ → $\sum_{a=q,g} J_a(0) = \frac{1}{2}$
  - D-terms $D_a(0)$ unknown but equally fundamental!
    - $D_a(t)$ GFFs encodes pressure and shear distributions

D-term from JLab DVCS

Experimental determination of DVCS D-term and extraction of proton pressure distribution
[Burkert, Elouadrhiri, Girod, Nature 557, 396 (2018)]

$$s(r) = -\frac{r}{2}\frac{d}{dr}\frac{1}{r}\frac{d}{dr}\widetilde{D}(r), \quad p(r) = \frac{1}{3}\frac{1}{r^2}\frac{d}{dr}r^2\frac{d}{dr}\widetilde{D}(r)$$

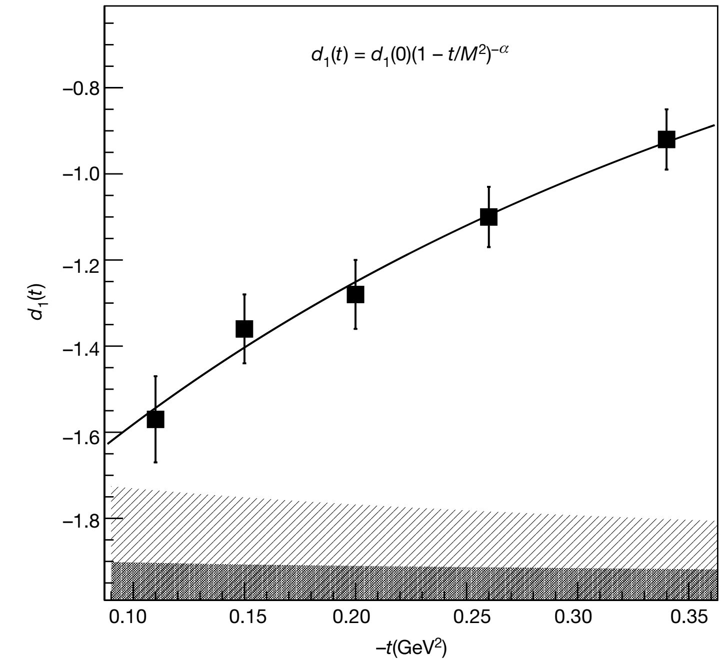
- Peak pressure near centre ~10^{35} Pascal, greater than pressure estimated for neutron stars
- Key assumptions: gluon D-term same as quark term, tripole form factor model, $D_u(t,\mu) = D_d(t,\mu)$

EXP + LQCD

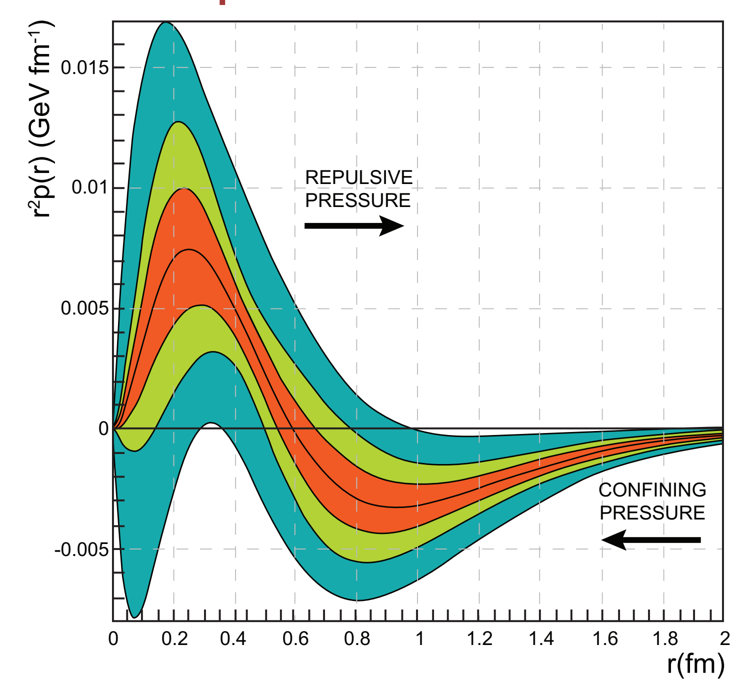
first complete pressure determination

[Shanahan, Detmold PRL 122 072003 (2019)]

DVCS (quark) D-term



Radial pressure distribution



Nucleon D-term GFFs from LQCD

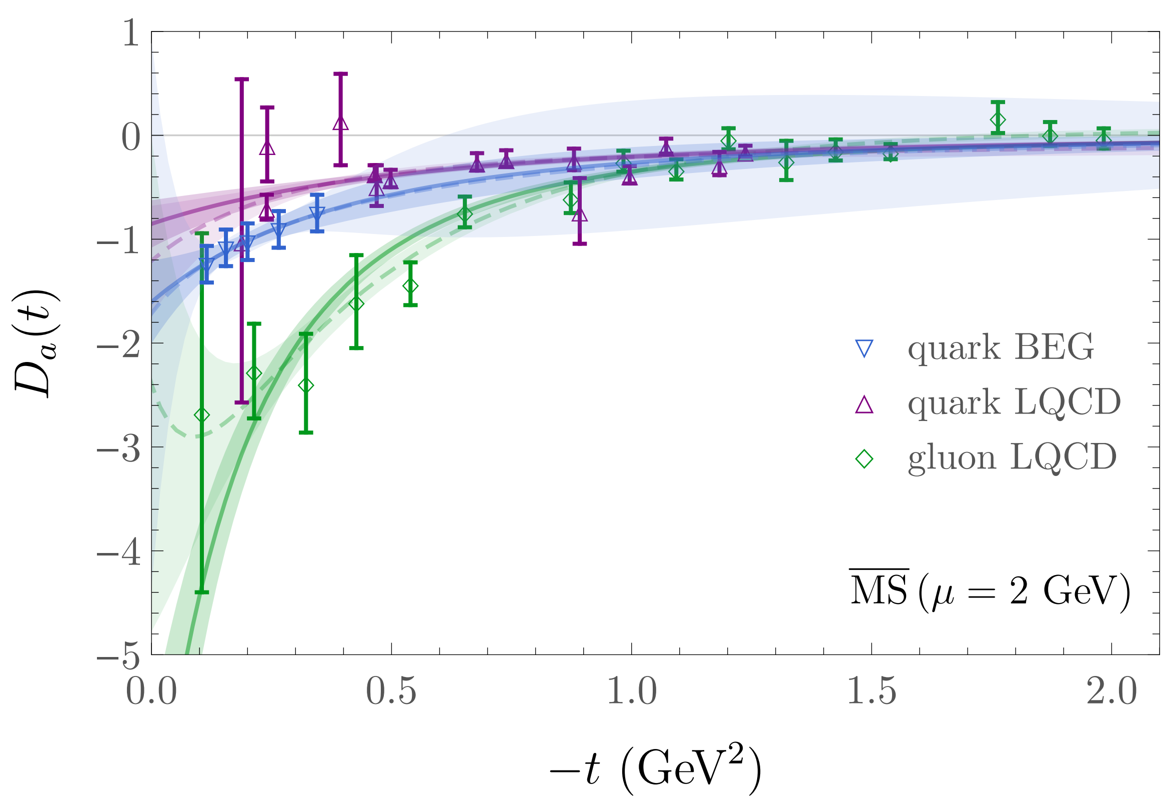
EXP + LQCD

first complete pressure determination

[Shanahan, Detmold PRL 122 072003 (2019)]

Key assumptions in pressure extraction from DVCS

- Gluon D-term same as quark term in magnitude and shape
  Factor of ~2 difference in magnitude, somewhat different t-dependence
- Tripole form factor model
  LQCD results consistent with ansatz, but more general form is less well constrained
- Isovector quark D-term vanishes
  $D_{u-d}(t) \sim 0$ *from other LQCD studies*



Gluon GFFs: Shanahan, Detmold, PRD 99, 014511

Quark GFFs: P. Hägler et al. (LHPC), PRD77, 094502 (2008)

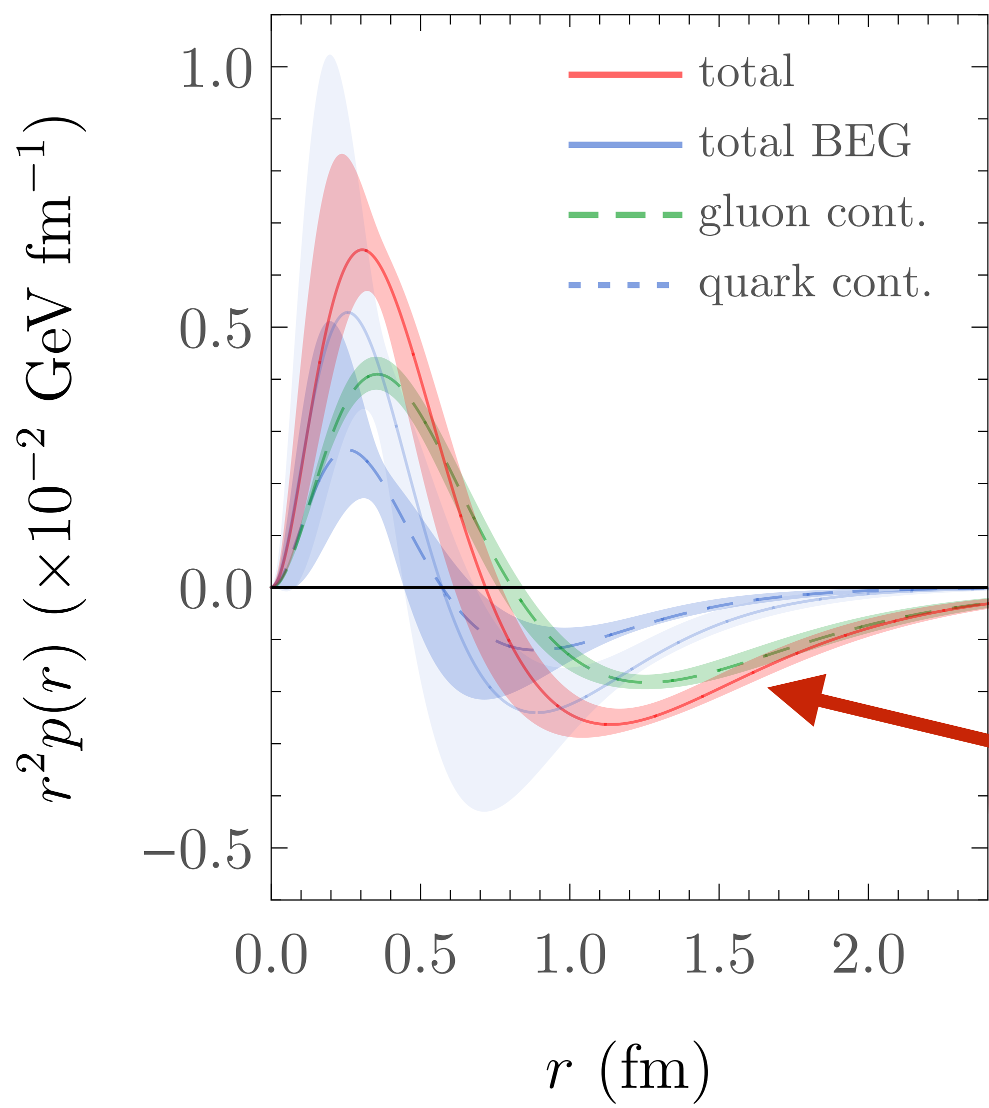
Expt quark GFFs (BEG): Burkert et al, Nature 557, 396 (2018)

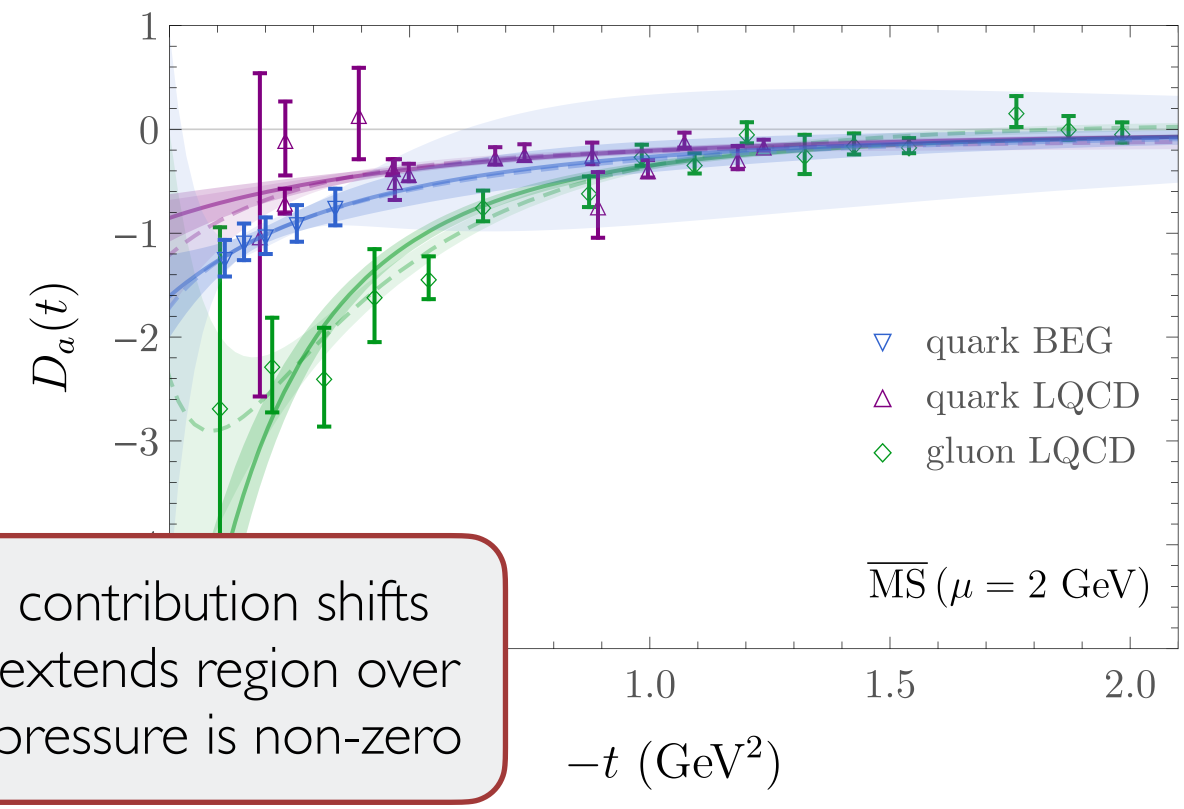
Proton pressure from LQCD

EXP + LQCD

first complete pressure determination

[Shanahan, Detmold PRL 122 072003 (2019)]





gluon contribution shifts peaks, extends region over which pressure is non-zero

Gluon GFFs: Shanahan, Detmold, PRD 99, 014511

Quark GFFs: P. Hägler et al. (LHPC), PRD77, 094502 (2008)

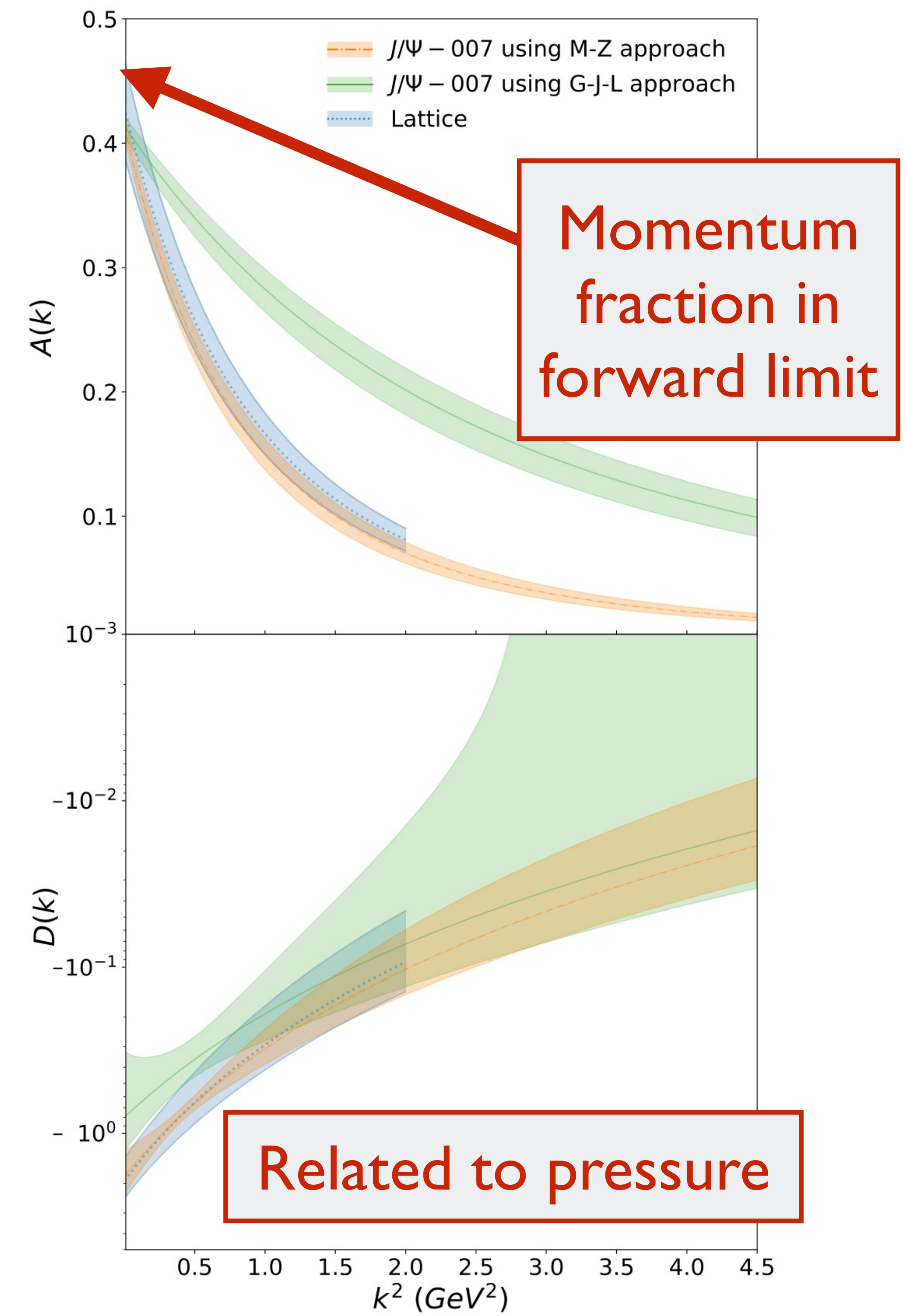
Expt quark GFFs (BEG): Burkert et al, Nature 557, 396 (2018)

Proton pressure from LQCD

First experimental constraint on gluon GFFs this year from J/ψ photoproduction

[Duran et al., arXiv:2207.05212]

- First extraction of nucleon quark and gluon D-term from experimental data is model dependent
- Earlier lattice prediction is crucial to distinguish between models based on
  - Holographic QCD (M-Z)
  - Vector meson dominance (G-J-L)

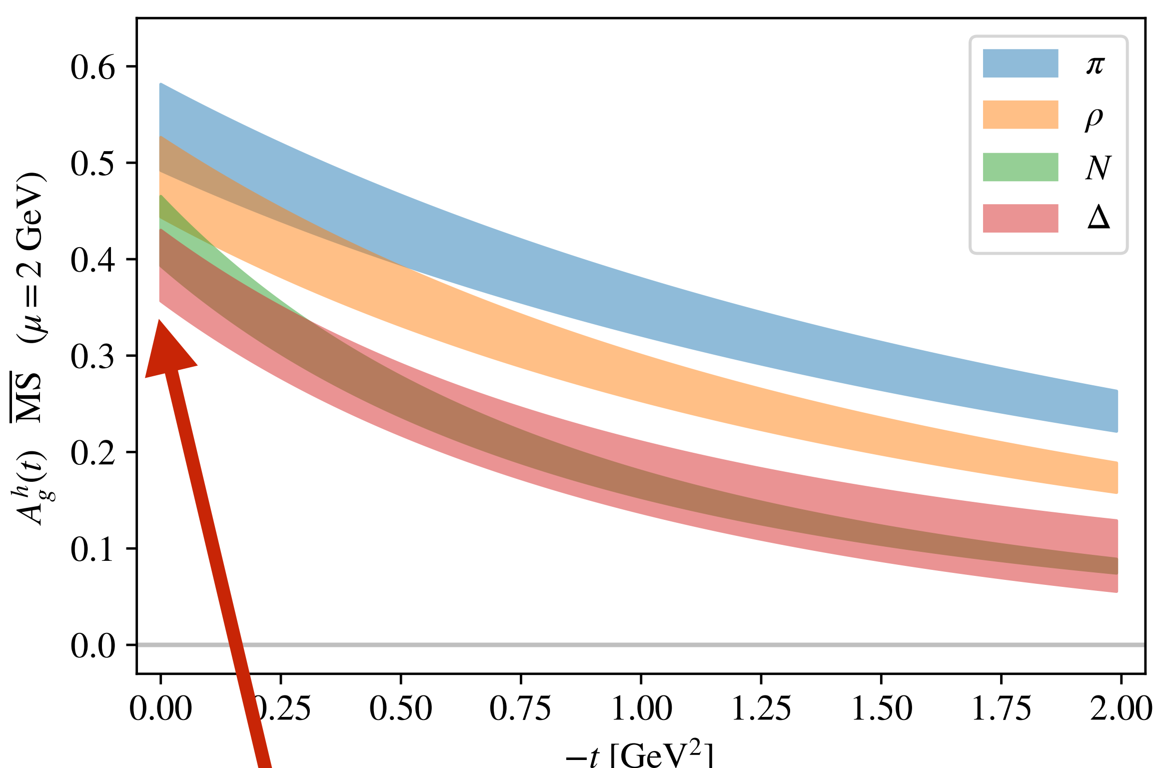


Lattice QCD: Pefkou, Hackett, Shanahan, PRD 105, 054509 (2022)

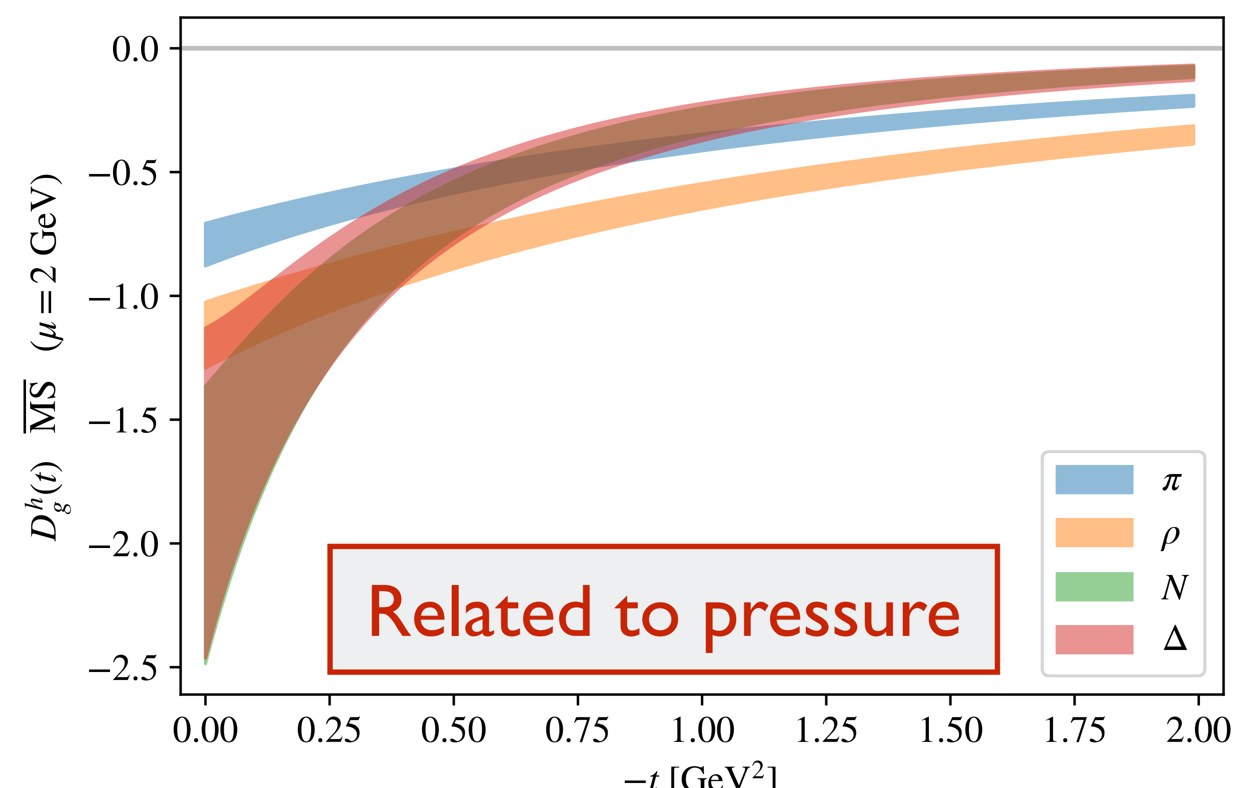
Experiment: Duran et al., arXiv:2207.05212 (2022)

Pressure distribution in other hadrons

- First comparisons of generalised gluon form factors (related to Fourier transforms of pressure distribution) for hadrons of different spin
- Qualitative differences between meson and baryon form factors
- Lattice QCD calculation with unphysically-heavy pion mass



Momentum fraction in forward limit



[Phys.Rev.D 105 (2022) 2107.10368 (2021)]

The gluon structure of the proton

First-principles QCD calculations

➡ QCD benchmarks and predictions ahead of experiment

Cover image from EIC whitepaper arXiv::1212.1701

Parton structure from LQCD

Future colliders will dramatically alter our knowledge of the gluonic structure of hadrons and nuclei

- Work towards a complete 3D picture of parton structure (moments, x-dependence of PDFs, GPDs, TMDs)

- First lattice QCD constraints on nuclear PDFs through their moments
- First lattice QCD calculations of the Collins-Soper kernel for TMD rapidity evolution
- First determination of gluon contributions to shear and pressure distributions in the proton and other hadrons

Lattice QCD calculations in hadrons and light nuclei will complement and extend understanding of fundamental structure of nature

Image Credit: 2018 EIC User's Group Meeting