Clean Time Crystals in Kicked Lieb-Liniger Model

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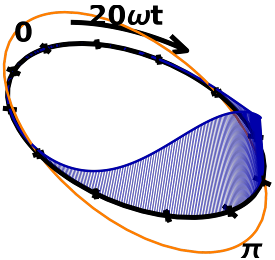
Clean Time Crystals in Kicked Lieb-Liniger Model

- **Why?**
  - Why Time Crystal?
  - Why Lieb-Liniger Model?
  - Why Periodically Kicked?
- **How?**
- **Results**

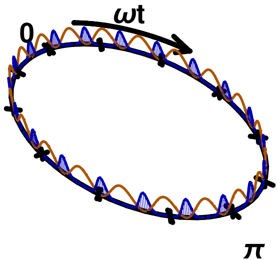
Clean Time Crystal

Closed quantum periodically driven system:

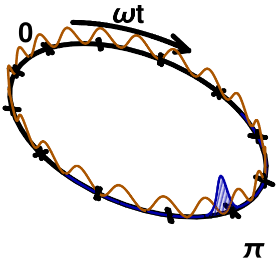
$$\widehat{H}(t) = \widehat{H}(t+T)$$



Quantum resonant states

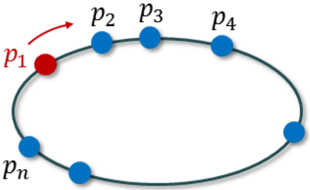


High order resonance



Spontaneous Symmetry Breaking

Lieb-Liniger Model



$$\hat{H} = \int_0^L dx \left[\psi^\dagger(x) \left(-\frac{\hbar \nabla^2}{2m} \right) \psi(x) + c\psi^\dagger(x)\psi^\dagger(x)\psi(x)\psi(x) \right]$$

For N particles:

$$H = -\sum_{j=1}^{N} \partial^2/\partial x_j^2 + 2c \sum_{1 \le i;j \le N} \delta(x_i - x_j)$$

with interaction strength $c = g_0(N-1)$.

Lieb-Liniger Model - It has analytical solutions!

$$\sum_{\pi} \mathsf{sign}(\pi) \exp\left(i \sum_{i=1}^{N} z_n \lambda_{\pi_n} \right) \prod_{j>k} (\lambda_{\pi_j} - \lambda_{\pi_k} - ic),$$

with λ being solutions of Bethe Equations.

$$\exp(i\lambda_j L) = \prod_{i \neq j}^{N} \frac{\lambda_j - \lambda_i + ic}{\lambda_j - \lambda_i - ic}$$

Lieb-Liniger

Integrable model:

$$P = \sum_{i=1}^{N} \lambda_i$$

$$E = \sum_{i=1}^{N} \lambda_i^2$$

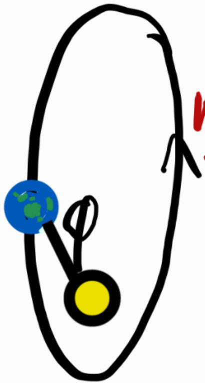
But also we realised that we can evaluate probability density and any potential terms:

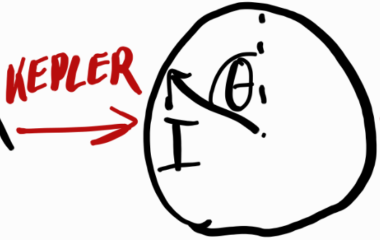
$$E_n = \langle \lambda | \sum_{i=1}^{N} \exp(inx_i) | \mu \rangle$$

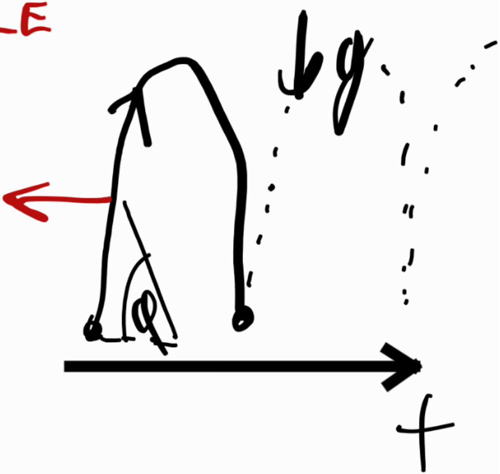
$$\rho(x) = \sum_{i=1}^{N} \exp(inx_i) \langle \lambda | E_n | \mu \rangle$$

in $O(N^4)$ instead of usual $O((N!)^2)$ (or smart $O((3^n))$.

ACTION-ANGLE



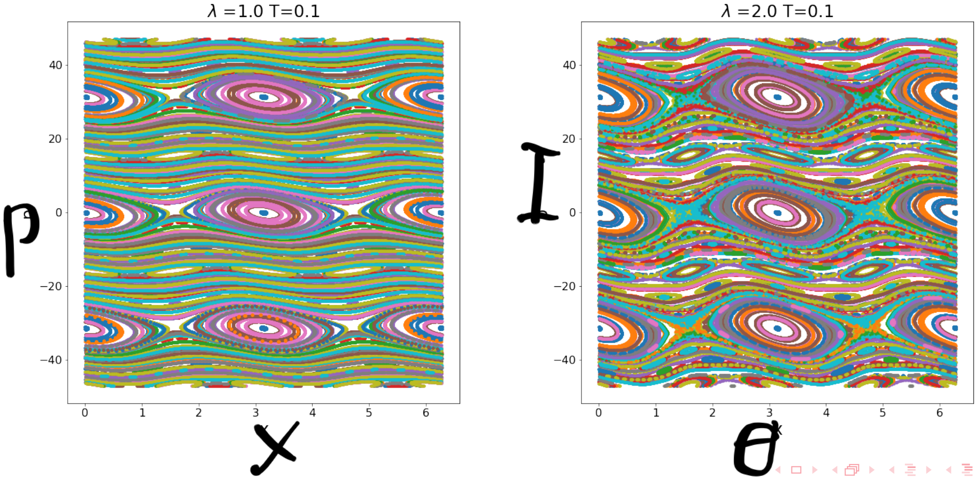




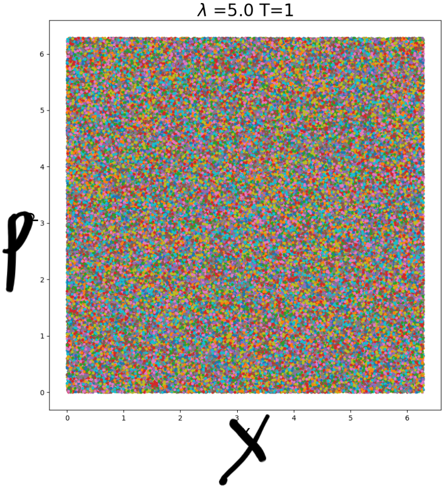
$$\begin{cases} \dot{I} = 0 \\ \dot{\theta} = \dfrac{\partial H}{\partial I} \end{cases}$$

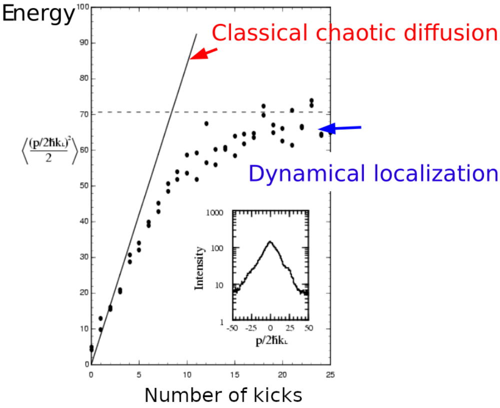
Kicked Rotator

$$H = -\sum_{j=1}^{N} \partial^2/\partial x_j^2 + F\sum_{j=1}^{N}\left(cos(2x_j)\sum_{m}\delta(t-mT)\right)$$



Kicked Rsotator





M. Raizen et al, PRL 75, 4598 (1995)

Interacting Kicked Rotator

$$H = -\sum_{j=1}^{N} \partial^2/\partial x_j^2 + 2c \sum_{1\leq i;j\leq N} \delta(x_i - x_j) + F \sum_{j=1}^{N} \left(cos(2x_j) \sum_m \delta(t - mT) \right)$$

$$H = H_{LL} + H_{kick}$$

Kicked rotor means single period propagator is very simple $t_{kick} + \epsilon^+ \rightarrow T + t_{kick} + \epsilon^+$:

$$U_T = \exp(-iH_{LL}T)\exp(-iH_{kick}).$$

We can obtain full Floquet solutions as eigenstates of:

$$-i \log(U_T)$$

Interacting kicked rotor

$$H = -\sum_{j=1}^{N} \partial^2/\partial x_j^2 + 2c \sum_{1\le i;j\le N} \delta(x_i - x_j) + F \sum_{j=1}^{N} \left(cos(2x_j) \sum_m \delta(t - mT) \right)$$

Different approaches:

$$H = H_{LL} + H_{kick}$$
$$H = H_p + H_x$$
$$H = H_{free} + H_{interaction}$$

Split Operator Method

$$H = -\sum_{j=1}^{N} \partial^2/\partial x_j^2 + 2c \sum_{1 \leq i;j \leq N} \delta(x_i - x_j) + F \sum_{j=1}^{N} \left(cos(2x_j) \sum_{m} \delta(t - mT) \right)$$

$$H = H_p + H_x$$

$$U(t + \Delta t) = \exp(-I\Delta t(H_x + H_p)) = \exp(-I\Delta t H_x)\exp(-I\Delta t H_p) + O(\Delta t^2)$$

Used because unitary transform from position to momentum representation:

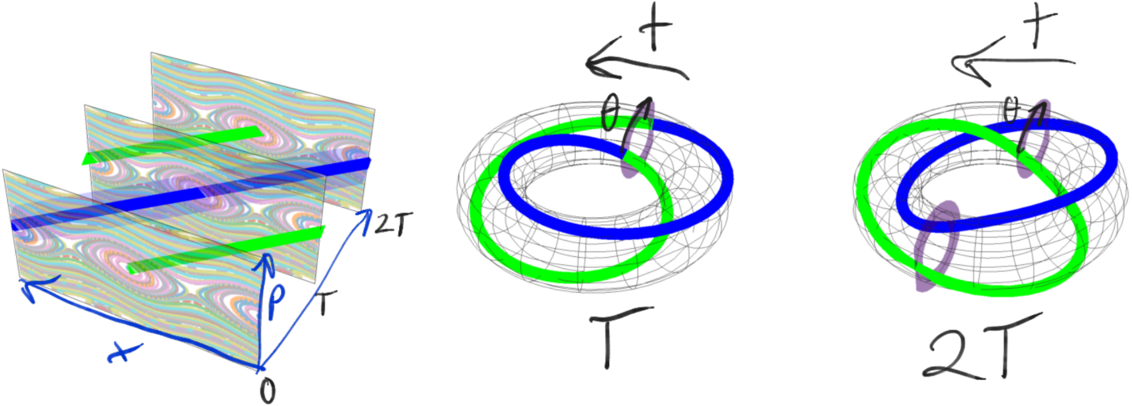
$$|x\rangle\langle p|\psi\rangle$$

is a Discrete Fourier Transform with computational complexity:

$$O(NlogN) < O(N^2),$$

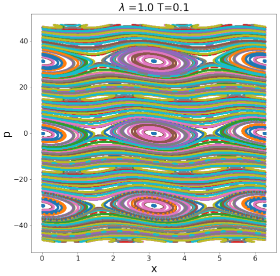
where N is the size of Hilbert space.

Poincare sections $H = H_{free}$



Torus should be deformed as $dI \wedge d\theta$ should be constant.

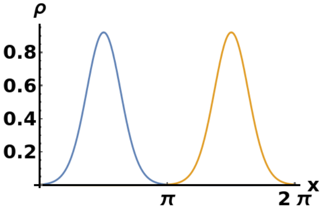
Quantum free: $H = H_{free}$



$$U_T = \exp(i\lambda cos(2\hat{x})) \cdot \exp(iT\hat{n}^2/2)$$

First right-moving band expansion (rotating frame):

$$H_{eff} = \frac{(n - \pi/T)^2}{2} + \frac{\lambda}{T} cos(2x).$$



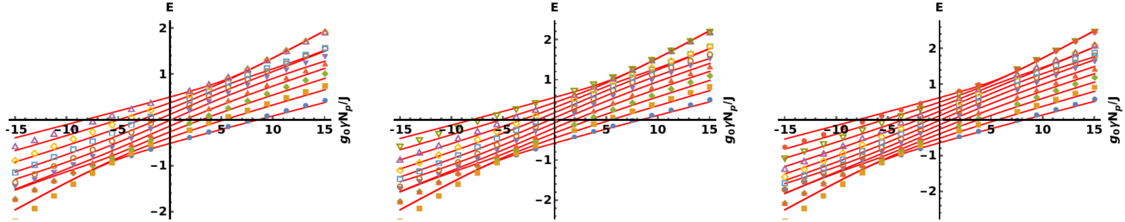
Two mode: $H = H_{free}^{truncated} + H_{interaction}$

$$J(a_1 a_2^\dagger + \text{h.c.}) + g_0\gamma \left(a_1^\dagger a_1^\dagger a_1 a_1 + a_2^\dagger a_2^\dagger a_2 a_2\right),$$

with:

$$J = \langle w_1 | H | w_2 \rangle \qquad \gamma = \frac{1}{T}\int\int dt dx |w_1|^4$$

w_1 - single particle localized state in well 1(left).



Lipkin-Meshkov-Glick

M mode system with *infinite* number of particles becomes a classical theory of a single particle on $SU(M)$.

F. Trimborn, D. Witthaut & H. J. Korsch, Phys. Rev. A 79, 013608 (2008)

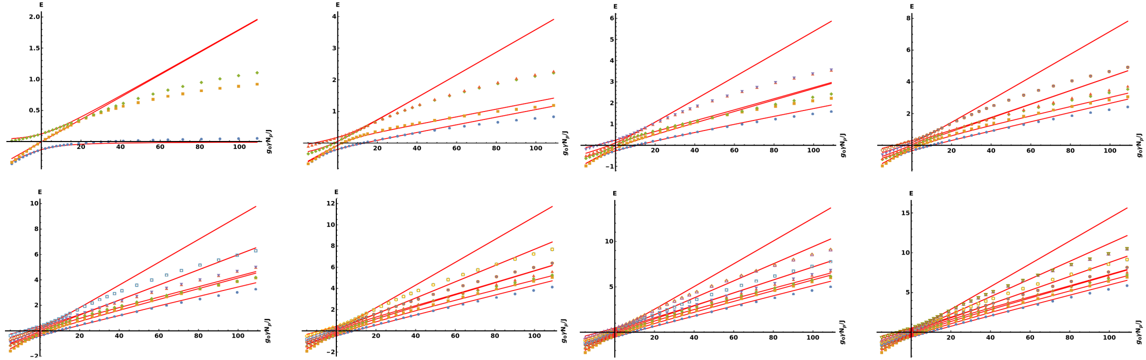
Lipkin-Meshkov-Glick:

Two mode BEC with a lot of particles it is equivalent to spin chain $(SU(2))$.

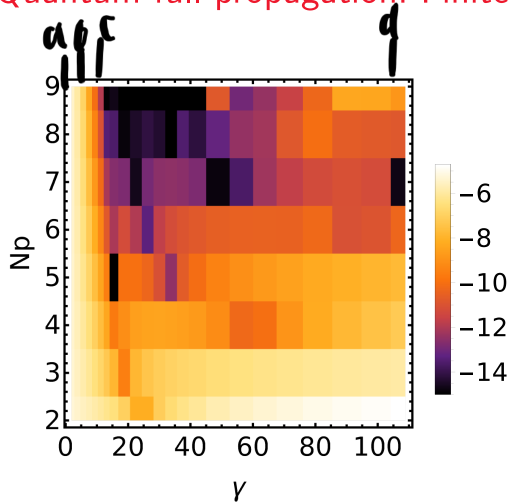
All time crystal systems are within the limit the same.

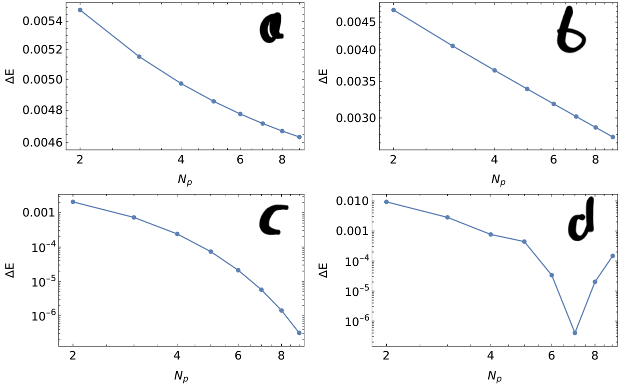
The same methodology of Bethe-Ansatz is used to solve fully integrable magnetism - Heisenberg models.

Eigenstates of Full Floquet Unitary: $H_{LL} + H_{kick}$

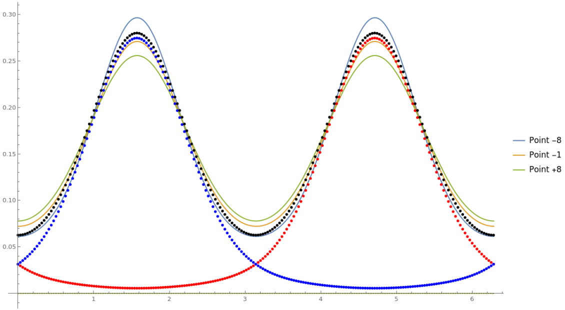


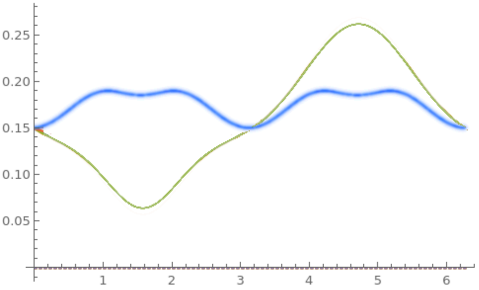
Quantum full propagation: Finite-size scaling





Quantum full propagation: Probability density





Conclusions

We have reviewed different approaches to time crystal like calculations:

$$H = -\sum_{j=1}^{N} \partial^2/\partial x_j^2 + 2c \sum_{1\le i;j\le N} \delta(x_i - x_j) + F\sum_{j=1}^{N}\left(cos(2x_j)\sum_m \delta(t-mT)\right)$$

- Almost time independent propagation:
  $H = H_{LL} + H_{kick}$
- Split operator:
  $H = H_p + H_x$
- Small interactions:
  $H = H_{free} + H_{interaction}$

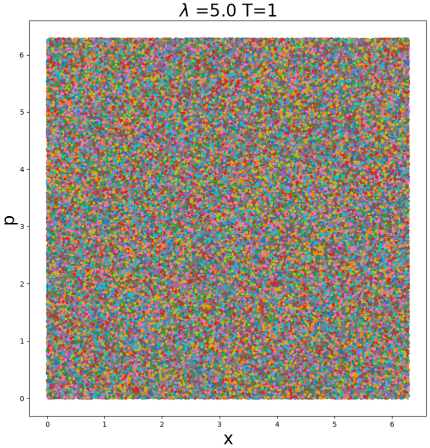
And we can be excited, that:

$H = H_{LL} + H_{kick}$

is an exact approach to kicked system and it is possible to numerically investigate $O(K^2 N_p^4)$.

Thank you for your attention!

Sidenote: Quantum chaos: $H = H_{free}$



$$H = \frac{p^2}{2} + \lambda cos(2x) \sum_{n=-\infty}^{\infty} \delta(t - nT)$$

then $I_n = Tp_n$ and $K = \lambda T$:

$$\begin{cases} I_{n+1} & = I_n + 2K sin(x+n) \\ x_{n+1} & = x_n + I(n+1) \end{cases}$$

So:

$$\langle p_{n+1}^2 \rangle \simeq \langle p_n^2 \rangle + 4\lambda^2 \langle \sin^2 x_n \rangle \simeq \langle p_n^2 \rangle + 2\lambda^2,$$

motion in momentum space is diffusive ($\langle p^2 \rangle$ increases linearly with time), with diffusion constant: $D = \frac{2\lambda^2}{T}$

Sidenote: Magnus expansion

For a time-periodic quantum Hamiltonian $H(t+T) = H(t)$, the **Floquet theorem**:

$$U(t,0) = P(t) \exp\left[-iH_F t\right], \tag{1}$$

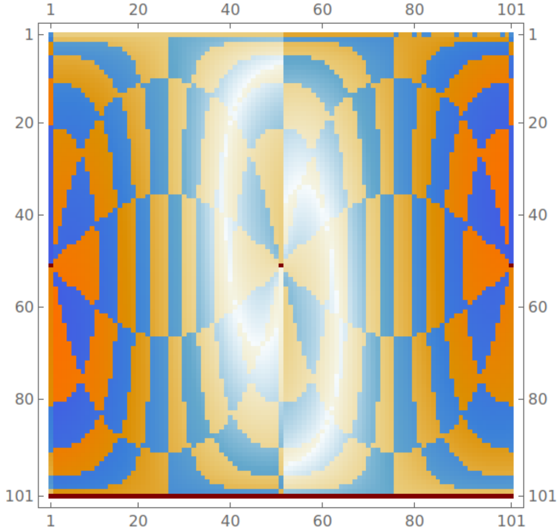
where H_F is a time-independent Floquet Hamiltonian, while $P(t)$ is a unitary time-periodic operator which fulfills $P(t+T) = P(t)$ and $P(0) = 1$.

$$H_F = -\frac{1}{iT} \log\left[U(T,0)\right] \tag{2}$$

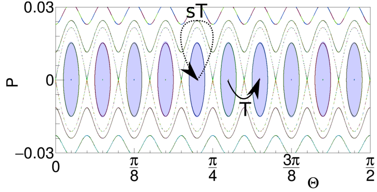
Usually it is not easy to calculate the right hand side of above, however, one can use the Magnus expansion of H_F in powers of $T/(2\pi) = 1/\omega$:

$$\begin{aligned} H_F^{(0)} &= \frac{1}{T}\int_0^T dt_1 H(t_1), \\ H_F^{(1)} &= \frac{1}{2Ti}\int_0^T dt_1 \int_0^{t_1} dt_2 \left[H(t_1), H(t_2)\right]. \end{aligned}$$

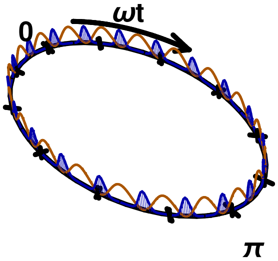
Mercator



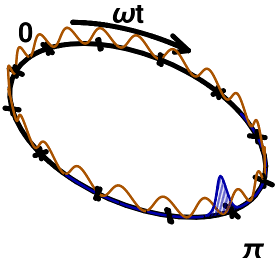
Clean Time Crystal



Classical resonance islands



Quantum resonant states



Spontaneous Symmetry Breaking

Closed quantum periodically driven system:

$$\widehat{H}(t) = \widehat{H}(t+T)$$

The field doing the periodic driving is infinitely intensive. TODO

One time scale T.

Open systems include dissipation and pumping terms adding additional time scales