Can nonlinear electromagnetic fields regularize black hole singularities?

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Nonlinear electrodynamics (NLE)

- Electromagnetic invariants: $\mathcal{F} = F_{ab}F^{ab}$, $\mathcal{G} = F_{ab}{*F}^{ab}$
- Maxwell's Lagrangian: $\mathscr{L} = -\frac{1}{4}\mathcal{F}$
- NLE Lagrangian: $\mathscr{L}(\mathcal{F}, \mathcal{G})$
- NLE theories:
  - $\mathcal{F}$-class: $\mathscr{L}(\mathcal{F})$
  - $\mathcal{F}\mathcal{G}$-class: $\mathscr{L}(\mathcal{F}, \mathcal{G})$

Generalised Maxwell's equations:

$d\mathbf{F} = 0$

$d * \mathbf{Z} = 0$, with $\mathbf{Z} = -4(\mathscr{L}_{\mathcal{F}}\mathbf{F} + \mathscr{L}_{\mathcal{G}}{*\mathbf{F}})$

Notation: $\mathscr{L}_x$ denotes $\partial_x \mathscr{L}$

Nonlinear electrodynamics (NLE)

- NLE energy-momentum tensor:

$$T_{ab} = -4\mathscr{L}_{\mathcal{F}}\widetilde{T}_{ab} + \frac{1}{4}T g_{ab},$$

$$\widetilde{T}_{ab} = \frac{1}{4\pi}\Big(F_{ac}\,F_b{}^c - \frac{1}{4}g_{ab}\mathcal{F}\Big)$$

- Maxwellian weak field (MWF) limit:

$$\mathscr{L}_{\mathcal{F}} \to -1/4, \quad \mathscr{L}_{\mathcal{G}} \to 0 \text{ as } (\mathcal{F},\mathcal{G}) \to (0,0)$$

- NLE+gravity Lagrangian: $\mathbf{L} = \frac{1}{16\pi}(\mathscr{L}^{(grav)} + 4\mathscr{L})\boldsymbol{\epsilon}$

Motivation

QFT

- Euler Heisenberg theory - 1-loop QED correction

$$\mathcal{L}^{(\mathrm{EH})} = -\frac{1}{4}\,\mathcal{F} + \frac{\alpha^2}{360 m_e^4}\left(4\mathcal{F}^2 + 7\mathcal{G}^2\right)$$

Resolution of point charge singularities

- Born-Infeld theory - phenomenological

$$\mathcal{L}^{(\mathrm{BI})} = b^2\left(1 - \sqrt{1 + \frac{\mathcal{F}}{2b^2} - \frac{\mathcal{G}^2}{16b^4}}\right)$$

Other NLE Lagrangians:

- ModMax theory

$$\mathcal{L}^{(\mathrm{MM})} = \frac{1}{4}\left(-\mathcal{F}\cosh\gamma + \sqrt{\mathcal{F}^2 + \mathcal{G}^2}\sinh\gamma\right)$$

- Power-Maxwell, exponential, logarithmic...

Bardeen black hole (1968)

- regular solution

$$ds^2 = -f(r)dt^2 + f^{-1}(r)dr^2 + r^2 d\Omega^2,$$
$$f(r) = 1 - \frac{2\mu r^2}{(r^2 + g^2)^{3/2}}$$

- corresponding NLE Lagrangian: [Ayón-Beato, García (2000)]

$$\mathcal{L} = \frac{3\mu}{g^3}\left(\frac{g\sqrt{2\mathcal{F}}}{2 + g\sqrt{2\mathcal{F}}}\right)^{5/2}$$

- $g = P \rightarrow$ magnetic charge

Bronnikov's regularity conditions

- spherically symmetric solutions
- $\mathcal{F}$-class Lagrangians with MWF limit
- regularity condition: curvature scalars must be bounded
- idea: translate curvature invariants to matter invariants

  $\rightarrow$ no globally regular spacetime if $Q \neq 0$

- goal: generalize result to $\mathcal{FG}$-class Lagrangians

EM and curvature invariants

EM invariants

- $4\pi^2 T_{ab} T^{ab} = \pi^2 T^2 + \mathscr{L}_{\mathcal{F}}^2 (\mathcal{F}^2 + \mathcal{G}^2)$
- $(T^n)^a{}_a$ gives no new independent EM invariants
  notation: $(X^n)^a{}_b = X^a{}_{c_1} X^{c_1}{}_{c_2} \cdots X^{c_{n-1}}{}_b$

Via Einstein's equation:

$$R - 4\Lambda = -8\pi T\,,$$
$$R_{ab}R^{ab} + 2\Lambda(2\Lambda - R) = (8\pi)^2\, T_{ab} T^{ab}$$

Bounded R and $R_{ab}R^{ab} \to$ bounded T, $\mathscr{L}_{\mathcal{F}}\mathcal{F}$ and $\mathscr{L}_{\mathcal{F}}\mathcal{G}$

Spherically symmetric spacetime

- static, spherically symmetric metric:
  $\mathrm{d}s^2 = -f(r)\,\mathrm{d}t^2 + \frac{1}{f(r)}\,\mathrm{d}r^2 + r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\varphi^2\right)$
- EM tensor: $\mathbf{F} = -E_r(r)\,\mathrm{d}t \wedge \mathrm{d}r + B_r(r)\,r^2 \sin\theta\,\mathrm{d}\theta \wedge \mathrm{d}\varphi$
- EM invariants: $\mathcal{F} = 2(B_r^2 - E_r^2)$, $\mathcal{G} = 4E_r B_r$
- source-free Maxwell's equations:

$$B_r = \frac{P}{r^2}$$

$$\mathscr{L}_{\mathcal{F}} E_r - \mathscr{L}_{\mathcal{G}} B_r = -\frac{Q}{4r^2}$$

Komar charges:

$Q = \frac{1}{4\pi}\oint_{\mathcal{S}} *\mathbf{Z}, \quad P = \frac{1}{4\pi}\oint_{\mathcal{S}} \mathbf{F}$

Electric case

$Q \neq 0,\ P = 0$

Theorem 1.

- $\mathcal{F}\mathcal{G}$-class NLE Lagrangian, MWF limit
  $\rightarrow$ R and $R_{ab}R^{ab}$ are not both bounded as $r \rightarrow 0$

Proof:

- Maxwell's equation (with $B_r = 0$) :
  $$\frac{\mathcal{F}}{r^4} = -\frac{8}{Q^2}\,(\mathcal{F}\mathscr{L}_\mathcal{F})^2,\ \mathscr{L}_\mathcal{F}^2 = -\frac{Q^2}{8\mathcal{F}r^4}$$
- if $\mathscr{L}_\mathcal{F}\mathcal{F}$ is bounded, $\mathcal{F} = O(r^4)$ as $r \rightarrow 0$
- $\mathscr{L}_\mathcal{F}$ is unbounded as $r \rightarrow 0$, contradiction with MWF limit
- note: $r \rightarrow 0$ limit coincides with weak field limit

Dyonic case

$Q \neq 0$, $P \neq 0$

- obstacle: we cannot test the MWF limit:

$$\mathcal{F} = 2\left(\frac{P^2}{r^4} - \frac{r^4}{16P^2}\,\mathcal{G}^2\right)$$

- no-go theorems can be proved for specific $\mathcal{F}\mathcal{G}$-class Lagrangians of the form:

$$\mathscr{L} = -\frac{1}{4}\mathcal{F} + h(\mathcal{F},\mathcal{G})$$

Magnetic case

$Q = 0,\ P \neq 0$

Bronnikov:

- $\mathcal{F}$-class Lagrangian
- $\lim_{\mathcal{F}\to\infty} \mathscr{L}(\mathcal{F})$ exists and is finite

  $\rightarrow$ magnetically charged solutions can be regular

Examples:

- $\mathscr{L} = -\mathcal{F}/(1 + 2\beta\mathcal{F})$
- $\mathscr{L} = -\mathcal{F}\exp(-\beta\mathcal{F})$
- ...

Problem: physical motivation, magnetic monopoles?

Theorems

	Dyonic	Magnetic
$\mathcal{F}$-class Lagrangians	✗	
quadratic Lagrangians $\mathscr{L} = -\frac{1}{4}\mathcal{F} + a\mathcal{F}^2 + b\mathcal{F}\mathcal{G} + c\mathcal{G}^2$	✗	✗
$\mathscr{L} = -\frac{1}{4}\mathcal{F} + h(\mathcal{G})$	✗	✗
$\mathscr{L} = -\frac{1}{4}\mathcal{F} + a\mathcal{F}^s\mathcal{G}^u,\ s, u \geq 0$	✗	
Born-Infeld Lagrangian	✗	✗
ModMax Lagrangian	✗	✗

Conclusion

- obstructions to the prospect of regularization:
  - singularity is present if $Q \neq 0$
  - physically motivated Lagrangians give no regular solutions

Open questions

- other $\mathcal{FG}$-class Lagrangians
- modified gravitational action
- nonminimal coupling of gravity and NLE
- other types of spacetimes

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References

[1] A. Bokulić, T. Jurić, I. Smolić, *Phys. Rev. D* 105, 024067 (2021)
[2] A. Bokulić, T. Jurić, I. Smolić, *Phys. Rev. D* 106, 064020 (2022)
[3] K. A. Bronnikov, *Phys. Rev. D* 63, 044005 (2001)
[4] J. Bardeen, *Proceeding of the International Conference GR5*, Tbilisi (1968)
[5] E. Ayón-Beato, A. García, *Phys. Lett.* B493, 149 (2000)