Dynamical branes on expanding orbifold and complex projective space

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arXiv:2211.13501 [hep-th]

[1] Introduction and our results

- We construct a dynamical p-brane solution on orbifold (the complex line bundle over CP^n space) as a solution.

- There are interesting properties when branes are located at an orbifold.
  (M.R. Douglas and G.W. Moore, hep-th/9603167)
  (M.R. Douglas, B.R. Greene and D.R. Morrison, Nucl. Phys.B 506 (1997) 84 [hep-th/9704151])

~ Orbifold singularities are resolved in the D-brane world volume theory.

~ One can expect that the spacetime itself becomes regular without any naked singularity once p-branes are placed at the orbifold singularities.

- This is important when one finds hints that the gravity theory giving a p-brane on the orbifold is known and well-defined supergravity, thus giving a handle on the strong coupling dynamics of string theory.

- Black hole on the Eguchi-Hanson space

(H. Ishihara, M. Kimura, K. Matsuno and S. Tomizawa,Phys. Rev. D 74 (2006) 047501 [hep-th/0607035])
(Hideki Ishihara, Masashi Kimura, Shinya Tomizawa, Class.Quant.Grav.23 (2006) L89 [hep-th/0609165])
(Chul-Moon Yoo, Hideki Ishihara, Masashi Kimura, Ken Matsuno, Shinya Tomizawa, Class.Quant.Grav. 25 (2008) 095017 [0708.0708 [gr-qc])

- Black holes on the complex line bundle over CP^n space

(M. Nitta, K. Uzawa, Eur.Phys.J.C 81 (2021) 6, 513, [arXiv:2011.13316 [hep-th]])

- (Static) Black p-brane on the orbifolds

(M. Nitta, K. Uzawa, JHEP 03 (2021) 018, [arXiv:2012.13285 [hep-th]])

[2] Dynamical p-brane on orbifolds

- Metric of dynamical p-brane in D dimensions

(Pierre Binetruy, Misao Sasaki, Kunihito Uzawa, Phys.Rev.D 80 (2009) 026001, 0712.3615 [hep-th])
(Kei-ichi Maeda, Nobuyoshi Ohta, Kunihito Uzawa, JHEP 06 (2009) 051, 0903.5483 [hep-th])
(Masato Minamitsuji, Nobuyoshi Ohta, Kunihito Uzawa, Phys. Rev. D 82 (2010) 086002)

$$ds^2 = h^a(x,\ y)\, q_{\mu\nu}\,(\mathrm{X})\, dx^\mu dx^\nu + h^b(x,\ y)\, u_{ij}\,(\mathrm{Y})\, dy^i dy^j,$$

orbifold

$$a = -\frac{D-p-3}{D-2}, \qquad b = \frac{p+1}{D-2},$$

- (p+2)-form field strength

$$F_{(p+2)} = d\left(h^{-1}\right) \wedge dt \wedge dx^1 \wedge \cdots \wedge dx^p$$

- It is straightforward to check that with such an ansatz the Bianchi identity is trivially satisfied.

- The field equation for the antisymmetric tensor becomes

(Kei-ichi Maeda, Nobuyoshi Ohta, Kunihito Uzawa, JHEP 06 (2009) 051)
(Masato Minamitsuji, Nobuyoshi Ohta, Kunihito Uzawa, Phys. Rev. D 82 (2010) 086002)

$$\partial_\mu \partial_i h = 0\,, \qquad \triangle_{\rm Y} h = 0\,,$$

$$\rightarrow \quad h(x,y) = h_0(x) + h_1(y)\,, \qquad \triangle_{\rm Y} h_1 = 0\,,$$

- Einstein equations

$$R_{\mu\nu}(\mathrm{X}) - h^{-1}D_\mu D_\nu h_0 - \frac{a}{2}h^{-1}q_{\mu\nu}\left(\triangle_{\mathrm{X}}h_0 + h^{-1}\triangle_{\mathrm{Y}}h_1\right) = 0\,,$$

$$R_{ij}(\mathrm{Y}) - \frac{b}{2}q_{\mu\nu}\left(\triangle_{\mathrm{X}}h_0 + h^{-1}\triangle_{\mathrm{Y}}h_0\right) = 0\,,$$

$$\Rightarrow \quad R_{\mu\nu}(\mathrm{X}) = 0\,, \quad R_{ij}(\mathrm{Y}) = \frac{1}{2}b\,(p+1)\,\lambda\,u_{ij}(\mathrm{Y})\,, \quad D_\mu D_\nu h_0 = \lambda\,q_{\mu\nu}(\mathrm{X})\,,$$

- The space Y is not Ricci flat, but the Einstein space such as CP^n if $\lambda \neq 0$, and the function h can be more non-trivial.

(Pierre Binetruy, Misao Sasaki, Kunihito Uzawa, Phys.Rev.D 80 (2009) 026001 0712.3615 [hep-th])
(Kei-ichi Maeda, Nobuyoshi Ohta, Kunihito Uzawa, JHEP 06 (2009) 051, arXiv: 0903.5483 [hep-th])

If we set

$$q_{\mu\nu}(\mathrm{X}) = \eta_{\mu\nu}(\mathrm{X}), \qquad D_\mu h_0 \neq 0, \qquad (D_\mu h_0)(D^\mu h_0) \neq 0,$$

the solution for h_0 is given by

$$h_0(x) = \frac{\lambda}{2} x_\mu x^\mu + \bar{a}_\mu x^\mu + \bar{a}$$

- When the space Y is Ricci flat like orbifold, the function h_0 is linear in the coordinates x^μ because of $\lambda=0$.

♦ Dynamical p-brane on the orbifold

cf) Black holes on Eguchi-Hanson space :
(H. Ishihara, M. Kimura and S. Tomizawa,
Class.Quant.Grav. 23 (2006) L89 [hep-th/0609165])

Eguchi-Hanson space – complex line bundle over CP^1 (2-sphere)

- p-brane on orbifold

$$u_{ij}(\mathrm{Y})dy^i dy^j = dr^2 + r^2 \left[\left\{ d\rho + \sin^2 \xi_{n-1} \left(d\psi_{n-1} + \frac{1}{2(n-1)}\omega_n \right) \right\}^2 + ds^2_{\mathbb{C}\mathrm{P}^{n-1}} \right]$$

If we impose $h_1 = h_1(r)$, the field equations become for

$$\triangle_{\mathrm{Y}} h_1(r) = 0\,, \quad \Rightarrow \quad h_1(r) = b_1 + \frac{b_2}{r^{D-p-3}}$$

the solution for $h(x, r)$ is given by

$$h(x, r) = \bar{a}_\mu x^\mu + b_1 + \frac{b_2}{r^{D-p-3}}$$

- There is a naked singularity at h=0.

(1) $r \to 0$: static p-brane solution.

(2) $r \to \infty$: asymptotically Kasner geometry

[3] Dynamical solution on the CP^n space

- Metric

CP^n space

$$ds^2 = h^a(x,\ y)\, q_{\mu\nu}\,(\mathrm{X})\, dx^\mu dx^\nu + h^b(x,\ y)\, u_{ij}\,(\mathrm{Y})\, dy^i dy^j ,$$

$$a = -\frac{D-p-3}{D-2}\,, \qquad b = \frac{p+1}{D-2}\,,$$

- CP^n is the Einstein space (NOT Ricci flat).

- Our universe depends on the quadratic function of time.

For the CP[1] space

$$ds^2_{\mathbb{C}\mathrm{P}^1} = \left(1+\tilde{r}^2\right)^{-2}\left(d\tilde{r}^2 + \tilde{r}^2 d\tilde{\theta}^2\right)$$

- Solution

$$h(x,\tilde{r},\,\tilde{\theta}) = \frac{\lambda}{2}x_\mu x^\mu + \bar{a}_\mu x^\mu + \tilde{c}_1 \ln\tilde{r} + \tilde{c}_2\,\tilde{\theta} + \tilde{c}_3$$

For the CP² space

$$ds^2_{\mathbb{CP}^2} = \left(1+\bar{\rho}^2\right)^{-2} d\bar{\rho}^2 + \frac{\bar{\rho}^2}{4}\left(1+\bar{\rho}^2\right)^{-2}\left(d\psi + \cos\theta d\phi\right)^2 + \frac{\bar{\rho}^2}{4}\left(1+\bar{\rho}^2\right)^{-1}\left(d\theta^2 + \sin^2\theta d\phi^2\right)$$

- Solution

$$h(x,\rho,\theta) = \frac{\lambda}{2} x_\mu x^\mu + \bar{a}_\mu x^\mu + \bar{c}_1\left(-\frac{1}{2\bar{\rho}^2} + \ln\bar{\rho}\right) + \bar{c}_2 \ln\tan\frac{\theta}{2} + \bar{c}_3$$

[4] Discussion and remarks

(1) Extension to dynamical p~brane

- Dynamical p~brane on the complex line bundle over CP^n space.

- We will be able to describe p~brane collision.

(2) Black hole

- Black hole on orbifolds (complex line bundle over CP^n space).

- Time dependent black hole.