Tidal response from scattering and the role of analytic continuation

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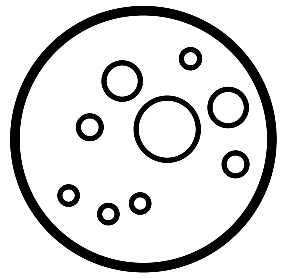
In collaboration with Tanja Hinderer and Jan Steinhoff

Based on arXiV:2108.03385

XV Black Holes Workshop 2022, Lisbon

Motivation

What is the Love number?



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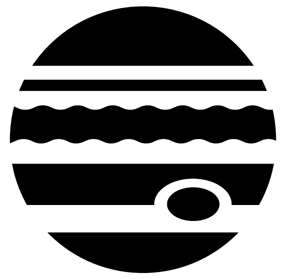
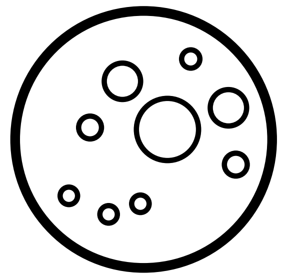
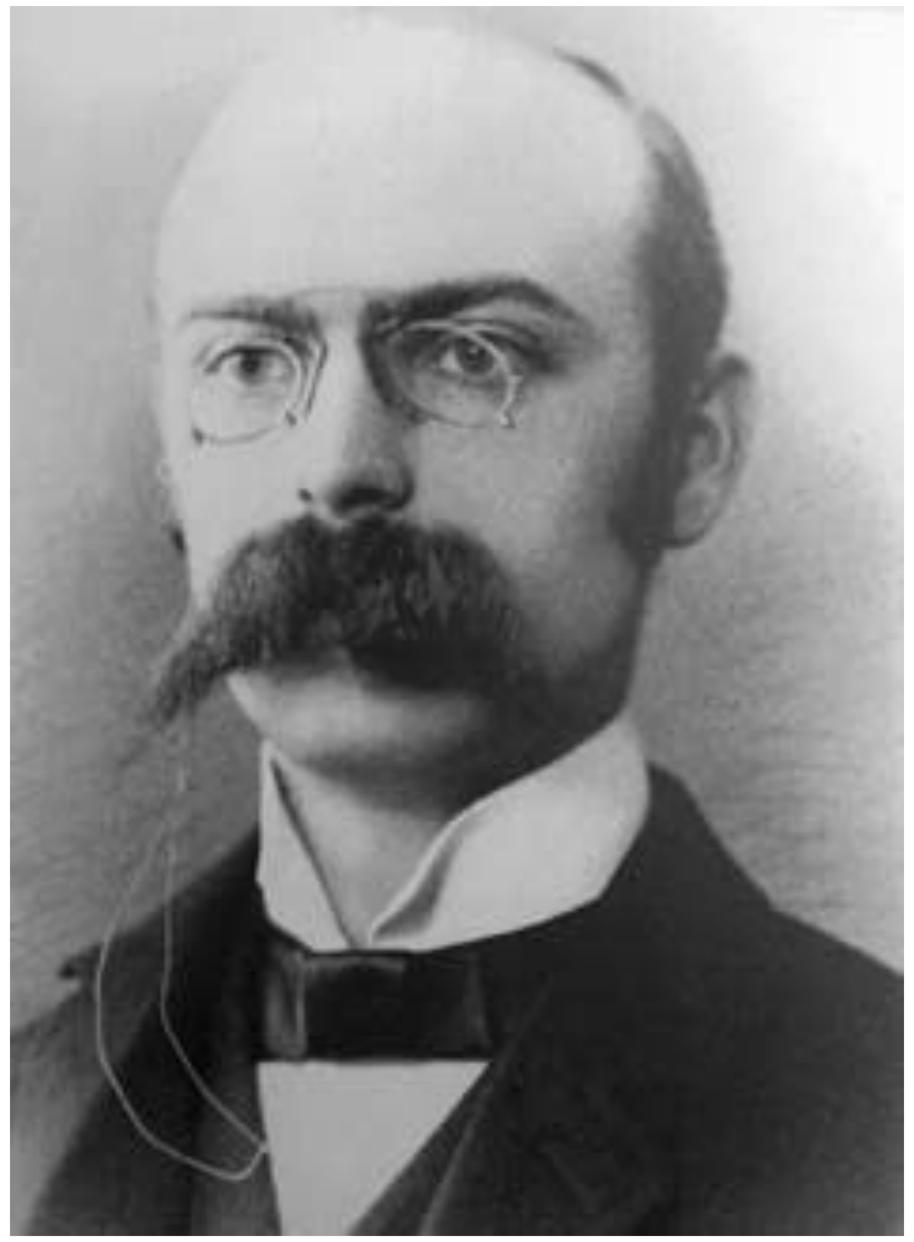
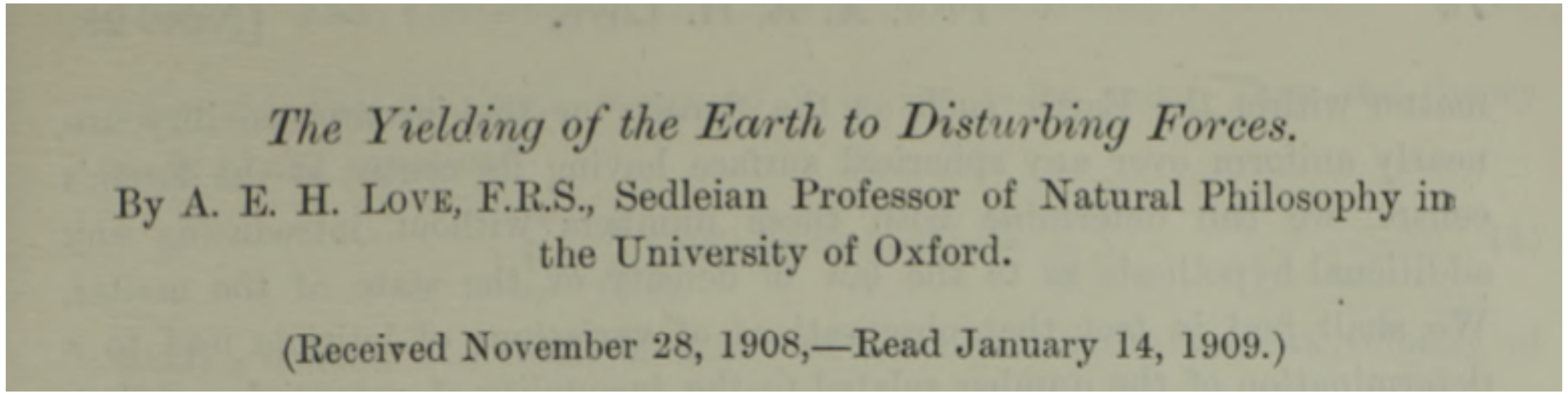






Augustus Edward Hough Love



The Yielding of the Earth to Disturbing Forces.

By A. E. H. Love, F.R.S., Sedleian Professor of Natural Philosophy in the University of Oxford.

(Received November 28, 1908,—Read January 14, 1909.)

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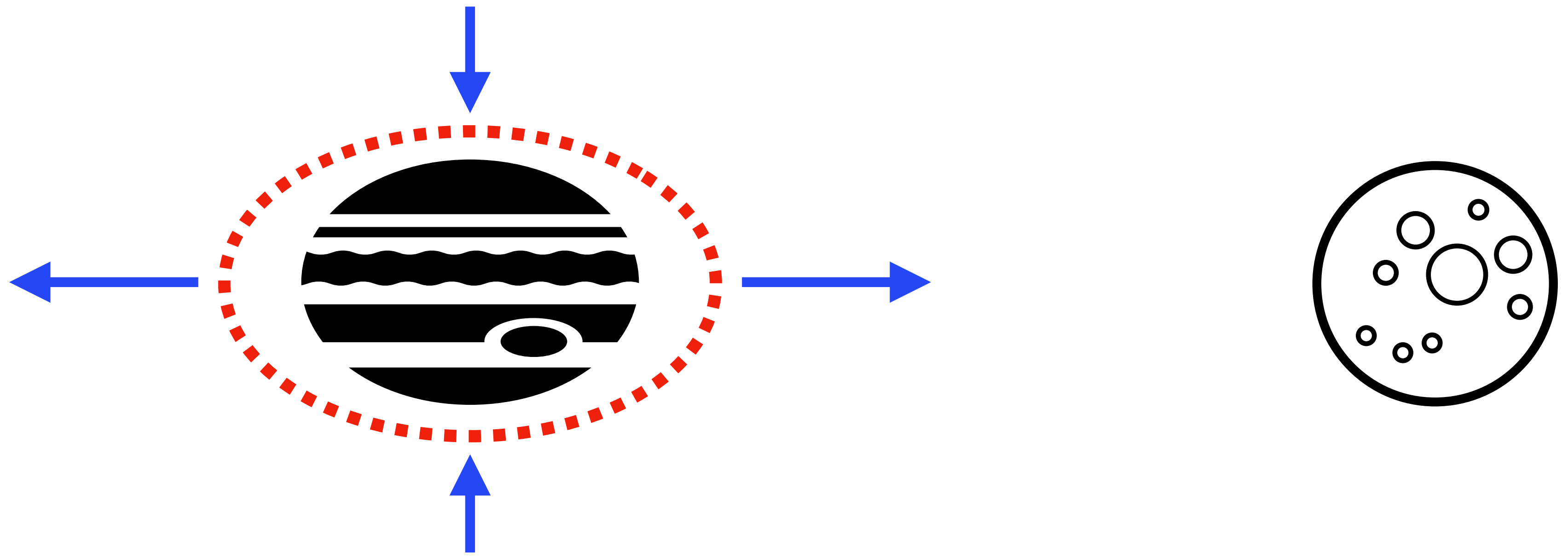
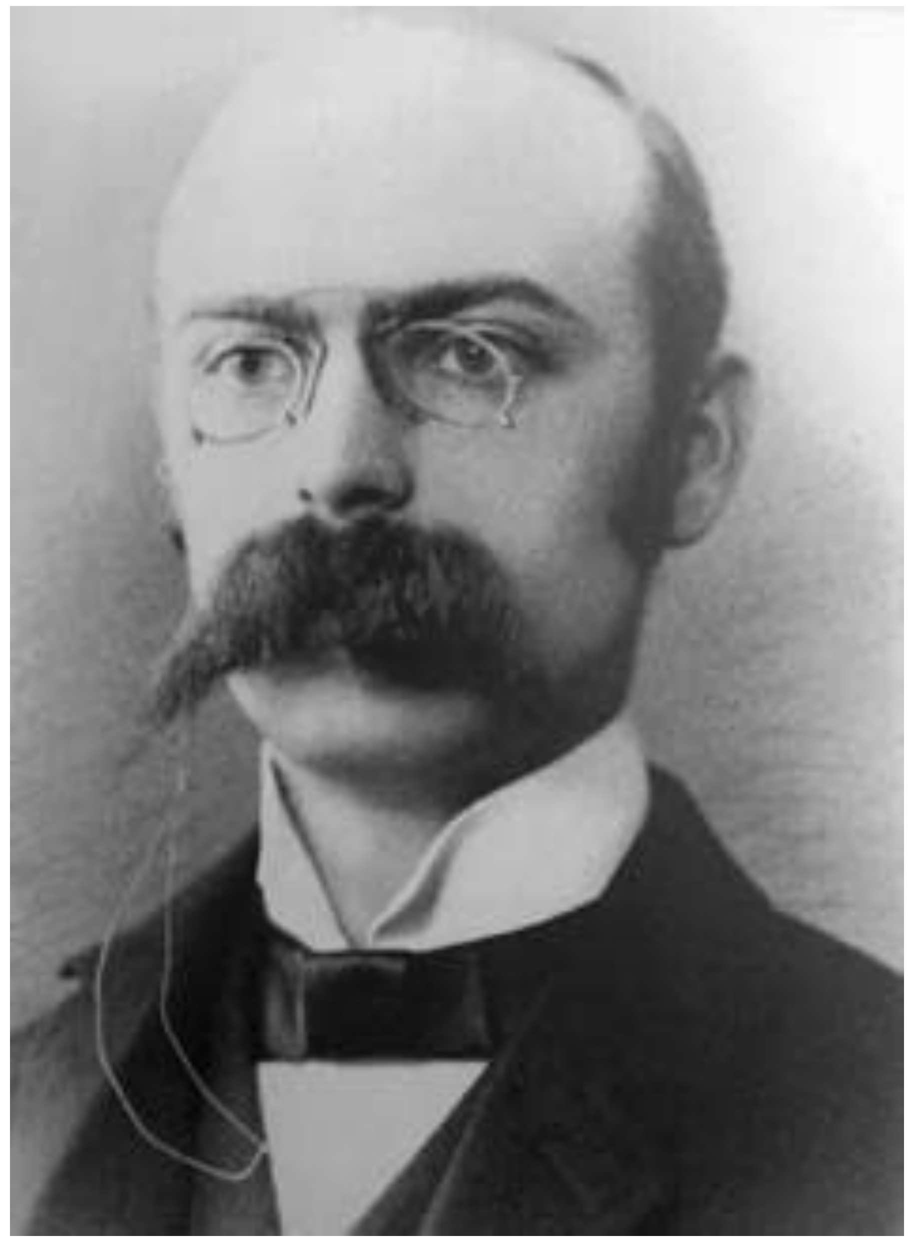


$$\text{Love number } \lambda = \frac{\text{Internal multipole moment } Q}{\text{External tidal field } \mathscr{E}}$$

(Or tidal deformability)



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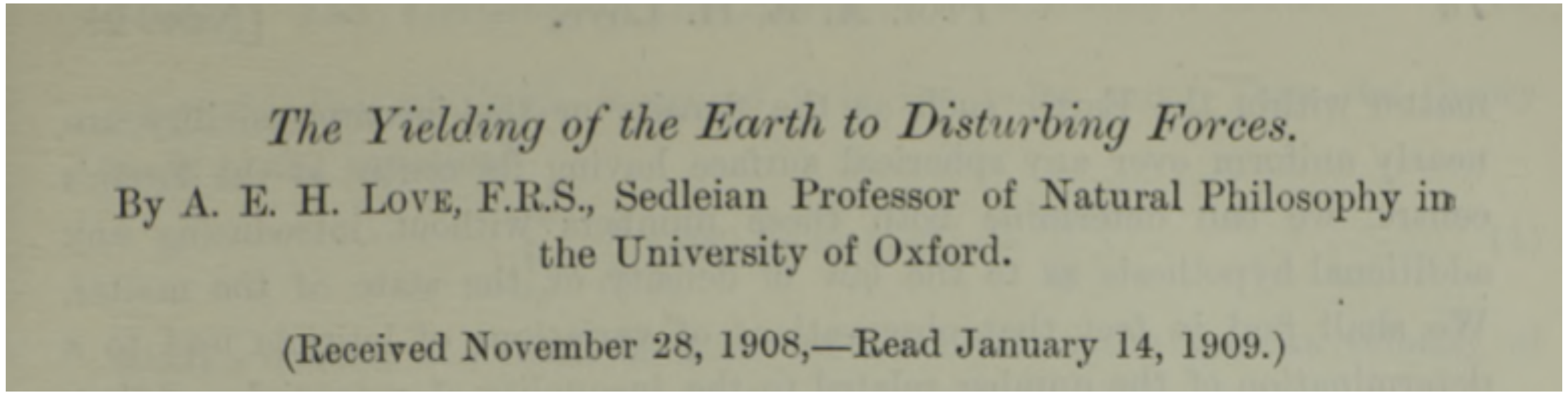

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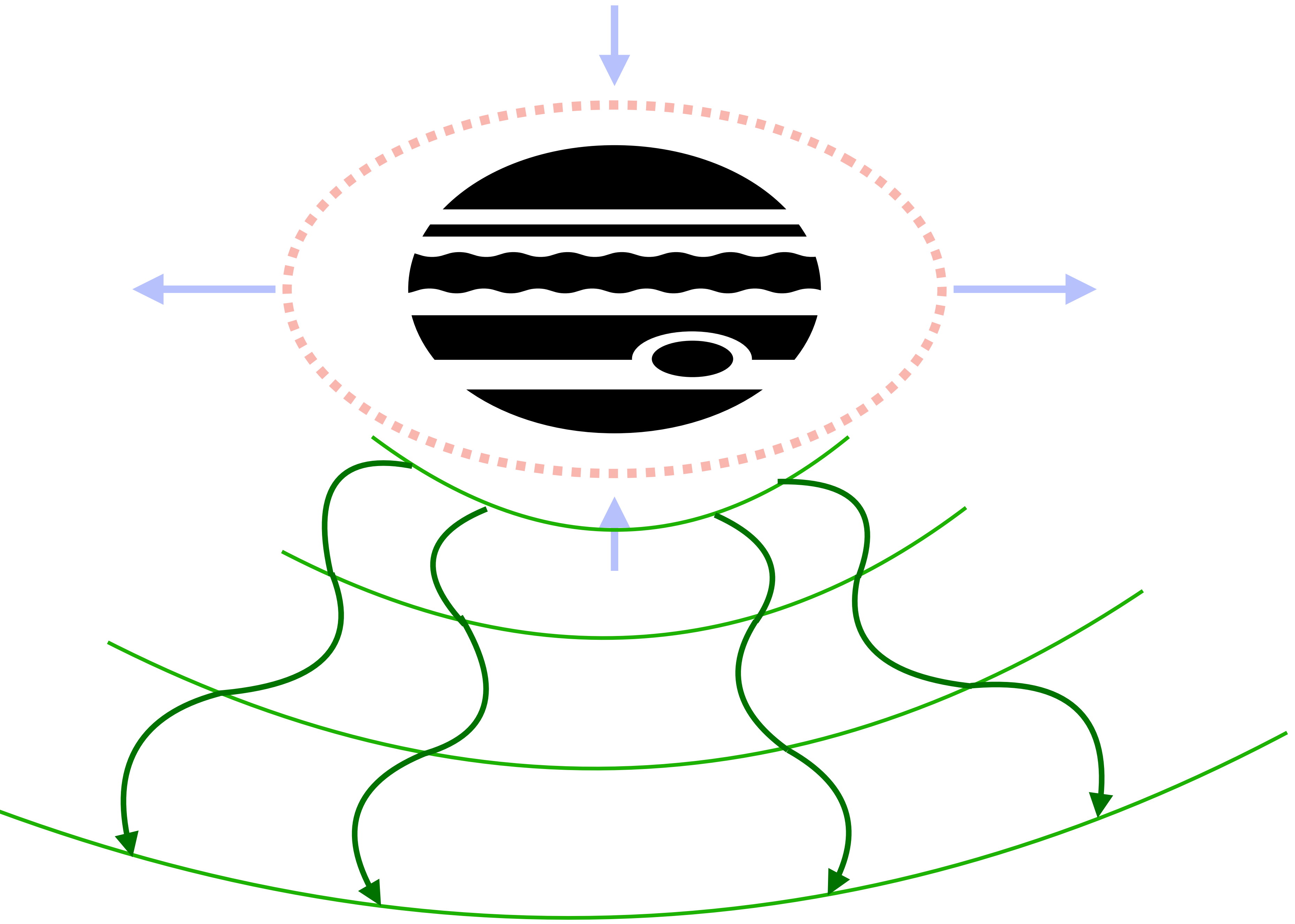
Why is it important?



1. Kind of object/theory

GR Black hole $\lambda_{BH} = 0$

Neutron star $\lambda_{NS} \neq 0$

Exotic object $\lambda_{?} = ?$

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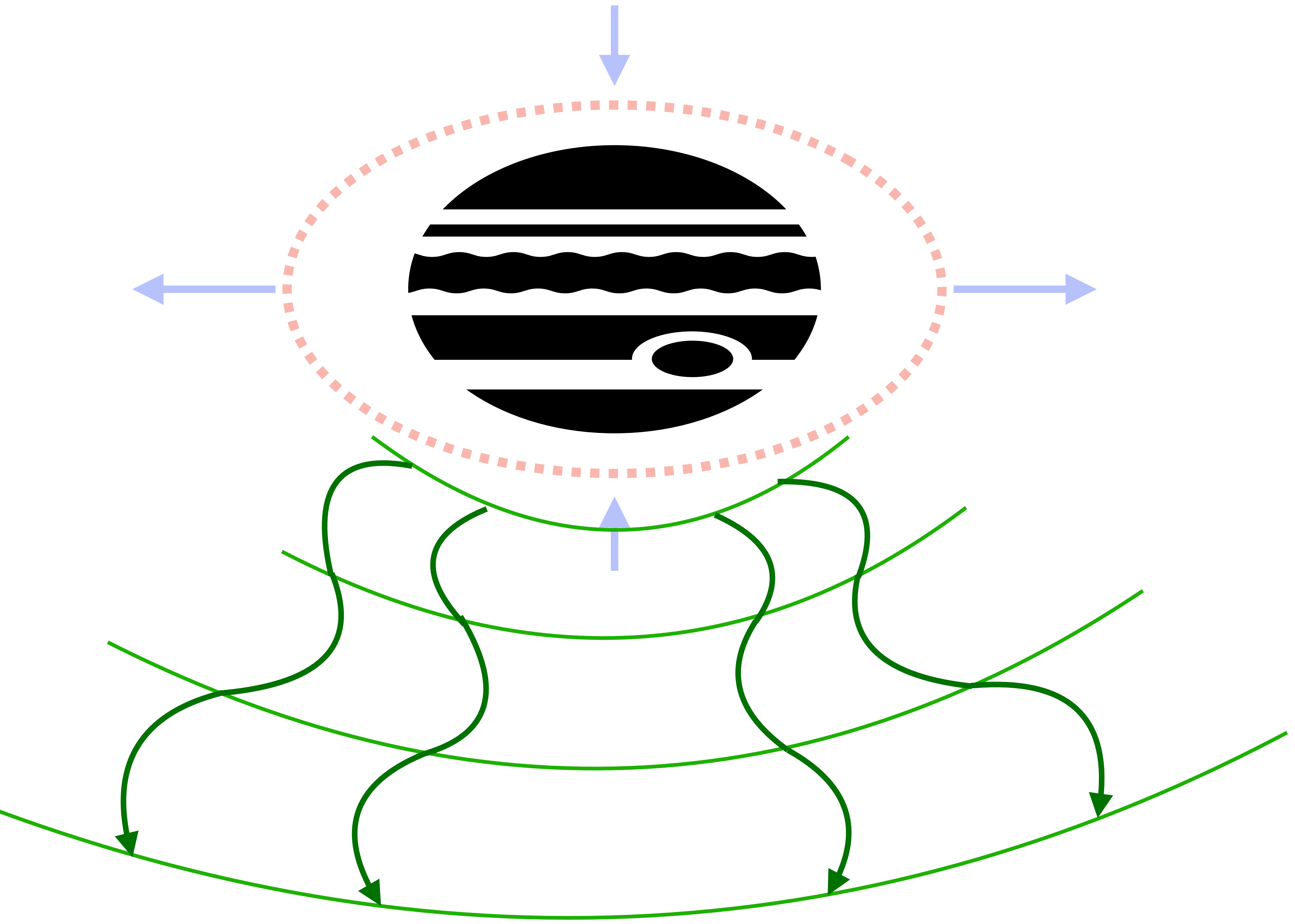

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2. EoS of the object

Neutron star core

QCD implications

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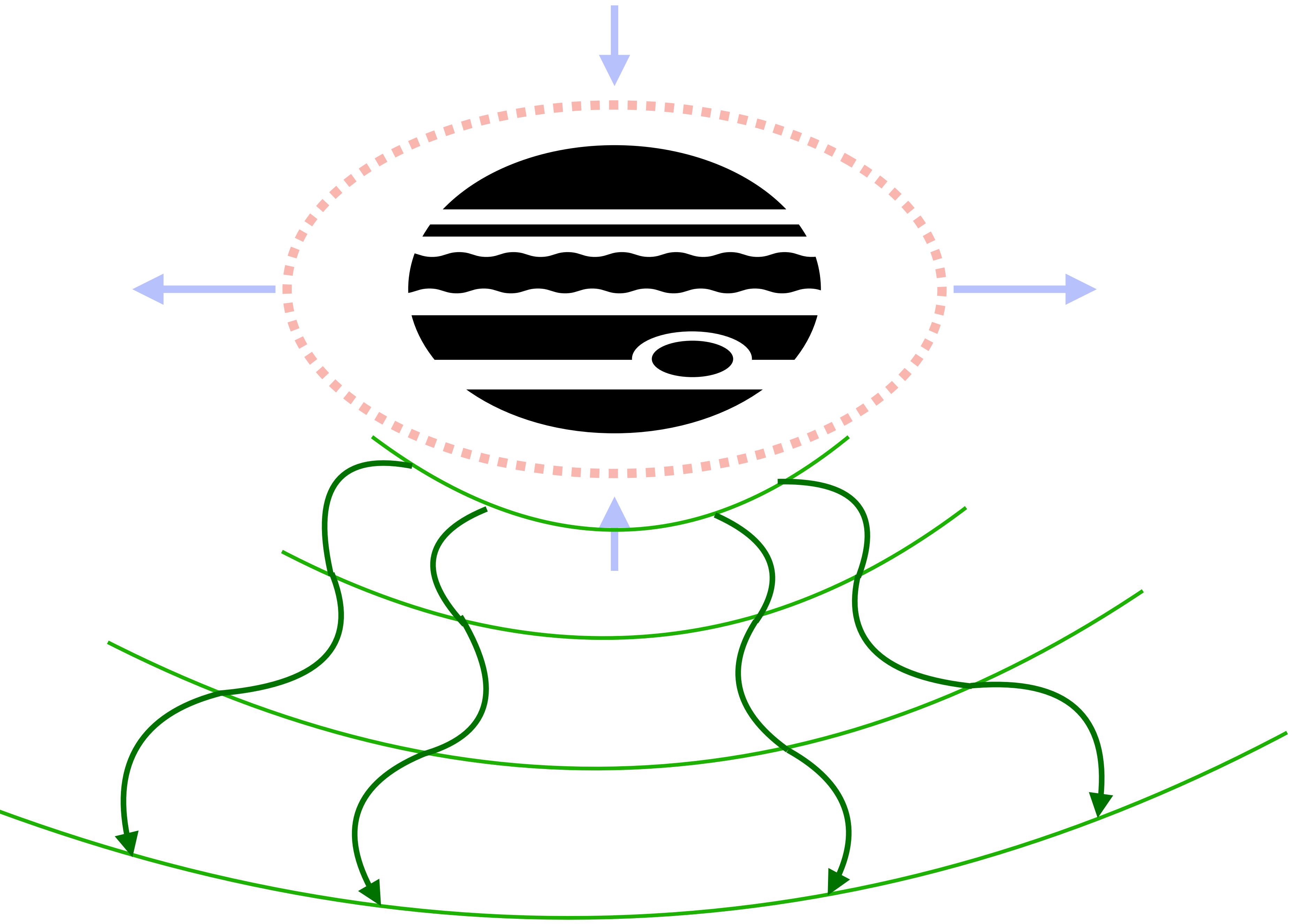

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QCD implications

3. Measurable in gravitational waves

First constrains from GW170817

Better sensitivity in future detectors

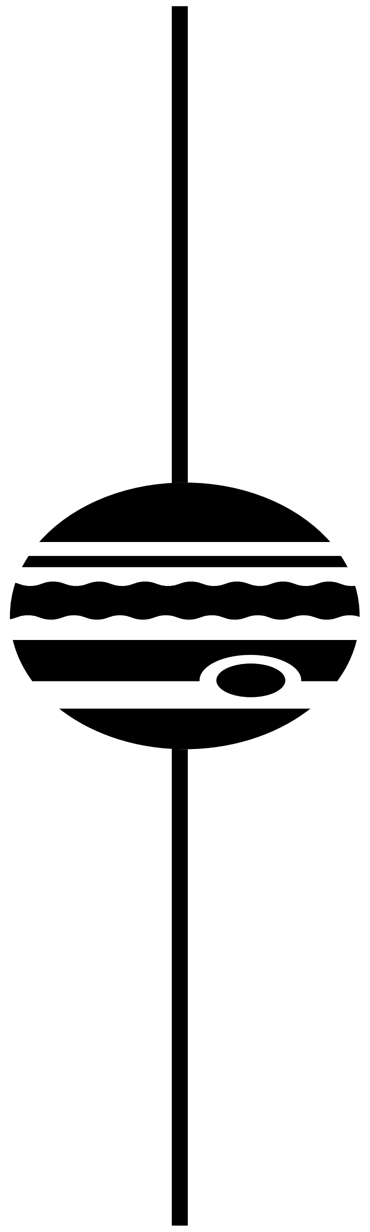
Need for accurate theory

Overview

- Challenge. **Relativistic** definition: spacetime multipoles.
- Distinguishing PN and tidal terms. Circular-orbit **Binding Energy.**
- Effective field theory set-up. Stationary approach and **new method**.
- Tidal response from scattering. **Scalar** perturbations and **Schwarzschild BH.**
- Summary and Outlook.

Challenge

Relativistic definition: spacetime multipoles



Worldline

Time-time component of the metric

$$g_{tt} = -(1 - 2U_{\text{eff}})$$

Multipole moments

Tidal moments

$$\lim_{r\to\infty} U_{\text{eff}} \sim \frac{M}{r} + \frac{3}{2}\frac{Q}{r^3} - \frac{1}{2}\mathcal{E} r^2 + \dots \qquad (\ell = 2)$$

Challenge

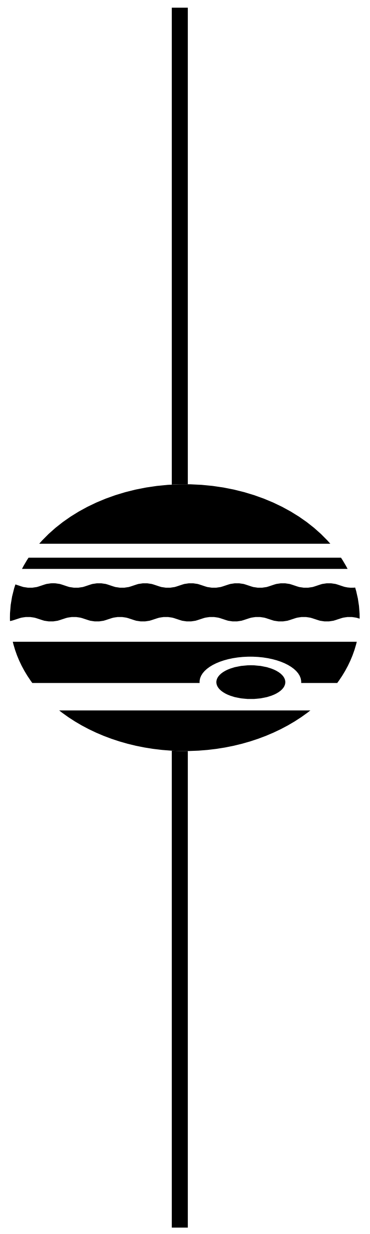
Relativistic definition: spacetime multipoles

Time-time component of the metric

$$g_{tt} = -\left(1 - 2U_{\mathrm{eff}}\right)$$

Coordinate dependent

$$\lim_{r\to\infty} U_{\mathrm{eff}} \sim \frac{M}{r} + \frac{3}{2}\frac{Q}{r^3} - \frac{1}{2}\mathscr{E}r^2 + \dots \qquad (\ell = 2)$$



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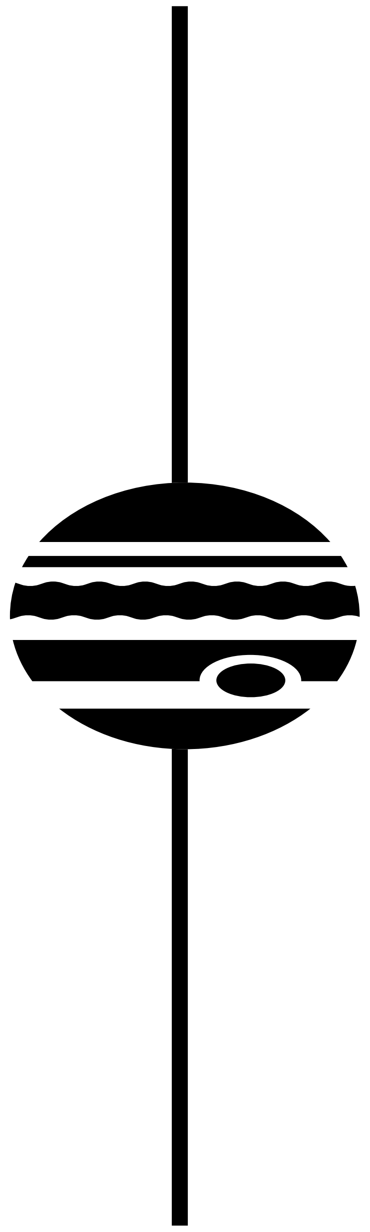
S. E. Gralla. "On the Ambiguity in Relativistic Tidal Deformability". In: Class. Quant. Grav. 35.8 (2018), p. 085002. arXiv: 1710.11096 [gr-qc].

Challenge

Relativistic definition: spacetime multipoles



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- Problems when two series overlap, e.g. post-Newtonian (PN) expansion of the tidal field:

$$\mathscr{E} \to \mathscr{E}^N \left[1 + \frac{\delta_{1\text{PN}}}{r} \ldots + \frac{\delta_{5\text{PN}}}{r^5} + \mathcal{O}\left(\frac{1}{r^6}\right)\right]$$

?

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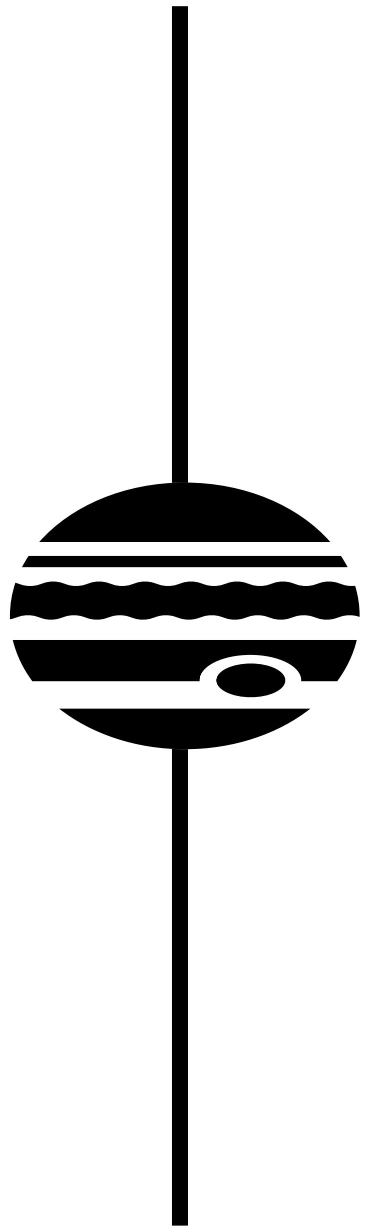

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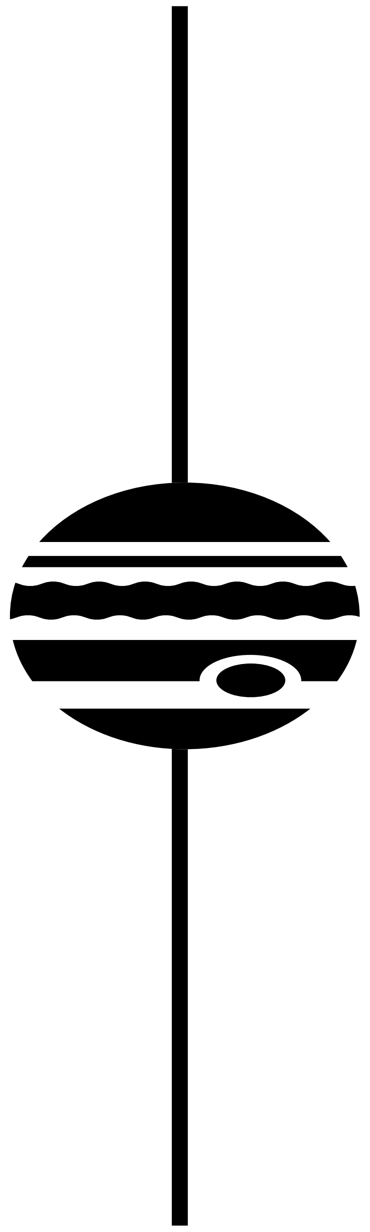

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- Even for gauge-invariant quantities: mixing of PN and multipolar expansion

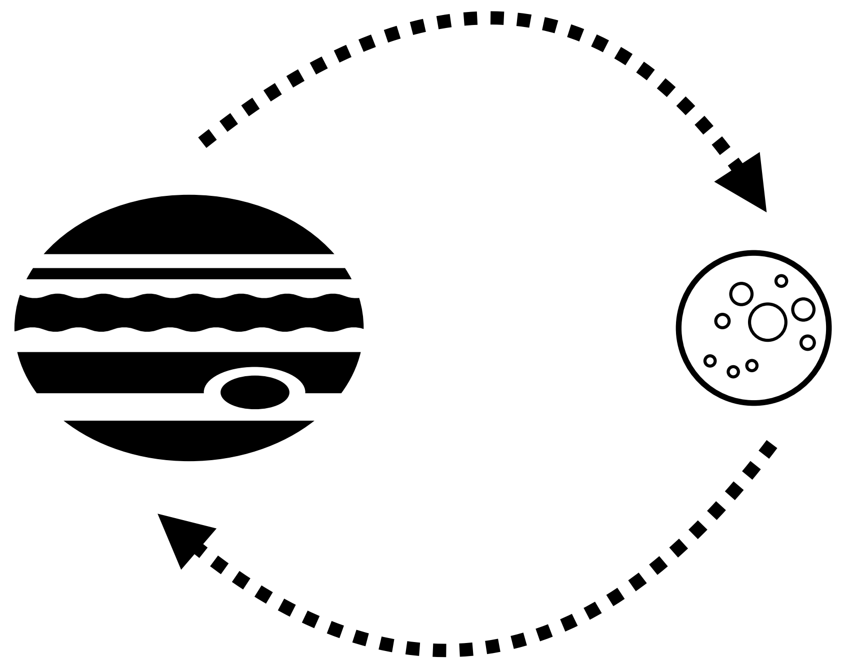
$$\frac{E(\Omega)}{E_0} \sim 1 + x\, c_{1\text{PN}} + x^2\, c_{2\text{PN}} + \ldots + x^5 \left(c_{5\text{PN}} + \lambda_{\ell=2}\, c^{\text{tidal}}_{\ell=2}\right) + \ldots$$

Binding energy of circular orbits

"PN" frequency parameter

Distinguishing PN and tidal terms

Circular-orbit binding energy



- d-(spacetime) dimensional theory
- Circular-orbit binding-energy

$$\frac{E(\Omega)}{E_0} \sim 1 + \sum_{\ell=2}^{\infty} x^{1+2\hat{\ell}} \lambda_\ell \, c^{\text{tidal}}_{\ell \hat{d}} + \sum_{n=2, n\in\mathbb{Z}^+}^{\infty} x^n \, c^{\text{PN}}_{n\hat{d}} + \dots$$

Distinguishing PN and tidal terms

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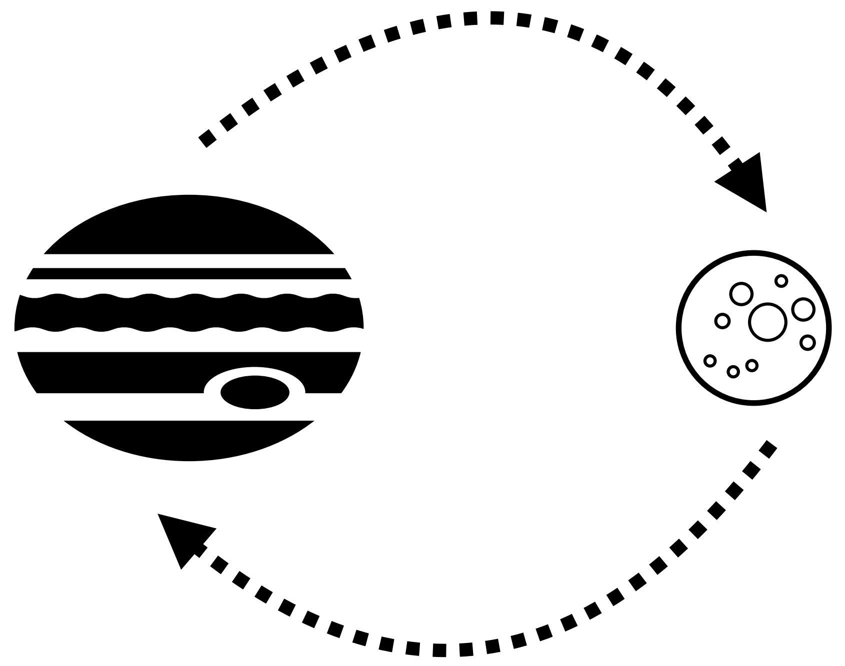

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$\hat{\ell} = \ell / \hat{d}$ $\hat{d} = d - 3$

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"PN" parameter

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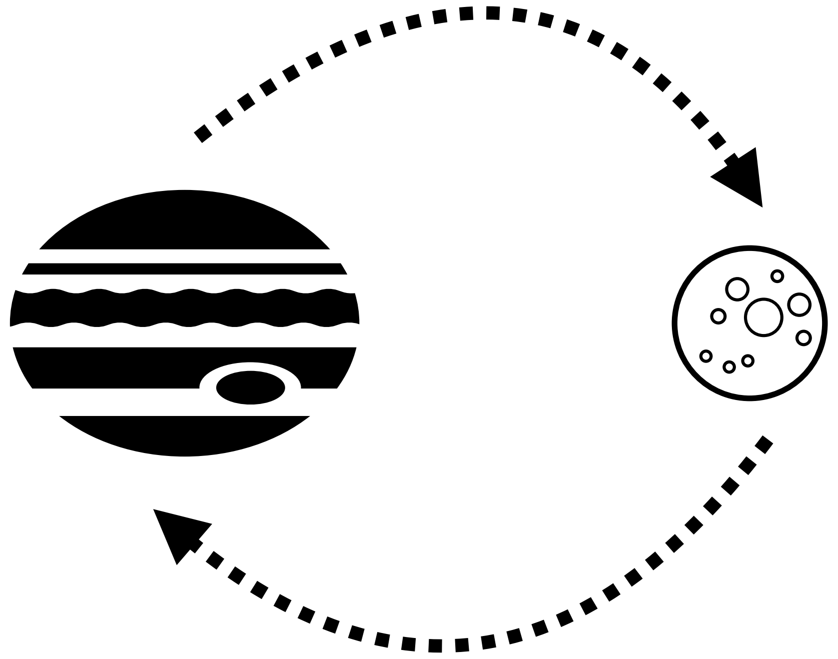

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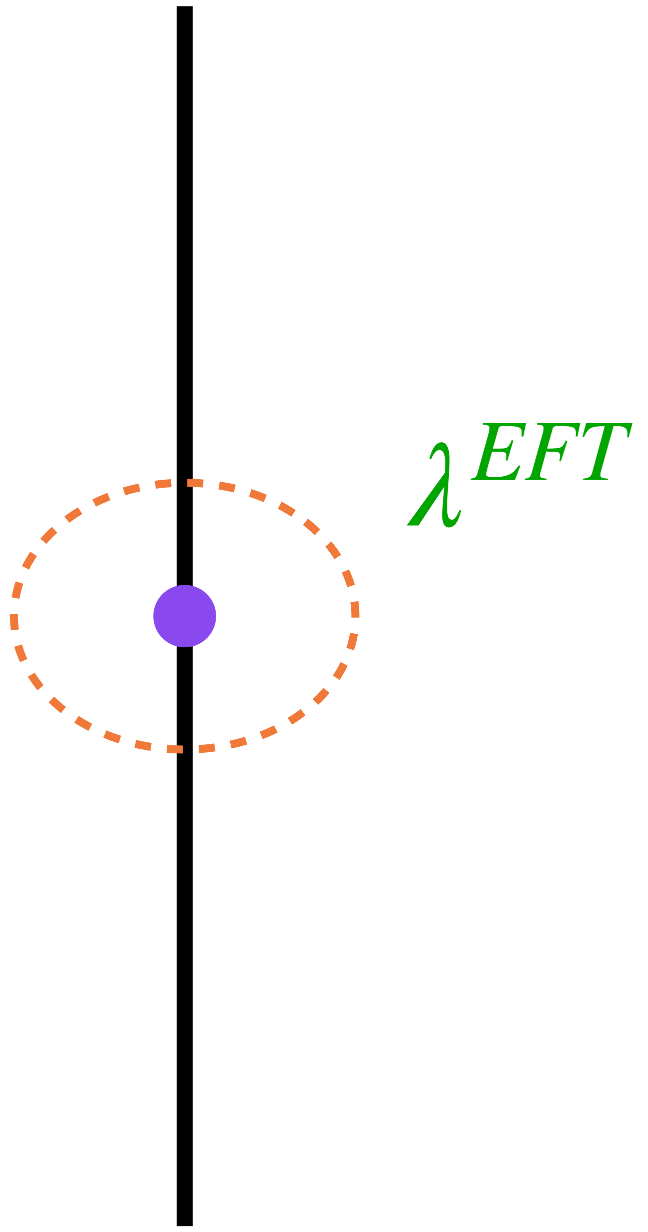
"PN" parameter

Analytic continuation (in multipolar order ℓ or spacetime dimension d)
disentangles multipolar and post-Newtonian terms

Effective Field Theory set-up

Usual stationary approach

Effective Field Theory



- Effective action:

$$S_{\rm EFT} = S_{\rm point\ particle} + S_{\rm finite\ size}$$

$$L_{\rm finite\ size} \propto Q\mathscr{E} \propto \lambda^{EFT}\mathscr{E}\mathscr{E}$$

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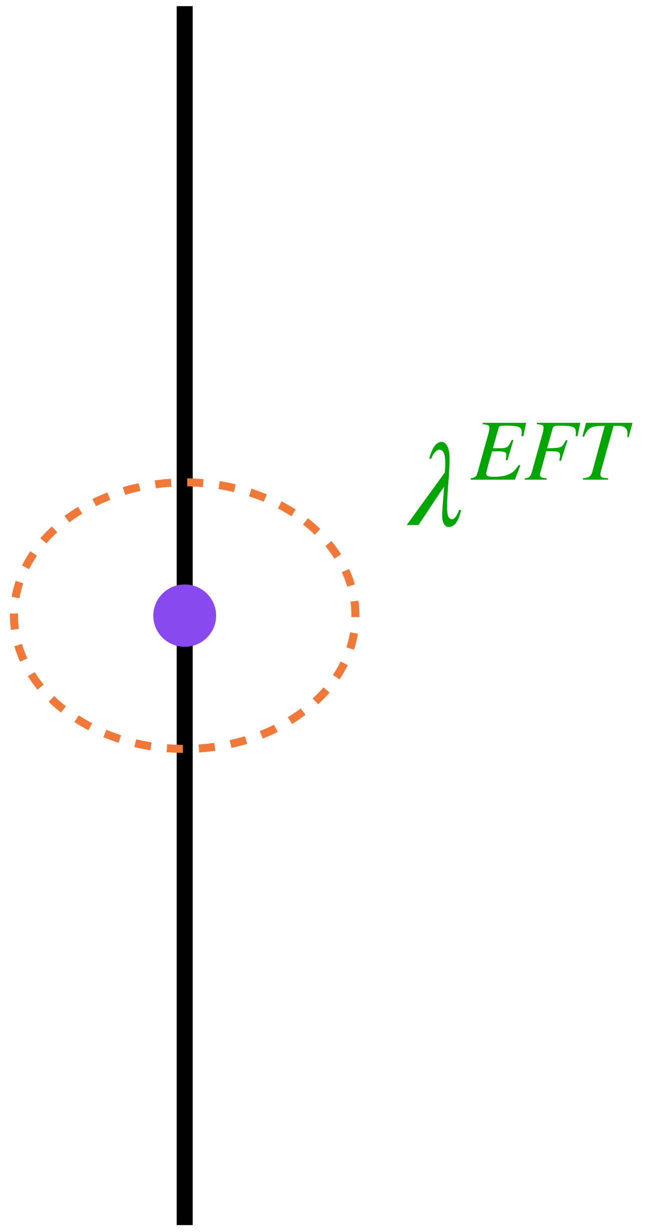

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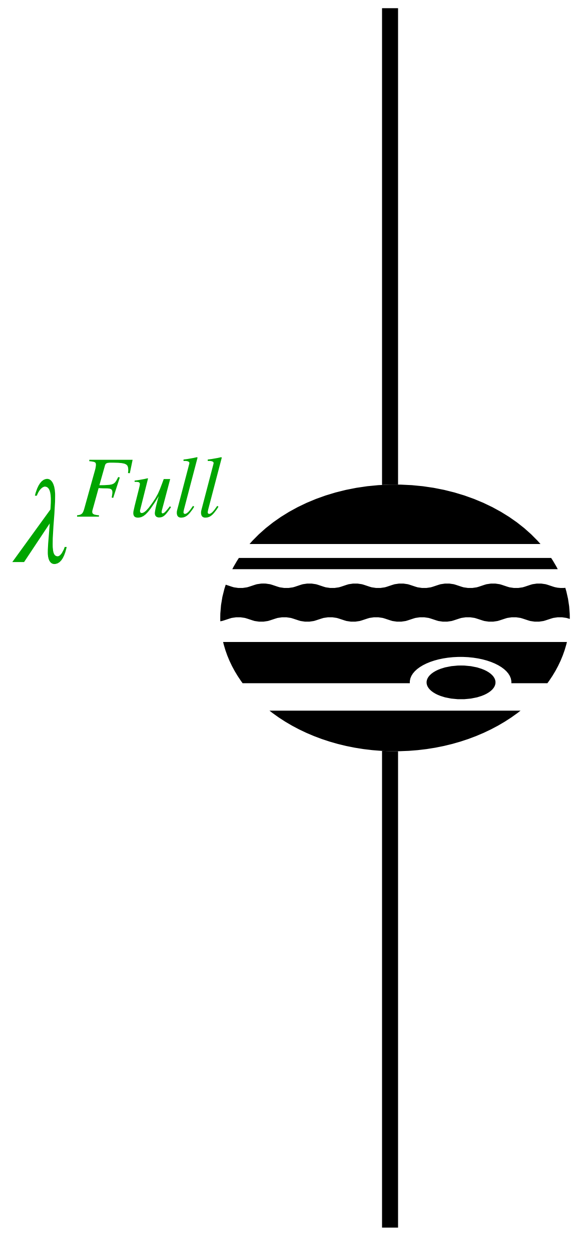
Full theory



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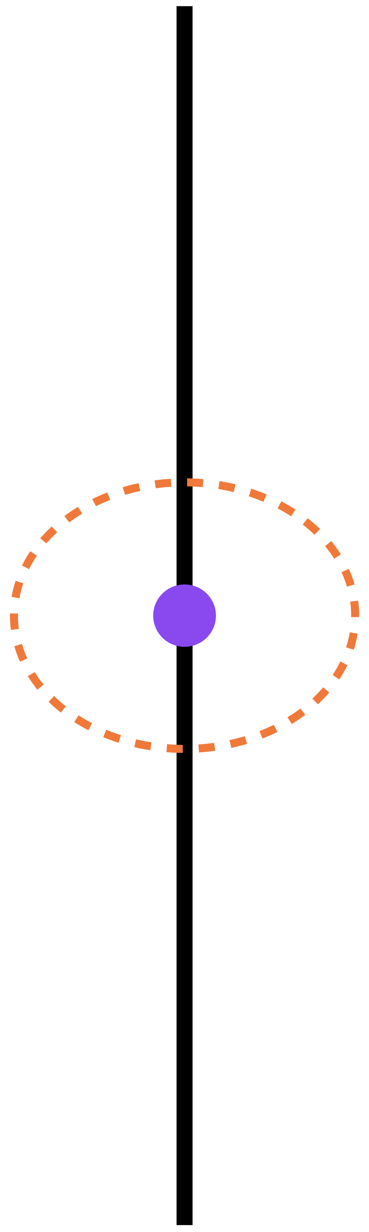
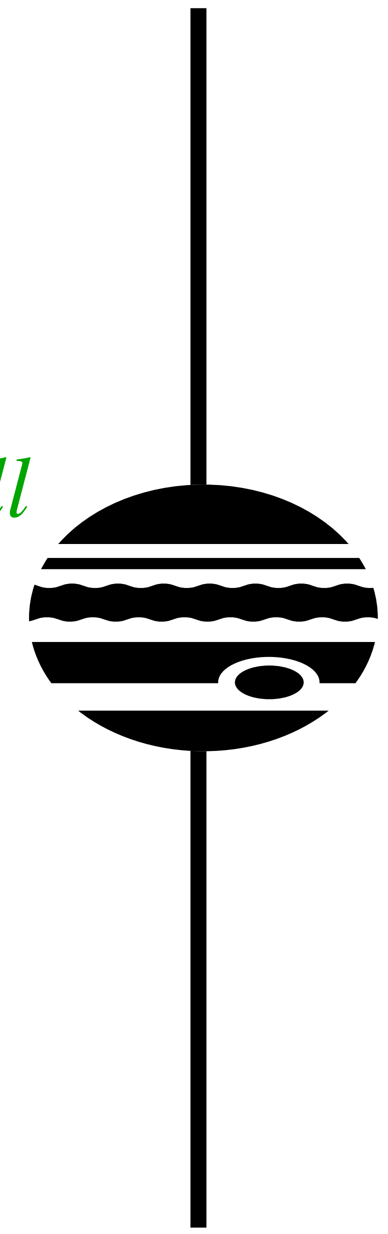
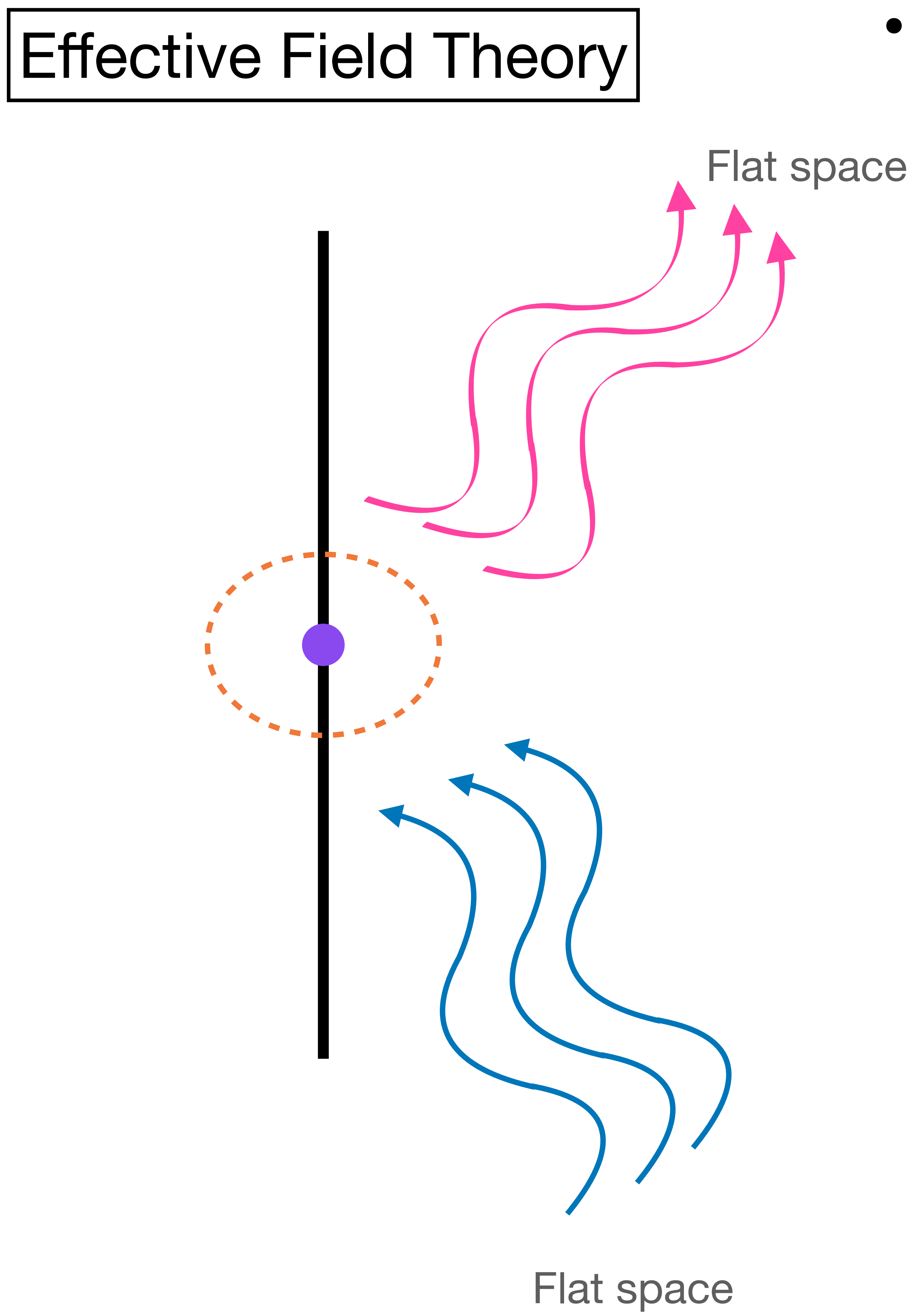


λ^{EFT} ⟵ Matching ⟶ λ^{Full}



Effective Field Theory set-up

New method: gauge-invariant scattering amplitudes



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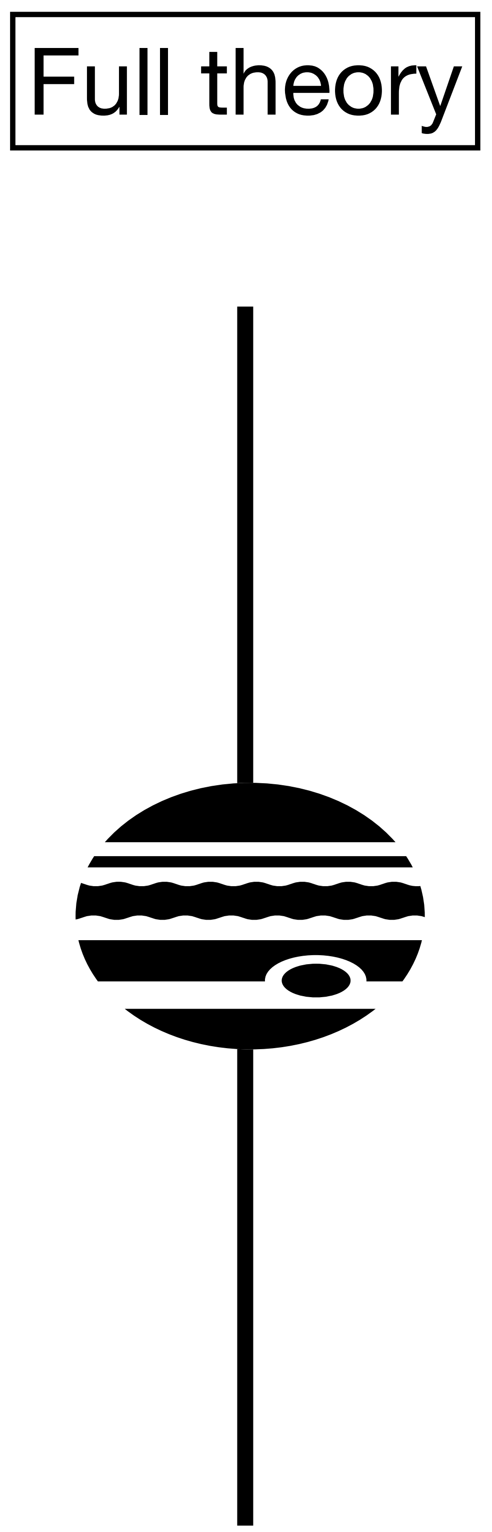

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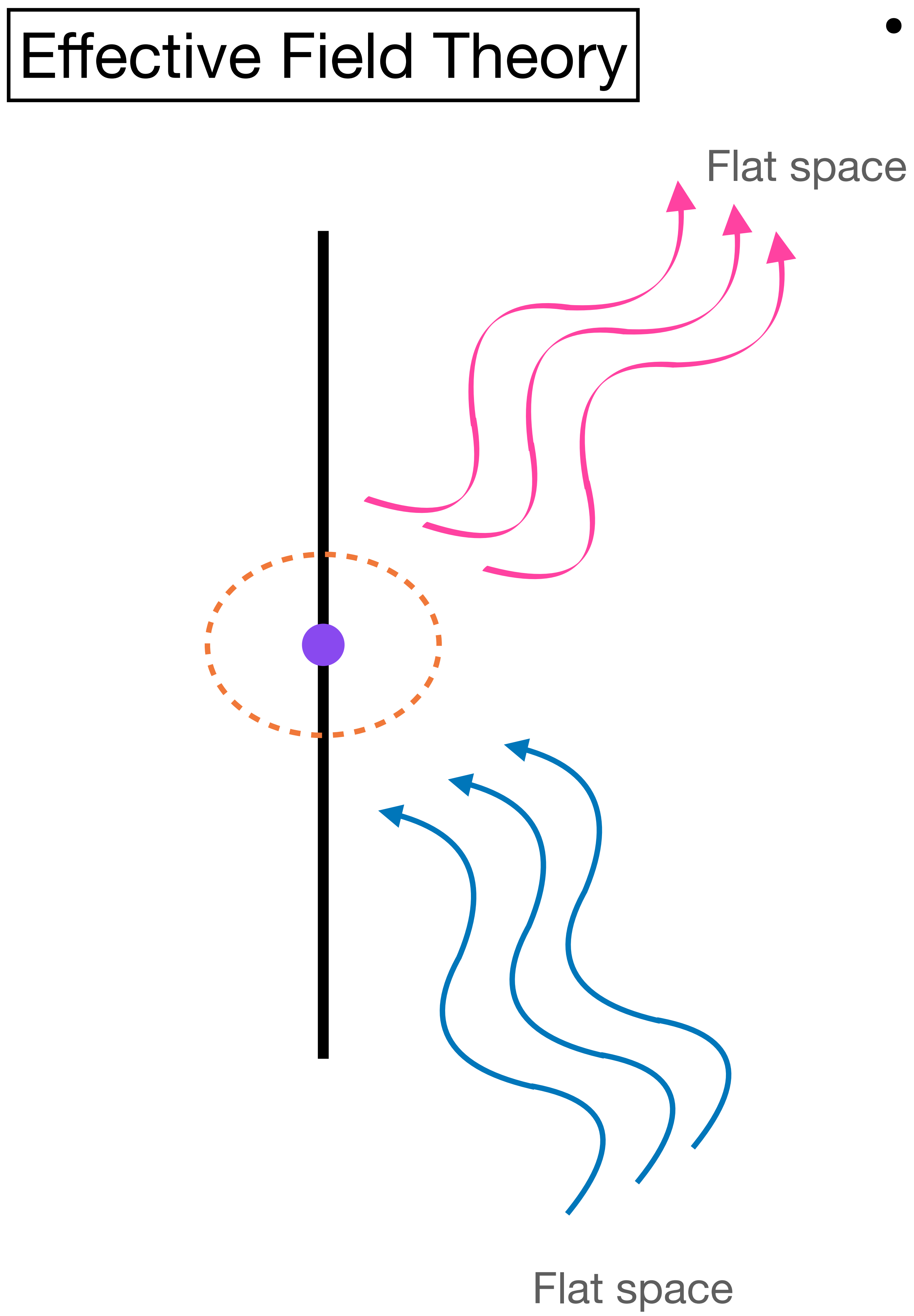

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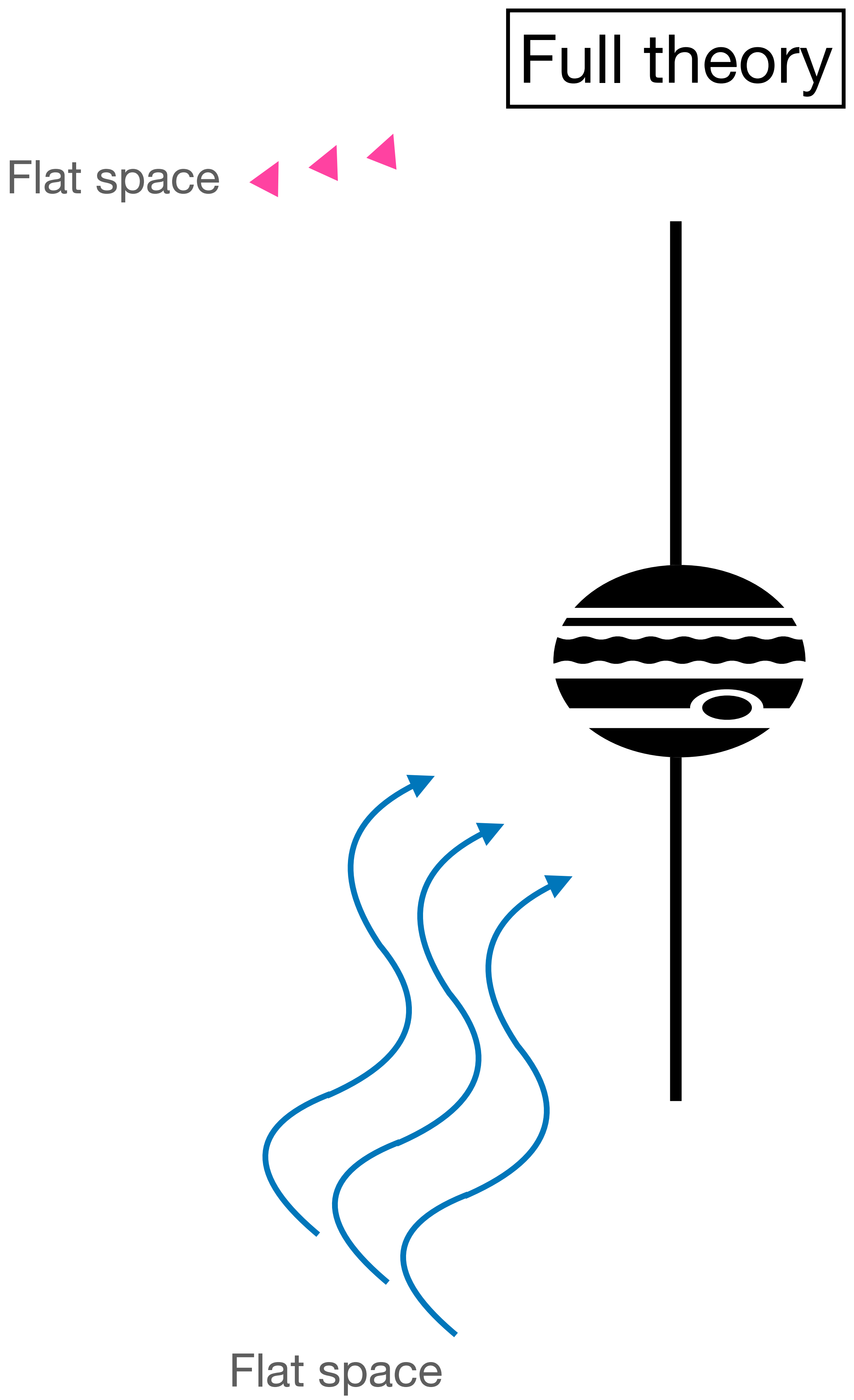

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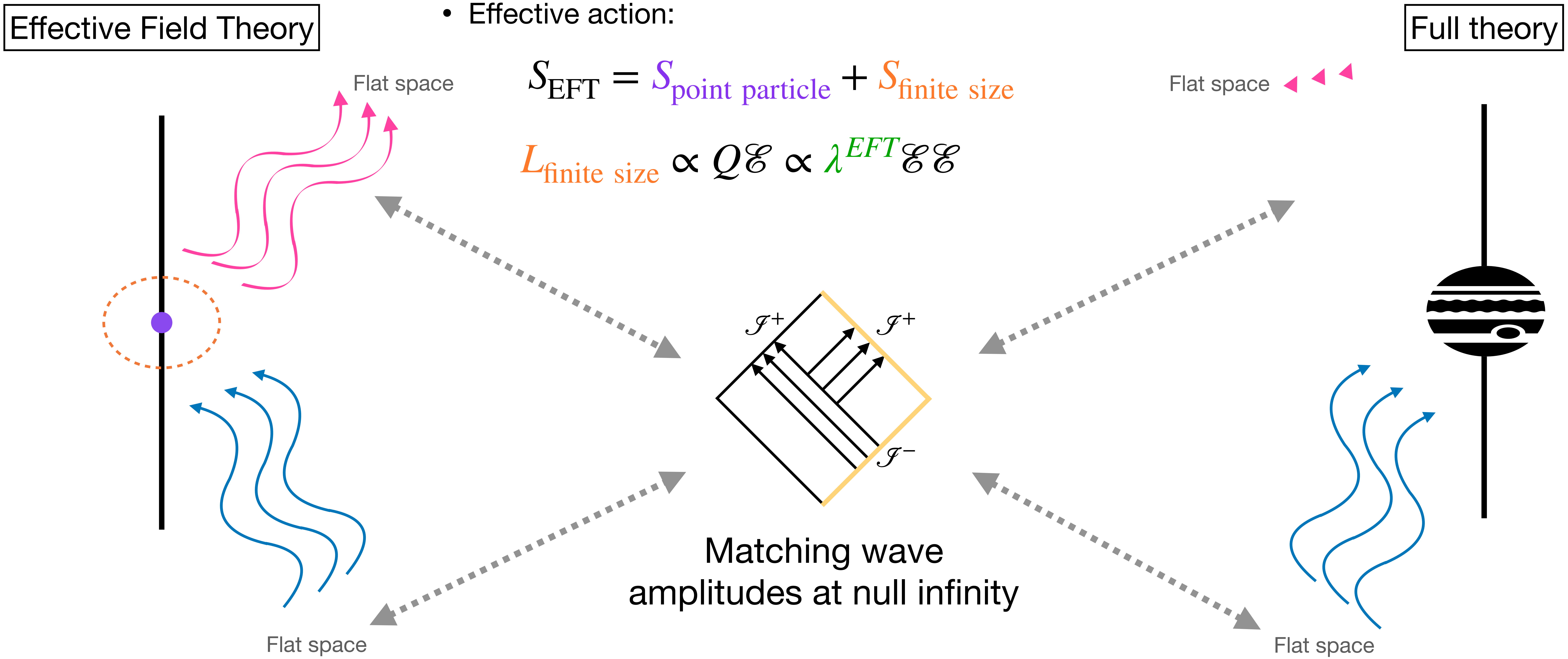


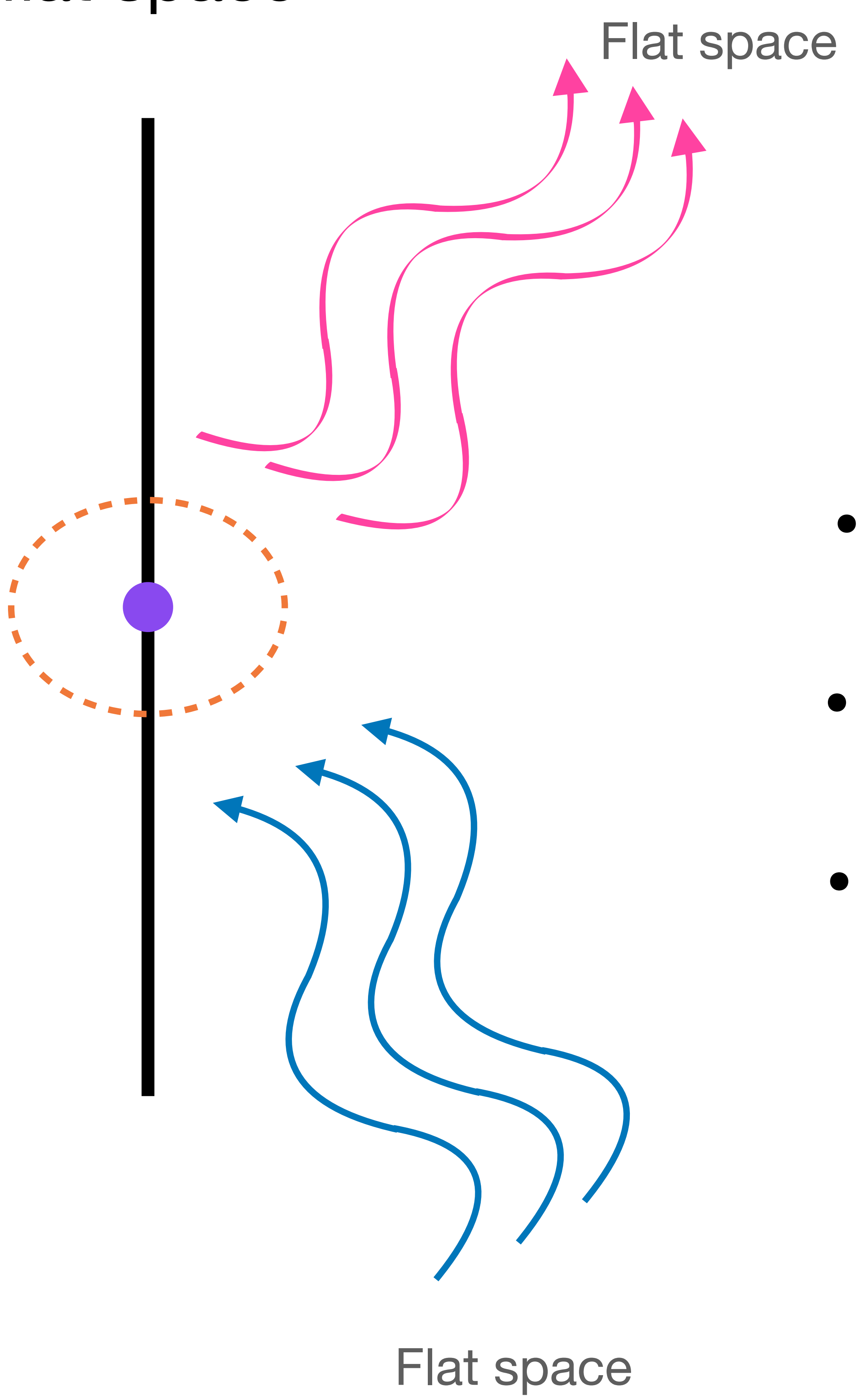
Tidal response from scattering

Example: scalar wave perturbations

Check our paper!

Effective Field Theory

Free massles scalar field in flat space



- Tidal response proportional to scattering amplitudes:

$$\lambda_\ell(\omega) = i\,\Xi_\ell \left[1 - \frac{2}{1+\frac{C_\ell^{\mathrm{in}}}{C_\ell^{\mathrm{out}}} e^{i\frac{\pi}{2}\left(\hat{d}+1\right)}} \right]$$

In and outgoing wave amplitudes
Depend on the nature of the object

- Valid for all frequencies
- Arbitrary ℓ and dimension
- No analytic continuation needed

Tidal response from scattering

Example: scalar wave perturbations of a Schwarzschild BH

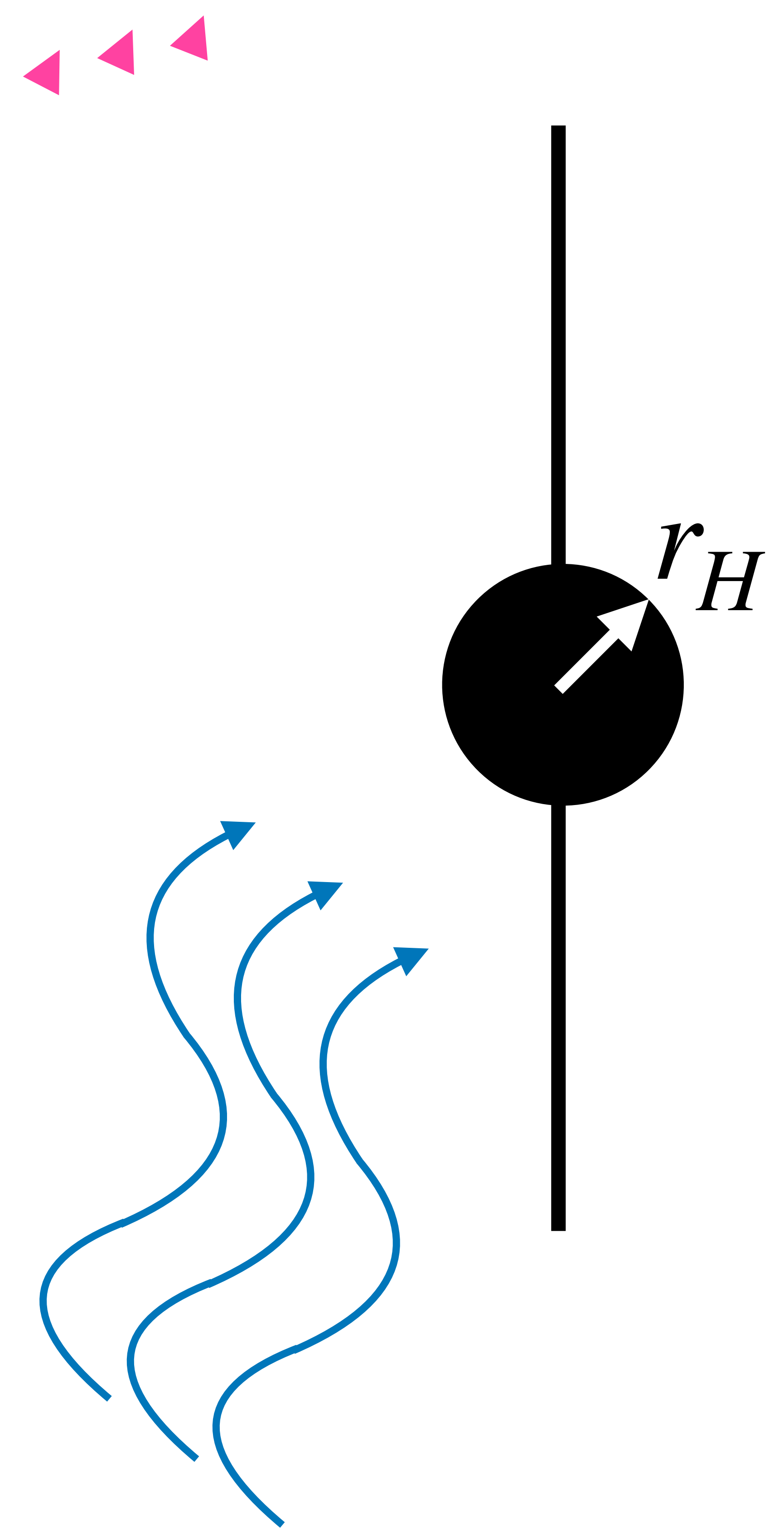
Check our paper!

- Analytic continuation to connect near-horizon to large distance behavior
- Tidal response of Schwarzschild BH from scattering amplitudes:

$$\lambda_{\hat{\ell}}(\omega) = C_{\hat{\ell},\hat{d}}\frac{\Gamma\left[\hat{\ell}+1+\frac{2ir_H\omega}{\hat{d}}\right]}{\Gamma\left[-\hat{\ell}-\frac{2ir_H\omega}{\hat{d}}\right]}r_H^{\hat{d}(2\hat{\ell}+1)}$$

- Information encoded:
  - Static Love number: $Re[\lambda_\ell(\omega \to 0)]$
  - Absorption cross section related to the imaginary part
- Checks:
  - Static Love number vanishes in 4D ✓
  - Recover $\ell = 0$ absorption cross section ✓

Schwarzschild black hole



D. N. Page. "Particle Emission Rates from a Black Hole: Massless Particles from an Uncharged, Nonrotating Hole". In: Phys. Rev. D 13 (1976), pp. 198–206.

Tidal response from scattering

Discussion

$$\lambda_\ell(\omega) = i\,\Xi_\ell \left[1 - \frac{2}{1 + \frac{C_\ell^{\rm in}}{C_\ell^{\rm out}} e^{i\frac{\pi}{2}\left(\hat{d}+1\right)}} \right]$$

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Full-frequency spectrum

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Full-frequency spectrum

Scattering states defined at flat-space null infinity ⟷ One-to-one identification to full theory

Tidal response from scattering

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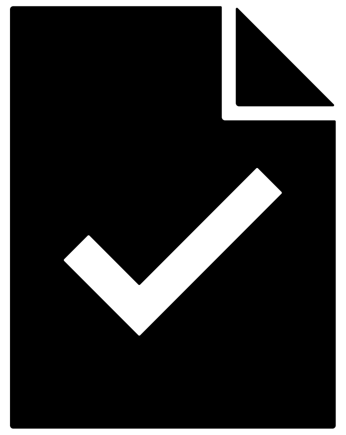
Full-frequency spectrum

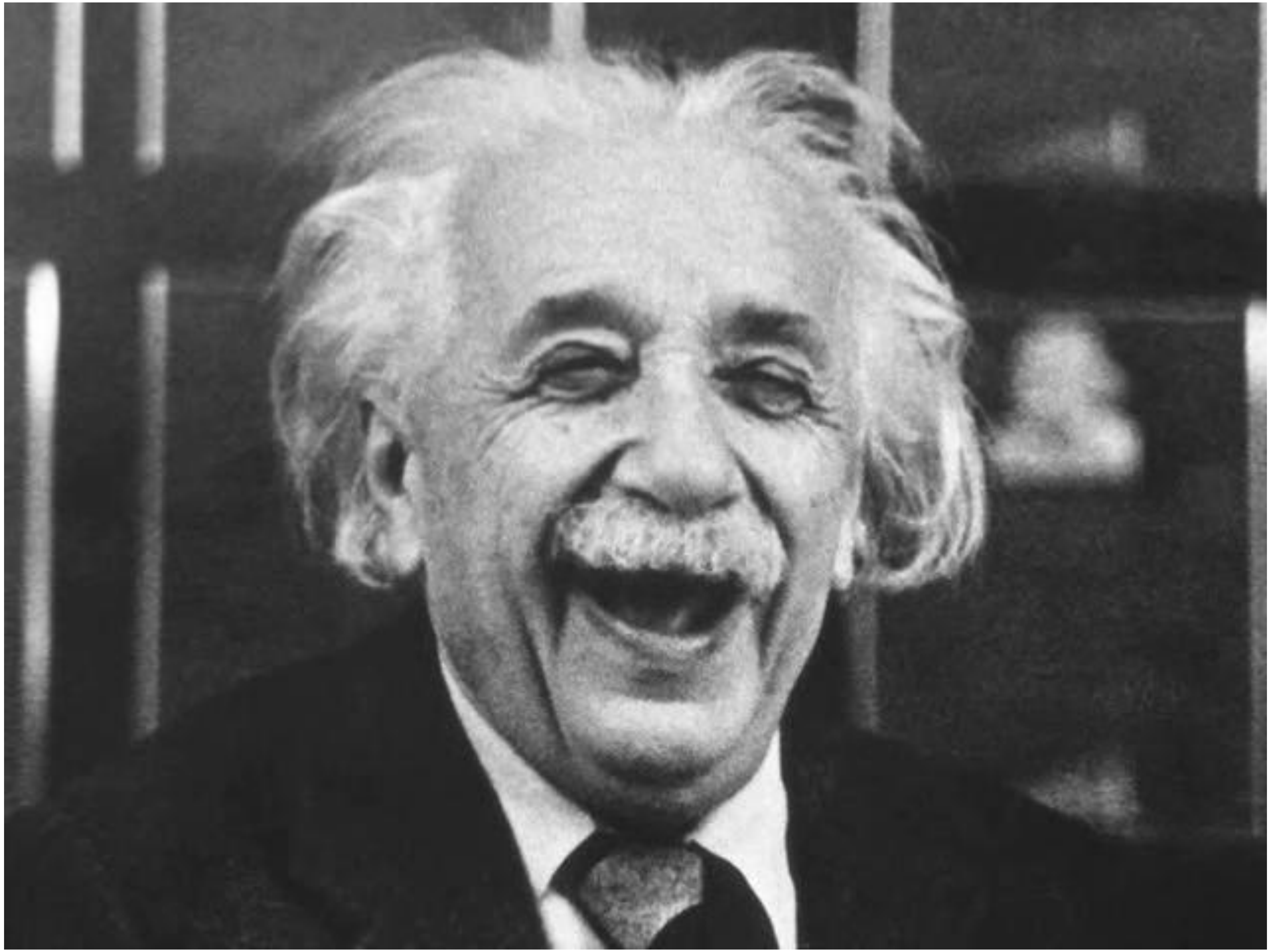
Recovers known results



Scattering states defined at flat-space null infinity ⟷ One-to-one identification to full theory

Summary and outlook

- Love number: body's **interior** and **spacetime**.
- High precision GWs require **accurate** theory.
- **Analytic continuation** to distinguish PN and tidal terms.
- Tidal response from **scattering**: gauge-invariant + frequency-dependent.
- **Scalar tidal response** as a proof of principle to establish the general method.
- Further steps: gravitational perturbations for different compact objects.



Thank you!

Check our paper!

arXiV:2108.03385

Backup slides

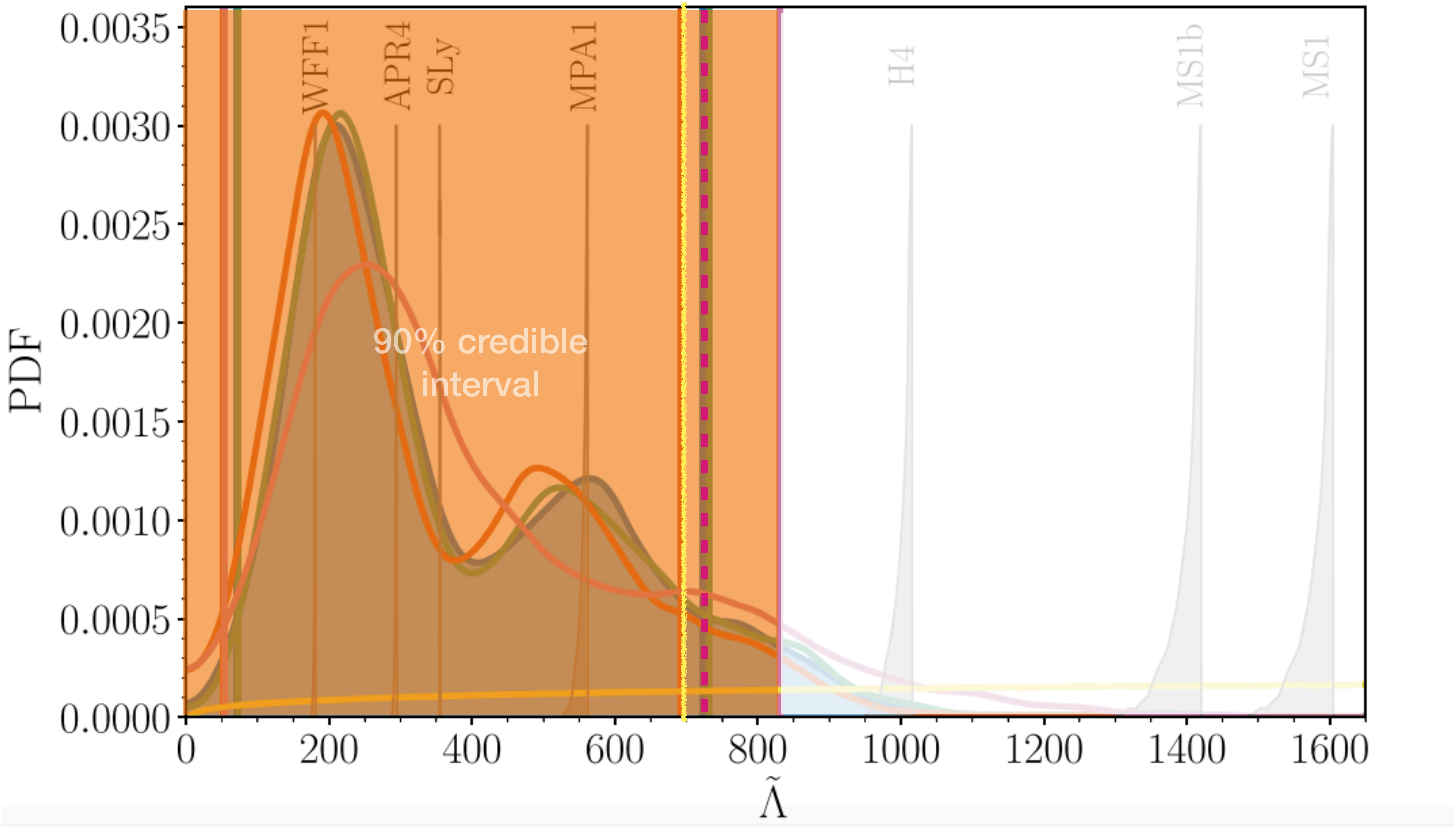


Figure taken from B. P. Abbott et al. “Properties of the binary neutron star merger GW170817”. In: Phys. Rev. X 9.1 (2019), p. 011001. arXiv: 1805.11579 [gr-qc].

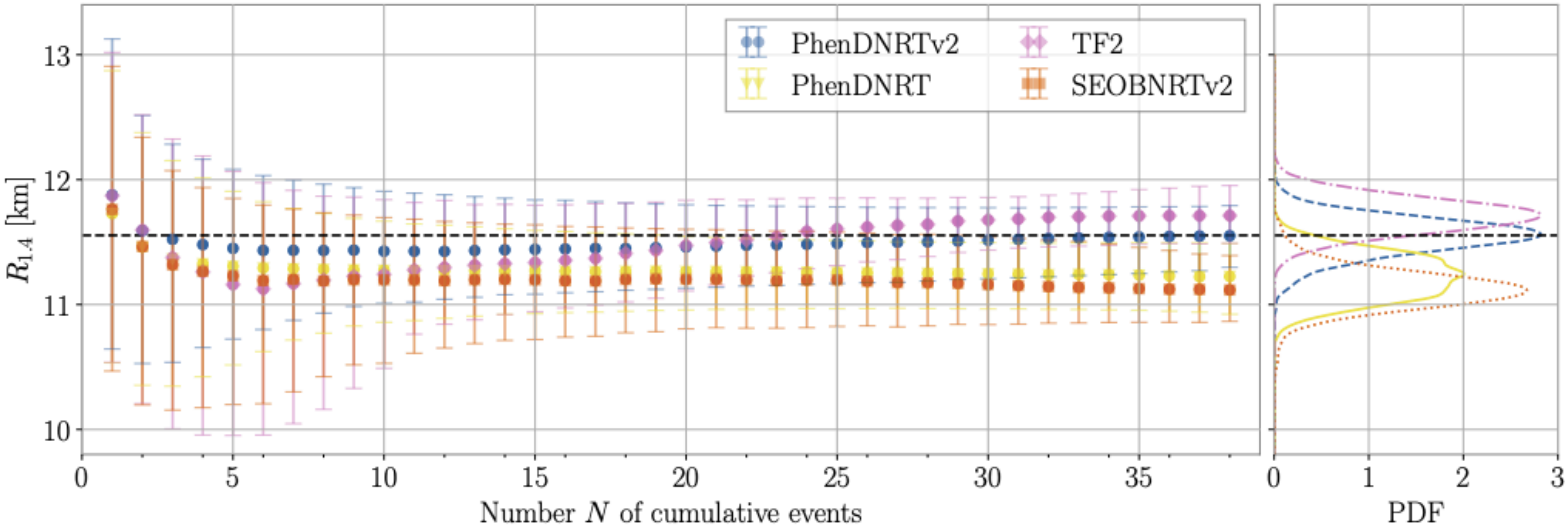


Figure taken from Kunert, Pang, Tews, Coughlin, Dietrich. "Quantifying modeling uncertainties when combining multiple gravitational-wave detections from binary neutron star sources”. In:Phys. Rev. D (2022). arXiv:2110.11835 [astro-ph.HE]

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- Tidal deformability

Tidal Love number

$$\lambda_\ell = \frac{2}{(2\ell-1)!!} k_\ell R^{2\ell+1}$$

- Dimensionless Love number

$$\Lambda_\ell = \frac{2}{(2\ell-1)!!} k_\ell C^{-2\ell-1} \qquad C \equiv \frac{GM}{c^2 R}$$

$$\Lambda_\ell = \frac{\lambda_\ell}{M^{2\ell+1}}$$

Challenge

Relativistic definition: spacetime multipoles

Time-time component of the metric

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Coordinate dependent

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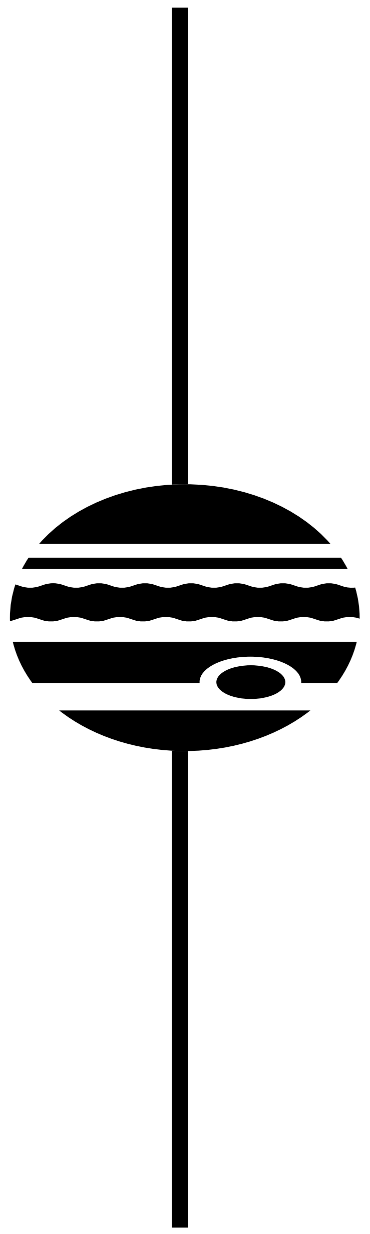

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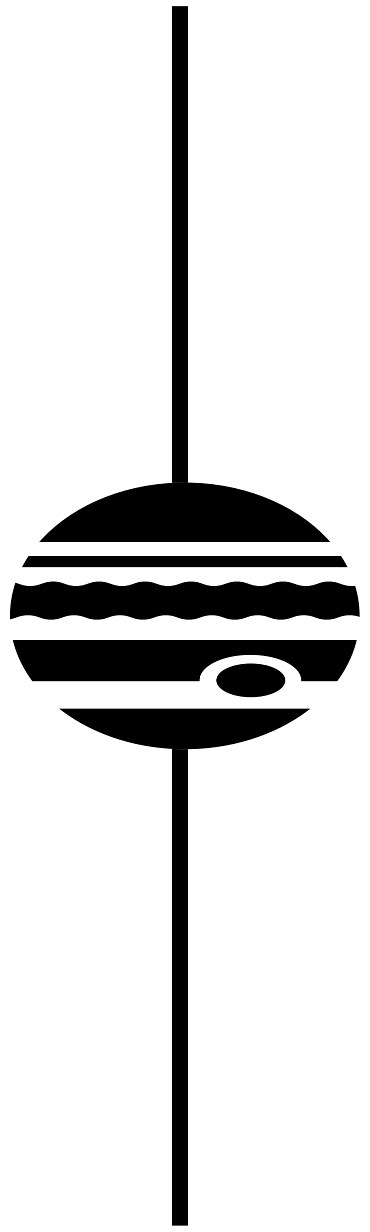
Coordinate dependent

$$\lim_{r\to\infty} U_{\text{eff}} \sim \frac{M}{r} + \frac{3}{2}\frac{Q}{r^3} - \frac{1}{2}\mathcal{E}r^2 + \dots \qquad (\ell = 2)$$



Worldline

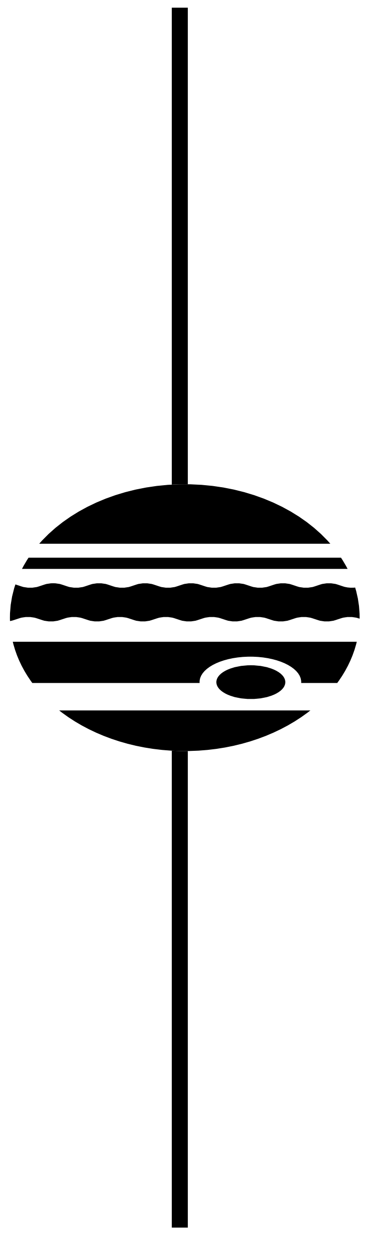
- Changing coordinates :

$$r \to r'\left[1 + N\left(\frac{M}{r'}\right)^5\right]$$

S. E. Gralla. "On the Ambiguity in Relativistic Tidal Deformability". In: Class. Quant. Grav. 35.8 (2018), p. 085002. arXiv: 1710.11096 [gr-qc].

Challenge

S.E. Gralla: On the Ambiguity in Relativistic Tidal Deformability



Time-time component of the metric

$$g_{tt} = -(1 - 2U_{\text{eff}})$$

Coordinate dependent

$$\lim_{r\to\infty} U_{\text{eff}} \sim \frac{M}{r} + \frac{3}{2}\frac{Q}{r^3} - \frac{1}{2}\mathscr{E}r^2 + \dots \qquad (\ell = 2)$$

- Changing coordinates :

$$r \to r'\left[1 + N\left(\frac{M}{r'}\right)^5\right] \qquad r^2 \to r'^2 + \frac{2NM^5}{r'^3}$$

S. E. Gralla. "On the Ambiguity in Relativistic Tidal Deformability". In: Class. Quant. Grav. 35.8 (2018), p. 085002. arXiv: 1710.11096 [gr-qc].

Challenge

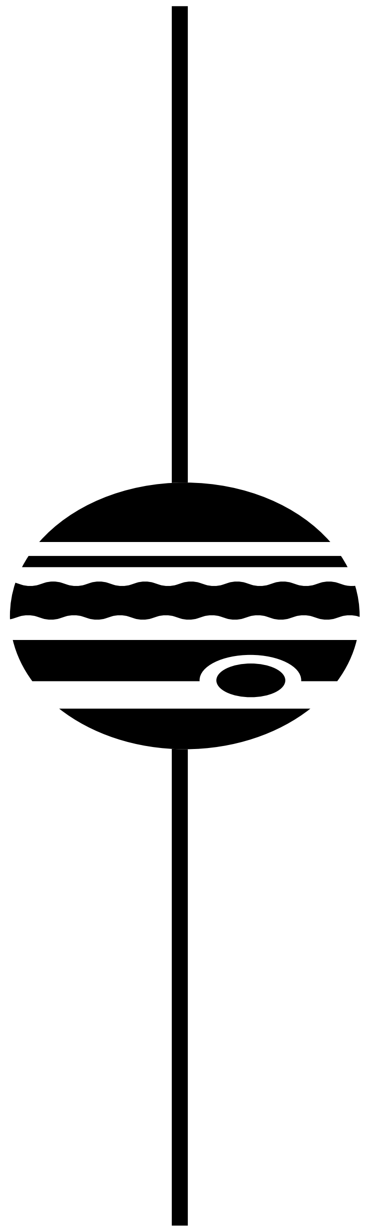
S.E. Gralla: On the Ambiguity in Relativistic Tidal Deformability

Time-time component of the metric

Coordinate dependent

$$g_{tt} = -(1 - 2U_{\text{eff}})$$

$$\lim_{r\to\infty} U_{\text{eff}} \sim \frac{M}{r'} + \frac{3}{2}\frac{Q - \frac{2\mathscr{E}NM^5}{3}}{r'^3} - \frac{1}{2}\mathscr{E}r'^2 + \dots \qquad (\ell = 2)$$



Worldline

- Changing coordinates :

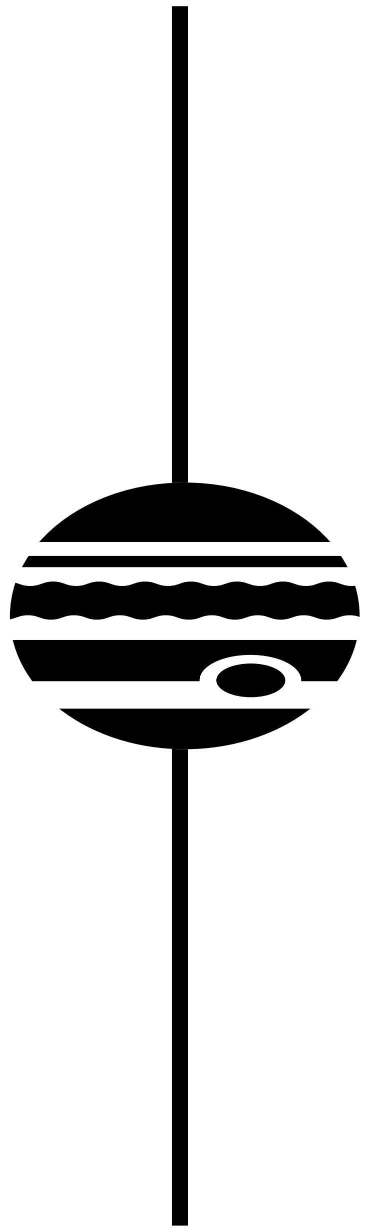
$$r \to r'\left[1 + N\left(\frac{M}{r'}\right)^5\right] \qquad r^2 \to r'^2 + \frac{2NM^5}{r'^3}$$

$$Q \to Q - \frac{2}{3}N\mathscr{E}M^5 \qquad \lambda' \to \lambda + \frac{2}{3}NM^5$$

Challenge

S. E. Gralla. "On the Ambiguity in Relativistic Tidal Deformability". In: Class. Quant. Grav. 35.8 (2018), p. 085002. arXiv: 1710.11096 [gr-qc].

S.E. Gralla: On the Ambiguity in Relativistic Tidal Deformability



- Adimensional tidal deformability

$$\Lambda = \frac{\lambda}{M^{2\ell+1}}$$

$$\Lambda' \rightarrow \Lambda + \frac{2}{3}N$$

- General coordinate transformation

$$r \rightarrow r'\left[1 + \sum_i \alpha_i \left(\frac{M}{r'}\right)^i\right] \qquad N = 2\alpha_1\alpha_4 + 2\alpha_2\alpha_3 + \alpha_5$$

- Estimation of the ambiguity

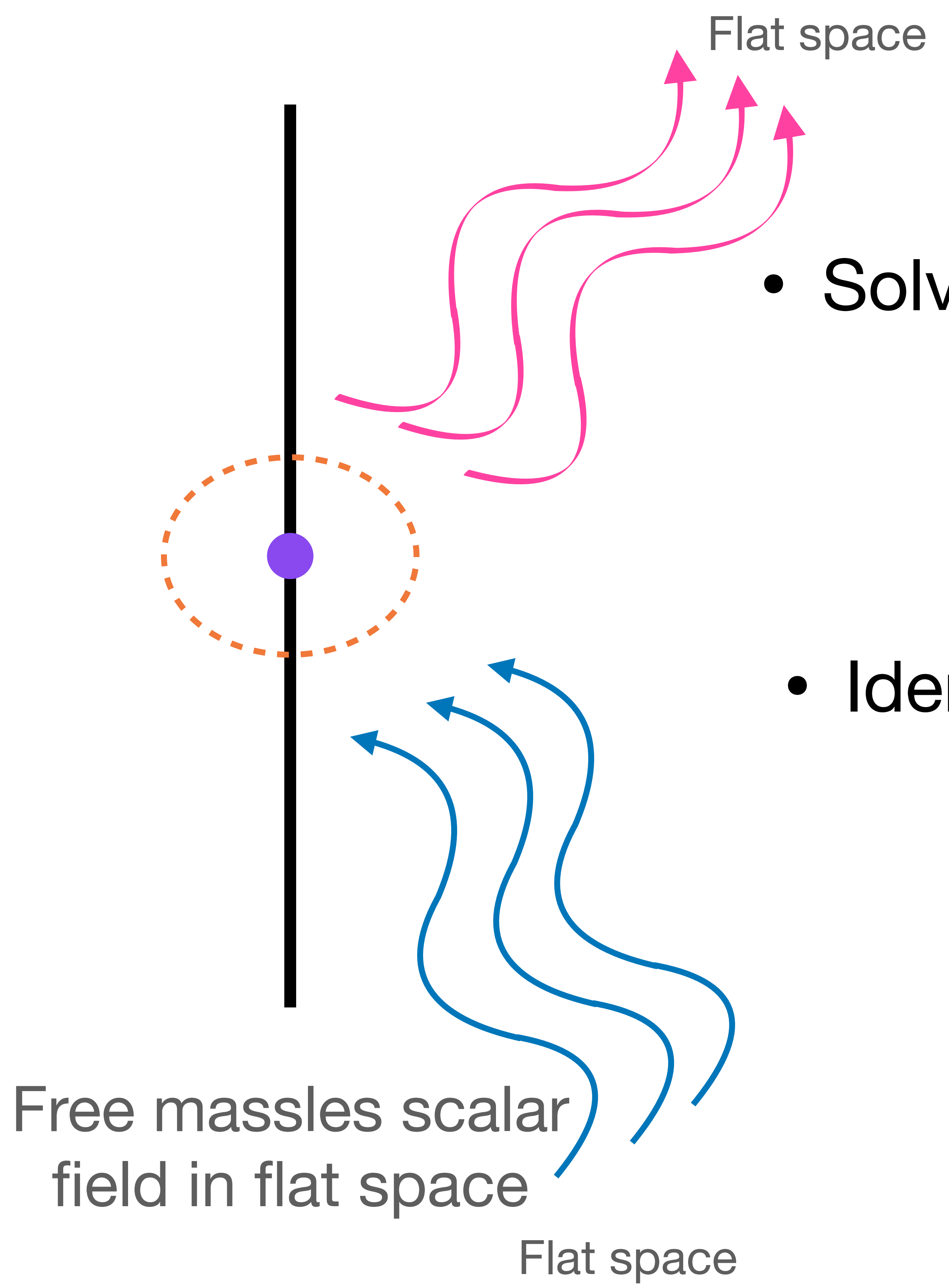
$$\alpha_i \sim \mathcal{O}(1) \qquad \Lambda' \rightarrow \Lambda + \mathcal{O}(0.1 - 10)$$

Tidal response from scattering

Example: scalar wave perturbations

Check our paper!

Effective Field Theory



- Equation of motion of the scalar field

$$\Box \phi \propto Q_L(\omega)$$

- Solve the source-free equation

$$\phi = \phi_{\mathrm{regular}} + \phi_{\mathrm{irregular}}$$

- Identify multipole and tidal moments in terms of scattering amplitudes

$$\Box \phi = \Box \phi_{\mathrm{irregular}} \propto \mathscr{A}_{\mathrm{irreg}} \quad \longleftrightarrow \quad \Box \phi \propto Q_L(\omega)$$

$$\mathscr{E}_L = \underset{r\to 0}{\mathrm{FP}}\, \partial_L \phi = \underset{r\to 0}{\mathrm{FP}}\, \partial_L \phi_{\mathrm{regular}} \propto \mathscr{A}_{\mathrm{regular}}$$

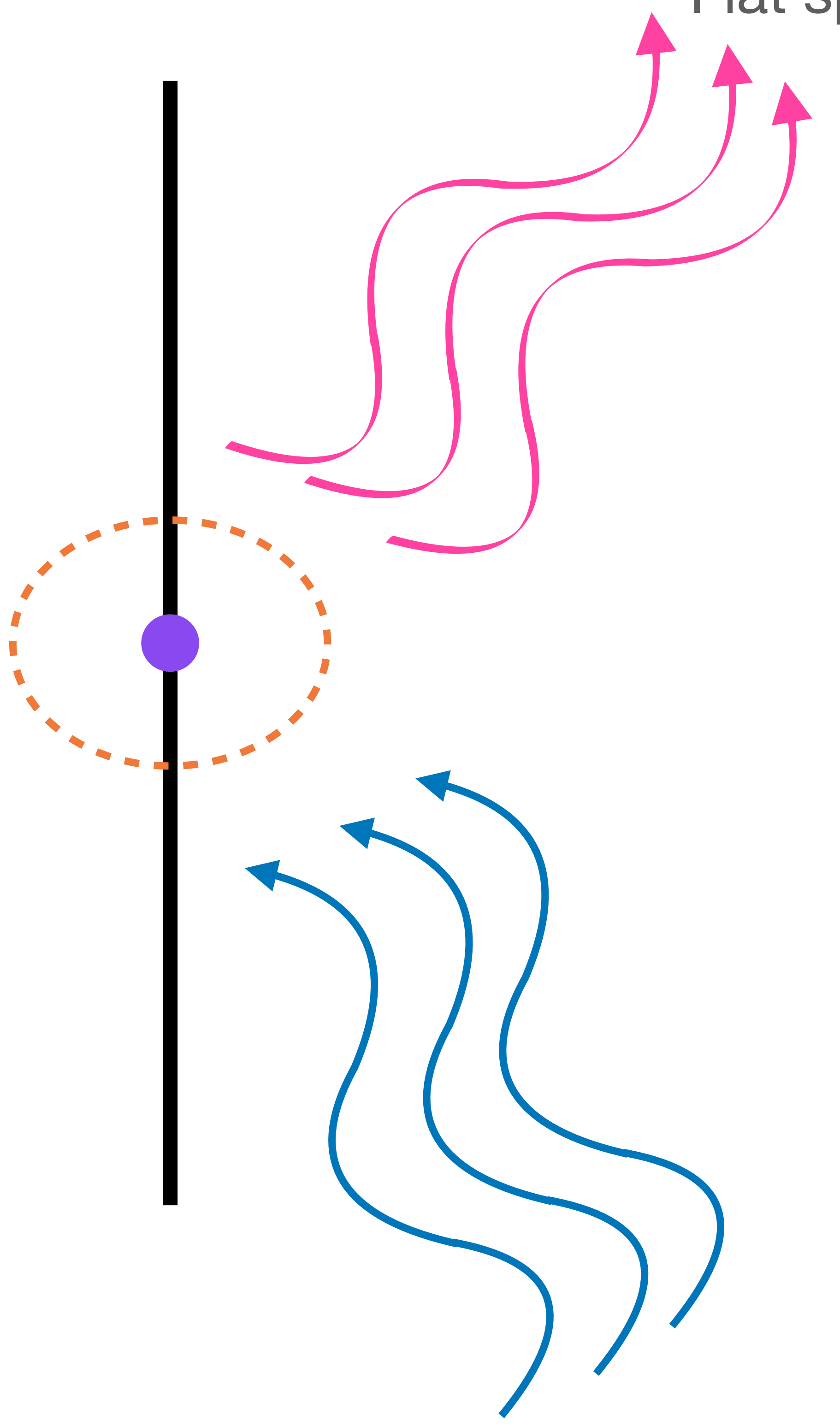
Tidal response from scattering

Example: scalar perturbations

Check our paper!

Effective Field Theory

Free massles scalar field in flat space



- Tidal response proportional to scattering amplitudes:

$$\lambda_\ell(\omega) = i\,\Xi_\ell \left[1 - \frac{2}{1 + \frac{C_\ell^{\text{in}}}{C_\ell^{\text{out}}} e^{i\frac{\pi}{2}\left(\hat{d}+1\right)}} \right]$$

In and outgoing wave amplitudes

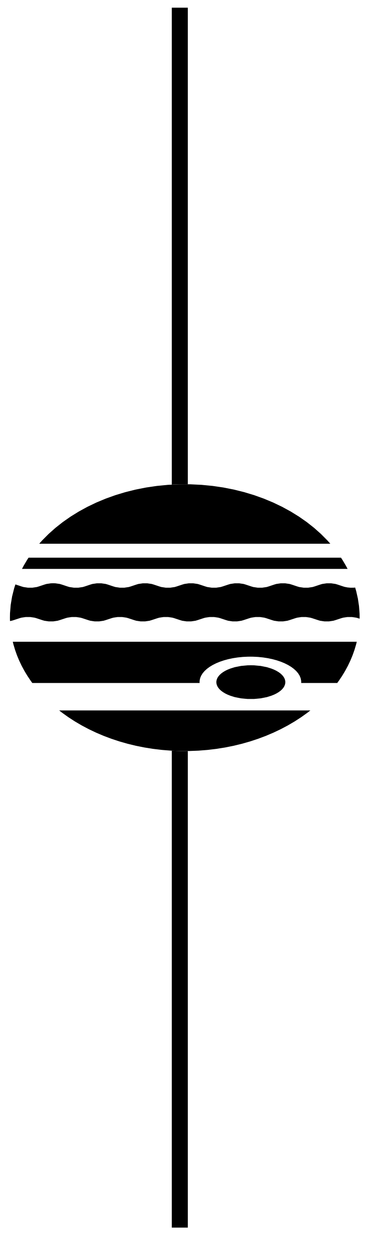
$$\Xi_\ell = -\frac{4\pi^{\hat{d}/2}}{2^\ell}\left(\frac{2}{\omega}\right)^{\hat{d}+2\ell}\Gamma\left(\frac{\hat{d}}{2}+\ell+1\right)$$

- Valid for all frequencies
- Arbitrary ℓ and dimension
- No analytic continuation needed

Problem

S. E. Gralla. "On the Ambiguity in Relativistic Tidal Deformability". In: Class. Quant. Grav. 35.8 (2018), p. 085002. arXiv: 1710.11096 [gr-qc].

Relativistic definition



Worldline

$$\lim_{r\to\infty} U_{\text{eff}} \sim \frac{M}{r} + \frac{3}{2}\frac{Q}{r^3} - \frac{1}{2}\mathcal{E}r^2 + \dots$$

Coordinate dependent

- Changing coordinates :

$$r' \to r\left[1 + N\left(\frac{M}{r}\right)^5\right] \qquad r'^2 \to r^2 + \frac{2NM^5}{r^3}$$

$$\lim_{r\to\infty} U_{\text{eff}} \sim \frac{M}{r} + \frac{3}{2r^3}\left(Q - \frac{2}{3}NM^5\mathcal{E}\right) - \frac{1}{2}\mathcal{E}r^2 + \dots$$

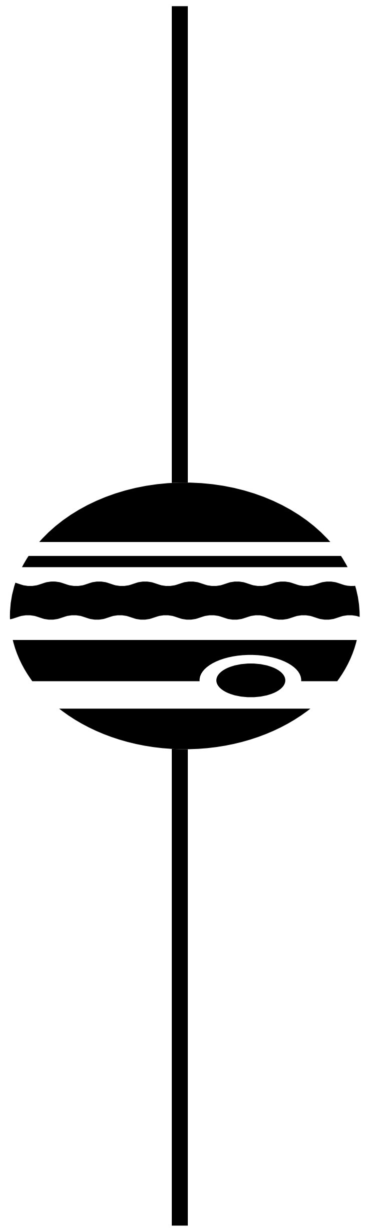
- Quadrupole and Love number change :

$$Q' \to Q - \frac{2}{3}NM^5\mathcal{E} \qquad \lambda' \to \lambda + \frac{2}{3}NM^5$$

Solution

Distinguishing powers

B. Kol and M. Smolkin .In: JHEP 02 (2012), p. 010. arXiv: 1110.3764 [hep-th].



- With analytic continuation

$$\lim_{r\to\infty} U_{\text{eff}} \sim \frac{M}{r} + \tilde{a}_\ell \frac{Q}{r^{\ell+1}} - \tilde{b}_\ell \mathscr{E} r^\ell + \dots$$

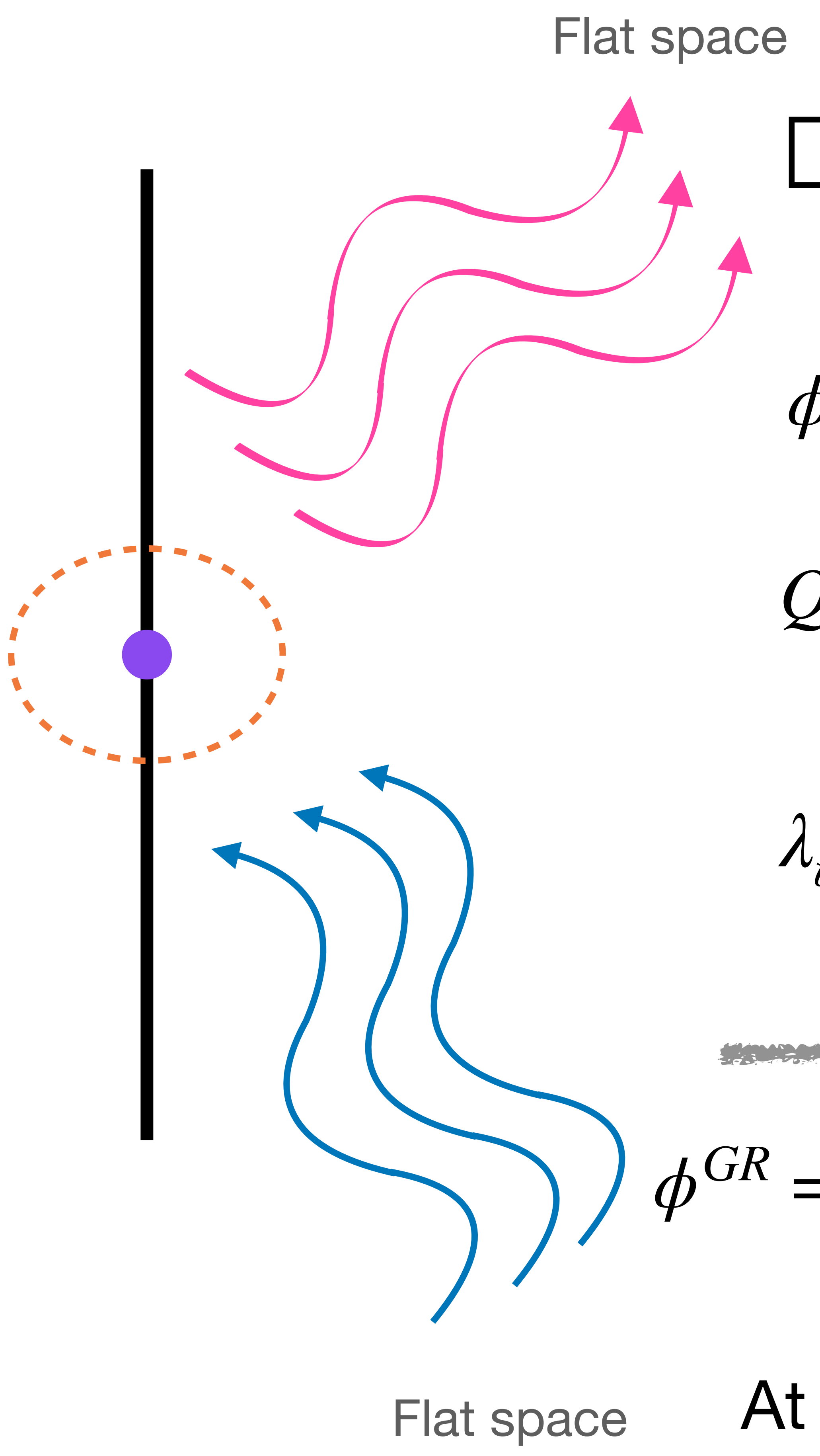
$$r^\ell \to r^\ell + \frac{2NM^5}{r^{5-\ell}}$$

$$\lim_{r\to\infty} U_{\text{eff}} \sim \frac{M}{r} + \tilde{a}_\ell \frac{Q}{r^{\ell+1}} - \tilde{b}_\ell \mathscr{E} r^\ell - \tilde{b}_\ell N M^5 \mathscr{E} r^{\ell-5} + \dots$$

- Unambiguous identification

Results

Tidal response from scattering



$$\Box \phi = \sum_{\ell=0}^{\infty} (-1)^{\ell} e^{i\omega t} Q^{L}(\omega) \partial_L \delta(x)$$

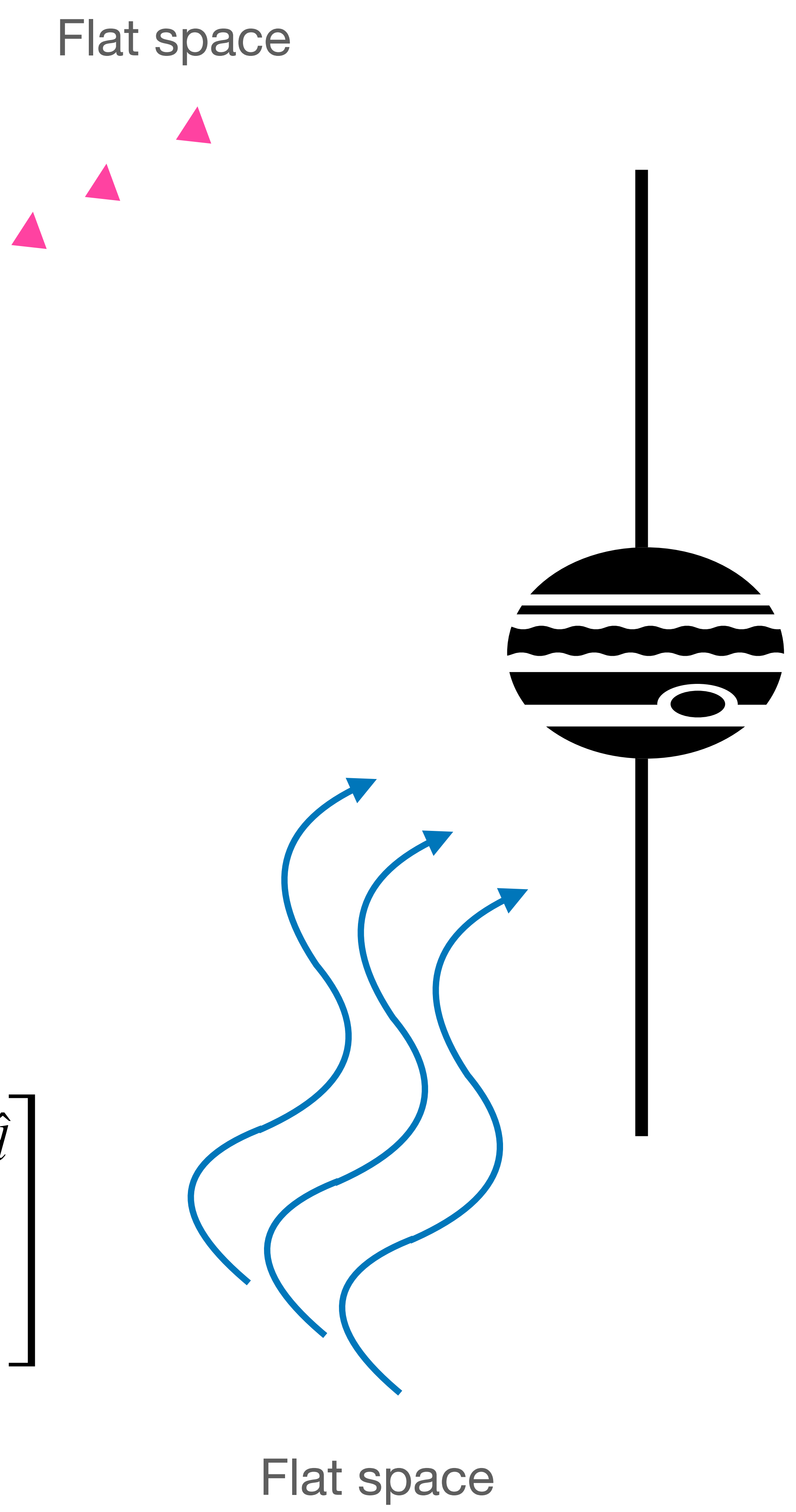
$$\phi^{EFT} = \sum_{\ell=0}^{\infty} e^{i\omega t} \sqrt{2\pi\omega}\; r^{-\hat{d}/2} \omega^{\ell} n_L (-1)^{\ell} \left(\hat{C}^{L}_{\rm reg} J_{\hat{d}/2+\ell}(\omega r) + \hat{C}^{L}_{\rm irreg} Y_{\hat{d}/2+\ell}(\omega r) \right) .$$

$$Q^{L}(\omega) = -\lambda_{\ell}(\omega) \underset{r\to 0}{\mathrm{FP}}\, \partial_L \phi(\omega)$$

$$\lambda_{\ell}(\omega) = -\frac{4\pi^{\hat{d}/2}}{2^{\ell}} \left(\frac{2}{\omega}\right)^{\hat{d}+2\ell} \Gamma\left(\frac{\hat{d}}{2} + \ell + 1\right) \frac{\hat{C}^{L}_{\rm irreg}}{\hat{C}^{L}_{\rm reg}}$$

$$\Box \phi = 0$$

$$\phi^{GR} = \sum_{\ell=0}^{\infty} e^{i\omega t} \sqrt{2\pi\omega}\; r^{-\hat{d}/2} \omega^{\ell} n_L (-1)^{\ell} \left(\hat{C}^{L}_{\rm reg} J_{\hat{d}/2+\ell}(\omega r) + \hat{C}^{L}_{\rm irreg} Y_{\hat{d}/2+\ell}(\omega r) \right) + \mathcal{O}\left[\left(\frac{r_0}{r}\right)^{\hat{d}} \right]$$



At ∞ :

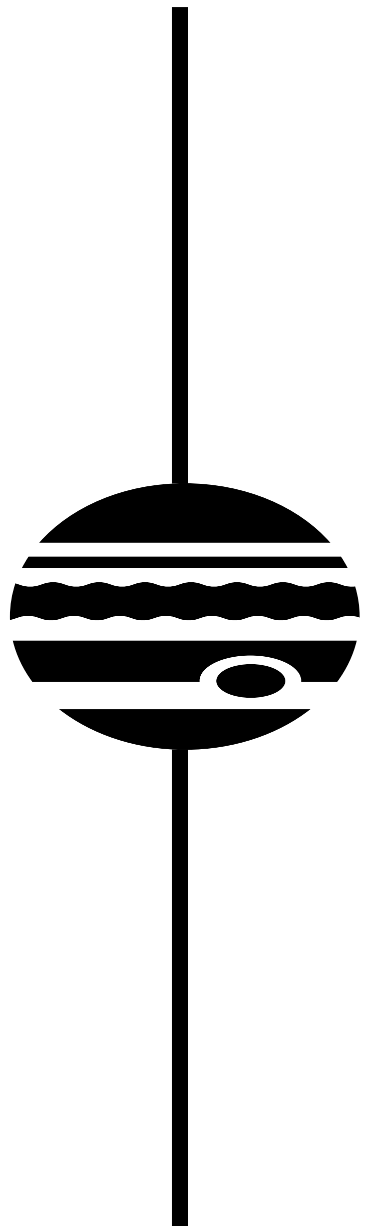
$$C^{EFT}_{\rm reg} = C^{GR}_{\rm reg} \qquad C^{EFT}_{\rm irreg} = C^{GR}_{\rm irreg}$$

Results

Tidal response from scattering

ℓ non-integer

$M\omega \ll 1$



- Near horizon solution at $r \to \infty$:

$$\phi^{GR}_{\rm Horizon} = C_{\rm horizon}\frac{\Gamma\left[-2l-1\right] r_0^{d(l+1)}\Gamma\left[1-\frac{2ir_0\omega}{d}\right]}{\Gamma\left[-l\right]\Gamma\left[-l-\frac{2ir_0\omega}{d}\right]} r^{-d(l+1)} + C_{\rm horizon}\frac{\Gamma\left[2l+1\right] r_0^{-dl}\Gamma\left[1-\frac{2ir_0\omega}{d}\right]}{\Gamma(l+1)\Gamma\left[l-\frac{2ir_0\omega}{d}+1\right]} r^{dl}$$

- Asymptotic solution at $r \to r_0$:

$$\phi^{GR}_{\infty} = A^{\infty}_{\rm irreg}\frac{(-1)^{dl+1}2^{d\left(l+\frac{1}{2}\right)+\frac{1}{2}}\omega^{-1/2(d-1)}\Gamma\left[d\left(l+\frac{1}{2}\right)\right]}{\sqrt{\pi}} r^{-d(l+1)} + A^{\infty}_{\rm reg}\frac{\sqrt{\pi}(-1)^{dl}2^{\frac{1}{2}-d\left(l+\frac{1}{2}\right)}\omega^{2dl+\frac{d}{2}+\frac{1}{2}}}{\Gamma\left[d\left(l+\frac{1}{2}\right)+1\right]} r^{dl}$$

- Matching:

$$\frac{A^{\infty}_{\ell\,\rm irreg}}{A^{\infty}_{\ell\,\rm reg}} = -\frac{\pi\left(\omega r_H/2\right)^{\hat{d}(2\hat{\ell}+1)}\Gamma(-2\hat{\ell}-1)\Gamma(b_\ell)\Gamma(a_\ell^+)}{\Gamma(-\hat{\ell})\Gamma(2\hat{\ell}+1)\Gamma(p)\Gamma(p+1)\Gamma(1-a_\ell^-)}$$

$$p = \frac{\hat{d}}{2}(2\hat{\ell}+1)$$

$$a_\ell^{\pm} = \hat{\ell} + 1 \pm \frac{2ir_H\omega}{\hat{d}}$$

Tidal response from scattering

Discussion

$$\lambda_\ell(\omega) = i\,\Xi_\ell \left[1 - \frac{2}{1 + \frac{C_\ell^{\rm in}}{C_\ell^{\rm out}} e^{i\frac{\pi}{2}\left(\hat{d}+1\right)}} \right]$$

Recovers known results

Full-frequency spectrum

Scattering states defined at flat-space null infinity ⟷ One-to-one identification to full theory

$\hat{d}$ → Simplifies calculations →

$$(r - r^*) \sim \begin{cases} -r_H \log(r^*) + \dots & d = 4 \\ \left(\frac{r_H}{r^*}\right)^{\hat{d}} r^* + \dots & d > 4 \end{cases}$$

→ Distinguishes PN and tidal terms

→ Not necessary to compute tidal response from scattering

$\hat{\ell}$ → Intrinsic of matched asymptotic expansions