Effective Models of Nonsingular Black Holes

Speaker: Andrea Pierfrancesco Sanna
Based on: Phys. Rev. D **106** (2022), arXiv:2204.09444
In collaboration with: Mariano Cadoni, Mauro Oi

INFN, Sezione di Cagliari, Università degli Studi di Cagliari

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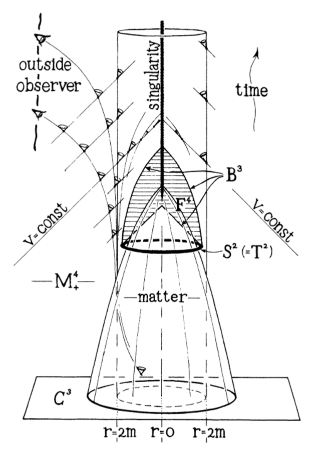
- Validity of Einstein's equations
- Energy conditions
- Global Hyperbolicity



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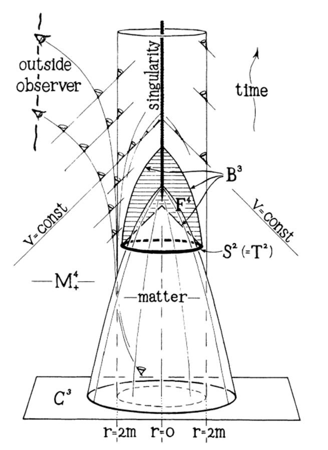

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Relaxing one (or more) assumptions allows to circumvent the theorem and obtain regular spacetimes[Carballo-Rubio et al. (2019)]

Common assumption: corrections to GR at the ultraviolet scale

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- Corpuscular gravity[Dvali & Gomez (2013), Dvali et al. (2021, 2022)]

A lesson from cosmology: the dark matter problem

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Schwarzschild-de Sitter solution

$$f(r) = 1 - \frac{R_S}{r} - \frac{r^2}{L^2}$$

R_S: baryonic-matter scale (**inner horizon**)

L: cosmological scale (**outer horizon**)

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<u>E ective model</u>: GR sourced by an *anisotropic* uid encoding *long-range quantum gravity e ects*[Cadoni et al. (2018-2021)]

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Main idea: use the same (*but reversed!*) principle adopted for the cosmological solution to address the singularity problem

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Solution:

$$e^{(r)} = e^{\quad (r)} \qquad A(r) = 1 \quad \frac{2GM(r)}{r}; \qquad M(r) = 4 \int_0^r \tilde{r}^2 \, d\tilde{r}$$

Imposing a de Sitter core

To eliminate the singularity: glue together a Schwarzschild patch at great distances from the center with a de Sitter patch near $r \to 0$

$$A(r) \xrightarrow{r \to 0} 1 - \frac{r^2}{\ell^2} \qquad\qquad A(r) \xrightarrow{r \to \infty} 1 - \frac{R_S}{r}$$

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ℓ is an additional quantum hair

Several models in the literature

- Bardeen model [Bardeen (1968)]
- Dymnikova model [Dymnikova (1992)]
- Hayward model [Hayward (2005)] [First proposed by Poisson & Israel (1988)]
- Gaussian-core model [Nicolini et al. (2006) ; Modesto et al. (2011)]
- ...
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In some models, the hair ℓ is an UV QG parameter (noncommutative geometry [Nicolini et al. (2006)], LQG effects [Modesto et al. (2011)], others [Rovelli et al. (2014) ; Frolov (2014)])

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$$A(r) = 1 - \frac{R_S}{\ell} F\left(\frac{r}{\ell}\right) \qquad \begin{cases} F(y) \sim y^2 & \text{for } y \to 0 \\ F(y) \sim \frac{1}{y} & \text{for } y \to \infty \end{cases}$$

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Extremal model `$_c$ R_S

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For other interesting thermodynamic features, see
Cadoni, Oi, **APS**, arXiv:2204.09444

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$$A(r) = 1 - \frac{2GMr^2}{(r+\ell)^3} \xrightarrow{r \gg GM} 1 - \frac{2GM}{r} + 6GM\frac{\ell}{r^2}$$

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$$\ell \quad \ell_P \qquad \Rightarrow \quad \frac{6GM\ell}{r^2} \quad 10^{\;\;53}$$

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(e.g. $\ell_c \simeq 0.296\,GM$)

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S2 Observational data

$\ell \quad 0{:}47\,GM$ with 95 % C. L.

[arXiv:2211.11585]

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THANKS FOR YOUR ATTENTION!