Signatures of Collapsing Spherically Symmetric Distributions of Dust

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Gravitational Collapse by Oppenheimer and Snyder

For a spherically symmetric contracting body of dust:

$$\mathrm{d}s^2 = -\mathrm{d}\tau^2 + \mathrm{e}^{\omega}\,\mathrm{d}R^2 + \mathrm{e}^{\lambda}\,\mathrm{d}\Omega^2 \quad \text{with} \qquad T^{\alpha\beta} = \rho\, u^{\alpha}\, u^{\beta}$$

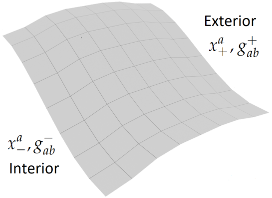
The solution is

$$\text{Interior:}\;\; \mathrm{d}s^2 = -\mathrm{d}\tau^2 + \xi^2(\tau, R_0)\left(\mathrm{d}R^2 + R^2\mathrm{d}\Omega^2\right)$$

$$\text{Exterior:}\;\; \mathrm{d}s^2 = -\mathrm{d}\tau^2 + \xi^{-1}(\tau, R)\left(\mathrm{d}R^2 + R^2\xi^3(\tau, R)\,\mathrm{d}\Omega^2\right)$$

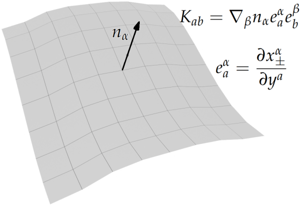
With $\xi(\tau, R) = \left[1 - \frac{3}{2}\frac{(2M)^{1/2}\tau}{R^{3/2}}\right]^{2/3}$ the scale factor. The interior is a flat FLRW metric and at $\tau_{coll} = \frac{2R^{3/2}}{3(2M)^{1/2}}$ there is collapse. Dust was shown to collapse.

We section spacetime and impose the metric to be some distribution



Continuity of the metric

$$[h_{ab}] = 0 \ , \quad h_{ab} = g_{\alpha\beta} e^\alpha_a e^\beta_b$$



Continuity of the (extrinsic) curvature dependent on the boundary

$$S_{ab} = -\frac{1}{8\pi}\left([K_{ab}] - [K]h_{ab}\right)$$

Choose interior and exterior spacetimes, conditions give the dynamics

Junction Conditions II

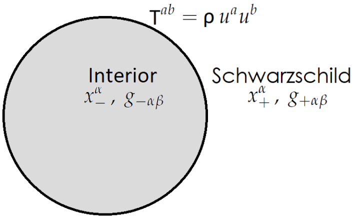
Assuming a non rotating, homogeneous, distribution of dust

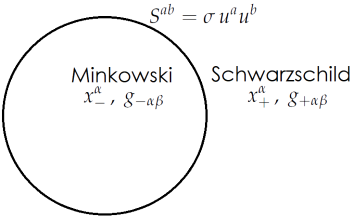
Massive sphere ($T^{\alpha\beta} = \rho\, u^\alpha\, u^\beta$):

$$[h_{ab}] = 0$$
$$[K_{ab}] = 0$$

Thin shell ($S^{ab} = \sigma\, u^a\, u^b$):

$$[h_{ab}] = 0$$
$$[K_{ab}] \neq 0$$





Collapsing Stars ($T^{\alpha\beta} = \rho\, u^\alpha\, u^\beta$)

Consider for the interior a FLRW metric with $k = 1$ and $a \in [0, A]$:

$$\mathrm{d}s^2 = \xi^2(\eta)\left[-\mathrm{d}\eta^2 + \frac{\mathrm{d}a^2}{1-a^2} + a^2\,\mathrm{d}\Omega^2\right]$$

And for the exterior a Schwarzschild metric:

$$\mathrm{d}s^2 = -\left(1-\frac{2M}{R}\right)\mathrm{d}T^2 + \left(1-\frac{2M}{R}\right)^{-1}\mathrm{d}R^2 + R^2\,\mathrm{d}\Omega^2$$

On the homogeneity of the star

$$[K_{ab}] = 0 \implies \dot{R}^2 = -A^2 + \frac{2M}{R}$$

Collapsing Bound Star

The interior parameterized solution

$$R(\eta) = \frac{R_0}{2}(1+\cos\eta)\ , \quad \tau(\eta) = \frac{R_0}{2}(\eta+\sin\eta)\ , \quad \xi(\eta) = \sqrt{\frac{R_0}{2M}}R(\eta)$$

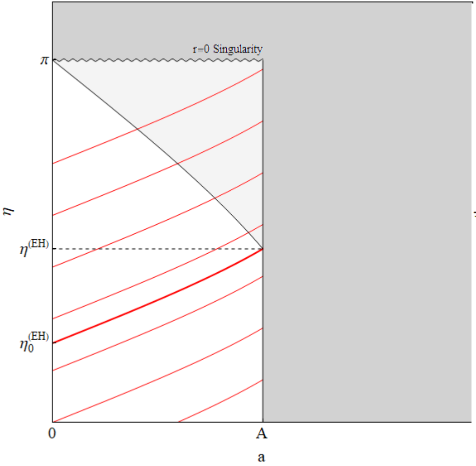
The exterior solution

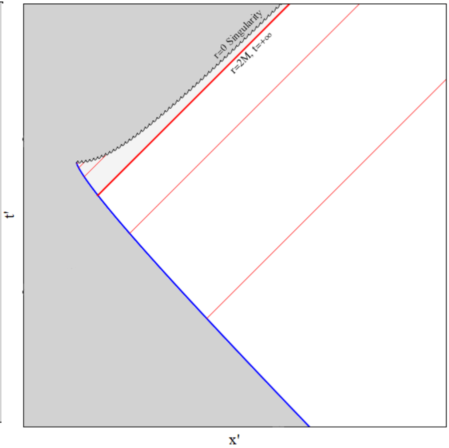
$$T(R) = \sqrt{F(R)} + 2M\ln\left|\frac{G(R)}{R-2M}\right| - \Delta\arccos\left(H(R)\right)$$

with $F(R)$, $G(R)$ and $H(R)$ regular functions of R, Δ a constant and $R_0 \equiv R(T=0)$.
The causal structure requires also,

$$\text{EH: } \xi(\eta_A^{EH})A = 2M\ , \quad \text{AH: } (\nabla g_{22})\cdot(\nabla g_{22}) = 0\ , \quad \text{LR} : \mathrm{d}s^2 = 0$$

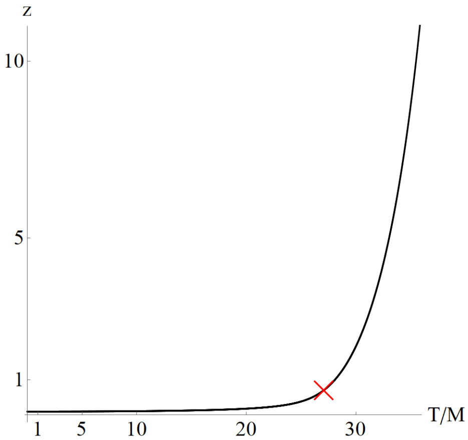
Causal Structure





Redshift

The exterior observer can observe the evolving redshift of light rays emitted from the star's surface



For $R_0 = 10^5 M$, the time to get to the photon sphere is:

$$M = 3M_\odot \implies t \approx 9\text{minutes.}$$

$$M = 100M_\odot \implies t \approx 5\text{hours.}$$

Collapsing Thin Shells ($S^{\alpha\beta} = \sigma\, u^\alpha\, u^\beta$)

The interior is Minkowski

$$\mathrm{d}s^2 = -\mathrm{d}t_-^2 + \mathrm{d}r_-^2 + r_-^2\,\mathrm{d}\Omega^2$$

and the exterior Schwarzschild

$$\mathrm{d}s^2 = -\left(1 - \frac{2M}{r_+}\right)\mathrm{d}t_+^2 + \left(1 - \frac{2M}{r_+}\right)^{-1}\mathrm{d}r_+^2 + r_+^2\,\mathrm{d}\Omega^2$$

The localized distribution of the shell gives

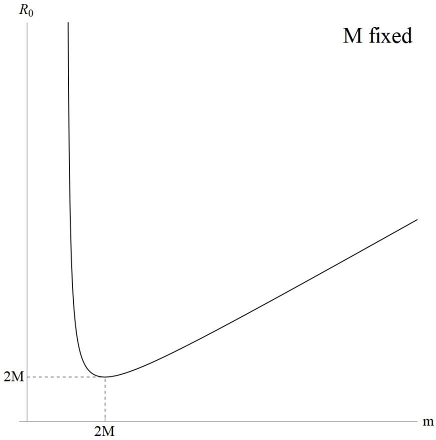
$$[K_{ab}] \neq 0 \implies M = m\sqrt{\dot{R}^2 + 1} - \frac{m^2}{2R}$$

With $m = 4\pi\sigma R^2$.

Collapsing Bound Shell

We choose $M < m$ from which

$$M = m\sqrt{\dot{R}^2 + 1} - \frac{m^2}{2R} \quad \xrightarrow[\dot{R}=0]{} R_0 = \frac{m^2}{2(m-M)}$$

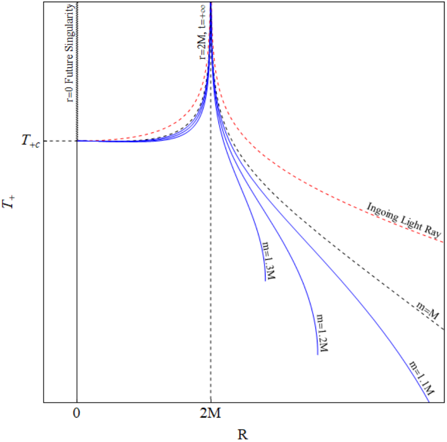


Two values of m ($m_<$,$m_>$) per R_0:

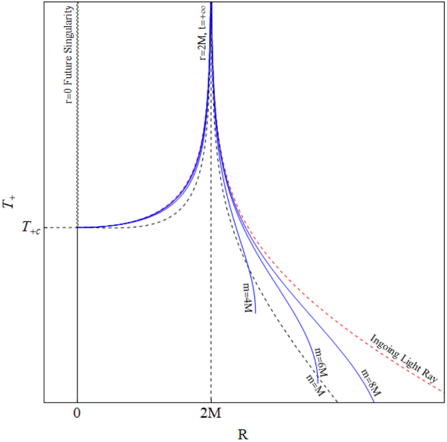
$$M < m_< < 2M$$
$$m_> > 2M$$

Bound Shell: Trajectories

$M < m < 2M$

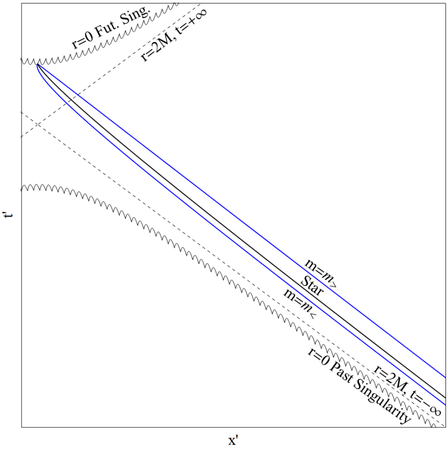


$m > 2M$

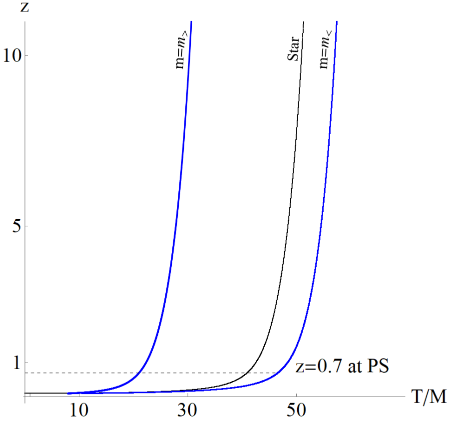


Star and Thin Shell Comparison

Trajectories



Redshift



- Junction conditions facilitate obtaining a description of spherically symmetric distributions of collapsing matter.
- Thin shells are simple and easy to solve but retain the general features of collapsing bodies.
- The interior observer can detect various events characteristic of black hole formation, but the exterior observer can also detect some signatures.

Thank you!