Dynamical branes on expanding orbifold and complex projective space

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[1] Introduction and our results

- We construct a dynamical p-brane solution on orbifold (the complex line bundle over CP^n space) as a solution.

- There are interesting properties when branes are located at an orbifold.
  (M.R. Douglas and G.W. Moore, hep-th/9603167)

- Orbifold singularities are resolved in the D-brane world volume theory.

- One can expect that the spacetime itself becomes regular without any naked singularity once p-branes are placed at the orbifold singularities.
• This is important when one finds hints that the gravity theory giving a p-brane on the orbifold is known and well-defined supergravity, thus giving a handle on the strong coupling dynamics of string theory.

• **Black hole on the Eguchi-Hanson space**
  (Hideki Ishihara, Masashi Kimura, Shinya Tomizawa, Class. Quant. Grav. 23 (2006) L89 [hep-th/0609165])
  (Chul-Moon Yoo, Hideki Ishihara, Masashi Kimura, Ken Matsuno, Shinya Tomizawa, Class. Quant. Grav. 25 (2008) 095017 [0708.0708 [gr-qc]])

• **Black holes on the complex line bundle over CP^n space**

• **(Static) Black p-brane on the orbifolds**
[2] Dynamical p-brane on orbifolds

- Metric of dynamical p-brane in D dimensions

(Kei-ichi Maeda, Nobuyoshi Ohta, Kunihito Uzawa, JHEP 06 (2009) 051, 0903.5483 [hep-th])

\[ ds^2 = h^a(x, y) g_{\mu\nu}(X) dx^\mu dx^\nu + h^b(x, y) u_{ij}(Y) dy^i dy^j, \]

\[ a = -\frac{D - p - 3}{D - 2}, \quad b = \frac{p + 1}{D - 2}, \]

- (p+2)-form field strength

\[ F_{(p+2)} = d (h^{-1}) \wedge dt \wedge dx^1 \wedge \cdots \wedge dx^p \]
• It is straightforward to check that with such an ansatz the Bianchi identity is trivially satisfied.

• The field equation for the antisymmetric tensor becomes

\[ \partial_\mu \partial_i h = 0, \quad \triangle_Y h = 0, \]

\[ \rightarrow h(x, y) = h_0(x) + h_1(y), \quad \triangle_Y h_1 = 0, \]

(Kei-ichi Maeda, Nobuyoshi Ohta, Kunihito Uzawa, JHEP 06 (2009) 051)
• Einstein equations

\[ R_{\mu\nu}(X) - h^{-1} D_{\mu} D_{\nu} h_0 - \frac{a}{2} h^{-1} q_{\mu\nu} \left( \Delta_X h_0 + h^{-1} \Delta_Y h_1 \right) = 0 , \]

\[ R_{ij}(Y) - \frac{b}{2} q_{\mu\nu} \left( \Delta_X h_0 + h^{-1} \Delta_Y h_0 \right) = 0 , \]

\[ \Rightarrow R_{\mu\nu}(X) = 0 , \quad R_{ij}(Y) = \frac{1}{2} b (p + 1) \lambda u_{ij}(Y) , \quad D_{\mu} D_{\nu} h_0 = \lambda q_{\mu\nu}(X) , \]

• The space Y is not Ricci flat, but the Einstein space such as \( \mathbb{CP}^n \) if \( \lambda \neq 0 \), and the function \( h \) can be more non-trivial.

If we set

\[ q_{\mu\nu}(X) = \eta_{\mu\nu}(X), \quad D_\mu h_0 \neq 0, \quad (D_\mu h_0)(D^\mu h_0) \neq 0, \]

the solution for \( h_0 \) is given by

\[ h_0(x) = \frac{\lambda}{2} x_\mu x^\mu + \bar{a}_\mu x^\mu + \bar{a} \]

- When the space \( Y \) is Ricci flat like orbifold, the function \( h_0 \) is linear in the coordinates \( x^\mu \) because of \( \lambda = 0 \).
Dynamical p-brane on the orbifold

cf) Black holes on Eguchi-Hanson space:

Eguchi-Hanson space – complex line bundle over CP$^1$ (2-sphere)

• p-brane on orbifold

$$u_{ij}(Y)dy^i dy^j = dr^2 + r^2 \left[ \left\{ d\rho + \sin^2 \xi_{n-1} \left( d\psi_{n-1} + \frac{1}{2(n-1)} \omega_n \right) \right\}^2 + ds_{\mathbb{C}P^{n-1}}^2 \right]$$
If we impose $h_1 = h_1(r)$, the field equations become for

$$\triangle_Y h_1(r) = 0, \quad \Rightarrow \quad h_1(r) = b_1 + \frac{b_2}{r^{D-p-3}}$$

the solution for $h(x, r)$ is given by

$$h(x, r) = a_{\mu} x^\mu + b_1 + \frac{b_2}{r^{D-p-3}}$$

• There is a naked singularity at $h=0$.

(1) $r \to 0$ : static p-brane solution.

(2) $r \to \infty$ : asymptotically Kasner geometry
[3] Dynamical solution on the $\mathbb{CP}^n$ space

- Metric

$$ds^2 = h^a(x, y) q_{\mu\nu}(X) dx^\mu dx^\nu + h^b(x, y) u_{ij}(Y) dy^i dy^j,$$

$$a = -\frac{D - p - 3}{D - 2}, \quad b = \frac{p + 1}{D - 2},$$

- $\mathbb{CP}^n$ is the Einstein space (NOT Ricci flat).

- Our universe depends on the quadratic function of time.
For the CP$^1$ space

\[ ds^2_{\text{CP}^1} = (1 + \tilde{r}^2)^{-2} \left( d\tilde{r}^2 + \tilde{r}^2 d\tilde{\theta}^2 \right) \]

• Solution

\[ h(x, \tilde{r}, \tilde{\theta}) = \frac{\lambda}{2} x_\mu x^\mu + \tilde{a}_\mu x^\mu + \tilde{c}_1 \ln \tilde{r} + \tilde{c}_2 \tilde{\theta} + \tilde{c}_3 \]
For the CP$^2$ space

$$
\begin{align*}
    ds_{\text{CP}^2}^2 &= (1 + \bar{\rho}^2)^{-2} \, d\bar{\rho}^2 + \frac{\rho^2}{4} \, (1 + \bar{\rho}^2)^{-2} \left( d\psi + \cos \theta \, d\phi \right)^2 \\
    &\quad + \frac{\rho^2}{4} \, (1 + \bar{\rho}^2)^{-1} \left( d\theta^2 + \sin^2 \theta \, d\phi^2 \right)
\end{align*}
$$

• Solution

$$
    h(x, \rho, \theta) = \frac{\lambda}{2} x_\mu x^\mu + \bar{a}_\mu x^\mu + \bar{c}_1 \left( -\frac{1}{2\rho^2} + \ln \bar{\rho} \right) + \bar{c}_2 \ln \tan \frac{\theta}{2} + \bar{c}_3
$$
[4] Discussion and remarks

(1) Extension to dynamical p-brane

• Dynamical p-brane on the complex line bundle over CP^n space.

• We will be able to describe p-brane collision.

(2) Black hole

• Black hole on orbifolds (complex line bundle over CP^n space).

• Time dependent black hole.