Australian Institute of Physics Congress 2022
Adelaide Convention Centre

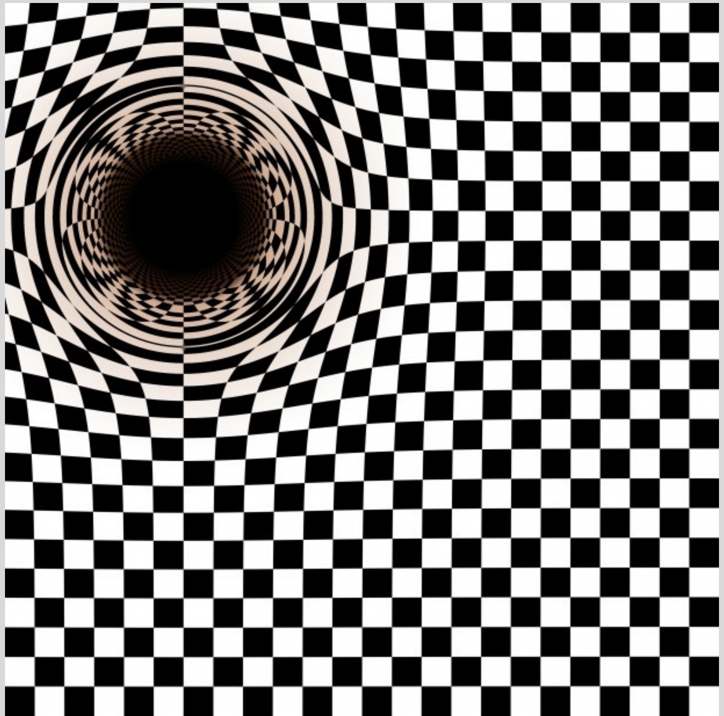
Australian National University

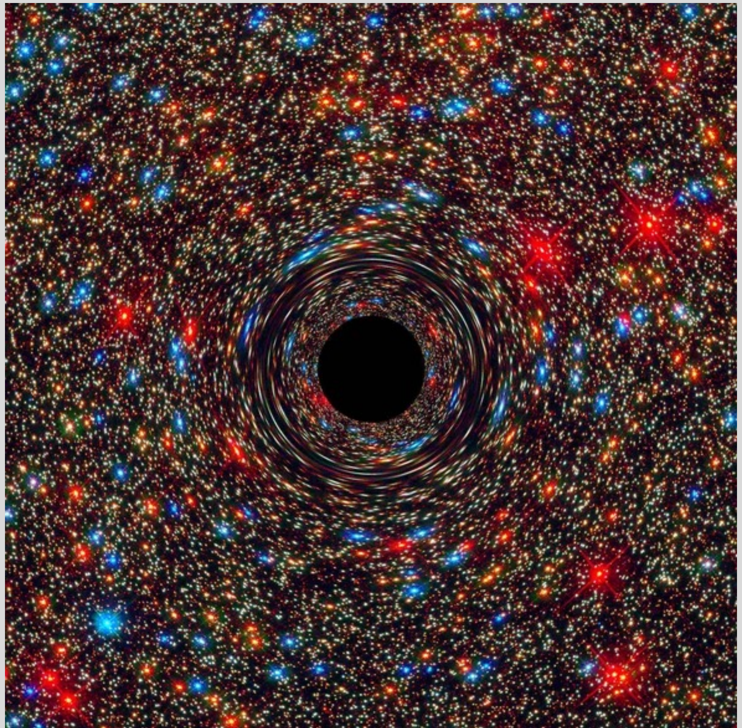
Walter Boas Medal 2022 Lecture

The Fabric of Space-Time

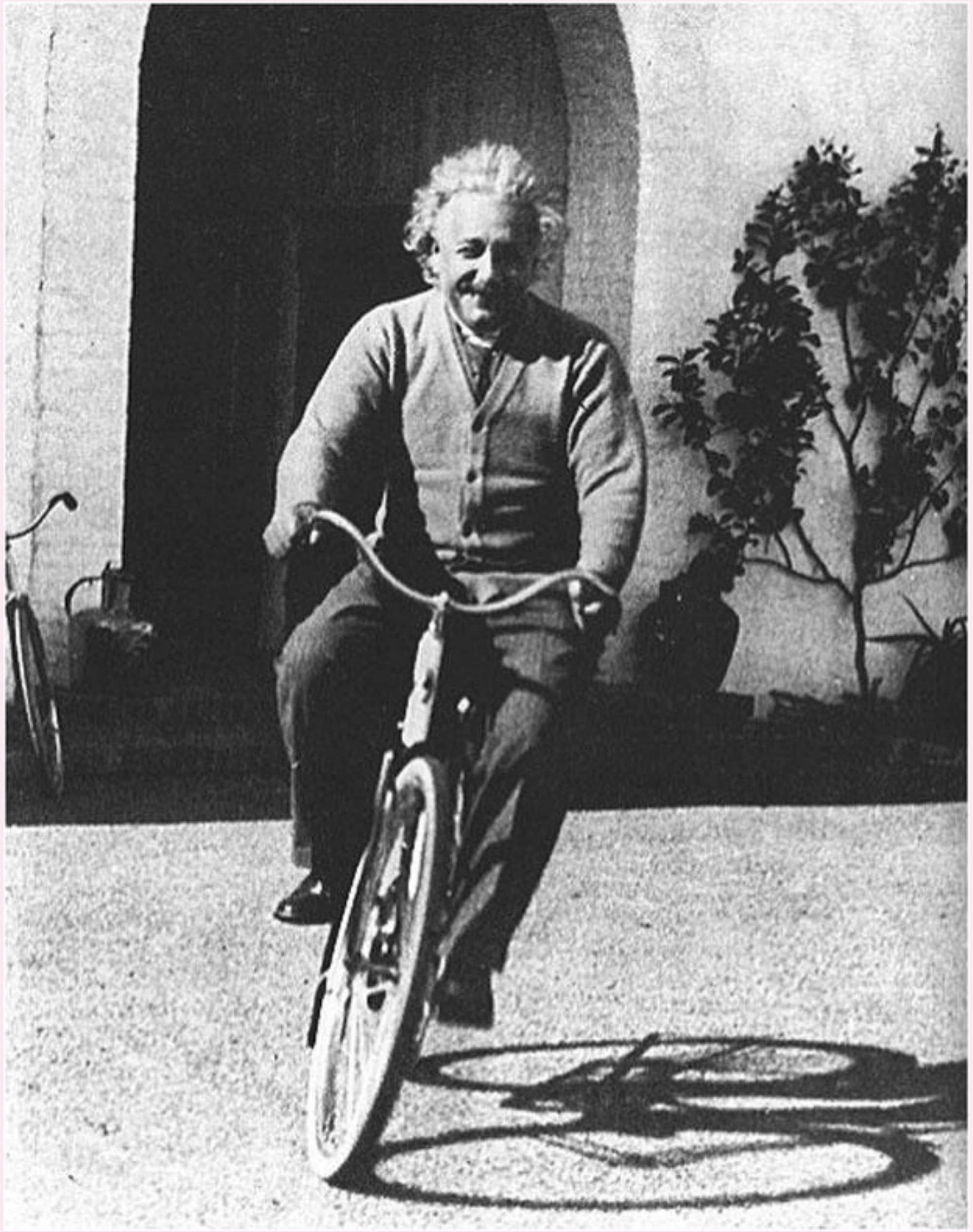
Distinguished Professor Susan M. Scott

Centre for Gravitational Astrophysics, Australian National University





Albert Einstein 1879–1955



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Theoretical Physicist

Photoelectric effect

Brownian motion

Special Relativity

The equivalence of mass and energy $E = mc^2$

General Relativity

General Relativity 1915

a unified description of gravity as a geometric property of space and time – **space-time**

THE EINSTEIN FIELD EQUATION

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} R \, g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

Geometry Mass, Energy

Matter warps space-time

Space-Time $(\mathcal{M}, g)$

A ***space-time*** $(\mathcal{M}, g)$ *is a 4-dimensional manifold* $\mathcal{M}$ *together with a Lorentzian* C^k *metric* g *on* $\mathcal{M}$

A C^k ***metric*** g *on a manifold* $\mathcal{M}$ *is a symmetric, non-degenerate* C^k *tensor field of type* $(0, 2)$ *on* $\mathcal{M}$

Lorentzian metric

$- + + +$

Exact Solutions of the Einstein Field Equations

Find a metric solution of the Einstein field equation corresponding to a physically reasonable form of matter

Isolated gravitating systems

Cosmologies

Karl Schwarzschild 1873–1916

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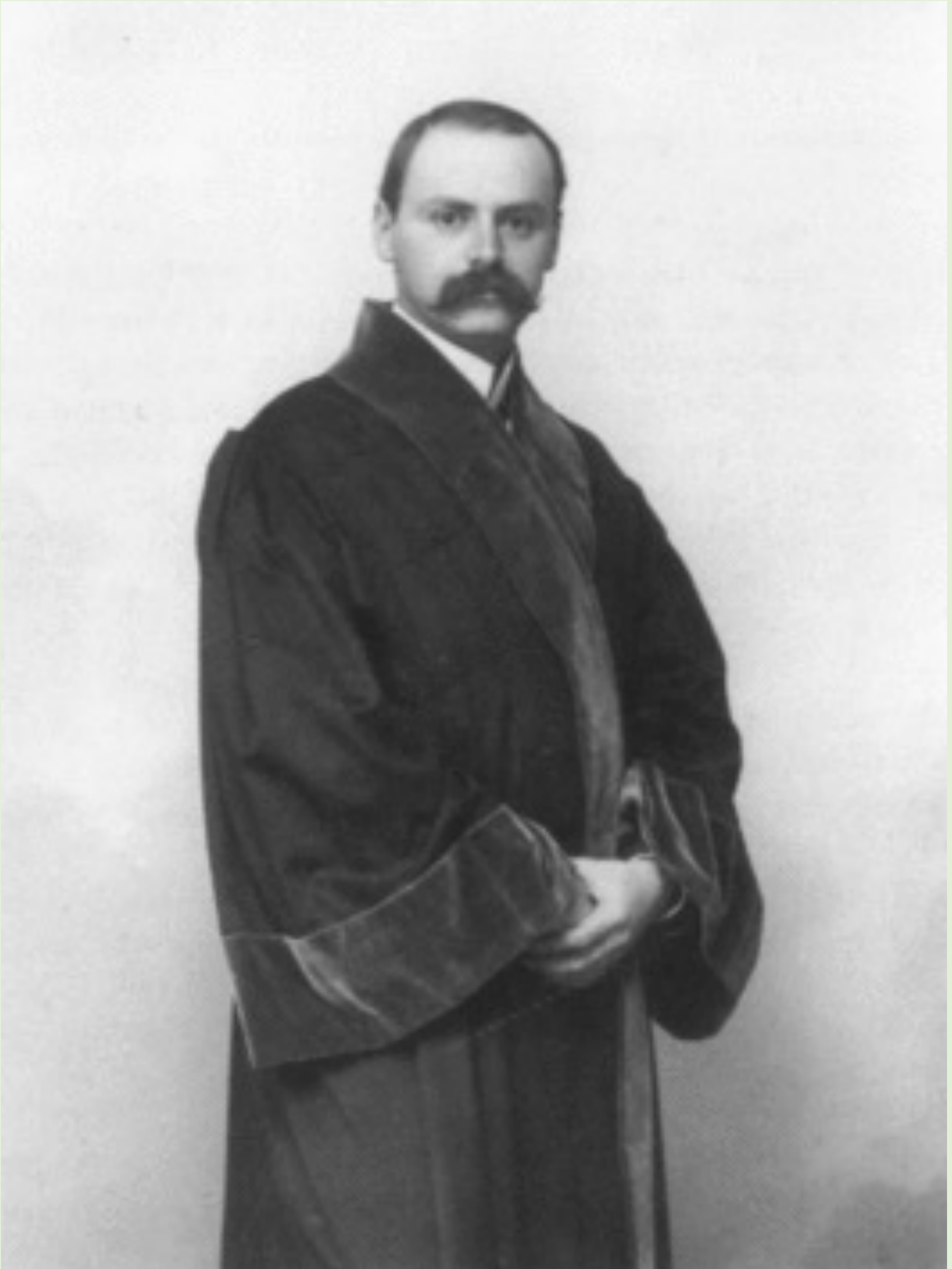
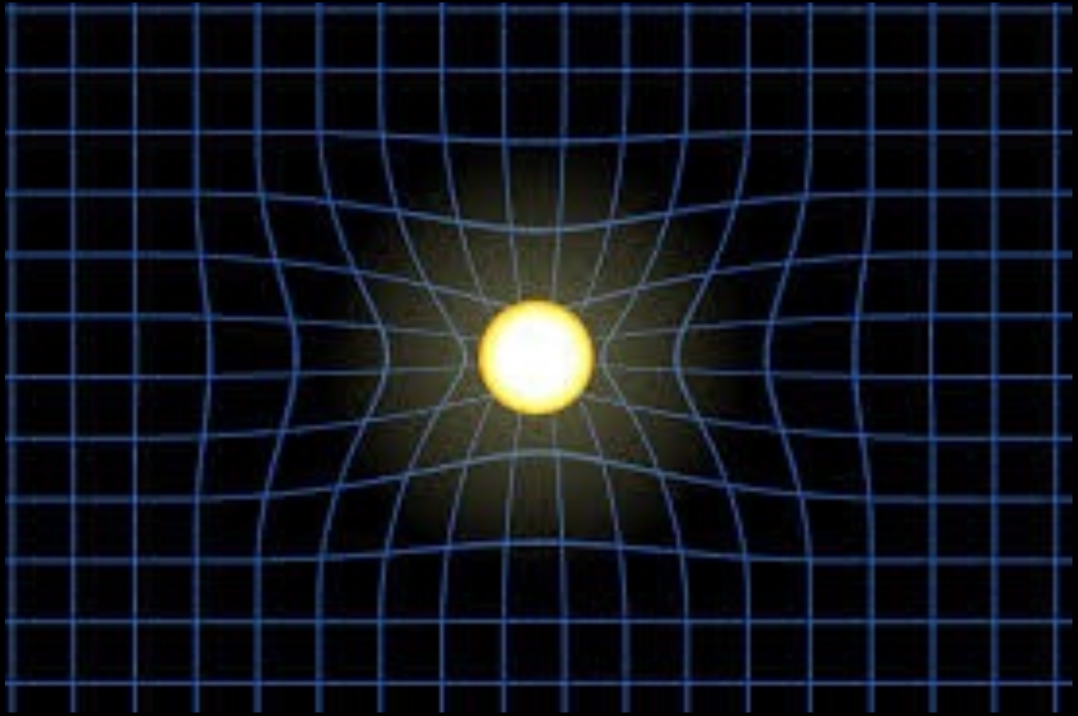
German physicist

He found the first exact solution of the Einstein field equation





Karl found the solution to the Einstein field equation for the gravitational field outside of a non-rotating spherically symmetric star

The Schwarzschild metric

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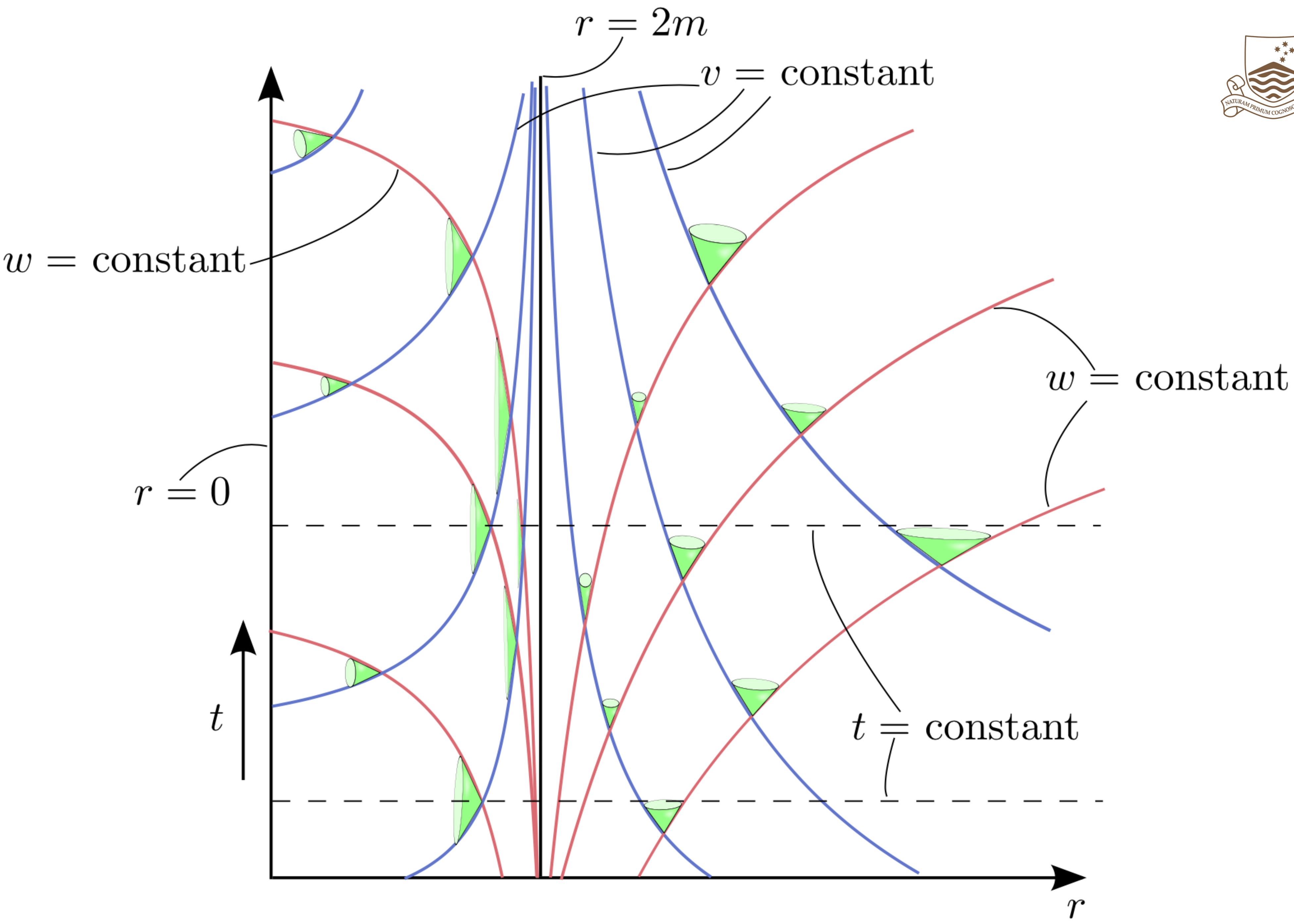
Spherical polar coordinates

$$ds^2 = -\left(1 - \frac{2m}{r}\right) dt^2 + \left(1 - \frac{2m}{r}\right)^{-1} dr^2 + r^2 \left(d\theta^2 + \sin^2\theta \, d\phi^2\right)$$

Problem: $1 - \frac{2m}{r} = 0$ when $r = 2m$

So the metric is singular at $r = 2m$!



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The enigma of radius r = 2m

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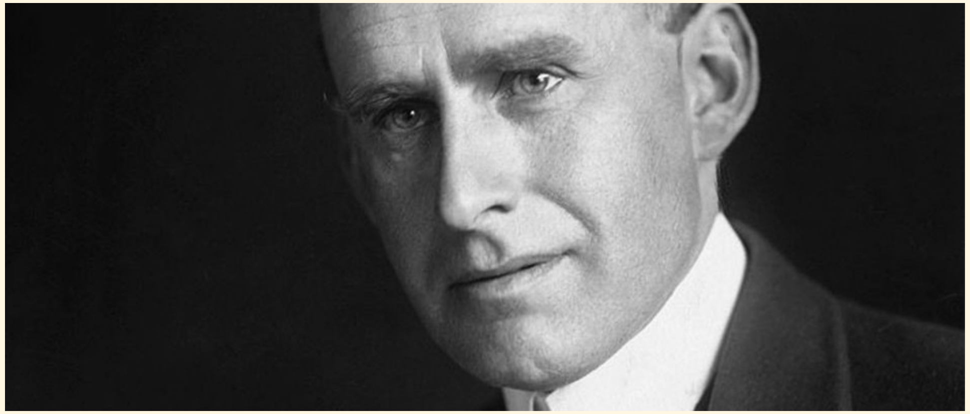
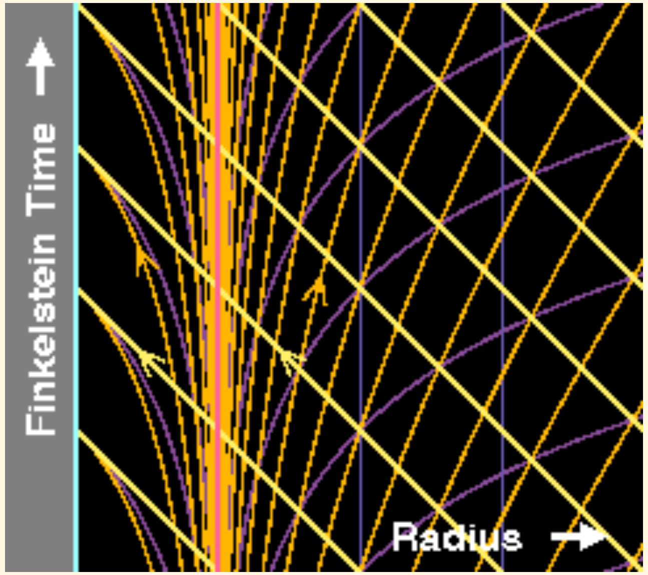
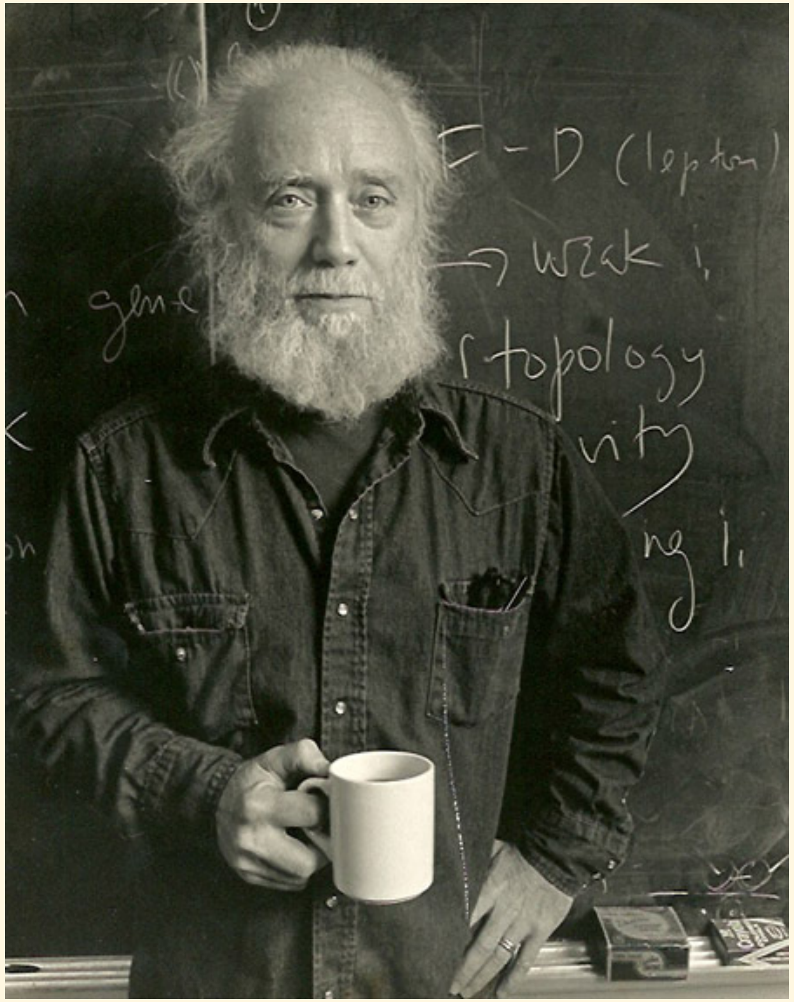
It took from 1915 to 1960 (45 years) for general relativity theorists to fully understand what was happening at radius r = 2m !!!

Arthur Stanley Eddington



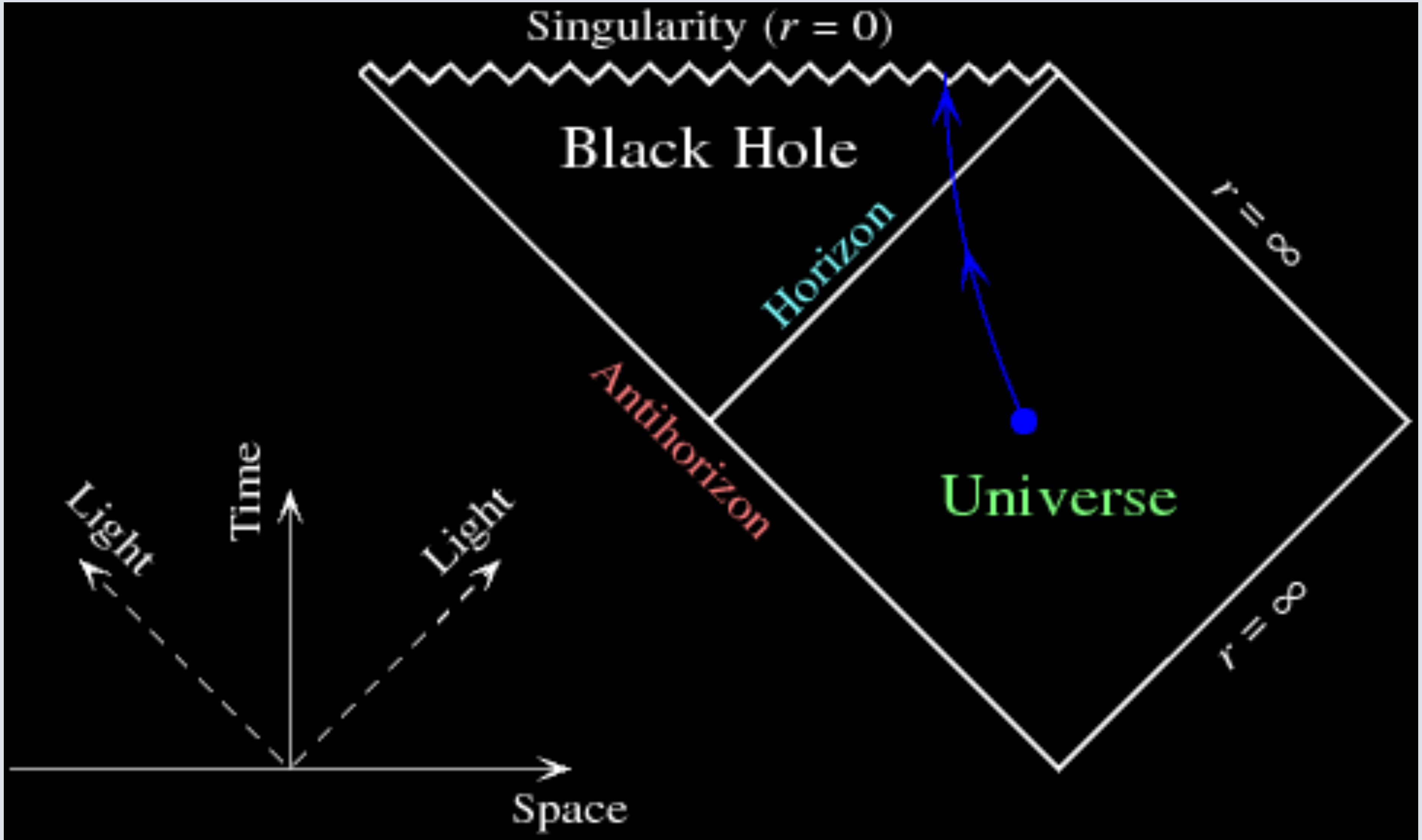


David Ritz Finkelstein

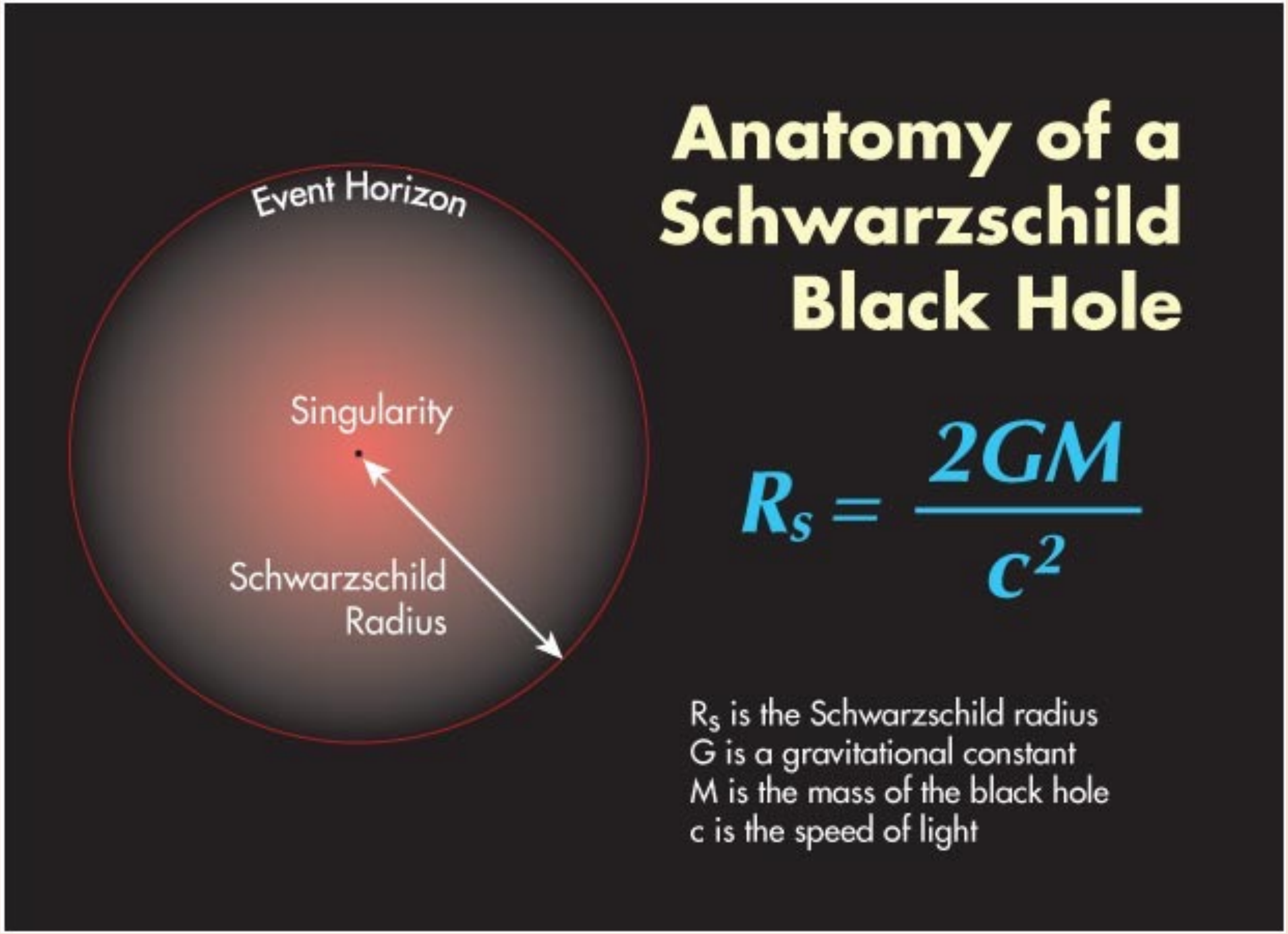


Penrose Diagram for Schwarzschild

Martin Kruskal George Szekeres 1960



A BLACK HOLE



To allow for stellar collapse, consider the Schwarzschild solution to be valid all the way into the centre of the star at $r = 0$ (1958)

THIS IS THE BLACK HOLE SOLUTION

What are singularities? 🤔

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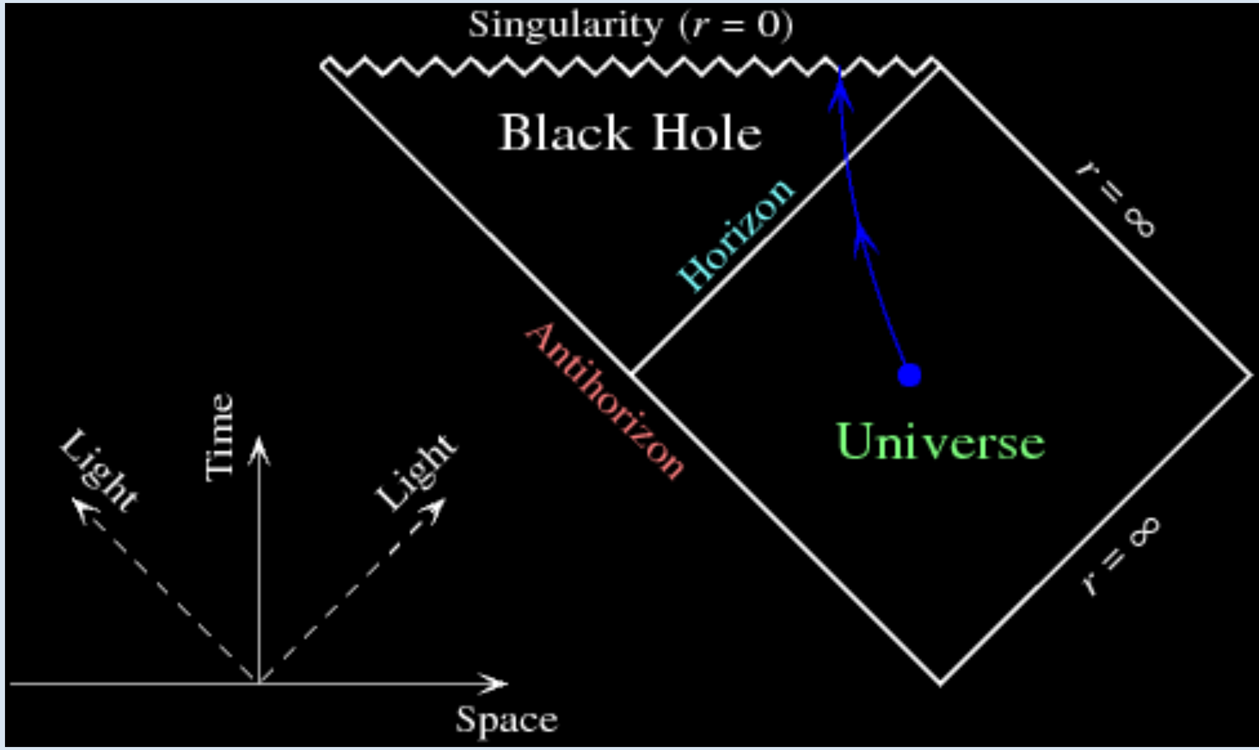
For exact solutions, *"places" on the edge of space-time where things go wrong*

Where is that "place" ??? ***The boundary***

In what way can things go wrong ???

These "places" are only physically important if an observer can reach there in finite proper time



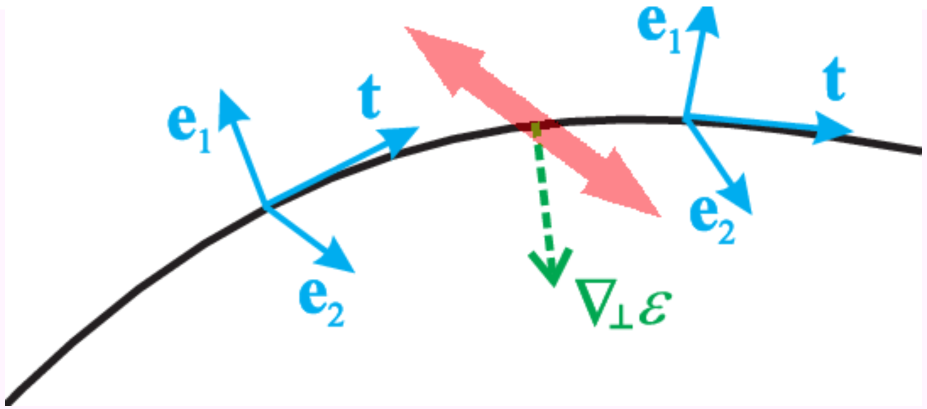
Broad classification of singularities

q is a singular boundary point of $(\mathcal{M}, g)$

The point $q \in \partial(\mathcal{M})$ (on the boundary of the manifold $\mathcal{M}$) is a C^k *curvature singularity* ($k \geqslant 0$) if there is a curve $\gamma(v)$ such that when an orthonormal tetrad $\{E_a(v)\}$ parallel propagated along $\gamma(v)$ is used as a basis, at least one curvature tensor component $R_{abcd;e_1 \ldots e_k}(v)$ does not behave in a C^0 way on $[0, v^+]$.

A *curvature singularity* will occur if some physical quantity (e.g. the density or pressure of a fluid) or some curvature tensor invariant (e.g. $R^{abcd}R_{abcd}$) is badly behaved as one approaches q.

The point $q \in \partial(\mathcal{M})$ is a C^k *quasiregular* singularity ($k \geqslant 0$) if it is not a C^k curvature singularity.



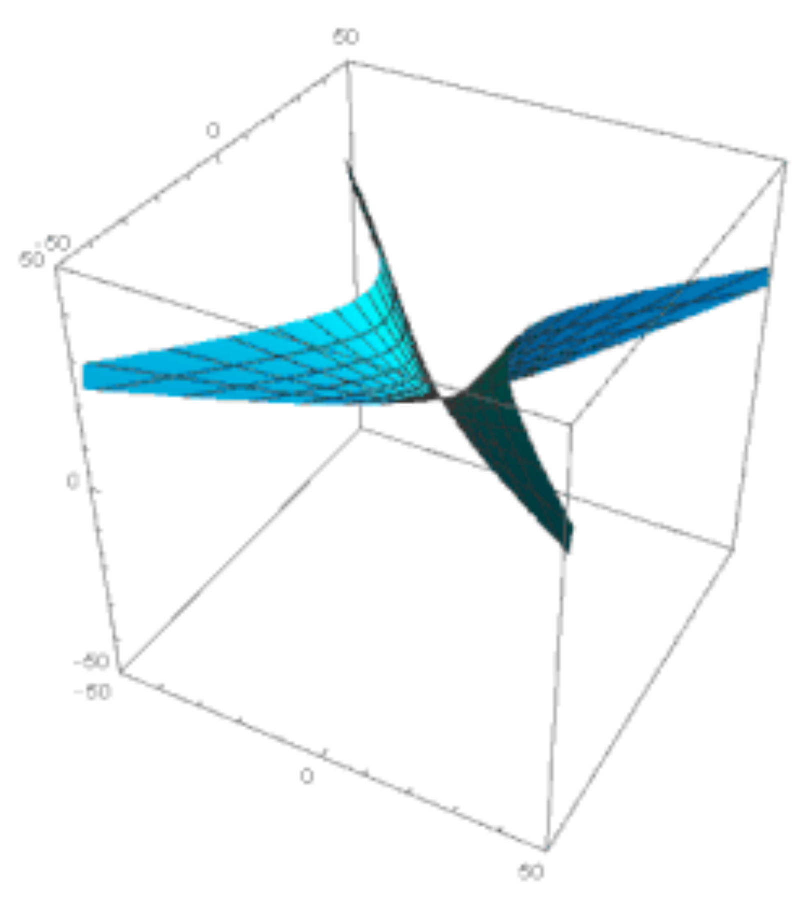
Further refinements

It can be useful to further refine the classification of a *curvature singularity*

Is it a *matter* singularity ? Ricci tensor is causing the problem
or
Is it a *conformal* singularity ? Weyl tensor is causing the problem

Are the relevant curvature components

unbounded ? a *divergent* singularity
or
bounded ? an *oscillatory* singularity



BKL cosmological singularity
Belinski–Khalatnikov–Lifshitz

Examples of curvature singularities

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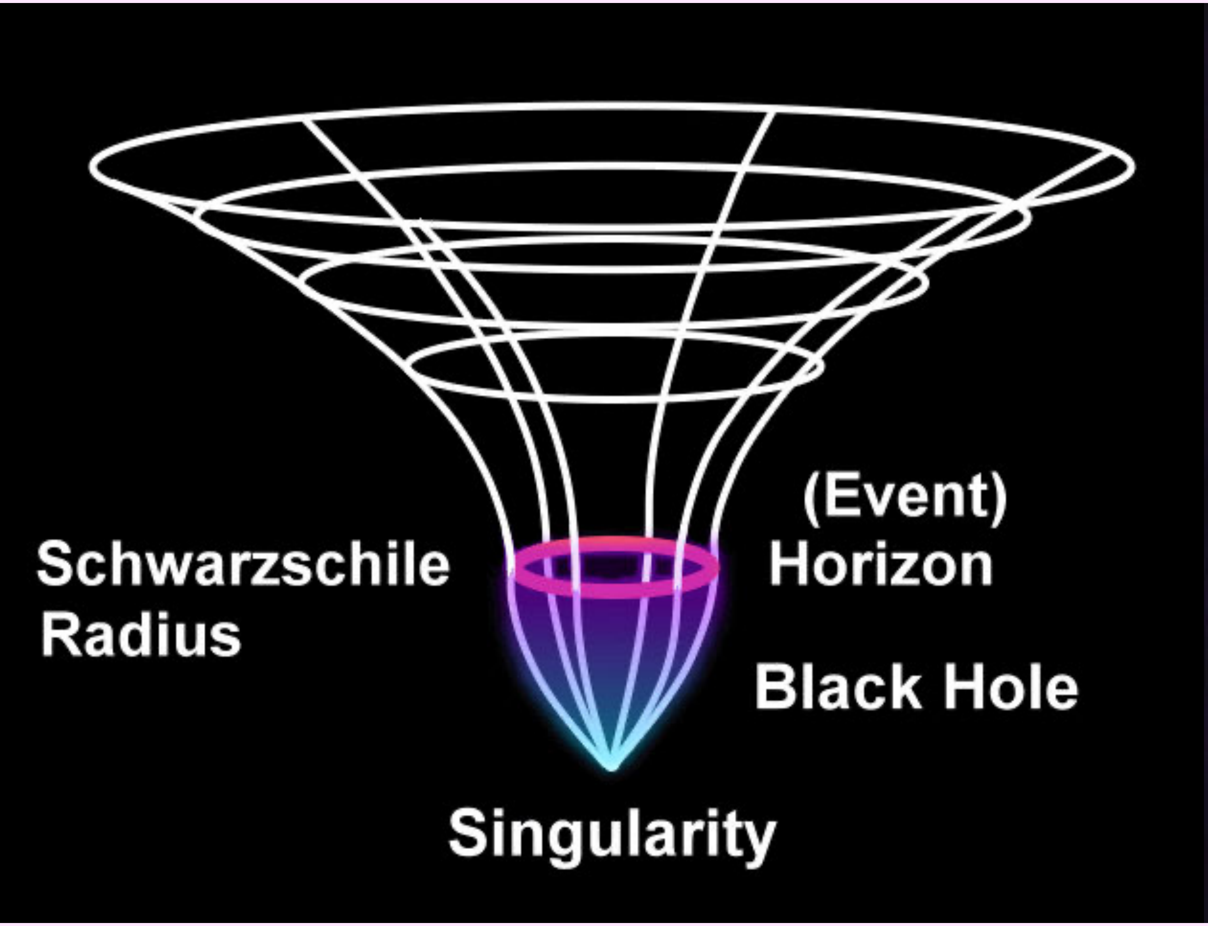
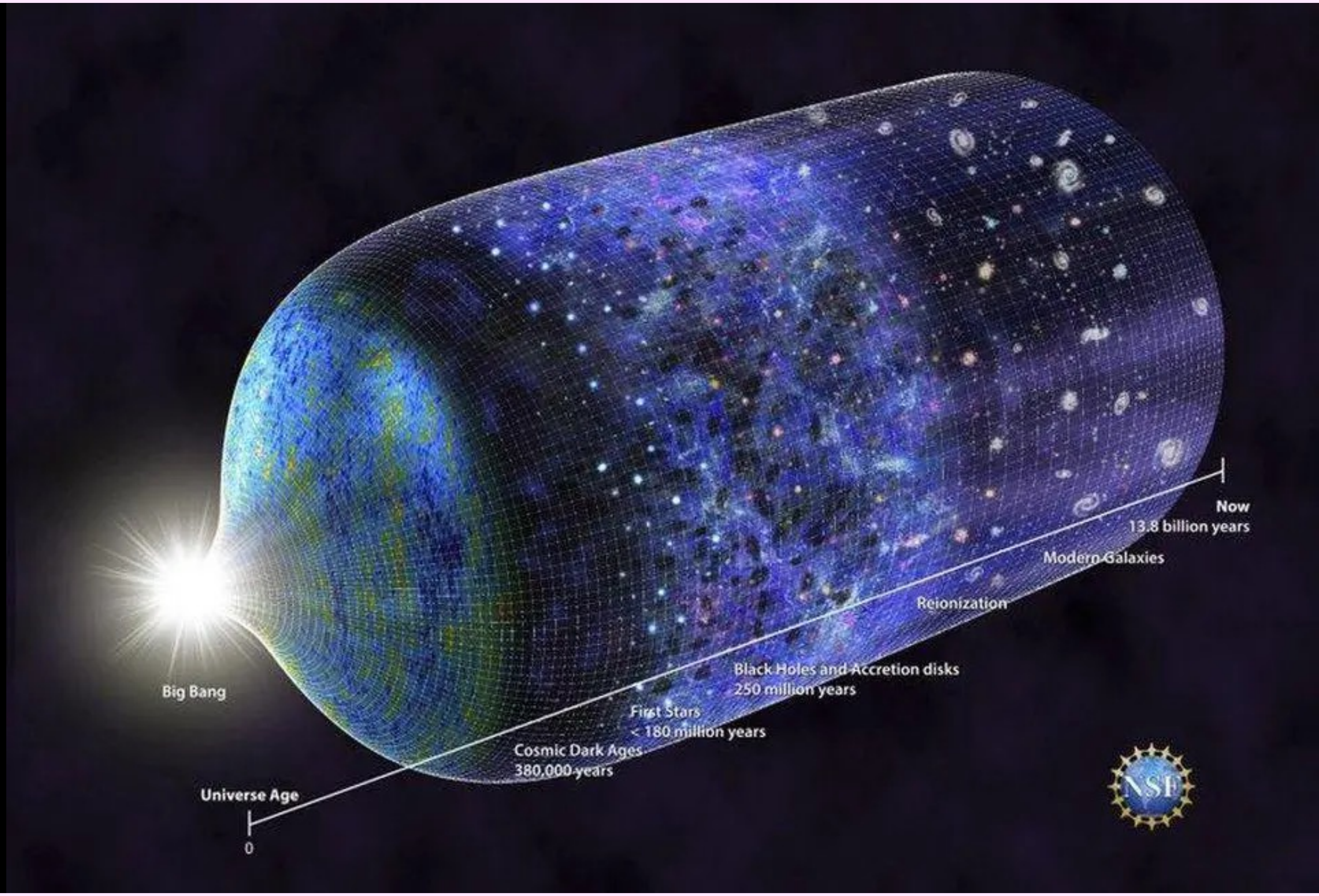




Schwarzschild singularity

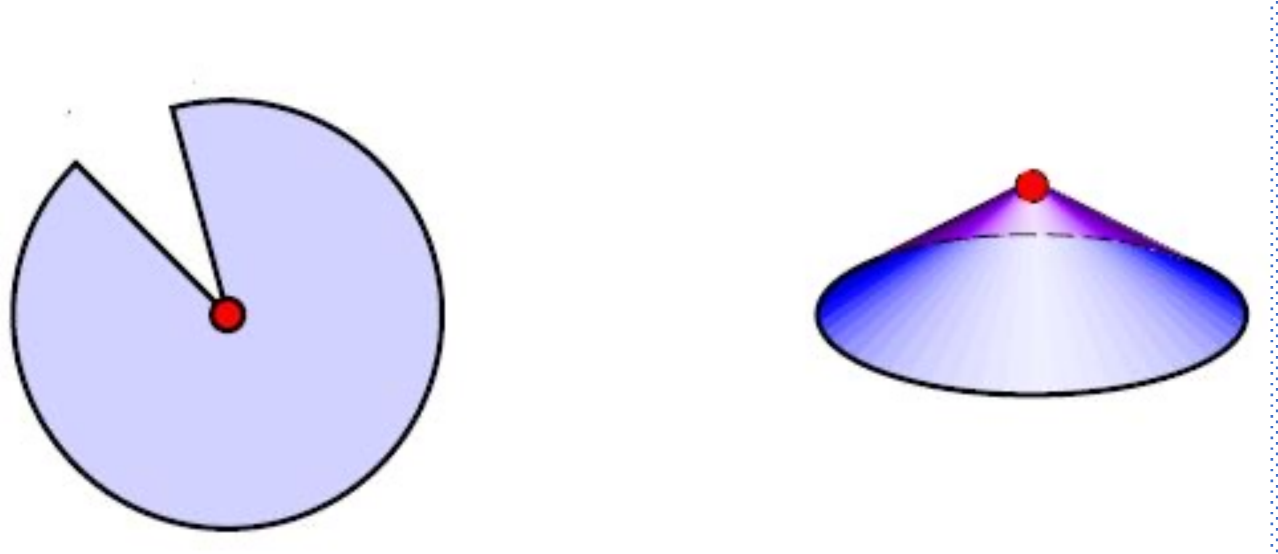
Friedmann-Robertson-Walker Big Bang singularity

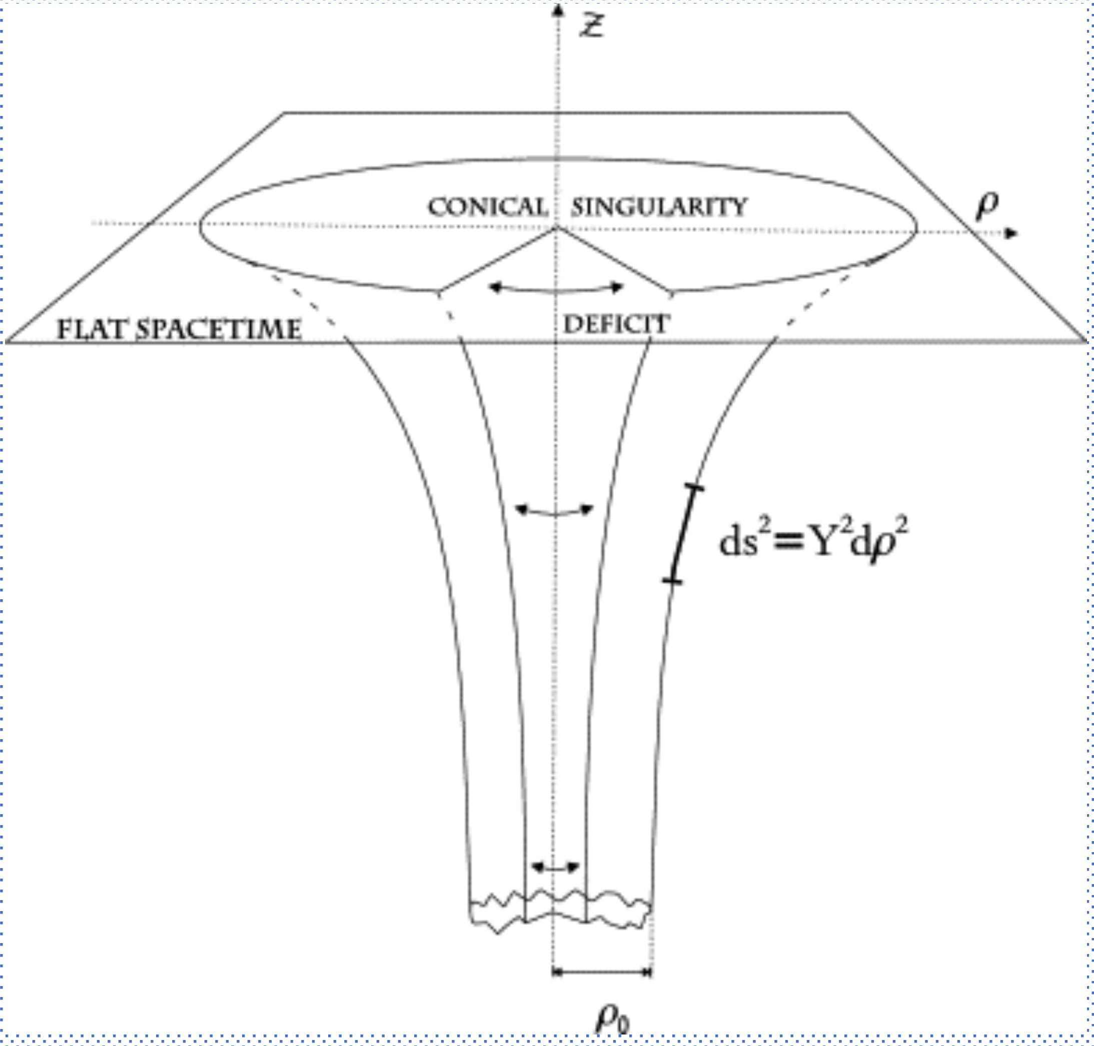
These curvature singularities are ***matter*** singularities and ***divergent*** singularities

Conical singularities

These are a type of *quasiregular* singularity

There is a geometric problem with the space-time
Something has been removed!





Chris Clarke: *every quasiregular singularity is locally extendible*

The situation gets worse 😱

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There is another type of singularity often encountered in new exact solutions

These singularities are not part of the standard classification

They are kind of a top-level "raw" singularity

Often exact solutions are not obtained in an optimal coordinate system

This can lead to the presence of singularities which are *jumbled*

They are called *directional singularities*

They were not well understood for many years

There was no precise definition of a directional singularity

It was a puzzle as to what to do with them … 🤔

Space-time needs a boundary

A boundary for space-time should include:

regions at *infinity*

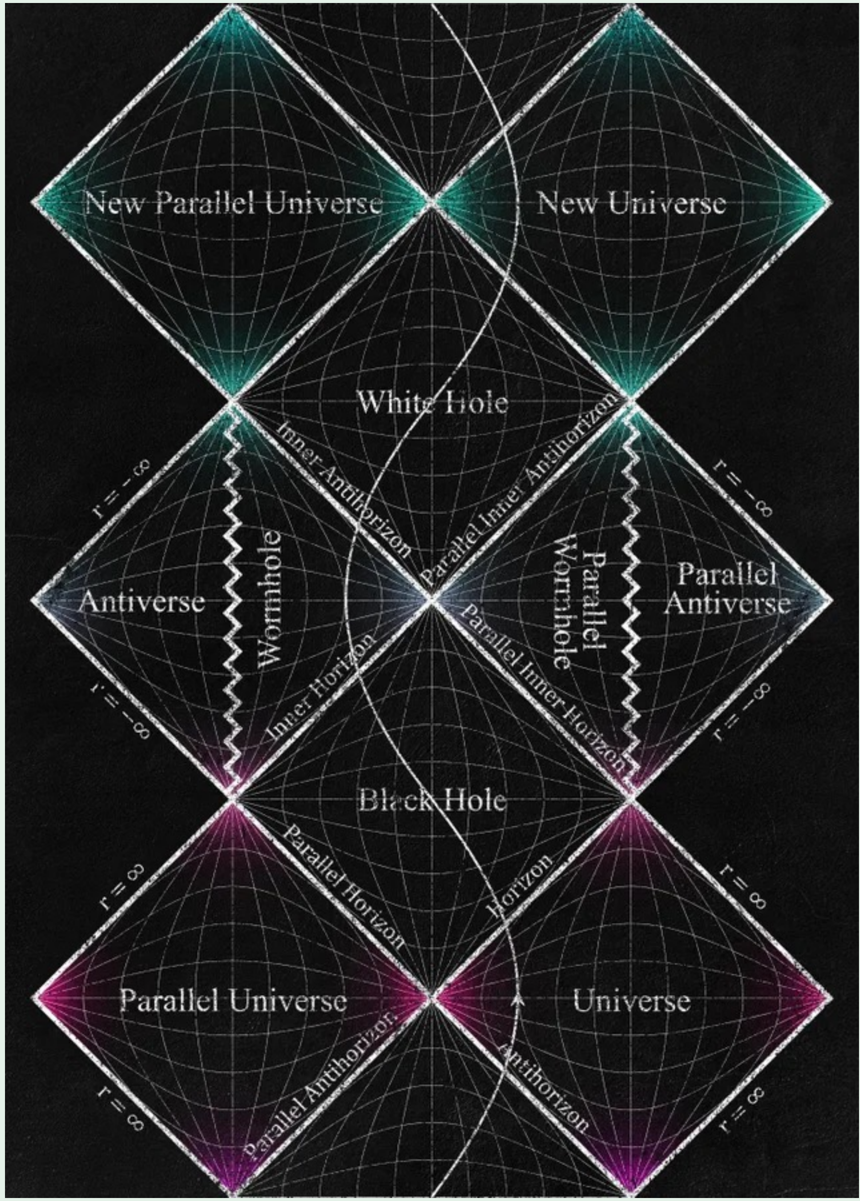
regular points through which the space-time could be extended

singularities – possibly including apparent and directional singularities

We will need to consider classes of curves

Which ones are important - geodesics, curves of bounded acceleration …

What is the best way of attaching a boundary to a space-time?



The Abstract Boundary (a-boundary)

Scott and Szekeres

Australian National University

We wanted a top-level solution

Where do you start when you have found a raw new exact solution in an arbitrary coordinate system?

We didn't want to require that the space-time is maximally extended

We didn't want to impose causality conditions on the space-time

We wanted the boundary to include everything – singularities, regular points, regions at infinity

Being able to choose the class of curves of interest was important

It was also important to build in flexibility with respect to differentiability

Ultimately, we wanted the a-boundary to be the go-to tool for pursuing singularities

Embeddings

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The central idea is to consider all possible embeddings of the space-time into larger manifolds of the same dimension

Each such embedding produces a boundary

Definition 9. An *enveloped manifold* is a triple $(\mathcal{M}, \widehat{\mathcal{M}}, \phi)$ where $\mathcal{M}$ and $\widehat{\mathcal{M}}$ are differentiable manifolds of the same dimension n and ϕ is a C^∞ embedding $\phi : \mathcal{M} \to \widehat{\mathcal{M}}$.

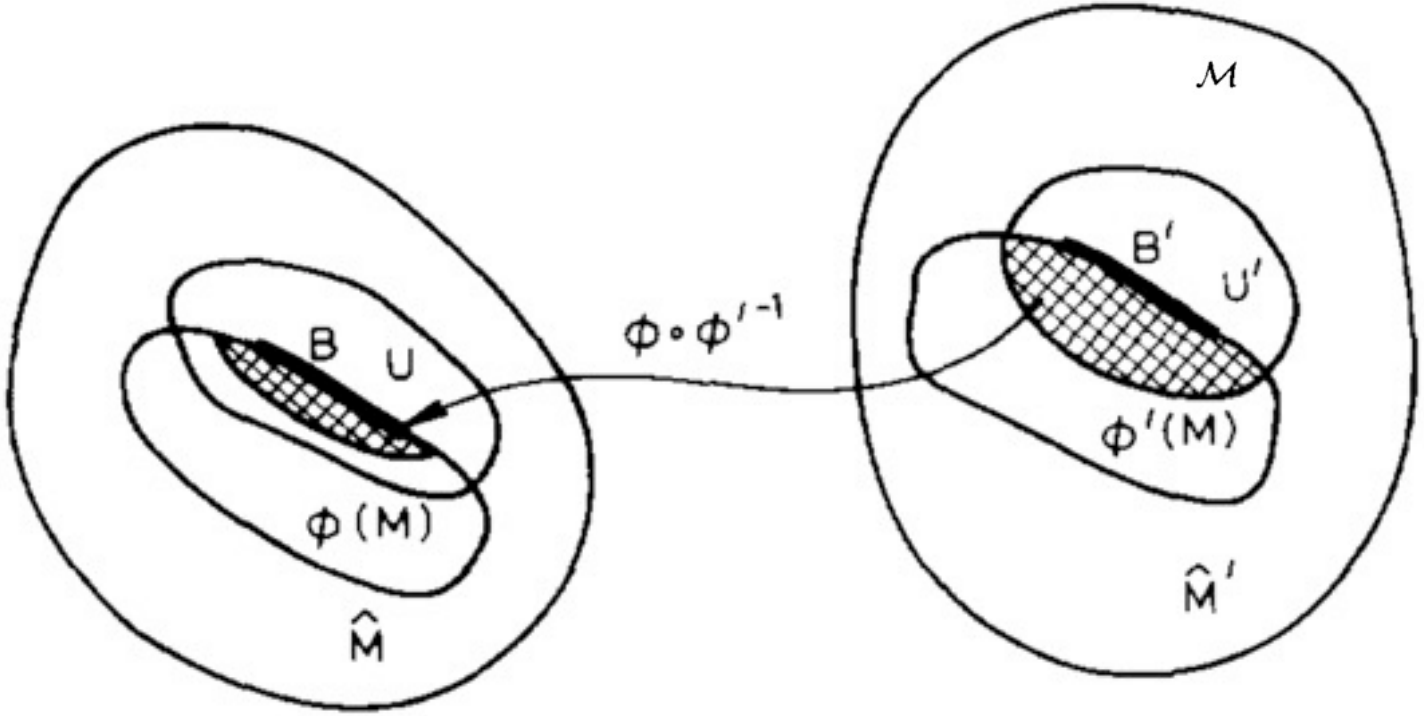
Boundary points in different embeddings can be "compared"

These comparisons lead to the notion of an ***optimal embedding*** and ***optimal boundary***

The covering relation

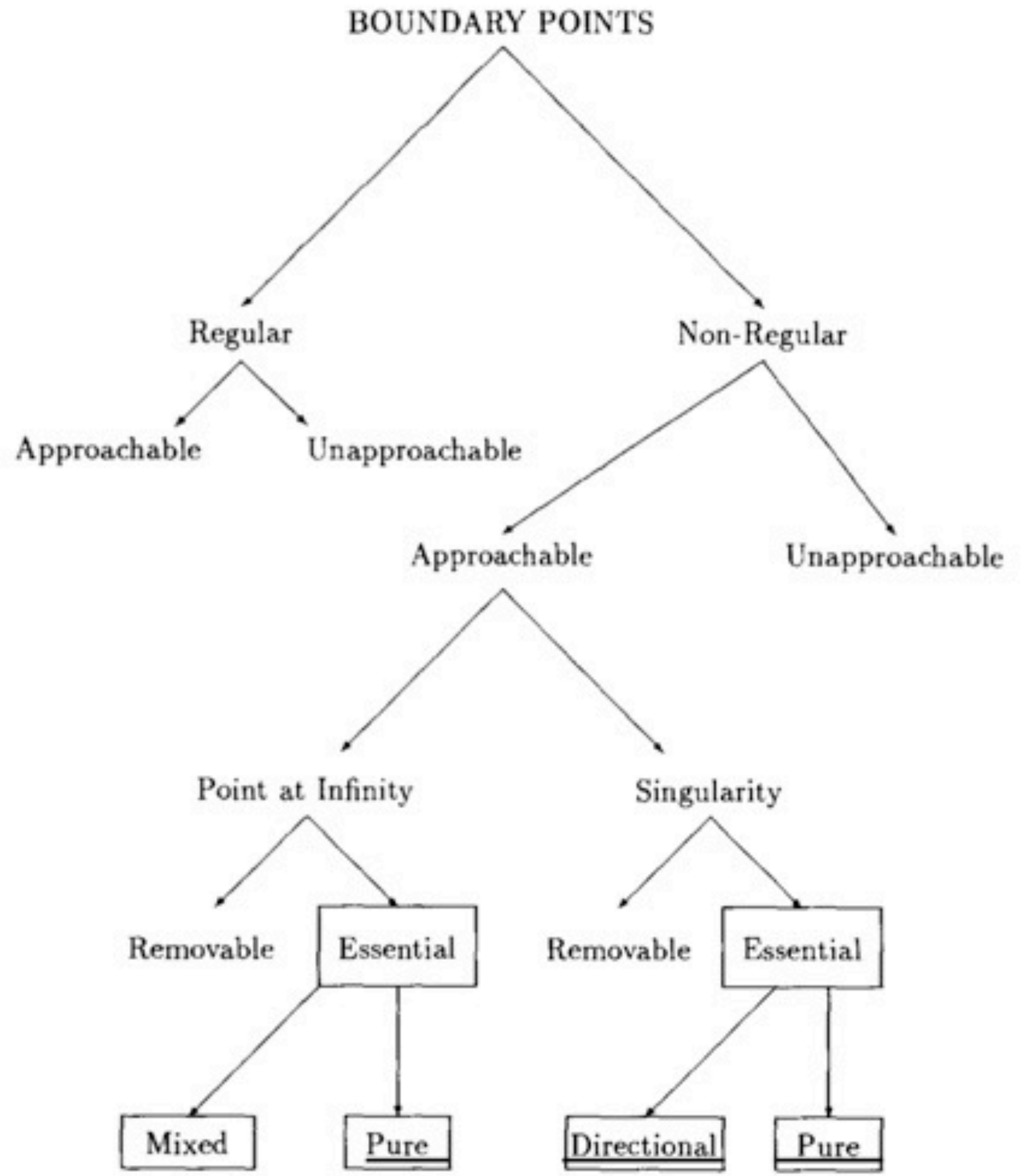
If B' is a boundary set of a second envelopment $(\mathcal{M}, \widehat{\mathcal{M}}', \phi')$ of $\mathcal{M}$ then we say B *covers* B' if for every open neighbourhood $\mathcal{U}$ of B in $\widehat{\mathcal{M}}$ there exists an open neighbourhood $\mathcal{U}'$ of B' in $\widehat{\mathcal{M}}'$ such that

$$\phi \circ \phi'^{-1} (\mathcal{U}' \cap \phi'(\mathcal{M})) \subset \mathcal{U} .$$



The set B covers the set B’

This says that one cannot get close to points of ***B’*** by a sequence of points from within $\mathcal{M}$ without at the same time approaching some point of ***B***



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Schematic classification of boundary points

Theorists wrap-up black hole story

Penrose's 1964 singularity theorem was revolutionary -> Nobel Prize in Physics 2020

Singularities are not an artefact arising from symmetry assumptions – they are a prediction of General Relativity

The singularity theorems involve three general elements:

1. an energy condition on the matter in the space-time
2. a causality condition on the space-time
3. a trapping condition – gravity is strong enough somewhere to trap a region

Black hole uniqueness theorems established – black holes characterised by their mass, angular momentum and charge

The first strong candidate for a black hole Cygnus X-1 was found in 1972

The singularity theorems are incomplete 😲

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The singularity theorems actually predict incomplete, causal geodesics – not singularities!

The existence of incomplete, causal curves poses a problem for any theory of space-time

If a timelike curve is incomplete in the future direction, then a particle moving on this world-line finds that its possible future suddenly comes to an end

If a timelike curve is incomplete in the past direction, then a particle moving on this world-line starts with no previous history

But WHAT do these incomplete, causal curves begin at or end at?

Is it a singularity? If so, *is it a curvature singularity*? And WHERE is it?

To answer these questions and "complete" the singularity theorems has been a major quest in General Relativity for more than half a century now 🏔️

The role of the Abstract Boundary

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Two topologies for a manifold together with its a-boundary

The attached point topology

R.A. Barry and S.M. Scott, “The attached point topology of the abstract boundary for spacetime”, *Classical and Quantum Gravity* **28**, 165003 (2011)

The strongly attached point topology

R.A. Barry and S.M. Scott, “The strongly attached point topology of the abstract boundary for space-time”, *Classical and Quantum Gravity* **31**, 125004 (2014)

Where do the incomplete, causal curves begin or end?

This very general result provides the answer

S.M. Scott and B.E. Whale, "**The Endpoint Theorem**", *Classical and Quantum Gravity* **38**, 065012 (2021)

Theorem 2.1 (The endpoint theorem). *Let $\mathcal{M}$ and $\mathcal{M}_\phi$ be smooth, connected, Hausdorff, paracompact manifolds of dimension n. If $(x_i)_{i\in\mathbb{N}}$ is a sequence of points in $\mathcal{M}$ without an accumulation point, then there exists an open embedding $\phi : \mathcal{M} \to \mathcal{M}_\phi$, such that $\partial\phi(\mathcal{M})$ is diffeomorphic to the $n-1$ dimensional unit ball and the sequence $(\phi(x_i))_{i\in\mathbb{N}}$ converges to some $y \in \partial\phi(\mathcal{M})$.*

We now have a **location** for the singularity theorems "singularities"

The Abstract Boundary can now be unleashed

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B.E. Whale, M.J.S.L. Ashley and **S.M. Scott**, "Generalizations of the abstract boundary singularity theorem", *Classical and Quantum Gravity* **32**, 135001 (2015)

Theorem 1.1. *Let $(\mathcal{M}, g)$ be a strongly causal, C^l maximally extended, C^k spacetime $(1 \leqslant l \leqslant k)$. Let $\mathcal{C}$ be the set of affinely parametrized causal geodesics in $\mathcal{M}$. There exists an incomplete curve in $\mathcal{C}$ if and only if the abstract boundary $\mathcal{B}(\mathcal{M})$ contains an abstract C^l essential singularity.*

Theorem 1.2 **(The abstract boundary singularity theorem).** *Let $(\mathcal{M}, g)$ be a future (past) distinguishing, C^l maximally extended, C^k spacetime $(1 \leqslant l \leqslant k)$ and let $\mathcal{C}$ be the family of generalized affinely parametrized continuous causal curves in $\mathcal{M}$. There exists an incomplete curve in $\mathcal{C}$ if and only if $\mathcal{B}(\mathcal{M})$ contains an abstract C^l essential singularity.*

The Singularity Theorems are now semi-completed

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The predicted incomplete, causal curves have an endpoint in an embedding

The singularities have a ***location***

The singularities are ***essential singularities***

They cannot be removed by switching to other embeddings

What remains to complete the singularity theorems?

To show that the predicted singularities are curvature singularities

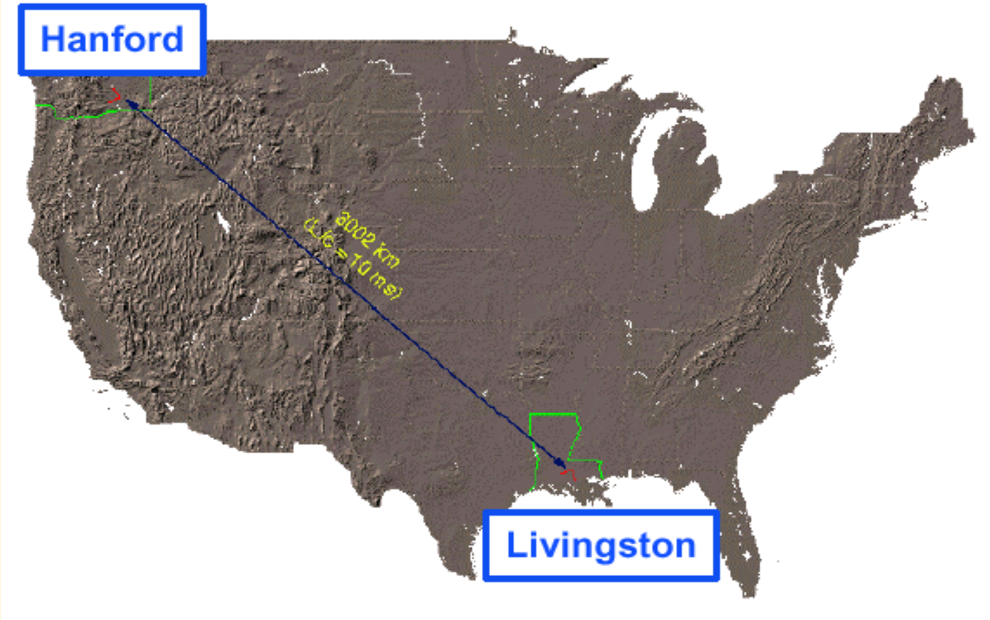
M.J.S.L. Ashley and S.M. Scott, "Curvature singularities and abstract boundary singularity theorems for space-time", *Contemporary Mathematics* **337**, 9–19 (2003)

History of Gravitational Wave theory and data analysis in Australia

- Commenced activities in 1997
- In 1999 established a formal collaboration with LIGO in data analysis
- contributed key components to the LIGO Data Analysis System
- pioneered the exchange of seismometer, power monitor and magnetometer data from the LIGO-Hanford, LIGO-Livingston and Virgo observatories. Searched for long and short timescale correlations in the merged environmental data
- First investigations into the merit of various global detector network configurations
- Investigated the possible effects of gravitational lensing on gravitational waves in various astrophysical scenarios
- Led the design and implementation of the search, using LIGO S5 data, for gravitational waves from Cassiopeia A
- Developed an innovative heterogeneous CPU/GPGPU/FPGA desktop computing system (the Chimera), built with commercial-off-the-shelf components
- ACIGA Data Analysis Cluster (ADAC)

Washington State

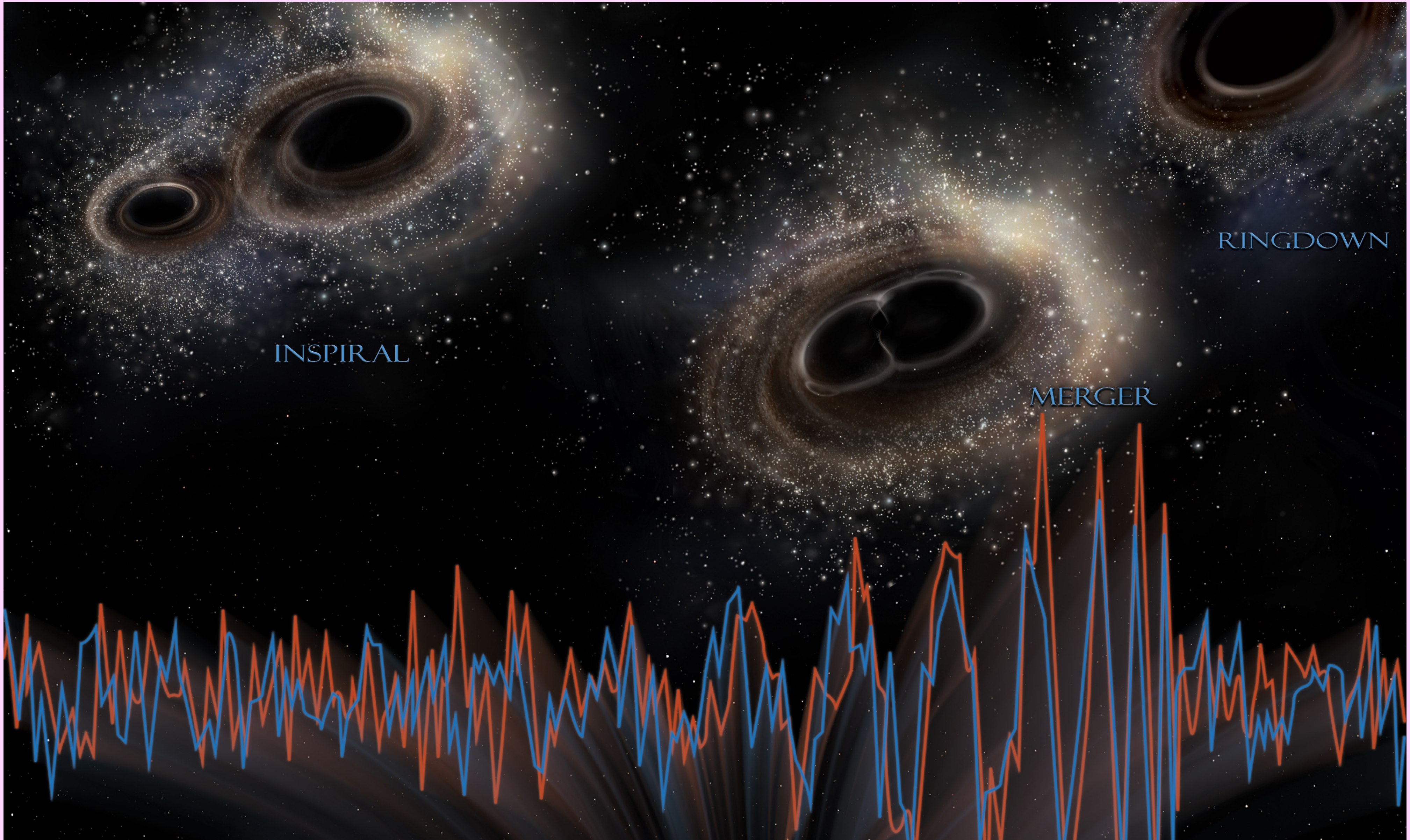
Louisiana



Direct Detection of Gravitational Waves

GW150914 1.3 billion years ago

36 and 29 solar mass black holes ➔ 62 solar mass black hole



- The first direct detection of a gravitational wave on Earth
- The first detection of a black hole binary system
- The first observation of black holes merging
- The first test of General Relativity when the gravitational field is very strong and highly dynamical during the black hole merger

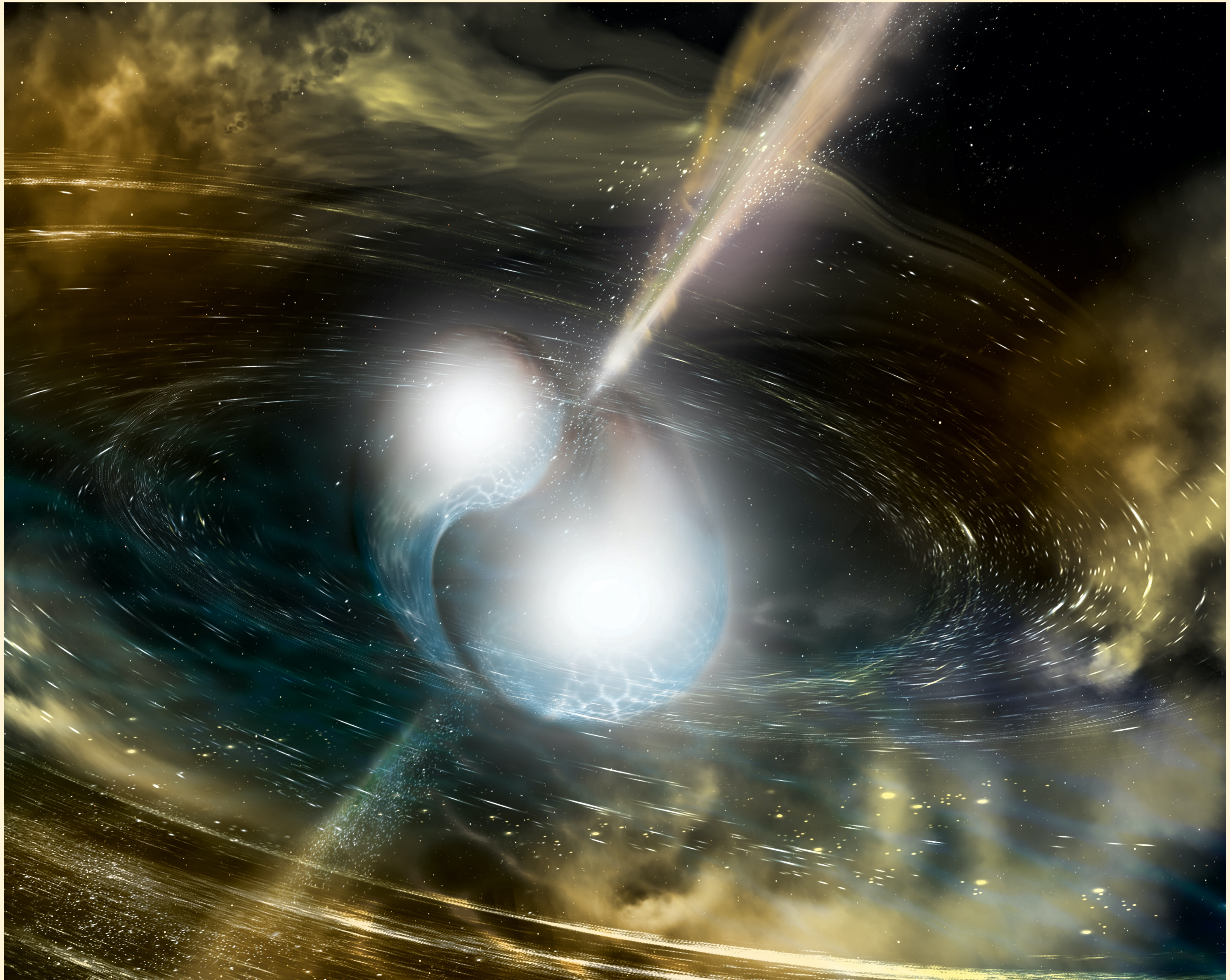
First detection of a binary neutron star system merger

GW170817

17 August 2017

First observation of a binary neutron star inspiral

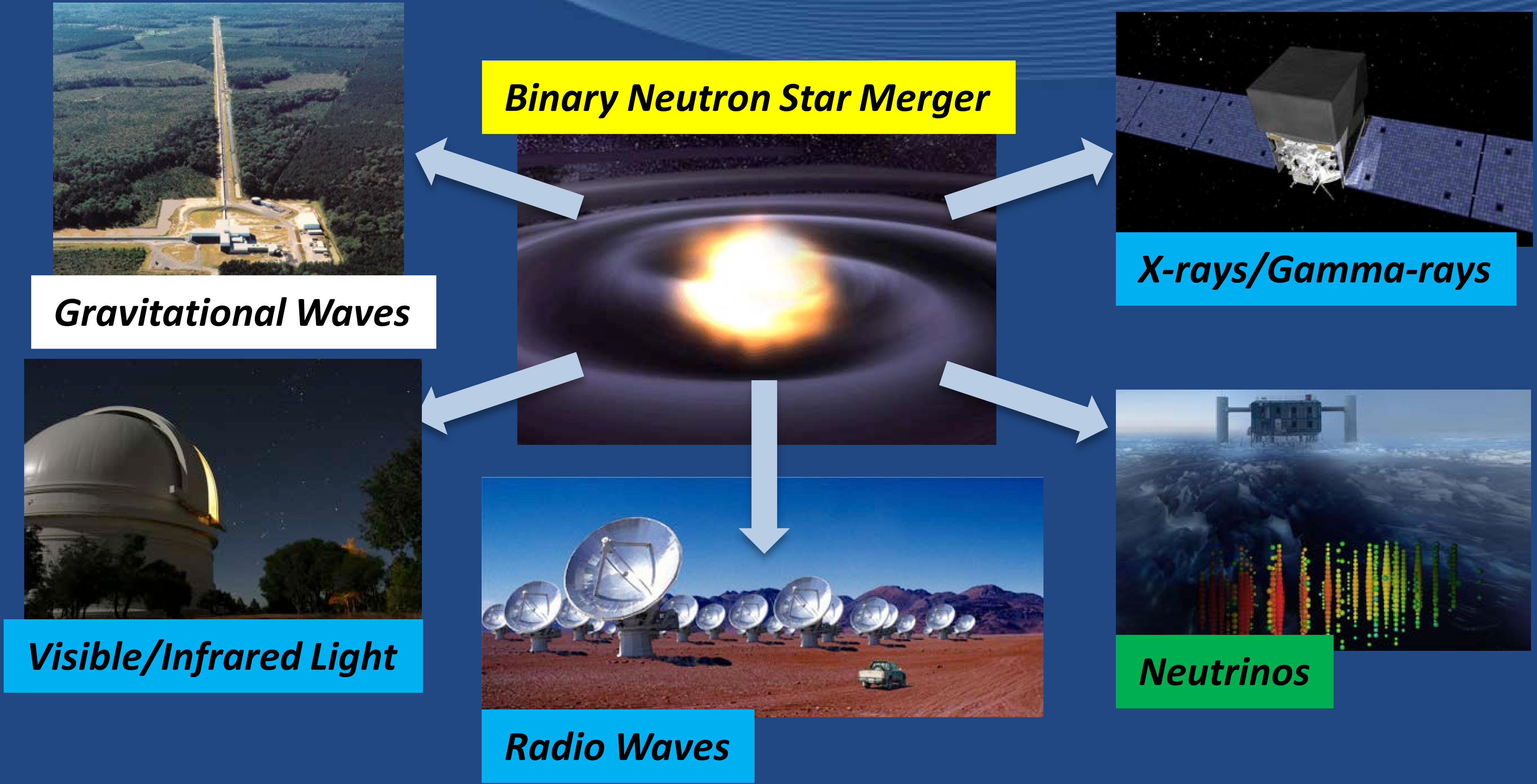
First direct evidence of a link between neutron star mergers and short gamma-ray bursts



Subsequent identification of transient counterparts across the electromagnetic spectrum

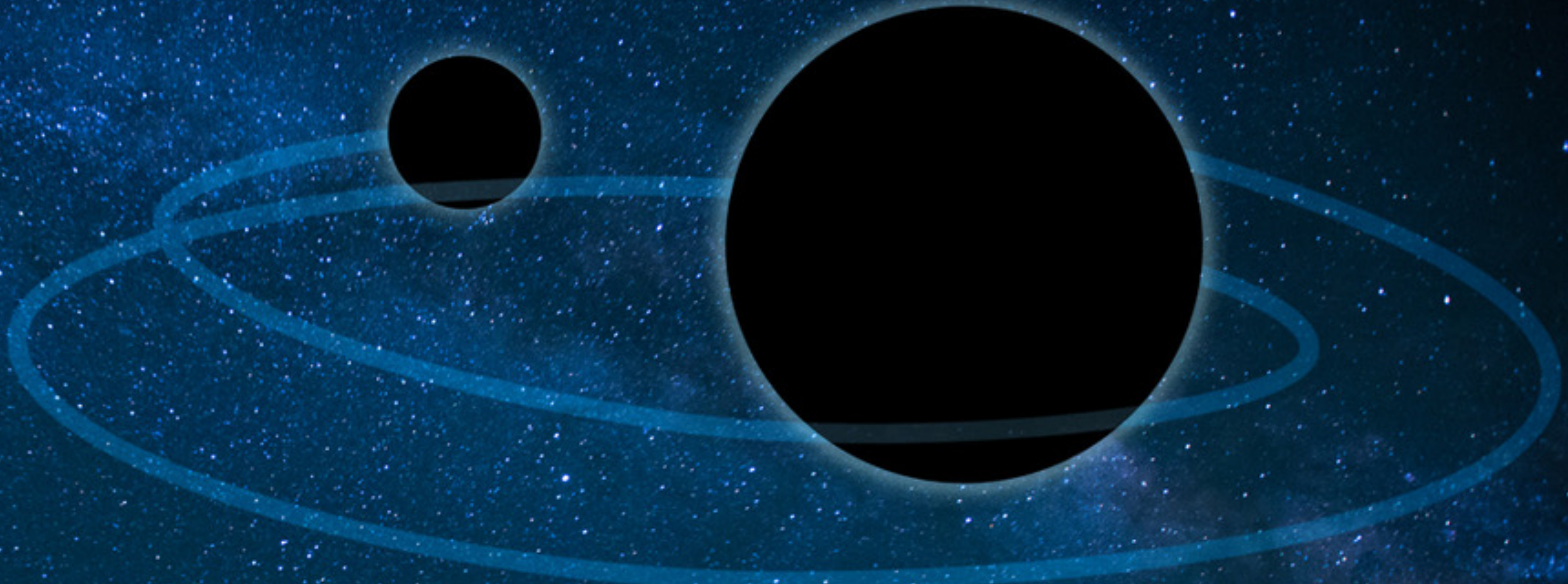
NSF/LIGO/Sonoma State University/
A. Simonnet

Multi-messenger Astronomy with Gravitational Waves



Recent Exceptional Detections

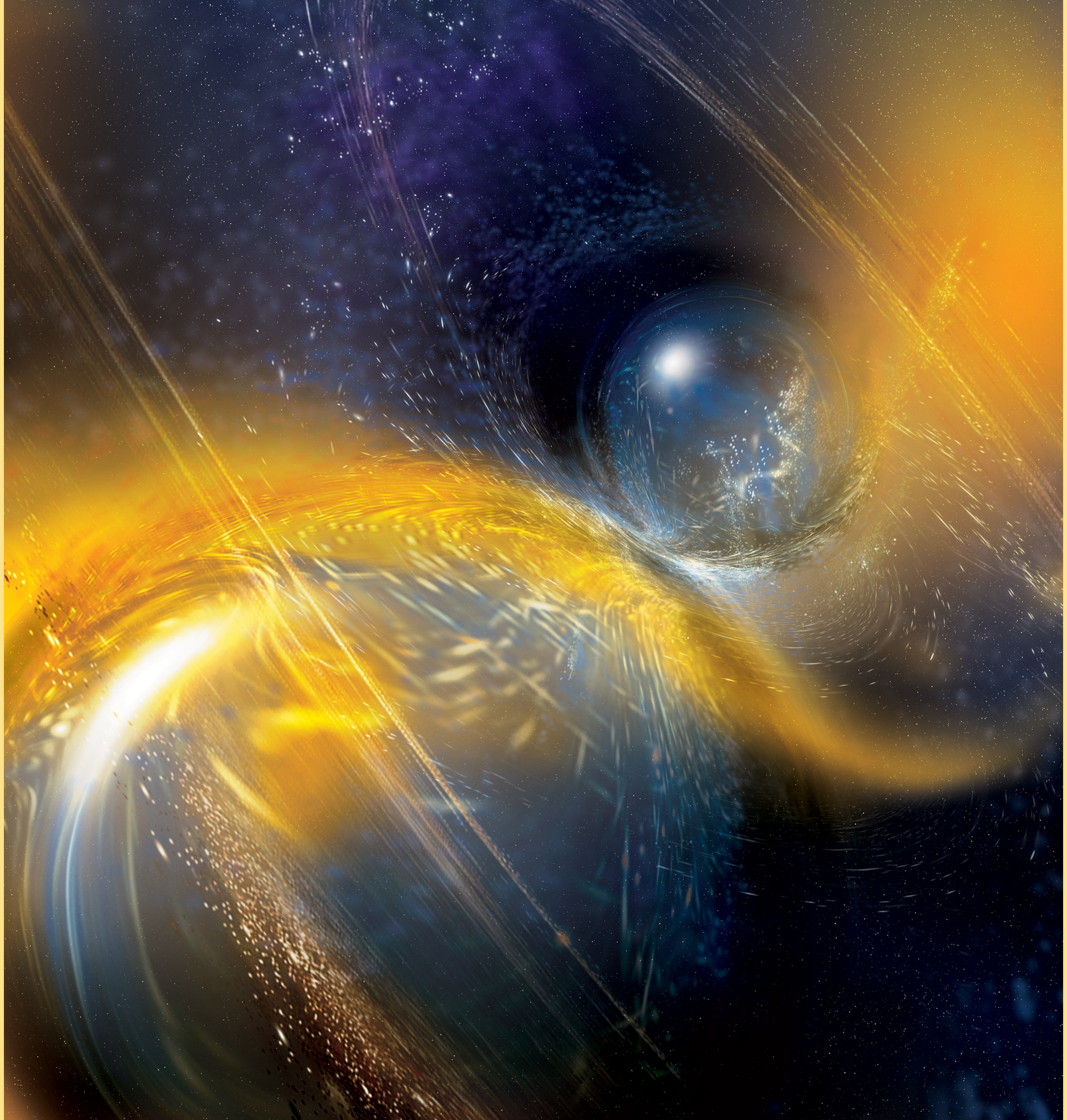
GW190412

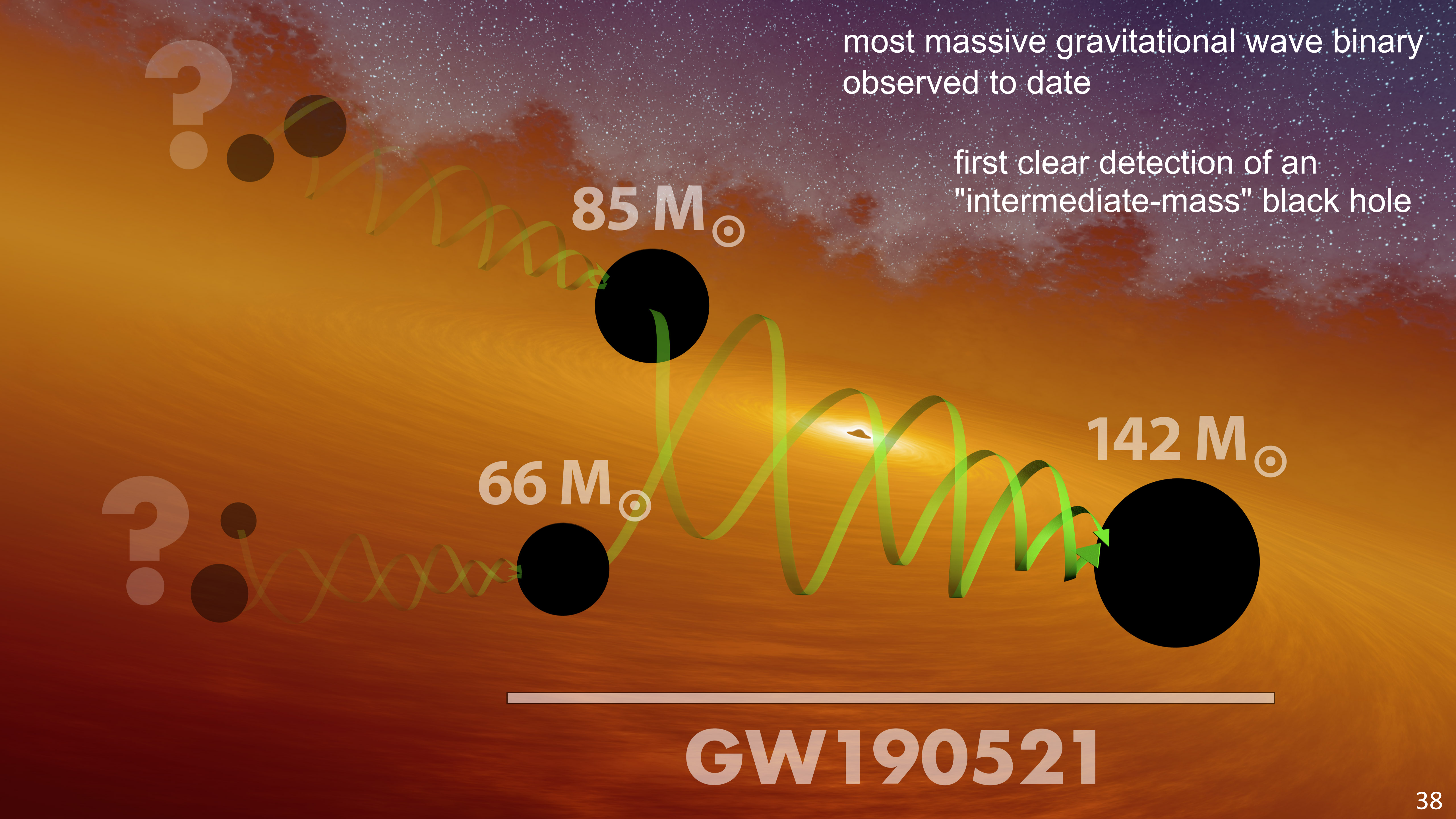


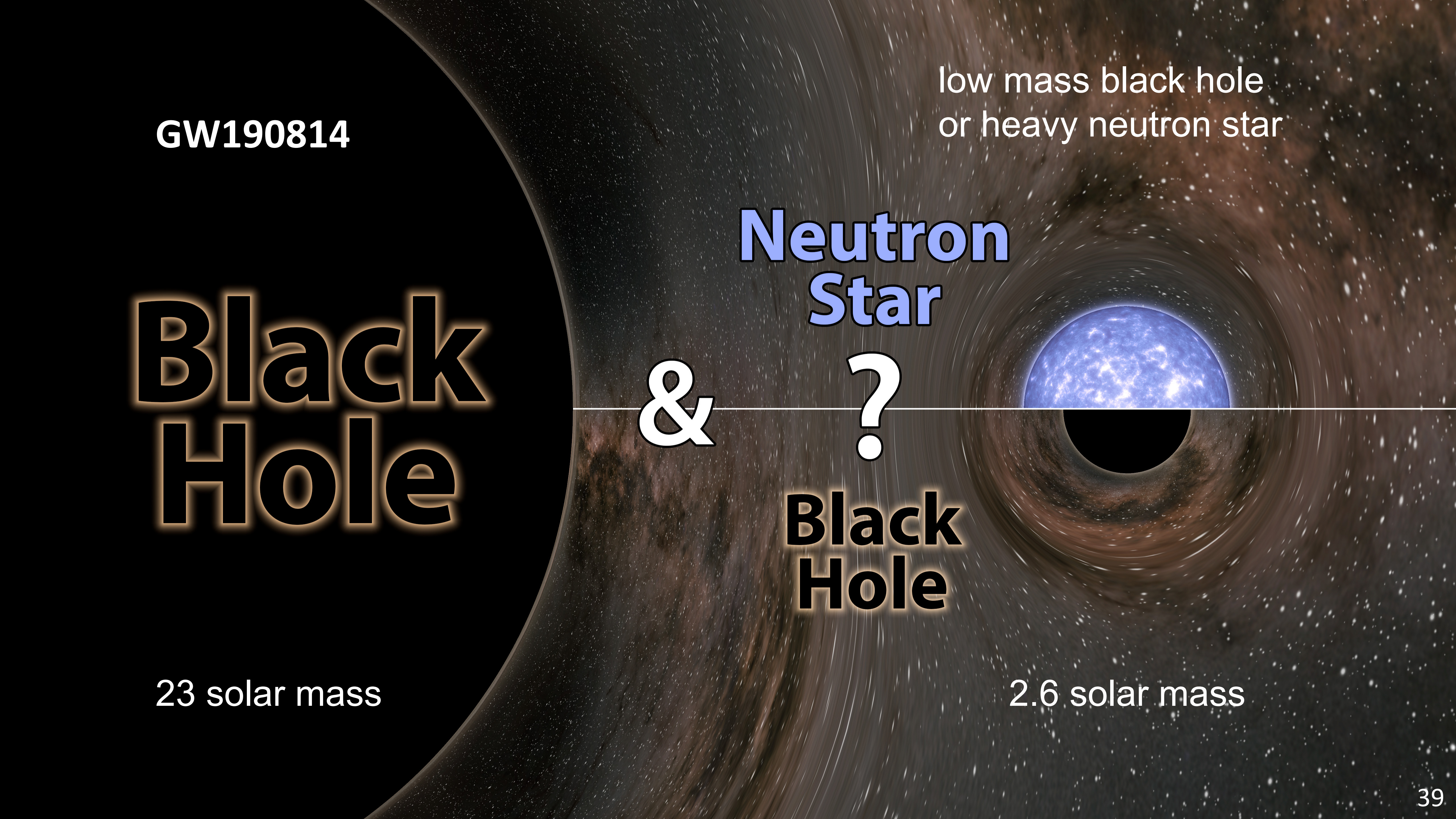
The first detection of a collision of a pair of very differently sized black holes
30 and 8 solar masses

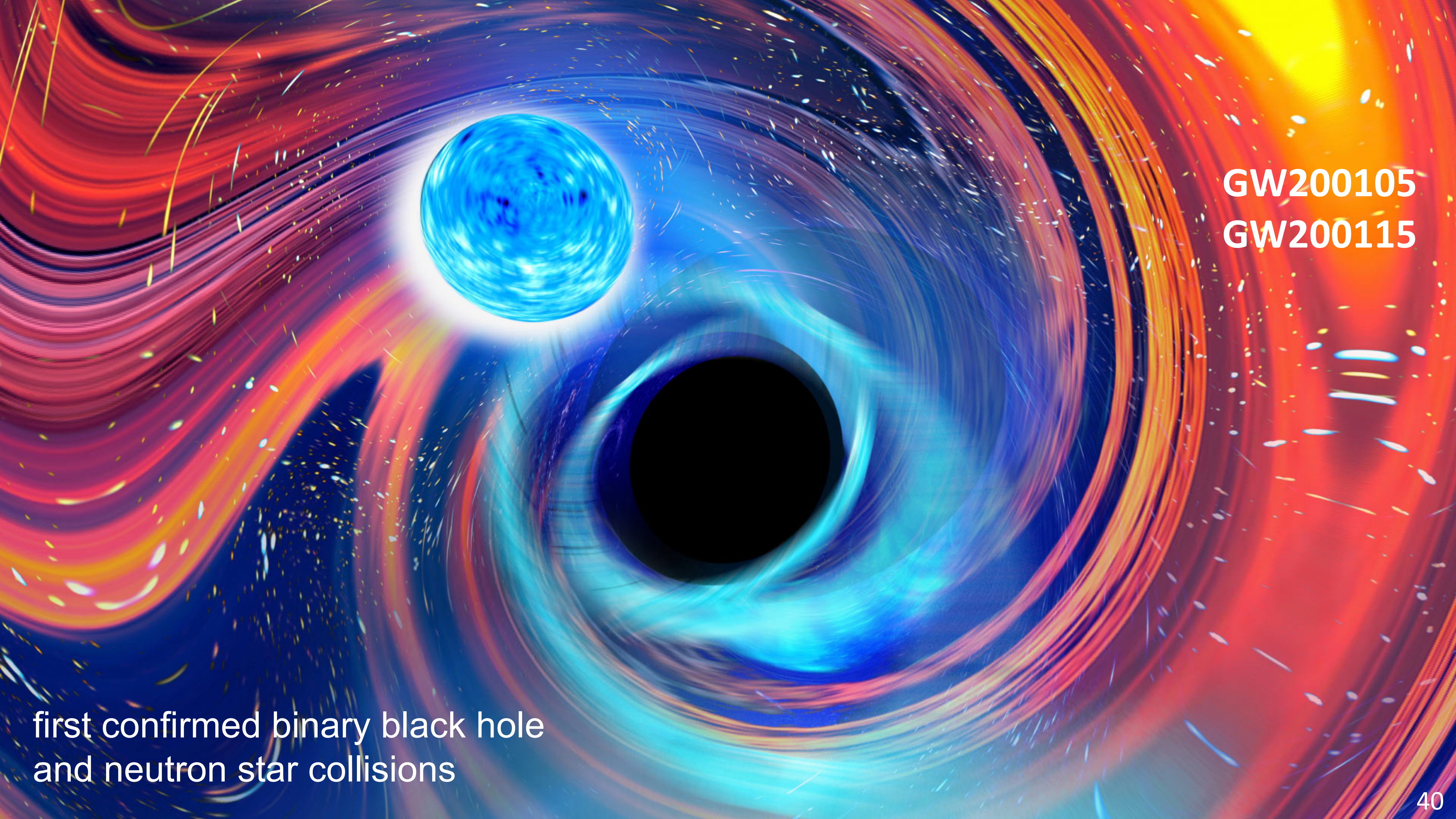
GW190425

Second binary
neutron star merger




most massive gravitational wave binary observed to date
first clear detection of an "intermediate-mass" black hole
85 M☉
66 M☉
142 M☉
GW190521

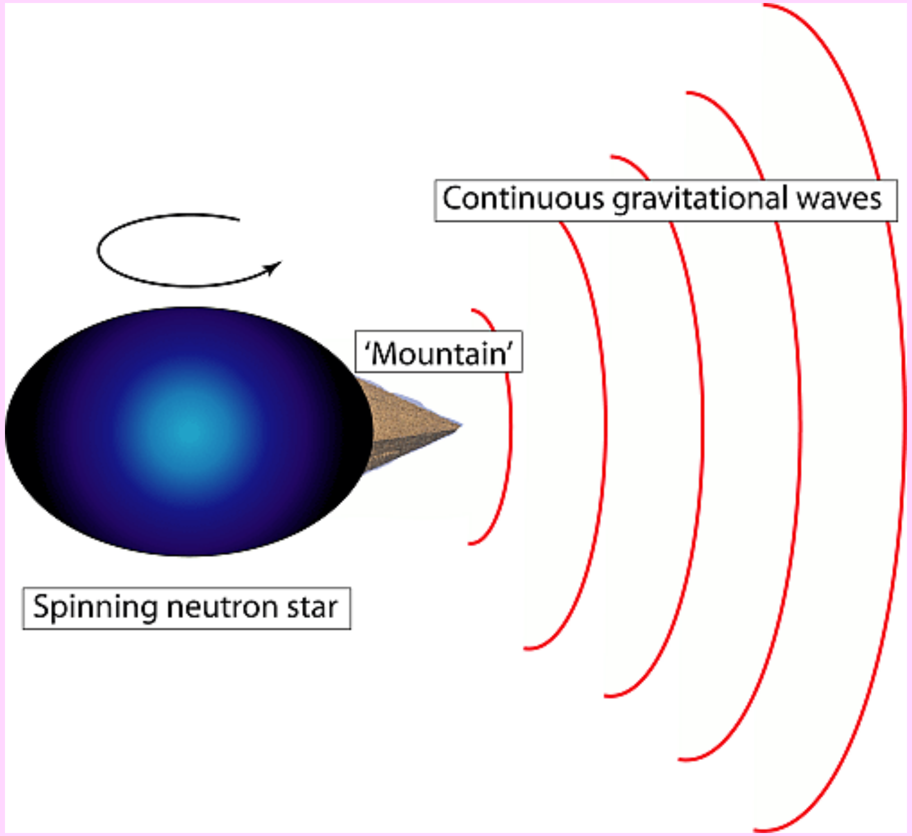

GW190814
low mass black hole
or heavy neutron star
Black Hole
&
Neutron Star
?
Black Hole
23 solar mass
2.6 solar mass



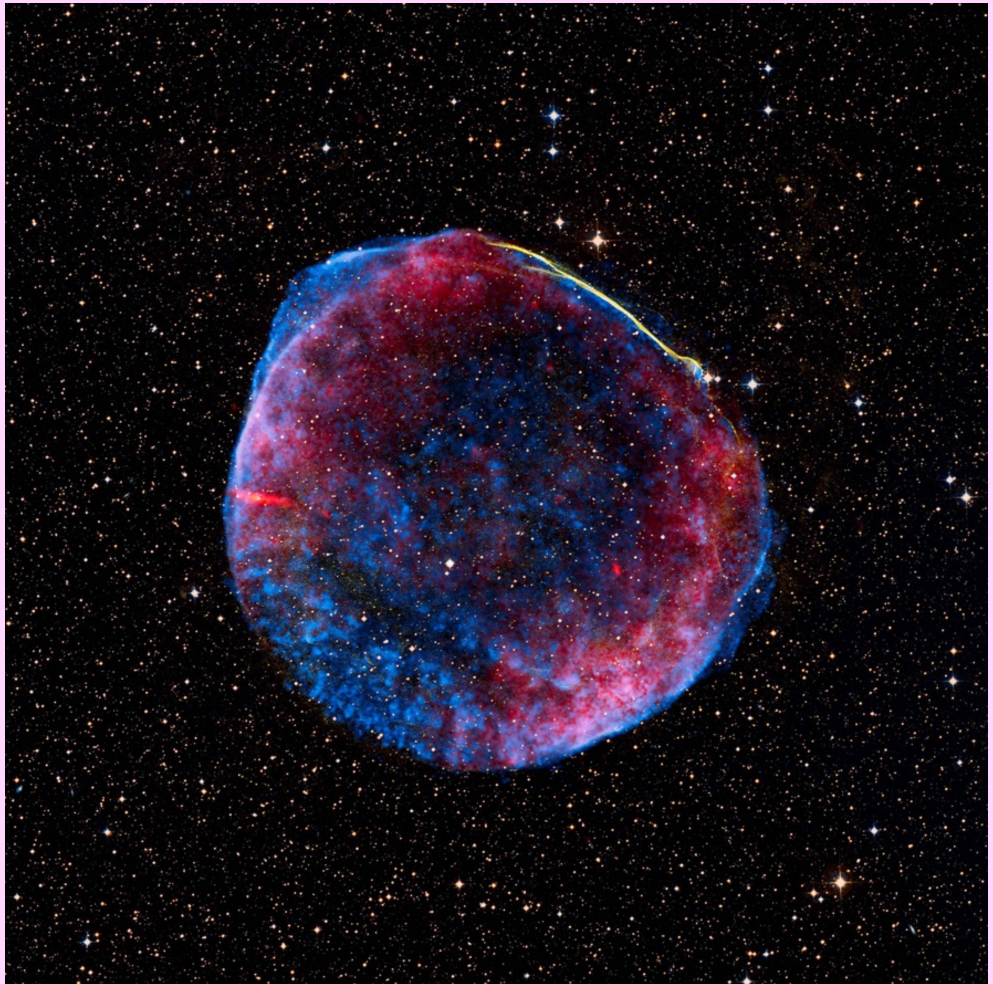
GW200105
GW200115

first confirmed binary black hole and neutron star collisions

Current work and the future



Understanding the ignition of a supernova explosion



Continuous gravitational waves from spinning neutron stars with millimetre size mountains

We expect the unexpected!

Inferring neutron star properties with continuous gravitational waves

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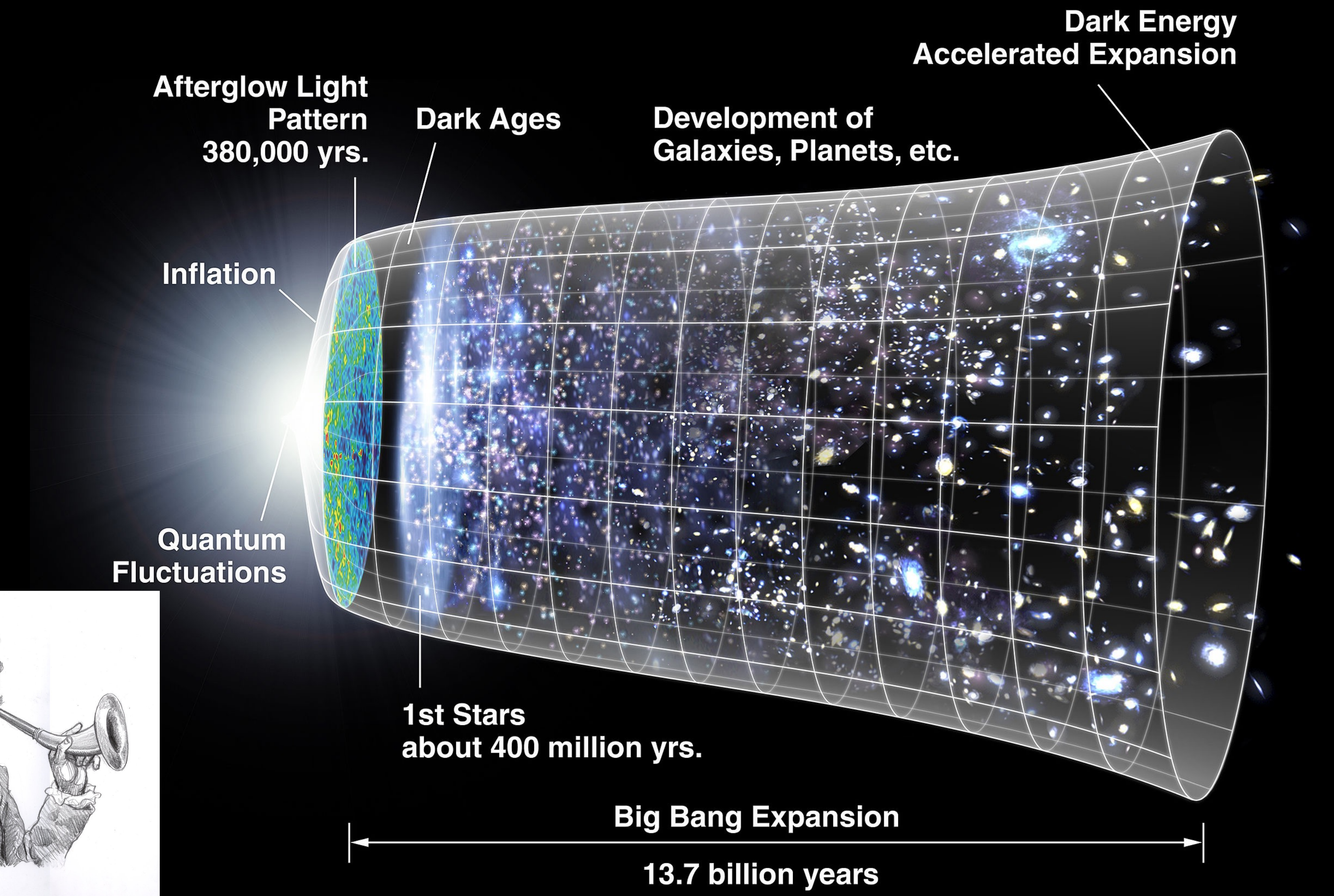
[2] *Centre for Gravitational Astrophysics, Australian National University, Canberra ACT 2601, Australia*
[3] *School of Physics, University of Melbourne, Parkville VIC 3010, Australia*

6 December 2022

ABSTRACT
Detection of continuous gravitational waves from rapidly-spinning neutron stars opens up the possibility of examining their internal physics. We develop a framework that leverages a future continuous gravitational wave detection to infer a neutron star's moment of inertia, equatorial ellipticity, and the component of the magnetic dipole moment perpendicular to its rotation axis. We assume that the neutron star loses rotational kinetic energy through both gravitational wave and electromagnetic radiation, and that the distance to the neutron star can be measured, but do not assume electromagnetic pulsations are observable or a particular neutron star equation of state. We use the Fisher information matrix and Monte Carlo simulations to estimate errors in the inferred parameters, assuming a population of gravitational-wave-emitting neutron stars consistent with the typical parameter domains of continuous gravitational wave searches. After an observation time of one year, the inferred errors for many neutron stars are limited chiefly by the error in the distance to the star. The techniques developed here will be useful if continuous gravitational waves are detected from a radio, X-ray, or gamma-ray pulsar, or else from a compact object with known distance, such as a supernova remnant.


Dark Energy
Accelerated Expansion
Afterglow Light
Pattern
380,000 yrs.
Dark Ages
Development of
Galaxies, Planets, etc.
Inflation
Quantum
Fluctuations
1st Stars
about 400 million yrs.
Big Bang Expansion
13.7 billion years

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LIGO

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ARC Centre of Excellence for Gravitational Wave Discovery