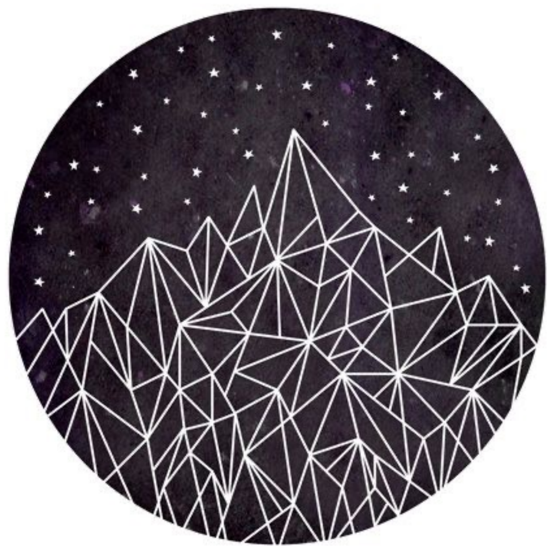
Polymeric quantisation of the interior of a Schwarzschild black hole



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AOS effective model

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- ✓ It provides an effective description of a Schwarzschild black hole (focus on the interior region).
- ✓ Gravitational degrees of freedom are described by means of Ashtekar-Barbero variables.

$$\{b, p_b\} = G\gamma, \qquad \{c, p_c\} = 2G\gamma.$$

- ✓ It is claimed to capture loop quantum corrections through the inclusion of two polymerisation parameters: $\delta_b,\ \delta_c$.
- ✓ The only nontrivial constraint is the effective Hamiltonian.

$$H_{\text{eff}}^{\text{AOS}} = \frac{L_o}{G}(O_b - O_c),$$

$$O_b = -\frac{1}{2\gamma}\left(\frac{\sin\delta_b b}{\delta_b} + \frac{\gamma^2\delta_b}{\sin\delta_b b}\right)\frac{p_b}{L_o}, \quad O_c = \frac{1}{\gamma}\frac{\sin\delta_c c}{\delta_c}\frac{p_c}{L_o}.$$

- ✓ The dynamics of the model leads to a replacement of the classical singularity with a regular transition surface.

Quantum kinematics

- ✓ We consider a Kantowski-Sachs cosmology (fit to describe the interior region).

$$H^{\mathrm{KS}} = \frac{L_o}{G}\left[O_b^{\mathrm{KS}}(b, p_b) - O_c^{\mathrm{KS}}(c, p_c)\right].$$

- ✓ We perform a phase-space extension: $\{\delta_b, p_{\delta_b}\} = 1, \ \{\delta_c, p_{\delta_c}\} = 1.$

- ✓ The additional canonical variables are bound by constraints that dictate their dependence on the rest of the phase space (constants of motion):

$$\Psi_b^{\mathrm{KS}} = \mathcal{K}_b(O_b^{\mathrm{KS}}, O_c^{\mathrm{KS}}) - \delta_b = 0,$$
$$\Psi_c^{\mathrm{KS}} = \mathcal{K}_c(O_b^{\mathrm{KS}}, O_c^{\mathrm{KS}}) - \delta_c = 0.$$

- ✓ We select the extended formalism as the starting point of the quantisation procedure.

$$\mathcal{N}_{\mu_b} = e^{i\mu_b b/2}$$

- ✓ We find a representation of the analogue of the holonomy-flux algebra using techniques inspired by LQC ⟶ polymeric representation.

$$\hat{p}_b|\mu_b, \mu_c\rangle = \frac{G\gamma\mu_b}{2}|\mu_b, \mu_c\rangle, \quad \hat{\mathcal{N}}_{\mu_b'}|\mu_b, \mu_c\rangle = |\mu_b + \mu_b', \mu_c\rangle.$$

$$\mathcal{H}_T^{\mathrm{kin}} = \mathcal{H}_{\mathrm{LQC}}^{\mathrm{kin}} \otimes L^2(\mathbb{R}, d\delta_b) \otimes L^2(\mathbb{R}, d\delta_c).$$

Quantum constraints

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- ✓ Next step: promote the constraints of the system to quantum operators.
- ✓ We need to find representations of the partial Hamiltonians.
- ✓ A regularisation is required in order to rewrite these objects in terms of holonomies of the connection, since the connection itself has no quantum analogue.
- ✓ A standard procedure can be shown to lead to regularised expressions that are identical to the AOS ones.
- ✓ Remarkably, only two geometrical operators need to be introduced:

$$\hat{\Omega}_b^{\delta_b} = \frac{1}{2\delta_b} |\hat{p}_b|^{1/2} \left[\widehat{\mathrm{sgn}(p_b)} \widehat{\sin \delta_b b} + \widehat{\sin \delta_b b}\, \widehat{\mathrm{sgn}(p_b)} \right] |\hat{p}_b|^{1/2}.$$

fixed and nonzero

- ✓ In each eigenspace of the operators that represent the parameters:

$$\hat{O}_b^{(\delta_b)} |\mu_b, \mu_c, \delta_b, \delta_c\rangle = -\frac{1}{2\gamma L_o} \left[\hat{\Omega}_b^{\delta_b} + \gamma^2 |\hat{p}_b| \left(\hat{\Omega}_b^{\delta_b} \right)^{-1} |\hat{p}_b| \right] |\mu_b, \mu_c, \delta_b, \delta_c\rangle,$$

$$\hat{O}_c^{(\delta_c)} |\mu_b, \mu_c, \delta_b, \delta_c\rangle = \frac{1}{\gamma L_o} \hat{\Omega}_c^{\delta_c} |\mu_b, \mu_c, \delta_b, \delta_c\rangle.$$

$$\hat{H}_{\mathrm{eff}}^{\mathrm{AOS}},\ \hat{\Psi}_b,\ \hat{\Psi}_c$$

Physical states

- ✓ The physical states of the theory must be annihilated by the constraint operators ⟶ quantum counterpart of their classical vanishing.

- ✓ We assume certain reasonable spectral properties for the quantum partial Hamiltonians (essential self-adjointness, discrete point spectrum, absolutely continuous continuous spectrum).

- ✓ We seek these states in the dual space of a dense subset of the kinematical Hilbert space, where any state can be decomposed in terms of the eigenfunctions of the partial Hamiltonian operators (spectral theorem).

- ✓ The quantum vanishing of the constraints imposes certain restrictions on the associated wave functions:

$$\psi(\delta_b, \delta_c, \rho_{\delta_b}, m) = \xi(m)\underbrace{\delta_D[\delta_b - \mathcal{K}_b(m,m)]}_{③}\underbrace{\delta_D[\delta_c - \mathcal{K}_c(m,m)]}_{②}\underbrace{\delta_D(m - \rho_{\delta_b})}_{①}.$$

- ✓ These generalised delta distributions can be integrated out, leading to physical states that are characterised by wave functions of the black hole mass with support on a very specific spectral subset:

① $\mathrm{Sp}_c \Rightarrow \mathrm{ISp}[\delta_b] \subset \mathrm{Sp}_b[\delta_b] \cap \mathrm{Sp}_c,$

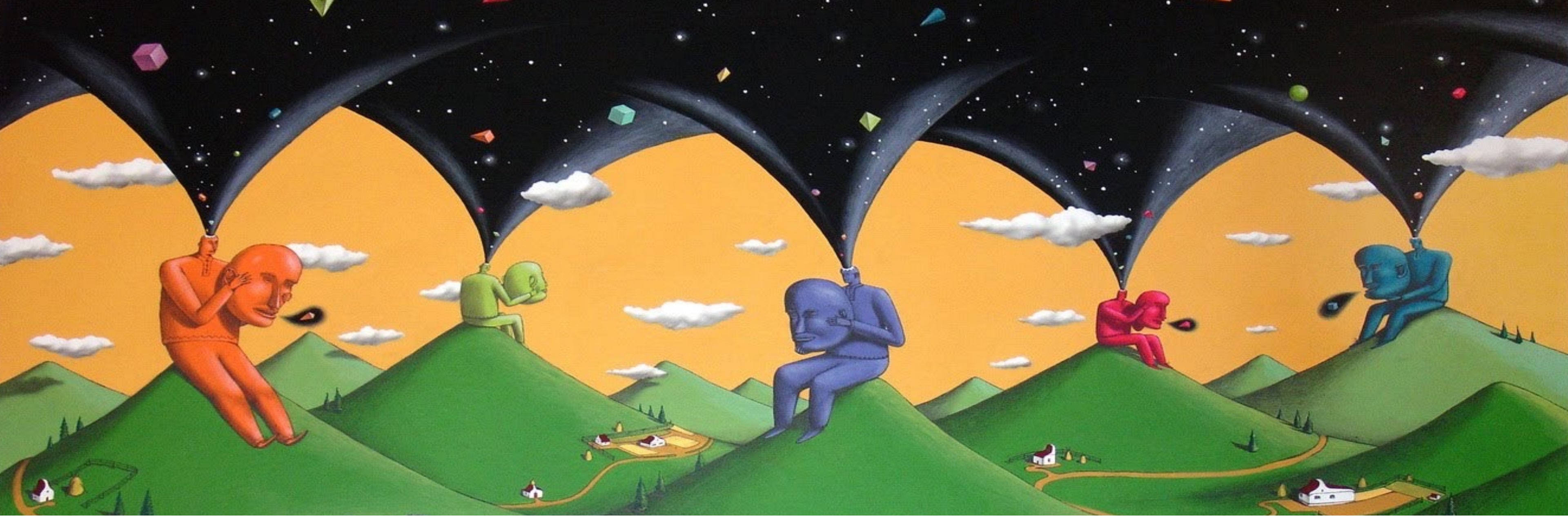
③ $\mathrm{ISp}[\delta_b] \Rightarrow \mathrm{CSp} = \{m \in \mathrm{ISp}[\mathcal{K}_b(m,m)]\}\,.$

$|\delta_b|, |\delta_c| \ll 1$

$|m| \gg m_{\mathrm{Pl}}$

Conclusions

- ✓ We have addressed the quantisation of the interior region of a Schwarzschild black hole using LQC techniques.
- ✓ We have started from a formalism based on an extension of a phase-space description of classical Kantowski-Sachs cosmologies.
- ✓ We have constructed a polymeric representation of the analogue of the holonomy-flux algebra.
- ✓ We have proposed a remarkably simple way to promote the partial Hamiltonians to quantum operators, which suffices to define all the constraints of the system quantum mechanically.
- ✓ We have assumed certain reasonable spectral properties for the partial Hamiltonian operators and sought the physical states of the theory.
- ✓ We have shown that they are characterised by wave functions of the black hole mass with support on a subset of the real line, the form of which is dictated by the spectral properties of the relevant geometrical operators.
- ✓ We have discussed the requirements for the physical states to be able to describe astrophysical black holes.



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