The effects of running gravitational coupling on three dimensional black holes

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Outline

- Introduction
- Classical solution
- Scale-dependent Idea
- Black hole solution
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Introduction

Starting point: the Einstein equations

$$\underbrace{R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}}_{\text{Geometry}} = \underbrace{-\Lambda g_{\mu\nu} + 8\pi G T_{\mu\nu}}_{\text{Matter}},$$

describe the Universe from (certain) small scales up to large scales (galaxy clusters and beyond).

	small		λ		large
Theory	???	$\cdots$	Consistent	$\cdots$	???
Observations	???	$\cdots$	Confirmed	$\cdots$	???

Introduction

Einstein's theory is still incomplete.

- Singularities inside black holes
- Cosmological constant problem
- EG does not describe small scale physics.

EG is usually considered as an **effective theory**. In order to obtain a complete description of our Universe, new physics is required.

Introduction

The simplest modification of the Einstein-Hilbert action is

$$I[\cdots] = \frac{1}{2\kappa}\int \mathrm{d}^D x\sqrt{-g}\;\mathrm{R}(\;\Gamma\;,\;\;g\;)+S_M,$$

G variable

Extra dimensions, Non-Conmutative space-time

Unimodular & Mimetic gravity

$f(R)$ $f(R_{\mu\nu}R^{\mu\nu})$

$f(T)$ $\cdots$

Massive gravity

Scalar-Vector-Tensor, Galileon

Introduction

Gravity + Quantum Mechanics

Quantum gravity

↓

Observables

1. Black holes
2. Cosmology

Introduction

$$\text{Approaches} \begin{cases} \text{Perturbative} \\ \text{Non perturbative} \\ \text{String Theory} \ldots \end{cases}$$

Particularly, in a QG **theory, the action is now scale dependent**

$$S \to \Gamma_k$$

Introduction

Quantum gravity allows us to get insights in the Black Hole theory...

At low energies, the resulting effective action of gravity shows us a **scale dependence**.

The couplings, which appearing in the effective action, evolve and depend on the scale, i.e., $G_0 \to G_k$ and $e_0 \to e_k$.

Classical Action ($J_0 = 0$)

The gravitational action in three dimensions is

$$I_0[g_{\mu\nu}] = \int d^3x\sqrt{-g}\left[\frac{1}{2\kappa_0}\Big(R - 2\Lambda_0\Big)\right],$$

which leads to

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = -\Lambda_0 g_{\mu\nu},$$

being Λ_0 and $\kappa_0 \equiv 8\pi G_0$ are the cosmological constant and the Einstein's constant respectively.

The line element for a non rotating black hole in (2+1) looks like:

$$ds^2 = -f_0(r)\,dt^2 + f_0(r)^{-1}\,dr^2 + r^2 d\phi^2,$$

Classical BTZ Solution ($J_0 = 0$)

With the solution given by

$$f_0(r) = -G_0 M_0 + \frac{r^2}{\ell_0^2}.$$

The entropy and temperature are:

$$S_0 = \frac{\mathcal{A}_H}{4G_0}\ , \qquad T_0 = \frac{1}{4\pi}\left|\frac{2M_0G_0}{r_0^+}\right|,$$

where $\Lambda_0 \equiv -1/\ell_0^2$ and M_0 is the mass of Black Hole.

Please, note that the horizon is given by the condition $f_0(r_H) = 0$

$$r_0^+ = \pm\sqrt{G_0 M_0}\ \ell_0,$$

Classical Rotating BTZ Solution

Considering a shift function

$$ds^2 = -f_0(r)\,dt^2 + f_0(r)^{-1}\,dr^2 + r^2\Big(N_0(r)dt + d\phi\Big)^2,$$

Where we have

$$f_0(r) = -G_0M_0 + \frac{r^2}{\ell_0^2} + \frac{G_0^2J_0^2}{4r^2},$$

$$N_0(r) = -\frac{G_0J_0}{2r^2}.$$

And J_0 is the angular momentum.

Action with Running Couplings

The gravitational action in three dimensions is

$$\Gamma[g_{\mu\nu}, k] = \int d^3x \sqrt{-g} \left[\frac{1}{2\kappa_k} \Big(R - 2\Lambda_k \Big) \right].$$

Thus, varying with respect to the metric field

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = -\Lambda_k g_{\mu\nu} + \kappa_k T_{\mu\nu},$$

where the **effective** energy-momentum tensor is given by

$$\kappa_k T_{\mu\nu} = \kappa_k T^m_{\mu\nu} - \Delta t_{\mu\nu}$$

being the new term:

$$\Delta t_{\mu\nu} = G_k \Big(g_{\mu\nu} \Box - \nabla_\mu \nabla_\nu \Big) G_k^{-1}$$

Beta function of $G(r)$

The beta function for the Newton's coupling in three dimensions is

$$\beta_g = \Big(1 + \eta_N\Big), \qquad \eta_N = \frac{B_1 g}{1 - g B_2}.$$

In the limit of $|B_1| >> |B_2|$ we obtain

$$G(r) = \frac{G_0}{1 + G_0 B_1 k(r)}, \qquad k(r) = \frac{\xi^2}{G_0 B_1 r^2}$$

And finally, the Newton's function is then

$$G(r) = G_0 \left[1 + \left(\frac{\xi}{r}\right)^2\right]^{-1}$$

Rotating Scale-dependent Solution

The line element is

$$ds^2 = -f(r)\,dt^2 + h(r)\,dr^2 + r^2\Big(N(r)dt + d\phi\Big)^2,$$

And we should solve the set $\{f(r), h(r), N(r), G(r), \Lambda(r)\}$

i) From gravitational beta function, we get $G(r)$

ii) From effective Einstein Equations, we obtain the remaining functions $\{f(r), h(r), N(r), \Lambda(r)\}$

Rotating Scale-dependent Solution

The unknown functions are

$$f(r) = -M(r) + \frac{r^2}{\ell_0^2} + \frac{J(r)^2}{4r^2}$$

$$M(r) = M_0\delta(r,\xi), \qquad J(r)^2 = J_0^2\delta(r,\xi)^2.$$

$$h(r) = \left[1 - \left(\frac{\xi}{r}\right)^2\right]^6 f(r)^{-1}$$

$$N(r) = N_0(r)\delta(r,\xi),$$

$$\delta(r,\epsilon) = -7 + \frac{2\xi^2}{r^2} - \frac{\xi^4}{3r^4} + \frac{8r^2}{\xi^2}\ln\left(1 + \frac{\xi^2}{r^2}\right).$$

Rotating Scale-dependent Solution

$$\Lambda(r) = \frac{1}{9\ell_0^2\xi^4\left(r^2-\xi^2\right)^6\left(\xi^2+r^2\right)^2}\Bigg[24\ell_0^2 r^8 \ln\left(1+\frac{\xi^2}{r^2}\right)\Bigg(2J_0^2\Big(\xi^8+6\xi^2r^6$$

$$-15\xi^4r^4-8\xi^6r^2\Big)-6J_0^2r^4\left(r^2-3\xi^2\right)\left(\xi^2+r^2\right)\ln\left(1+\frac{\xi^2}{r^2}\right)$$

$$+3M_0\xi^2r^4\left(r^2-3\xi^2\right)\left(\xi^2+r^2\right)\Bigg)+\xi^4\Bigg(-9r^{12}\left(r^2-3\xi^2\right)\left(\xi^2+r^2\right)$$

$$-\left(-3\xi^6+12r^6-39\xi^2r^4+14\xi^4r^2\right)\times J_0^2\ell_0^2(-\xi^6+12r^6+3\xi^2r^4$$

$$+2\xi^4r^2)-12\ell_0^2M_0r^8(\xi^6+6r^6-15\xi^2r^4-8\xi^4r^2)\Bigg)\Bigg].$$

Black Hole Thermodynamics

We found the black hole temperature and the entropy by using the standard relations:

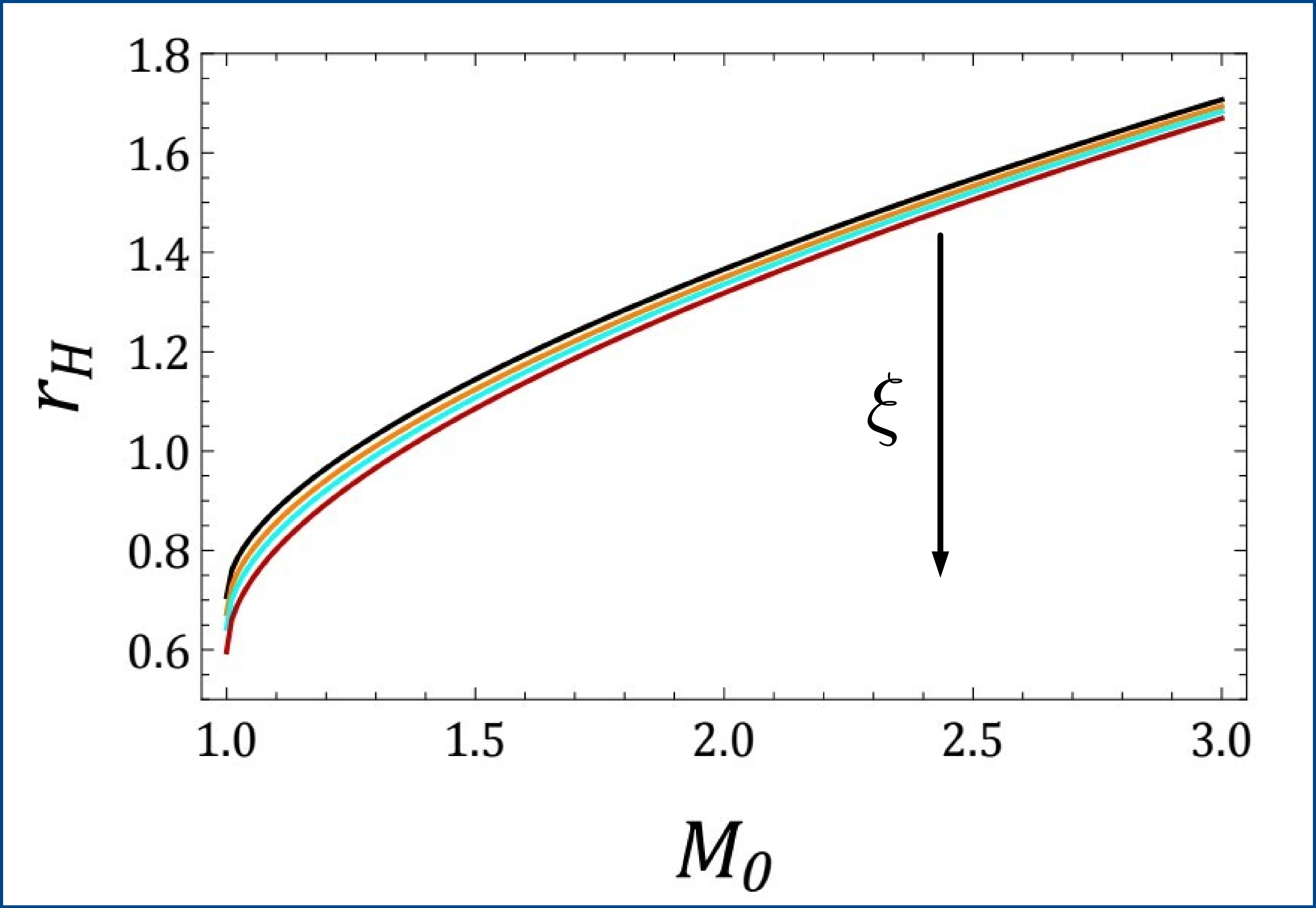
$$T_H(r_H) = \frac{1}{4\pi}\left|\frac{2M_0 G(r_H)}{r_H}\Delta\right|.$$

$$S_H(r_H) = \frac{\mathcal{A}_H(r_H)}{4G(r_H)} = S_0(r_H)\left[1+\left(\frac{\xi}{r_H}\right)^2\right].$$

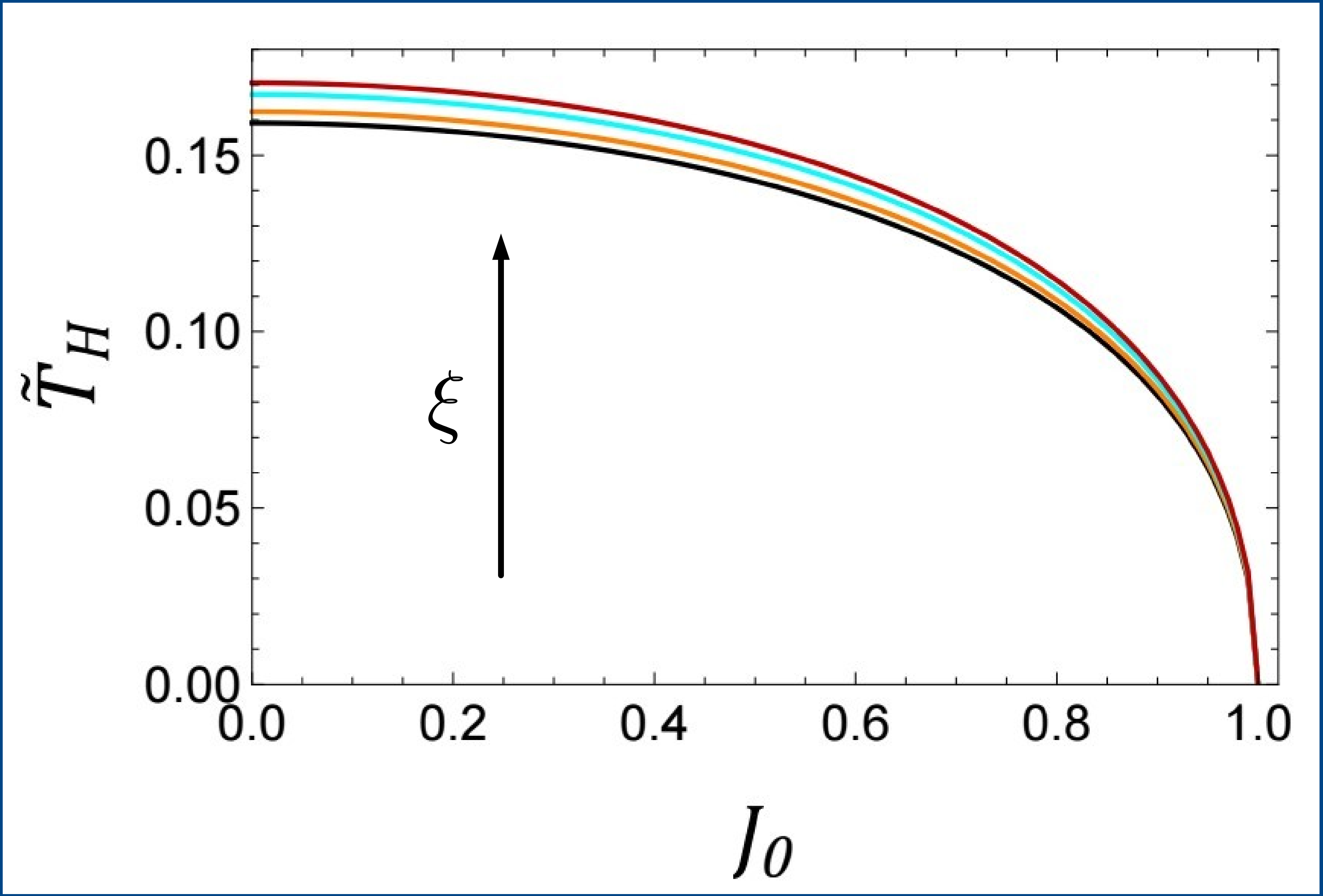
where

$$\Delta = \sqrt{1-\left(\frac{J_0}{M_0\ell_0}\right)^2}.$$

Numerical Results A



Numerical Results B



Take home messages

1- Scale-dependence in 2+1 dimensions slightly modifies the classical black hole solution.

2- From the beta function for the gravitational coupling, we obtain the explicit form of Newton's coupling

3- Classical and scale-dependent black hole solutions have the same critical angular momentum.

4- Rotating scale-dependent solution converges to the classical one when $\xi \to 0$