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Yukawa Casimir Wormholes

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The traversable wormhole metric

M. S. Morris and K. S. Thorne, Am. J. Phys. 56, 395 (1988).

$$ds^2 = -\exp\left(-2\phi(r)\right)dt^2 + \frac{dr^2}{1 - \frac{b(r)}{r}} + r^2 d\theta^2 + r^2 \sin^2\theta\, d\varphi^2$$

b(r) is the shape function

ϕ*(r)* is the redshift function

$$r \in [r_0, +\infty)$$

Condition

$$b_\pm(r_0) = r_0$$

$$b_\pm(r) < r$$

Proper radial distance

$$l(r) = \pm\int_{r_0}^{r} \frac{dr'}{\sqrt{1 - b_\pm(r')/r'}}$$

$$\lim_{r\to\infty} b_\pm(r) = b_\pm \qquad \text{Appropriate asymptotic}$$

$$\lim_{r\to\infty} \phi_\pm(r) = \phi_\pm \qquad \text{limits}$$

Einstein Field Equations

Orthonormal frame

$$b'(r) = 8\pi G \rho c^2 r^2$$

$$\phi'(r) = \frac{b + 8\pi G p_r r^3}{2r^2\left(1 - b(r)/r\right)} \qquad \tau(r) = -p_r$$

$$p'_r(r) = \frac{2}{r}\left(p_t(r) - p_r(r)\right) - \left(\rho(r) + p_r(r)\right)\phi'(r)$$

Exotic Energy

$$\rho(r) + p_r(r) < 0 \quad r \in [r_0, r_0 + \varepsilon] \iff b'(r) < b(r)/r \quad r \in [r_0, r_0 + \varepsilon]$$

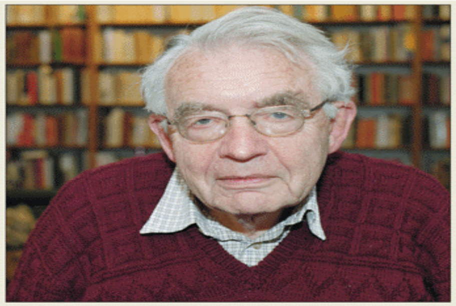
Flare-Out Condition

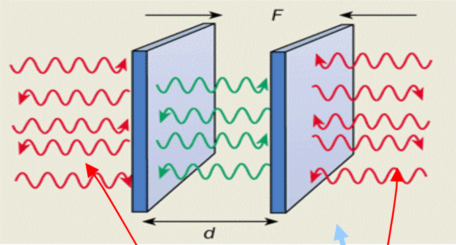
Candidate → Casimir Energy

Casimir Effect

H.B.G.Casimir and D.Polder,
Phys. Rev., 73, 360, 1948

Hendrik Casimir 1909-2000





(ZPE) responsible for the Casimir effect. This was predicted by Casimir [1] and confirmed experimentally in the Philips laboratories†. This is induced when the presence of electrical conductors distorts the zero-point energy of the quantum electrodynamics vacuum. Two parallel conducting surfaces, in a vacuum environment, attract one another by a very weak force that varies inversely as the fourth power of the distance between them. This kind of energy is a purely quantum effect; no real particles are involved, only virtual ones. The difference between the stress–energy computed in the presence and in the absence of the plates with the same boundary conditions gives

$$\Delta\langle T^{\mu\nu}\rangle = \langle T^{\mu\nu}\rangle^{p}_{\mathrm{vac}} - \langle T^{\mu\nu}\rangle_{\mathrm{vac}} = \frac{\pi^2}{720a^4}\begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -3 \end{pmatrix}. \qquad (1)$$

It is evident that separately, each contribution coming from the summation over all possible resonance frequencies of the cavities is divergent and devoid of physical meaning but the *difference* between them in the two situations (with and without the plates) is well defined. Note that the energy density

$$\rho = E/V = \Delta\langle T^{00}\rangle = -\frac{\pi^2}{720a^4}$$

Very Tiny

$$S = 1cm^2$$
$$a = 1\mu m$$

F$\simeq 1.7 \times 10^{-7} N$

Only wavelength less than d

Any wavelength is possible

Take seriously the Casimir Energy → State of the Art

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \quad \longrightarrow \quad G_{\mu\nu} = \frac{8\pi G}{c^4} \langle T_{\mu\nu} \rangle^{Ren}$$

See also

M.S. Morris, K.S. Thorne, U. Yurtsever (Caltech). 1988. 4 pp. Published in Phys.Rev.Lett. 61 (1988) 1446-1449

M. Visser, Lorentzian Wormholes: From Einstein to Hawking (American Institute of Physics, New York), 1995.

$$\rho(a) = -\frac{\hbar c \pi^2}{720 a^4} \qquad p_r(a) = -3\frac{\hbar c \pi^2}{720 a^4} \qquad p_t(a) = \frac{\hbar c \pi^2}{720 a^4} \quad \longrightarrow \quad b(r) = r_0 - \frac{\pi^3}{270 a^4}\left(\frac{\hbar G}{c^3}\right)\left(r^3 - r_0^3\right),$$

This is not a TW because there is no A. Flatness. It is Asymptotically de Sitter

It can be transformed into a TW with the junction condition method matching the solution with the Schwarzschild metric at some point r=c

Take seriously the Casimir Energy→ State of the Art

R.G. Eur.Phys.J.C 79 (2019) 11, 951 ArXiV: 1907.03623 [gr-qc].

When $\omega = \frac{r_0^2}{r_1^2}$ → Traversable Wormhole

$$r_1^2 = \frac{\pi^3 l_p^2}{90}$$

$$\phi(r) = \frac{1}{2}(\omega - 1)\ln\left(\frac{r(\omega+1)}{(\omega r + r_0)}\right) \xrightarrow{\omega = 3} \phi(r) = \ln\left(\frac{4r}{3r + r_0}\right)$$

Planckian

$$b(r) = \left(1 - \frac{1}{\omega}\right) r_0 + \frac{r_0^2}{\omega r} \xrightarrow{\omega = 3} b(r) = \frac{2}{3} r_0 + \frac{r_0^2}{3r}$$

SET $T_{\mu\nu} = \left(\frac{r_0^2}{3\kappa r^4}\right)\left[diag(-1,-3,1,1) + \left(\frac{6r}{3r + r_0}\right) diag(0,0,1,1)\right]$

Other Profile → Generalized Absurdly Benign TW

R.G. *Eur.Phys.J.C* 80 (2020) 12, 1172ArXiV: 2008.05901 [gr-qc]

Identify the Casimir Energy Density

$$b(r) = \frac{1}{r_0^{\alpha-1}}\left[r_0 - \frac{\rho_0\kappa}{3\alpha}\left(r^3 - r_0^3\right)\right]^{\alpha}, \qquad \alpha > 1; \qquad \Phi(r) = 0; \qquad r_0 \le r \le \bar{r}$$

$$b(r) = 0, \qquad \Phi(r) = 0; \qquad r \ge \bar{r},$$

Close to the throat $b\left(r\right) \simeq r_0\left(1 - \frac{r_0 l_P^2 \pi^3}{90 a^4}\left(r - r_0\right)\right)$

Plate separation *nm* →

$$r_0 = \frac{3}{\pi}\sqrt{\frac{10}{\pi}}\frac{a^2}{l_P} \simeq 1.7 \times 10^{17} m$$

Absurdly Benign Traversable Wormhole →

$$b(r) = r_0\left(1 - \left(\frac{r - r_0}{a}\right)\right)^2, \qquad \Phi(r) = 0; \qquad r_0 \le r \le r_0 + a$$

$$b(r) = 0, \qquad \Phi(r) = 0; \qquad r \ge r_0 + a.$$

Plate separation *pm* →

$$r_0 \simeq 10^{11} m.$$

Yukawa-Casimir Wormholes

Motivations

Yukawa in 1935 proposed a potential to describe nonrelativistic strong interactions between nucleons

$$V(r) = -\frac{\alpha}{r}\exp(-\mu r)$$

Yukawa corrections to the Newton potential → $V(r) = -\frac{Gm_1 m_2}{r}\left(1+\alpha\exp(-\mu r)\right)$

such deviations are important also for the Casimir effect

- Nuclear Physics
- Yukawa constraints on the recent measurement of the Casimir force

(Bordag,Gillies and Mostepanenko Phys.Rev.D56:6-10,1997 arXiv:hep-th/9705101)

- Van der Waals forces described in a Yukawa form

(*Milonni The Quantum Vacuum: An Introduction to Quantum Electrodynamics)*

- *Black Holes in Modified Gravity (MOG)*

(*J.W. Moffat, arXiv:1412.5424 [gr-qc])*

- *Modified Theory of Gravity*
- *Many other contexts!!!*

Yukawa-Casimir Wormholes

Eur.Phys.J.C **81**, Article number: 824 (2021) arXiv:2107.09276 [gr-qc]

$$b(r) = \frac{r_0^2}{r} \quad EB\ Wormhole \qquad \Longleftrightarrow \qquad b(r) = \lim_{\mu \to 0} \frac{r_0^2}{r} e^{-\mu(r-r_0)}$$

Yukawa Wormhole

$$\rho(r) = -\frac{r_0^2}{8\pi G r^4} e^{-\mu(r-r_0)}(1+\mu r) \longrightarrow -\frac{1+\mu r_0}{8\pi G r_0^2}$$

$$p_r(r) = -\frac{r_0^2}{8\pi G r^4} e^{-\mu(r-r_0)} \longrightarrow -\frac{1}{8\pi G r_0^2}$$

$$p_t(r) = \frac{r_0^2}{16\pi G r^4} e^{-\mu(r-r_0)}(2+\mu r) \longrightarrow \frac{2+\mu r_0}{16\pi G r_0^2}$$

On the throat

$$M(r) = \frac{r_0^2 \exp\left(-\mu\left(r-r_0\right)\right)}{2Gr} \to 0 \text{ when } r \to \infty$$

Zero Mass Wormhole

But $\mp \frac{\pi r_0}{4G} \lessgtr M^P(r) \lessgtr 0 \ when\ r \to \infty$

$$\pm \frac{\pi r_0}{4G} \gtrless E_G(r) \gtrless 0$$

Total Energy

Out of the throat $b(r)$ and $b'(r) \to 0$ for $\mu \to \infty$

Superposing Yukawa-Casimir Wormholes

Eur.Phys.J.C **81**, Article number: 824 (2021) arXiv:2107.09276 [gr-qc]

$$b(r) = r_0\left(\alpha \exp\left(-\mu\left(r-r_0\right)\right) + (1-\alpha)\left(\frac{r_0}{r}\right)^c \exp\left(-\nu\left(r-r_0\right)\right)\right)$$

$$\omega(r) = \frac{r_0\left(\alpha \exp\left(-\mu\left(r-r_0\right)\right) + (1-\alpha)\left(\frac{r_0}{r}\right)^c \exp\left(-\nu\left(r-r_0\right)\right)\right)}{r\left(r_0\mu\alpha \exp\left(-\mu\left(r-r_0\right)\right) + (1-\alpha)\left(\frac{r_0}{r}\right)^c \exp\left(-\nu\left(r-r_0\right)\right)\left(\nu r_0 + c\frac{r_0}{r}\right)\right)}$$

Zero Tidal Forces

$$\Phi(r) = 0$$

$$r_0 = \frac{3}{\pi}\sqrt{\frac{10}{\pi}}\frac{a^2}{l_P} \simeq 1.7 \times 10^{17} m$$

Plate separation *pm*

$$r_0 \simeq 10^{11} m.$$

Superposing Yukawa-Casimir Wormholes

Reverse Procedure

$$\rho(r)=\frac{r_0\rho_C}{r}\left(\alpha\exp\left(-\mu\left(r-r_0\right)\right)-\left(1-\alpha\right)\exp\left(-\nu\left(r-r_0\right)\right)\right)$$

$$b(r)=r_0+\frac{r_0\left(1+\nu r_0\right)\left(\alpha-1\right)\rho_C\kappa}{\nu^2}+\frac{\alpha r_0\kappa\rho_C\left(1+\mu r_0\right)}{\mu^2}$$
$$-\frac{r_0\kappa\rho_C\left(\nu r+1\right)\left(\alpha-1\right)e^{-\nu\left(r-r_0\right)}}{\nu^2}-\frac{\alpha r_0\kappa\rho_C\left(\mu r+1\right)e^{-\mu\left(r-r_0\right)}}{\mu^2}$$

Zero Tidal Forces

$$\Phi(r)=0$$

If $\alpha=\dfrac{\mu^2\left(\kappa\nu r_0\rho_C+\rho_C\kappa-\nu^2\right)}{\left(\mu^2\nu r_0+\mu\nu^2 r_0+\mu^2+\nu^2\right)\rho_C\kappa}.$

$$\Rightarrow\quad b(r)=r_0\frac{\left(e^{-\nu(r-r_0)}\left(\rho\kappa\left(1+\mu r_0\right)+\mu^2\right)\left(\nu r+1\right)-e^{-\mu(r-r_0)}\left(\rho\kappa\left(1+\nu r_0\right)-\nu^2\right)\left(\mu r+1\right)\right)}{\left(\nu r_0+1\right)\mu^2+\mu\nu^2 r_0+\nu^2}$$

Superposing Yukawa-Casimir Wormholes

Reverse Procedure

$$\rho(r)=\frac{r_0\rho}{r}\left(\alpha\exp\left(-\mu\left(r-r_0\right)\right)-\left(1-\alpha\right)\exp\left(-\nu\left(r-r_0\right)\right)\right)$$

$$\kappa=\frac{8\pi G}{c^4}$$

On the throat

$$\omega\left(r_0\right)=\frac{\left(1+\nu r_0\right)\mu^2+\left(1+\mu r_0\right)\nu^2}{2\nu^2 r_0^2\mu^2+r_0^3\rho\kappa\nu\mu\left(\nu-\mu\right)+r_0^2\rho\kappa\left(\nu^2-\mu^2\right)}.$$

Zero Tidal Forces

$$\Phi(r)=0$$

$$\mu=\frac{m}{r_0};\qquad \nu=\frac{n}{r_0}\qquad \text{and}\qquad r_0=\frac{x}{\sqrt{\rho\kappa}};\qquad m,n\in\mathbb{R}_+$$

$$\omega\left(r_0\right)=1$$

Plate separation *pm*

$$r_0\simeq 10^{11}m.$$

$$\frac{\left(1+n\right)m^2+\left(1+m\right)n^2}{\left(2n^2m^2+x^2nm\left(n-m\right)+x^2\left(n^2-m^2\right)\right)}=1,$$

Superposing Yukawa-Casimir Wormholes

Reverse Procedure

$$\rho(r)=\frac{r_0\rho}{r}\left(\alpha\exp\left(-\mu\left(r-r_0\right)\right)-\left(1-\alpha\right)\exp\left(-\nu\left(r-r_0\right)\right)\right)$$

$$\kappa=\frac{8\pi G}{c^4}$$

Solution

$$x=\frac{\sqrt{\left(2n^2-n-1\right)m^2-n^2m-n^2}}{\sqrt{\left(\left(n+1\right)m+n\right)\left(m-n\right)}}.$$

Zero Tidal Forces

$$\Phi\left(r\right)=0$$

$$m_\pm=n\frac{1\pm\sqrt{9n^2-4n-4}}{2\left(2n^2-n-1\right)}$$

$$n\geq\frac{2}{9}\left(1+\sqrt{10}\right)\simeq 0.92495.$$

$$r_0\simeq x\times 10^{17}m.$$

Conclusions and Perspectives

- Casimir energy is the only source of exotic matter that can be generated in laboratory.
- Traversable wormholes can be sustained by Casimir Energy.
- The Wormhole is traversable in principle but not in practice.
- Generalized Absurdly Benign Traversable Wormholes seem to have the right properties for traversability together with Yukawa–Casimir wormholes → Need to be carefully investigated
- Including quantum fluctuations.
- Adding Extra Sources
- TW relevant for GW as BH mimickers.

Thank You for Your Attention

Outlook