Study of impact ionization coefficients in Low Gain Avalanche Diodes

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Outline

- Input data for the new parametrization.
  - TCAD simulation of CNM-12916 and HPK2 LGADs (Splits: 1, 2, 3 and 4).
  - Existing impact ionization models: Massey, Van Overstraeten and Okuto-Crowell.
  - Gain measurements as a function of the electric field and temperature with IR-laser.
- Fitting the parameters of these three models to our data outside TCAD.
  - Solve the impact ionization equation in C++.
- Error evaluation.
- Study of the breakdown voltage as a function of the temperature.
- Summary and next steps.

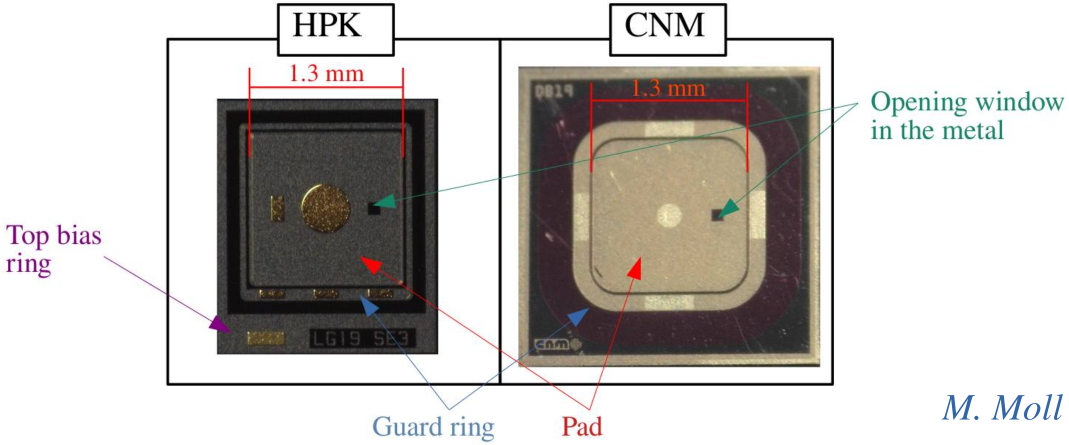
This presentation is an update from the work presented in the last "*40th RD50 workshop (CERN)*":

M. Moll et al, "TCAD simulation of impact ionization in non-irradiated LGADs"

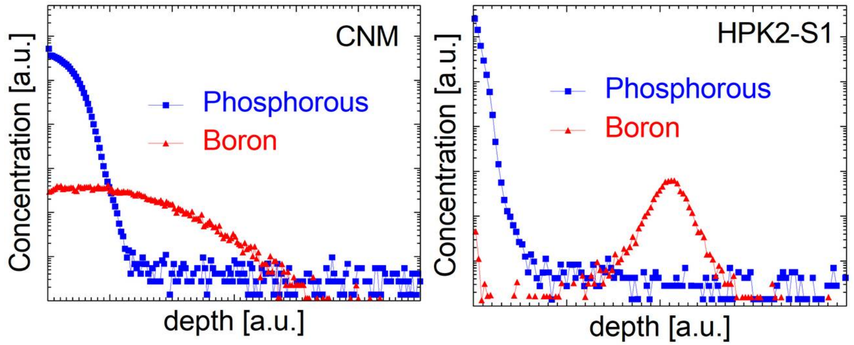
E. Curras et al, "New impact ionization parameters"

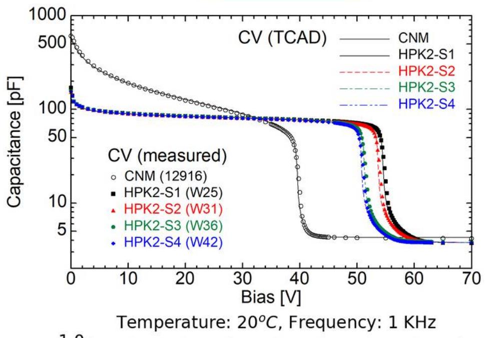
Input data: Electric fields from TCAD

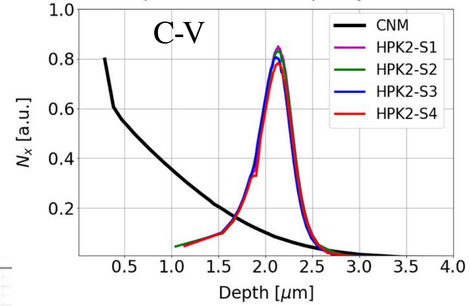
- **HPK** LGADs and PAD detectors:
  - HPK prototype 2 sensors: HPK2.
  - Thickness: 50 um.
  - 4 different splits: S1 (W25), S2 (W31), S3 (W36) and S4 (W42) (from higher to lower gain).
  - Deep gain layer implantation.
- **CNM** LGADs and PAD detectors:
  - CNM run 12916.
  - Thickness: 50 um.
  - Only one split.
  - Shallow gain layer implantation.



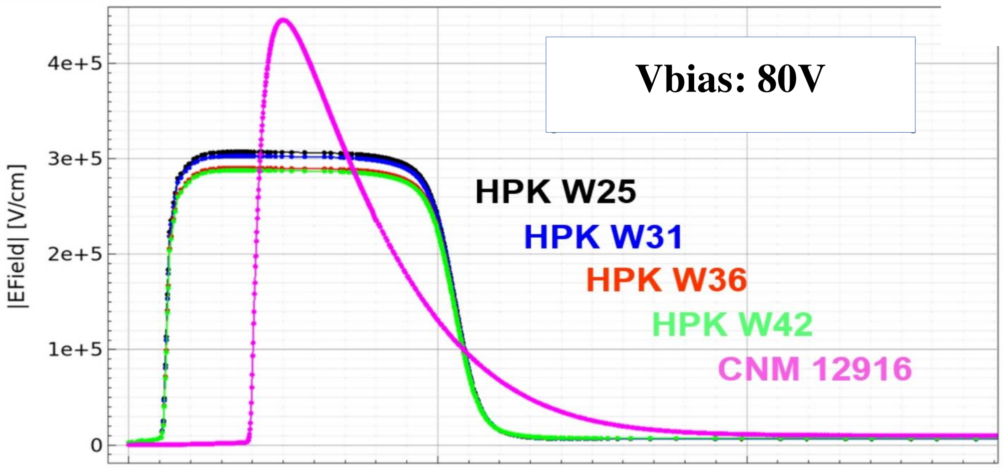
SIMS measurements







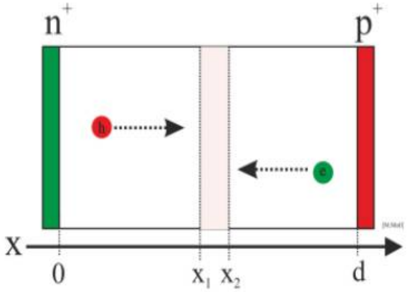
Electric field profiles extracted from TCAD



M. Moll et al, "TCAD simulation of impact ionization in non-irradiated LGADs" 40th RD50 Workshop.

Impact ionization equation

- Consider multiplication of electrons and holes



charge generation [As/cm³]

$$-\frac{dJ_n}{dx} = \alpha_n J_n + \alpha_p J_p + g(x) = \frac{dJ_p}{dx}$$

$\alpha_n(E,T)$ (e^-) $\alpha_p(E,T)$ (h^+)

$$J = J_n(x) + J_p(x)$$

$$-\frac{dJ_n}{dx} = (\alpha_n - \alpha_p) J_n + \alpha_p J + g(x) = \frac{dJ_p}{dx}$$
$$= (\alpha_p - \alpha_n) J_p + \alpha_n J + g(x)$$

- Solution for the total current:

Gain equation implemented in C++

$$J = M_n J_n(d) + M_p J_p(0) + \int_0^d g(x) M(x) dx$$

with

$$M(x) = \frac{\exp\left(-\int_x^d (\alpha_n - \alpha_p)\, d\eta\right)}{1 - \int_0^d \alpha_n \exp\left(-\int_\xi^d (\alpha_n - \alpha_p)\, d\eta\right) d\xi}$$

See also: A. Howard et al, "Determining the Impact Ionisation Parameters for LGADs" 39th RD50 Workshop.

Impact ionization models "$\alpha_{n,p}$ = f(E,T)":

All these three models have a different formulation for the temperature dependence.

Massey:

$$\alpha_{n,p}(E,T) = A_{n,p}\, exp\left(-\frac{C_n + D_{n,p}\cdot T}{E}\right)$$

6 parameters: $A_{n,p}$, $C_{n,p}$ and $D_{n,p}$

Van Overstraeten:

$$\alpha_{n,p}(E,T) = \gamma\, A_{n,p}\, exp\left(-\gamma\,\frac{B_{n,p}}{E}\right),$$

$$\text{with}\quad \gamma = \frac{tanh\left(\frac{\hbar\omega_{op}}{2kT_o}\right)}{tanh\left(\frac{\hbar\omega_{op}}{2kT}\right)},$$

5 parameters: $A_{n,p}$, $B_{n,p}$ and $\hbar\omega_{op}$

Most abundant optical phonon energy

Okuto-Crowell:

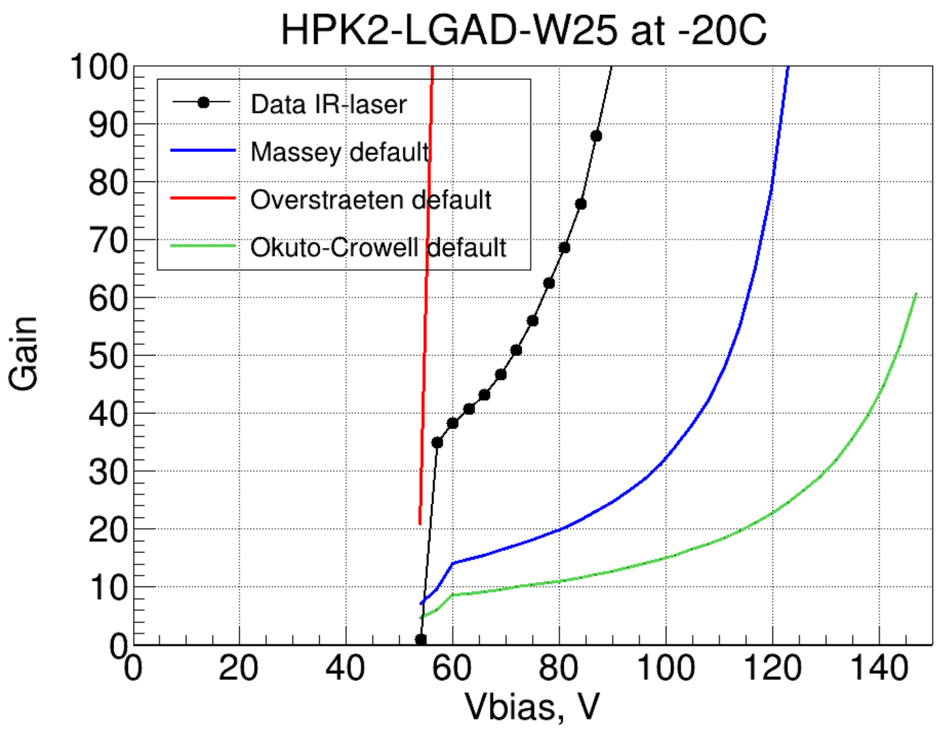
8 parameters: $A_{n,p}$, $B_{n,p}$, $C_{n,p}$ and $D_{n,p}$

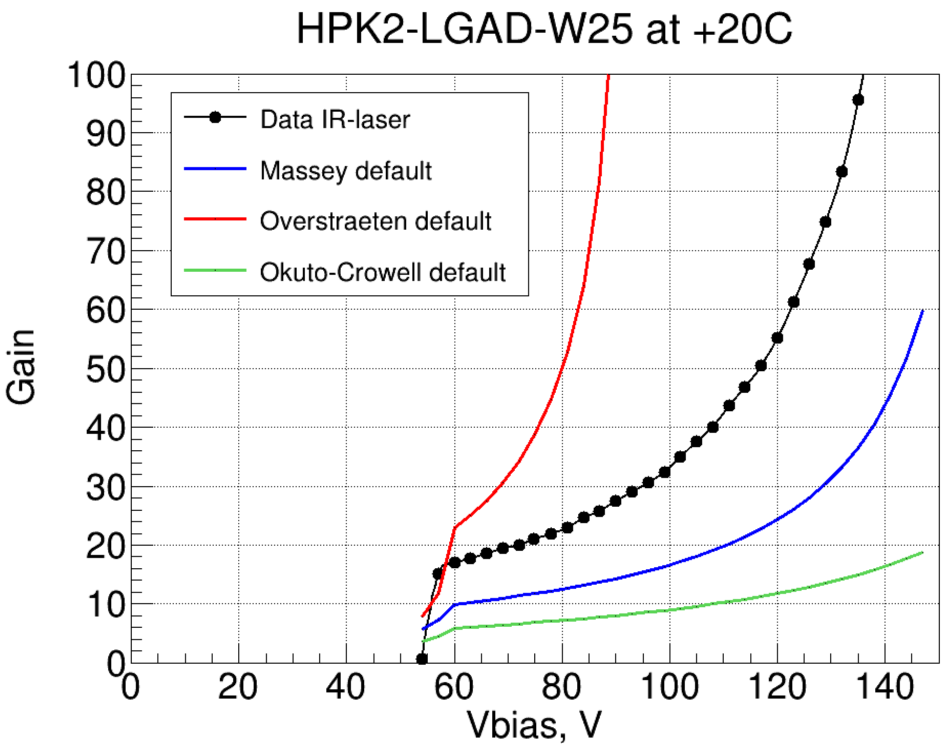
$$\alpha_{n,p}(E,T) = A_{n,p}(1 + C_{n,p}\cdot\tilde{T})E\, exp\left(-\left(\frac{B_{n,p}(1 + D_{n,p}\cdot\tilde{T})}{E}\right)^2\right)$$

with $\tilde{T} = T - 300\,K$ and eight parameters $A_{n,p}$,$B_{n,p}$, $C_{n,p}$ and $D_{n,p}$

Impact ionization models "$\alpha_{n,p} = f(E,T)$":

- **Motivation**: measured and simulated gain using these models with the default parametrization disagree significantly. For this particular example:
  - Overstraeten overestimates the gain.
  - Massey and Okuto-Crowell underestimate the gain.





Gain curves measured in TCT (IR-laser: 1064 nm)

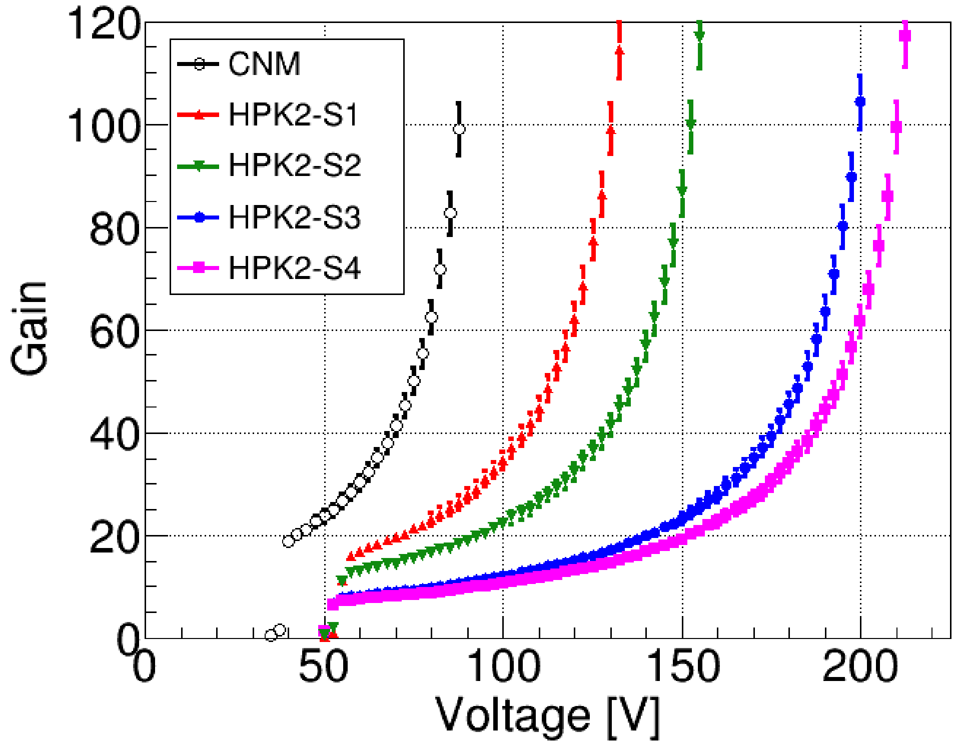
- 4 different temperatures:
  - -15ºC, 0ºC, 20ºC and 40ºC
- IR-laser intensity ~1 MIP
- IR-laser spot **~80 um**

- ~5% error in the gain measurements
- Gain is evaluated:

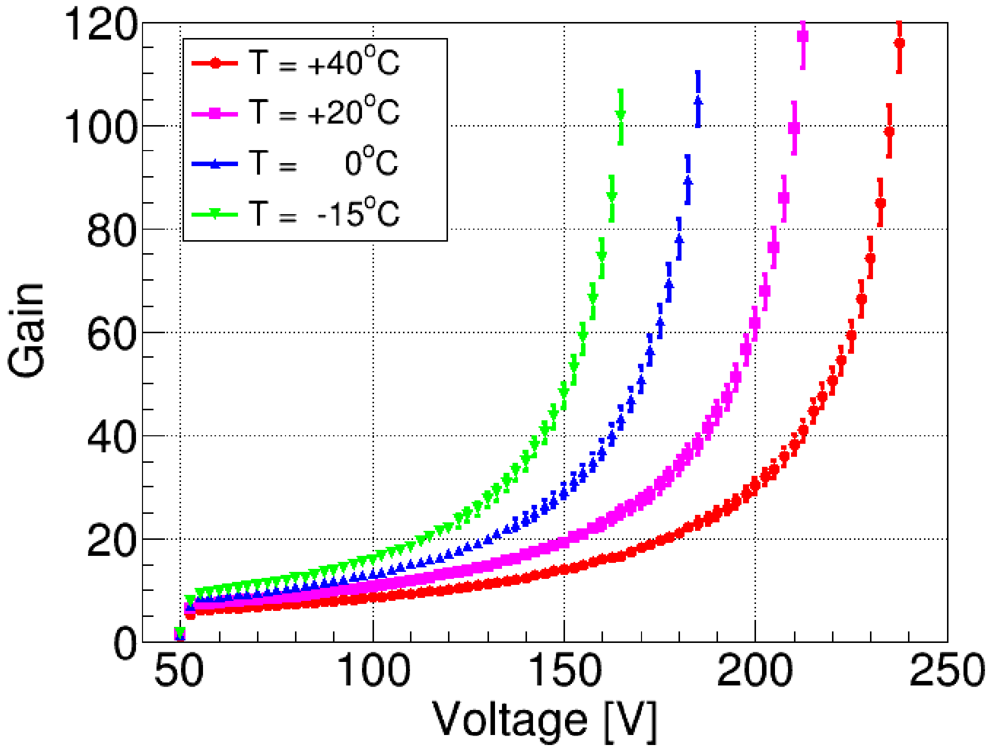
$$G(V,T) = \frac{Q_{LGAD}(V,T)}{Q_{PIN}(100V,T)^{*}}$$

* Q_{PIN} measured at 100V, (V_{FD} ~ 10 V)

IR-Laser, temperature = 20ºC

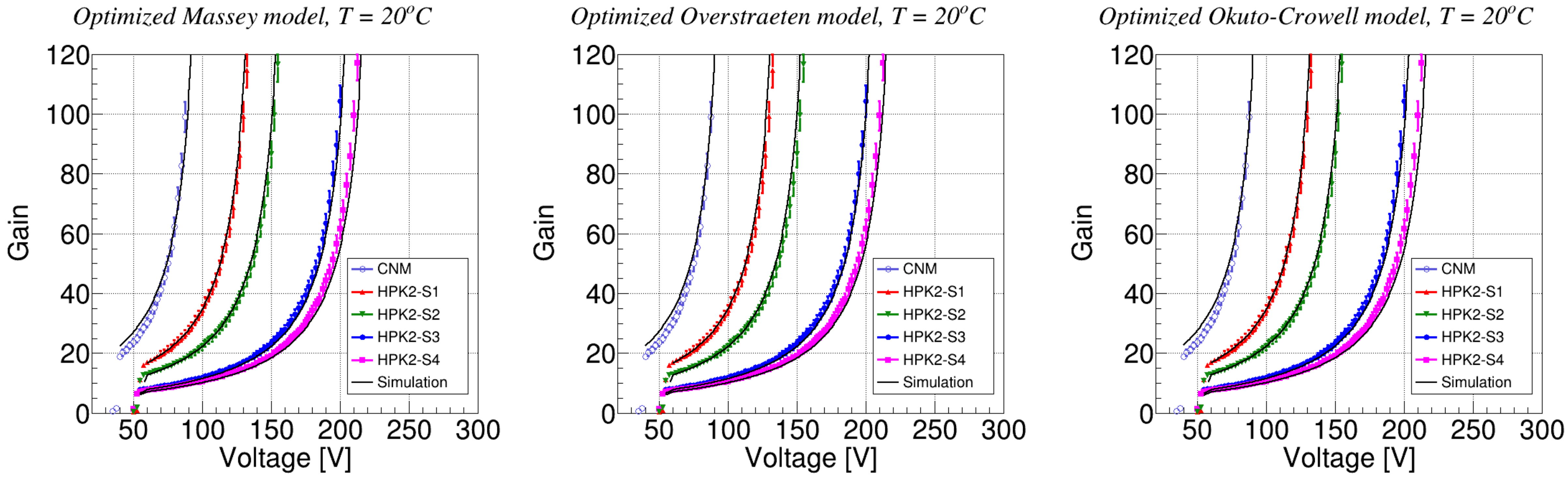


IR-Laser, HPK2-S4 LGAD



Tuning the parameters: results for 20ºC

Data for **all temperatures and all LGADs fitted together**.



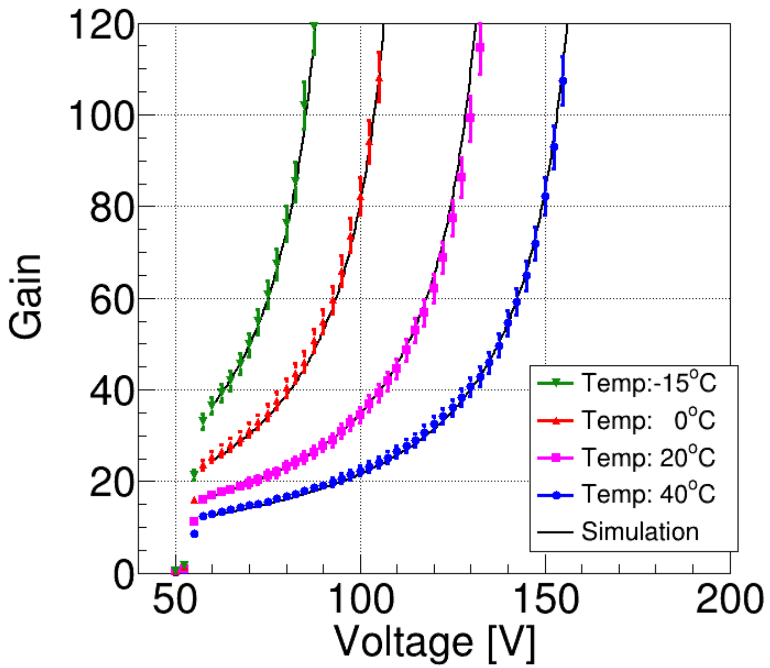
After tuning the parameters with the measured data, we get a good agreement for all the LGADs at 20ºC with the three models.

The three models work well after tuning the parameters.

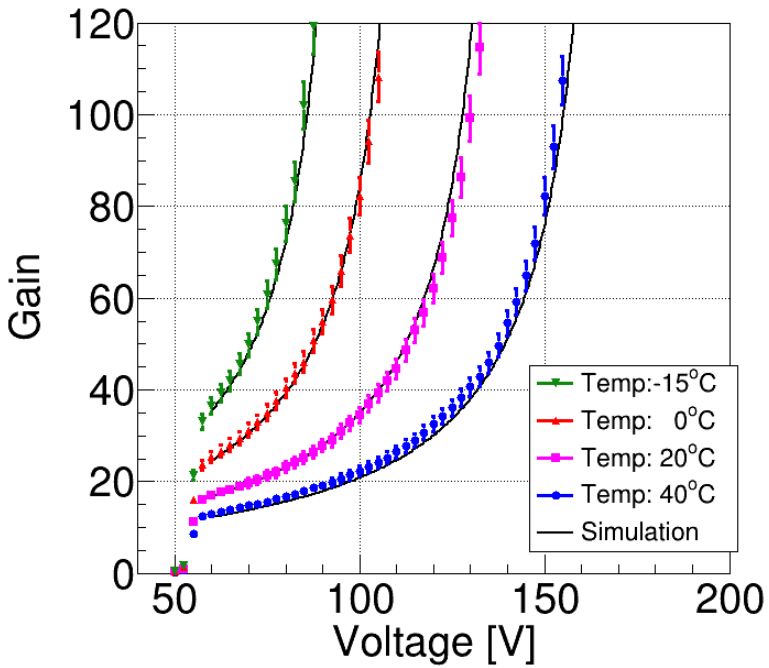
Tuning the parameters: results for different temperatures

Data for **all temperatures and all LGADs fitted together**.

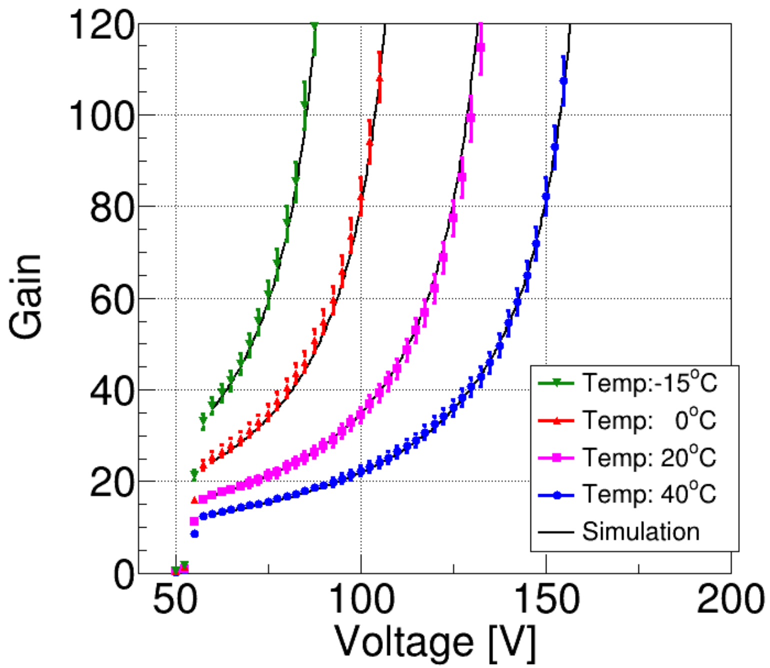
Optimized Massey model, HPK2-S1



Optimized Overstraeten model, HPK2-S1



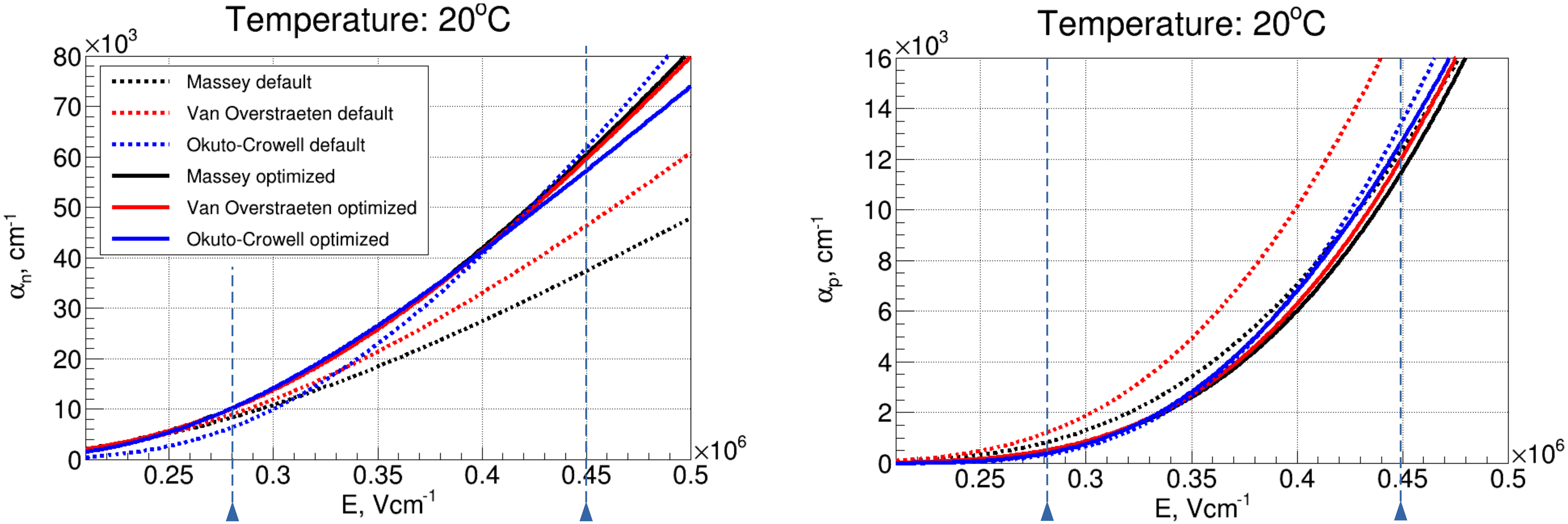
Optimized Okuto-Crowell model, HPK2-S1



After tuning the parameters with the measured data, we get a good agreement with all the samples for the four measured temperatures with the three models: here we show HPK2-S1 LGAD as an example

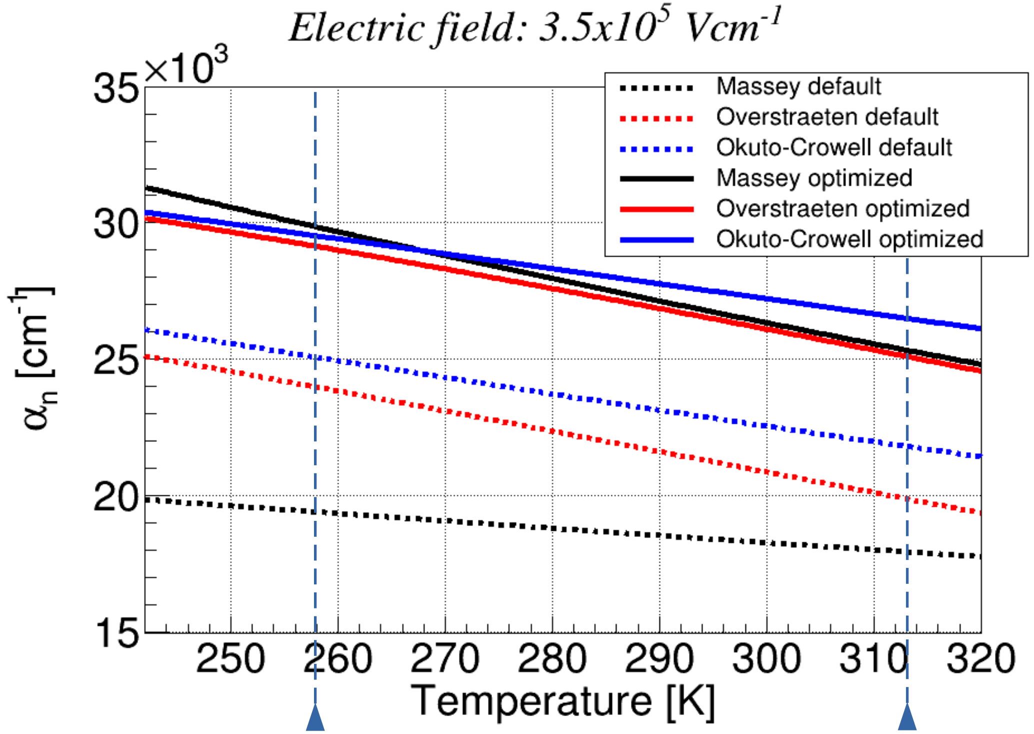
The three models work well after tuning the parameters.

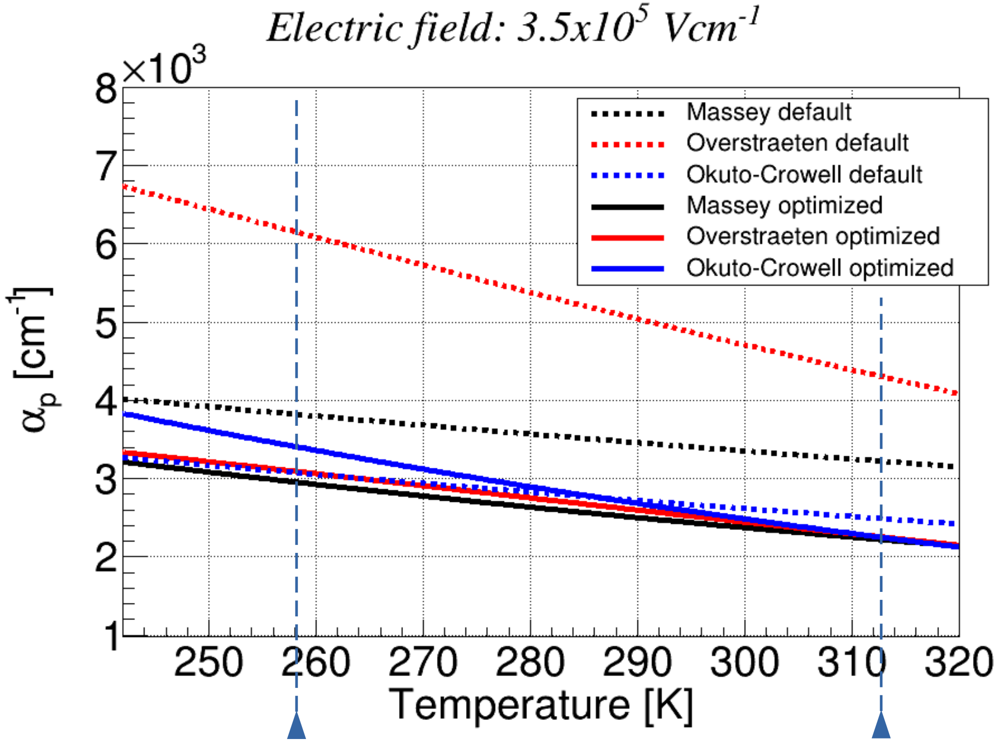
Alpha(α_n) and beta(α_p) comparison: at 20ºC



- All models give similar impact ionization coefficients as a function of the electric field, within an error of ~10% in the range of studied electric fields ($2.8\text{x}10^5$ Vcm^{-1}– $4.5\text{x}10^5$ Vcm^{-1}).
  - The simulated gain after the optimization of the parameters is almost the same.
- This is not the case with the default parametrizations:
  - Massey and Okuto-Crowell models underestimate the gain as α_n(E) is too small.
  - Van Overstraeten is overestimating the gain as α_p(E) is too high.

Alpha(α_n) and beta(α_p) comparison: at 3.5×10^5 Vcm^{-1}





- All models give similar impact ionization coefficients as a function of the temperature, within an error of ~10% in the range of studied temperatures (258 K – 313 K).
  - All methods use a different formulation for the temperature dependence of the impact ionization coefficients.
  - The simulated gain after the optimization of the parameters is almost the same.
- This is not the case with the default parametrizations:
  - All the models underestimate α_n(T).
  - Van Overstraeten and Massey are overestimating α_p(T), but not Okuto-Crowell.

New parameters after the optimization

	Massey default		Massey optimized	
Parameter	electrons	holes	electrons	holes
A (cm^{-1})	4.43×10^5	1.13×10^6	1.186×10^6	2.250×10^6
C ($V \cdot cm^{-1}$)	9.66×10^5	1.71×10^6	1.020×10^6	1.851×10^6
D ($V \cdot cm^{-1} K^{-1}$)	4.99×10^2	1.09×10^3	1.043×10^3	1.828×10^3

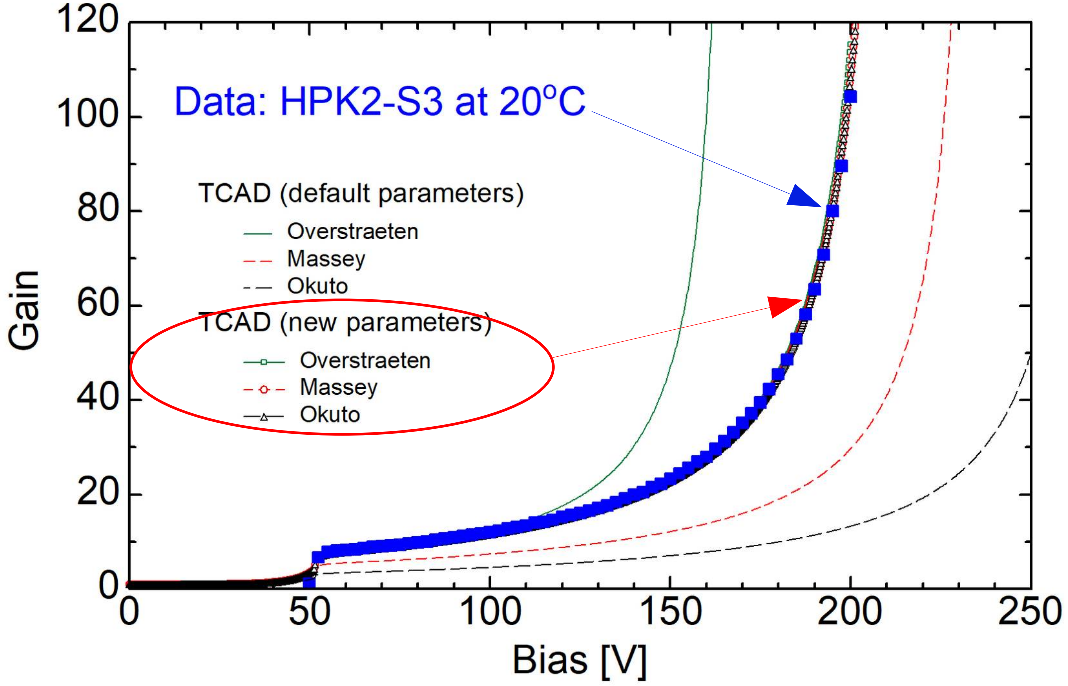
	Van Overstraeten default		Van Overstraeten optimized	
Parameter	electrons	holes	electrons	holes
A (cm^{-1})	7.03×10^5	1.582×10^6	1.149×10^6	2.519×10^6
B ($V \cdot cm^{-1}$)	1.231×10^6	2.036×10^6	1.325×10^6	2.428×10^6
$\hbar\omega_{op}$ (eV)	0.063	0.063	0.0758	0.0758

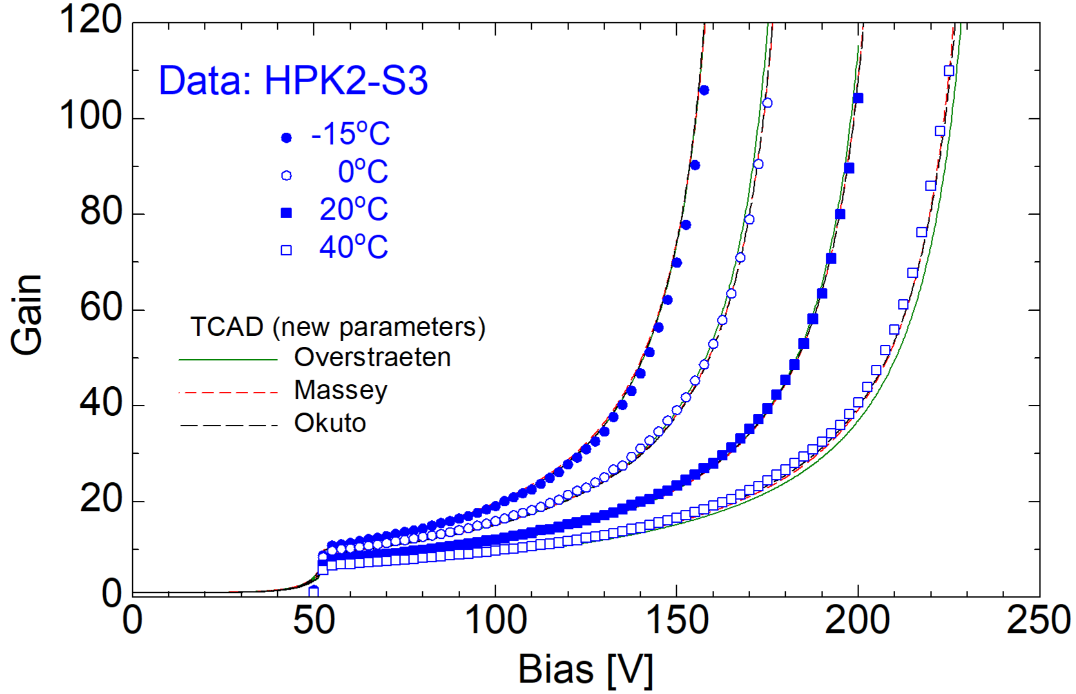
	Okuto-Crowell default		Okuto-Crowell optimized	
Parameter	electrons	holes	electrons	holes
A (V^{-1})	0.426	0.243	0.289	0.202
B ($V \cdot cm^{-1}$)	4.81×10^5	6.53×10^5	4.01×10^5	6.40×10^5
C (K^{-1})	3.05×10^{-4}	5.35×10^{-4}	9.03×10^{-4}	-2.20×10^{-3}
D (K^{-1})	6.86×10^{-4}	5.67×10^{-4}	1.11×10^{-3}	8.25×10^{-4}

- In general, all the parameters change significantly after the optimization.
  - A small variation in some parameters can change the simulated gain drastically.
  - Same others are less critical.
- The value of the residuals of the gain, using the least squares method with the **default** parameters are:
  - Massey : 30.4
  - Van Overstraeten: 5.39×10^4
  - Okuto-Crowell: 38.3
- The value of the residuals of the gain, using the least squares method with the **optimized** parameters are:
  - Massey: 0.52
  - Van Overstraeten: 0.68
  - Okuto-Crowell: 0.57

New parametrization works well in **TCAD**

C++ and TCAD give the same results !





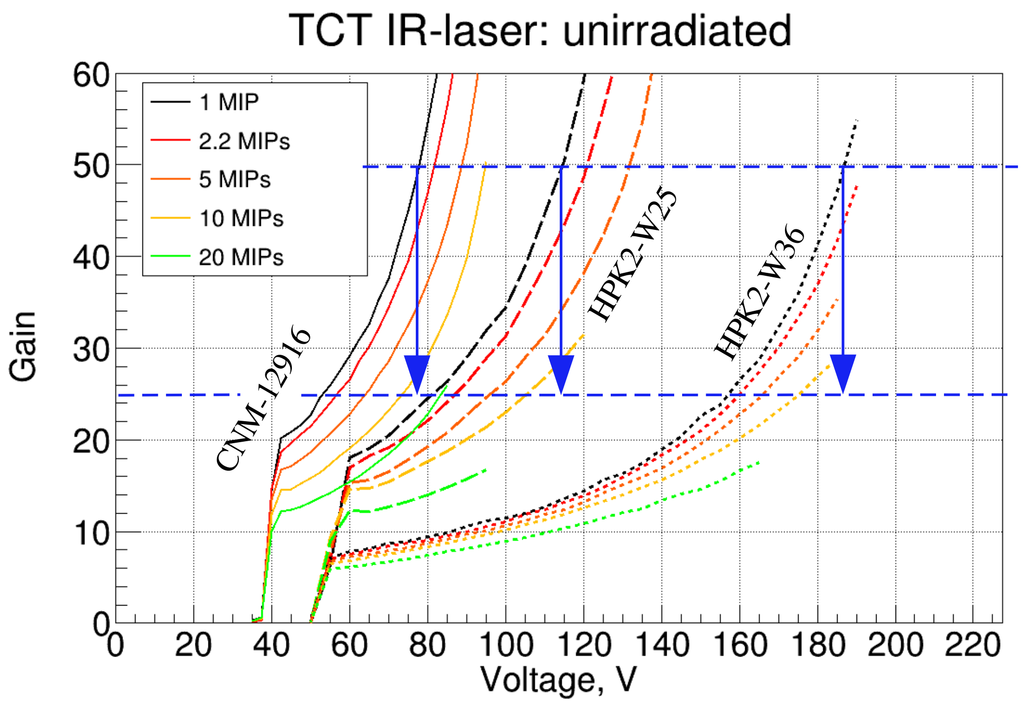
- The three models give identical results after the optimization.
- With the default parametrization, there is no agreement between the measured gain and the simulated one:
  - Van Overstraeten is overestimating it.
  - Massey and Okuto-Crowell underestimating it.

- The three methods give good agreements between the simulated gain and the measured one at the four different temperatures studied.

Possible sources of error in the method

- Experimental data, error in the measurement of the gain:
  - Bias voltage.
  - Temperature.
  - IR-laser fluctuations.
- Gain variation between "identical" samples.
- Gain reduction mechanism.

~ 5% error in the measured gain



Gain reduction mechanism

For all the samples, depending how the gain is measured, we can have a big difference:

- For a gain of 50 at 1 MIP the gain drops more than 50% for 20 MIPs.

First reported at:

"E.Curras et al, 16th (Virtual) "Trento" Workshop on Advanced Silicon Radiation Detectors"

Further readings:

"E. Curras et al, "Gain reduction mechanism observed in Low Gain Avalanche Diodes"

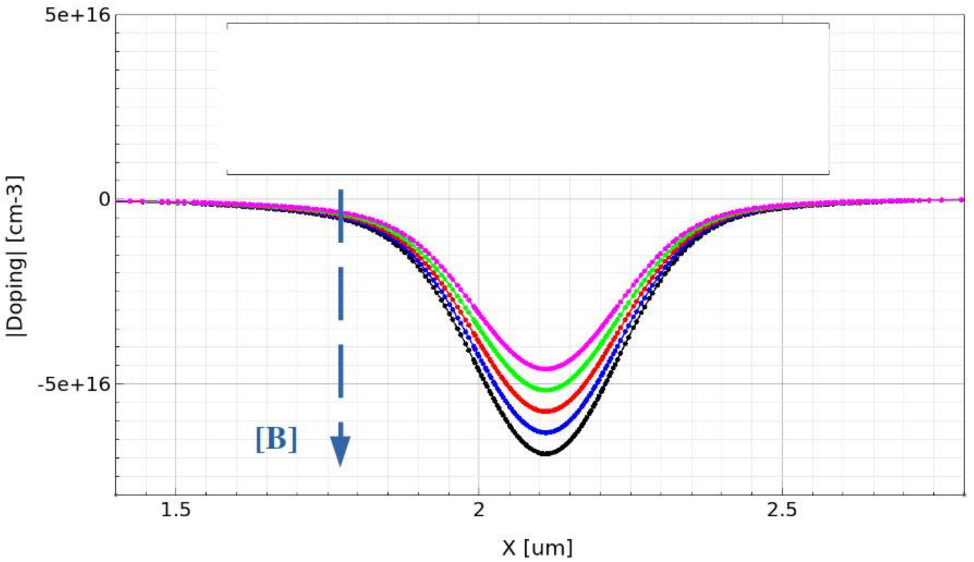
"G. Kramberger et al, "Gain dependence on free carrier concentration in LGADs"

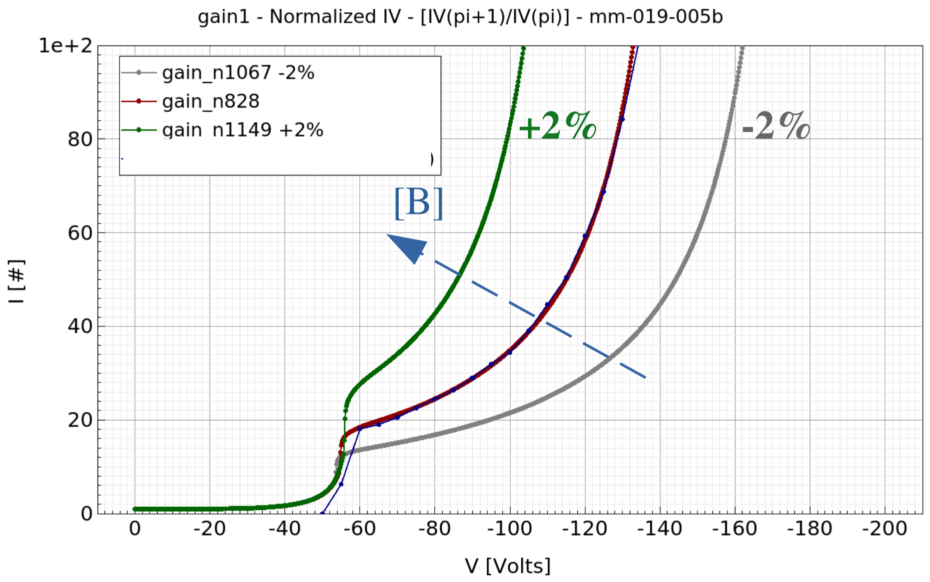
Possible sources of error in the method

- Variation of the gain layer parameters:
  - Variation of the gain layer doping concentration.
  - Variation of the gain layer position in depth.
- Tiny variations in the V_{GL} between "identical" samples leads to a significant differences in the gain curves.
- These errors can be minimized with the C-V characteristics.

} Very difficult to estimate the error

Variation of the gain layer doping concentration [B]



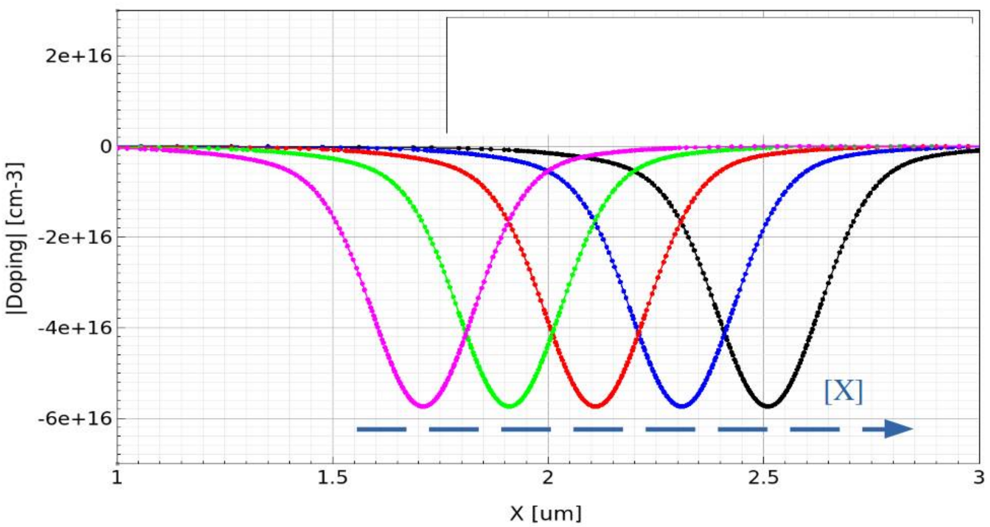


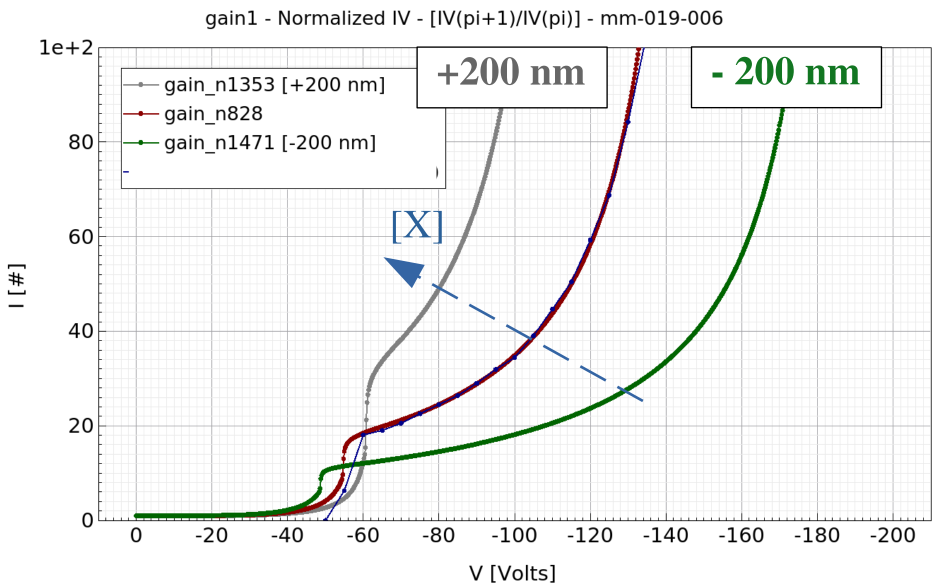
Possible sources of error in the method

- Variation of the gain layer parameters:
  - Variation of the gain layer doping concentration.
  - Variation of the gain layer position in depth.
- Tiny variations in the V_{GL} between "identical" samples leads to a significant differences in the gain curves.
- These errors can be minimized with the C-V characteristics.

} Very difficult to estimate the error

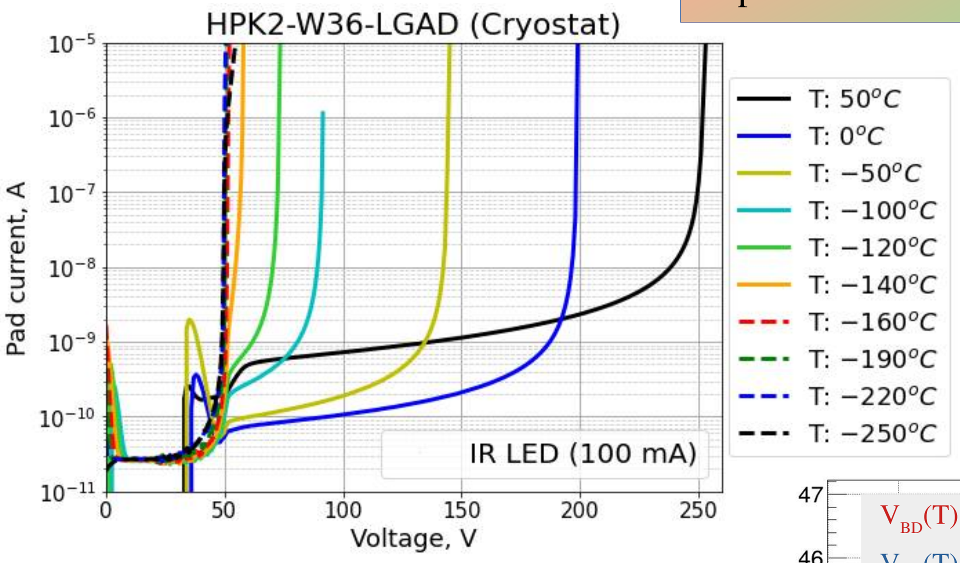
Variation of the gain layer position in depth [X]

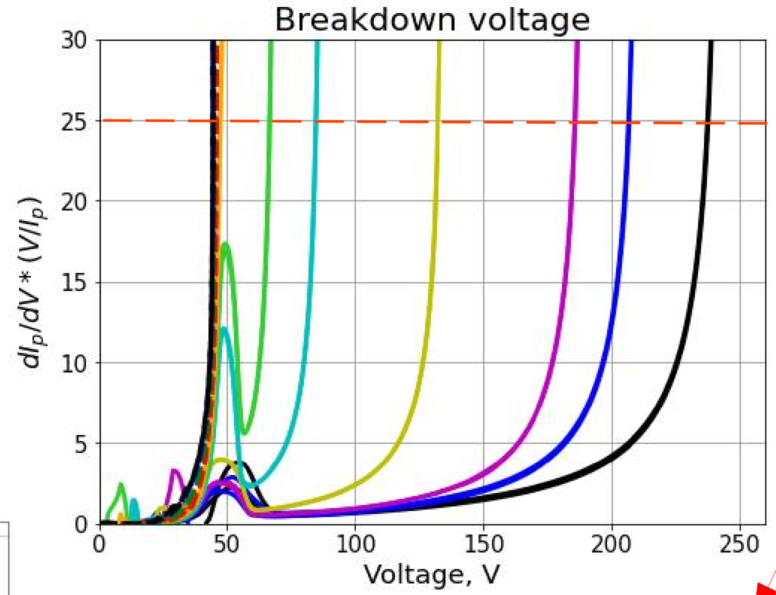




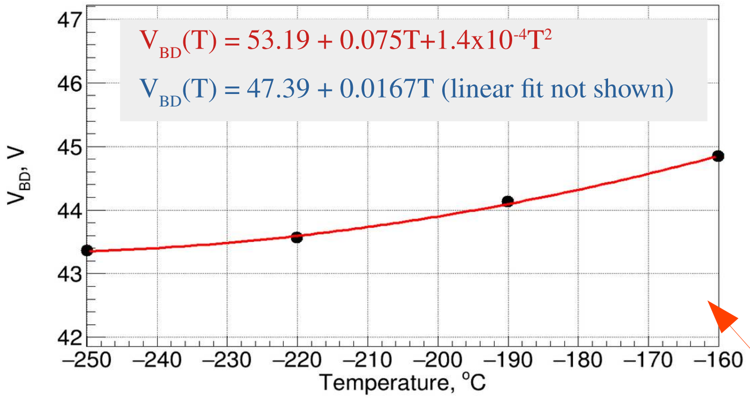
Breakdown voltage vs temperature (HPK2 LGADs)

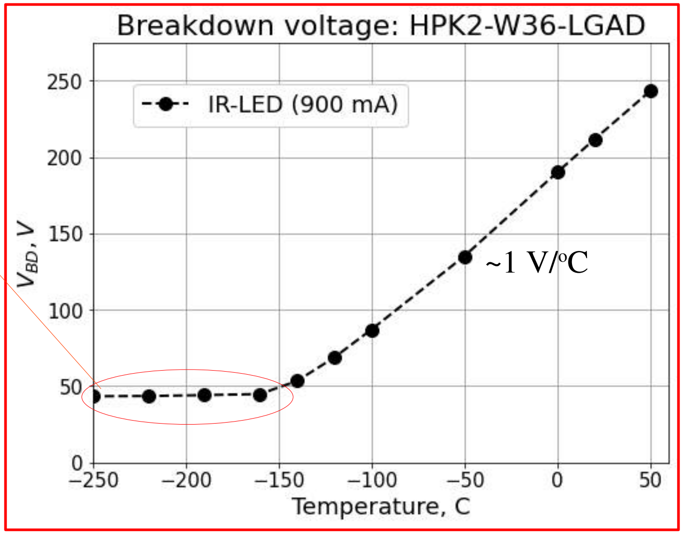
IR-LED (940 nm)
Top illumination





Threshold





- **Can we model the breakdown behavior using the new impact ionization parameters?**
  - $\alpha_n(T)$ and $\alpha_p(T)$ given by the optimized **Massey** model.
  - Using a simplified GL electric field profile: const E in the GL.

Breakdown voltage vs temperature (HPK2 LGADs)

Gain equation implemented in C++

$$M(x) = \frac{\exp\left(-\int_x^d (\alpha_n - \alpha_p)\, d\eta\right)}{1 - \int_0^d \alpha_n \exp\left(-\int_\xi^d (\alpha_n - \alpha_p)\, d\eta\right) d\xi}$$

if **E = const** →

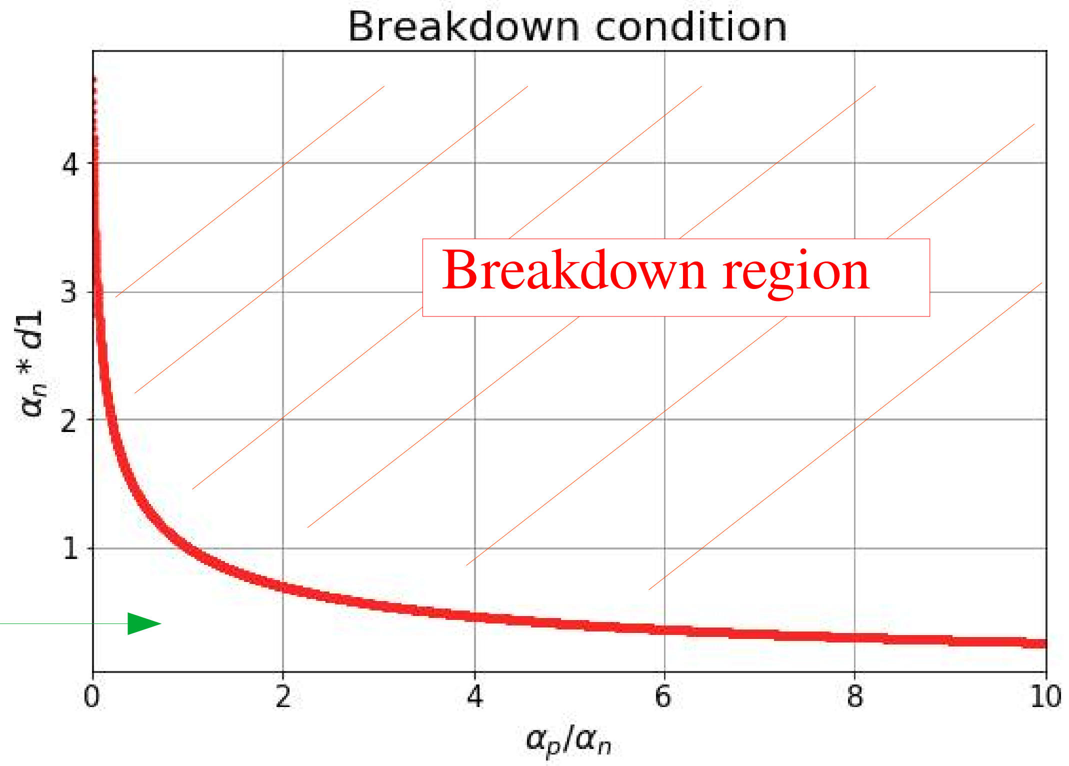
Gain equation for constant electric field

$$M_n = \left[1 - \frac{\alpha_n}{\alpha_n - \alpha_p}\left(1 - \exp\left(-d_1\left(\alpha_n - \alpha_p\right)\right)\right)\right]^{-1}$$

The breakdown condition depends on the values of:

- α_n(T).
- α_p(T).
- d_1 (GL thickness).

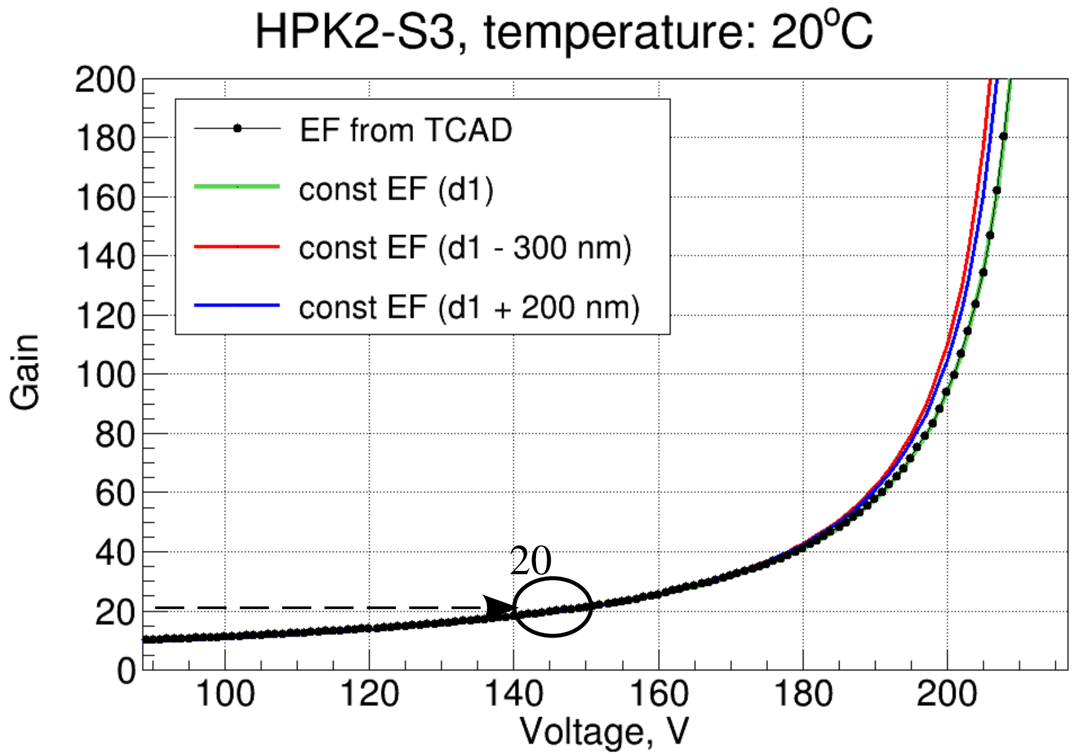
Working point region



Breakdown voltage vs temperature (HPK2 LGADs)

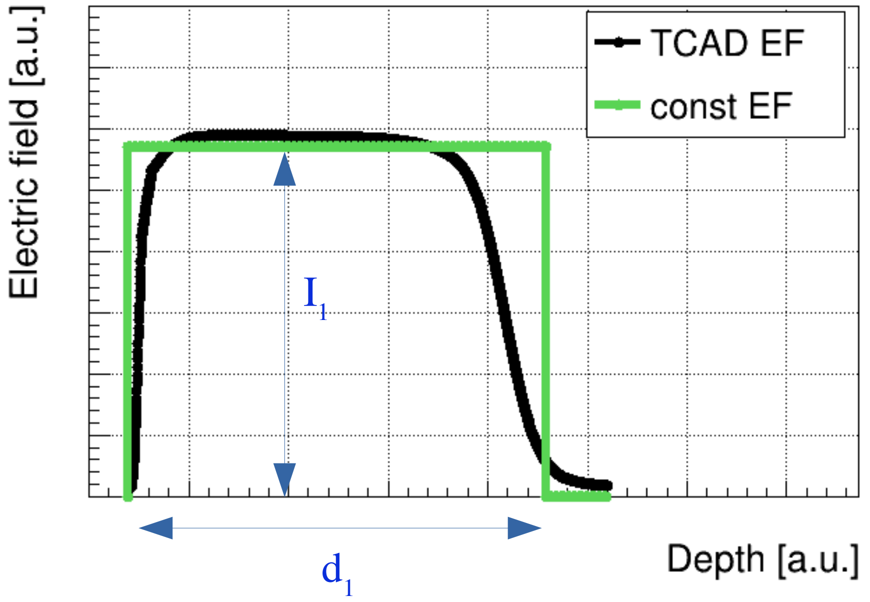
E = const
Gain model = Massey opt.
Sensor = HPK2-LGAD-S3

For the HPK2-LGADs: same gain curve using a **constant electric field profile** in the gain layer!



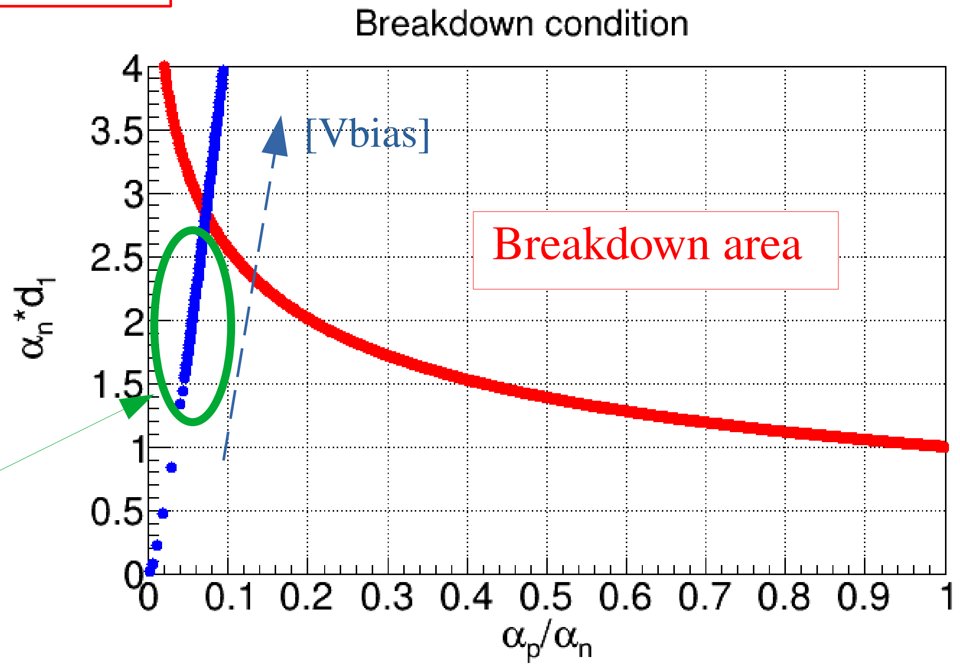
It is possible to get the same gain curve as the one simulated with the EF profile from TCAD using a const EF.

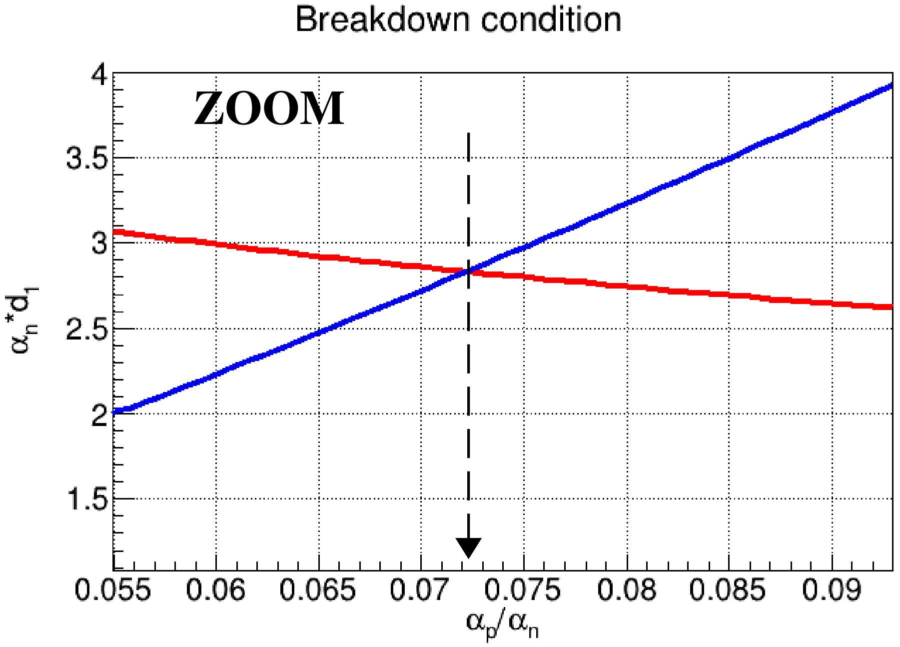
- At 146 V the gain is fixed to 20, this is how I_1 is tuned
- Combinations of I_1 and d_1 have to be properly chosen:
  - If d_1 is increased or decreased the gain curves are different. They show an early breakdown.
- With the CNM-LGADs is not possible to find any combination of d_1 and I_1 that gives the same gain curve.



Breakdown voltage vs temperature (HPK2 LGADs)

E = const
Gain model = Massey opt.
Sensor = HPK2-LGAD-S3





$$\frac{\alpha_p}{\alpha_n} = 0.0725 \xrightarrow{\text{Massey}} \frac{A_p exp(-\frac{C_p + D_p T}{E})}{A_n exp(-\frac{C_n + D_n T}{E})} = 0.0725 \longrightarrow E = \frac{C_n - C_p}{ln(0.0725\frac{A_n}{A_p}))} + \frac{D_n - D_p}{ln(0.0725\frac{A_n}{A_p}))}T$$

Solving the equation for the Massey model, using the optimized parameters, we can obtaining the expression for the E(T) in the breakdown condition:

$$E = 2.47 \times 10^5 + 2.34 \times 10^2 T$$

Breakdown voltage vs temperature (HPK2 LGADs)

E = const
Gain model = Massey opt.
Sensor = HPK2-LGAD-S3

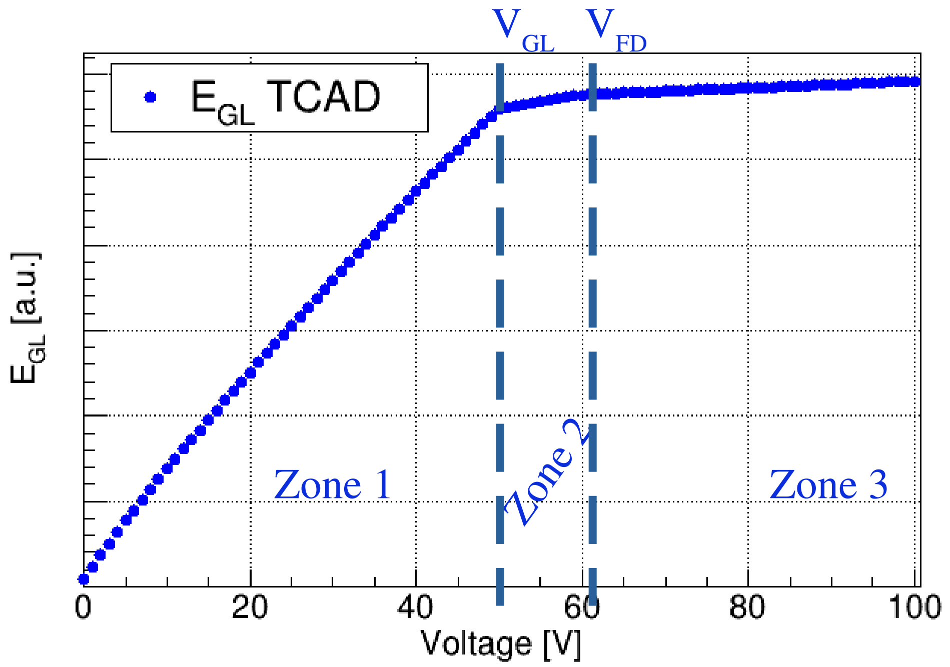
$$E = 2.47 \times 10^5 + 2.34 \times 10^2 T$$

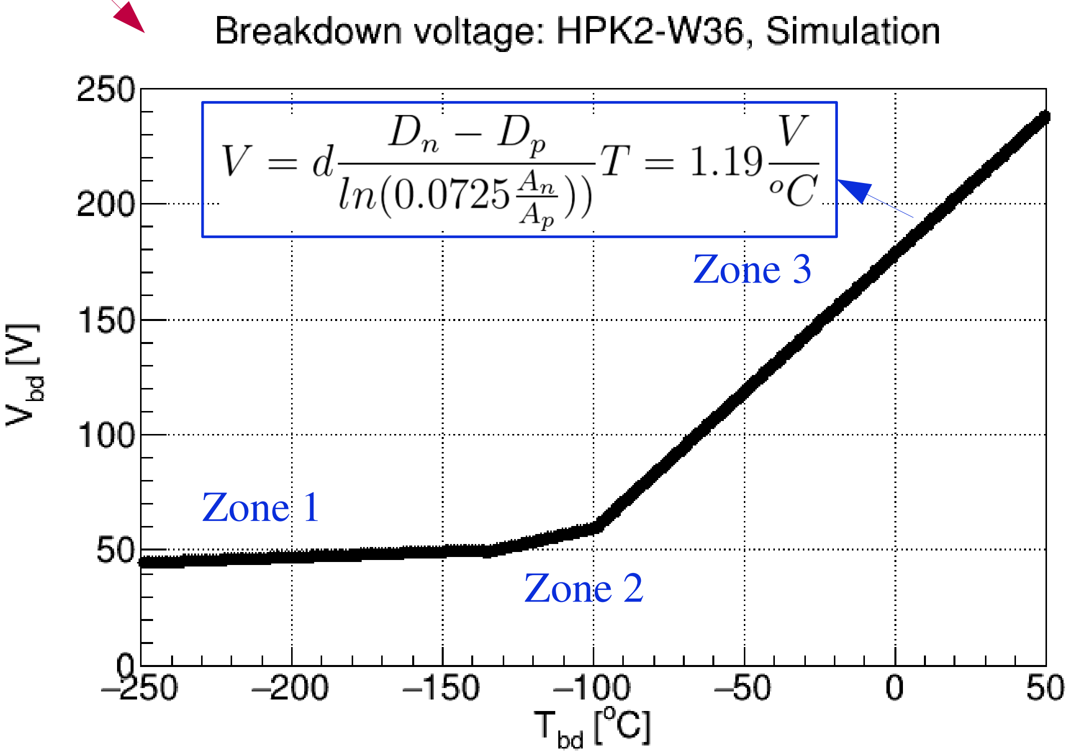
Zone 1 (0 V – V_{GL}) → $E = C_1 + 6.26 \times 10^3 V - 16.07 V^2$

Zone 2 (V_{GL} – V_{FD}) → $E = C_2 + 8.04 \times 10^2 V$

Zone 3 (V_{FD} – V_{BD}) → $E = C_3 + 1.96 \times 10^2 V$ → **1/d** [cm^{-1}]V: being *d*, the **active thickness** of the LGAD

E(T) in breakdown



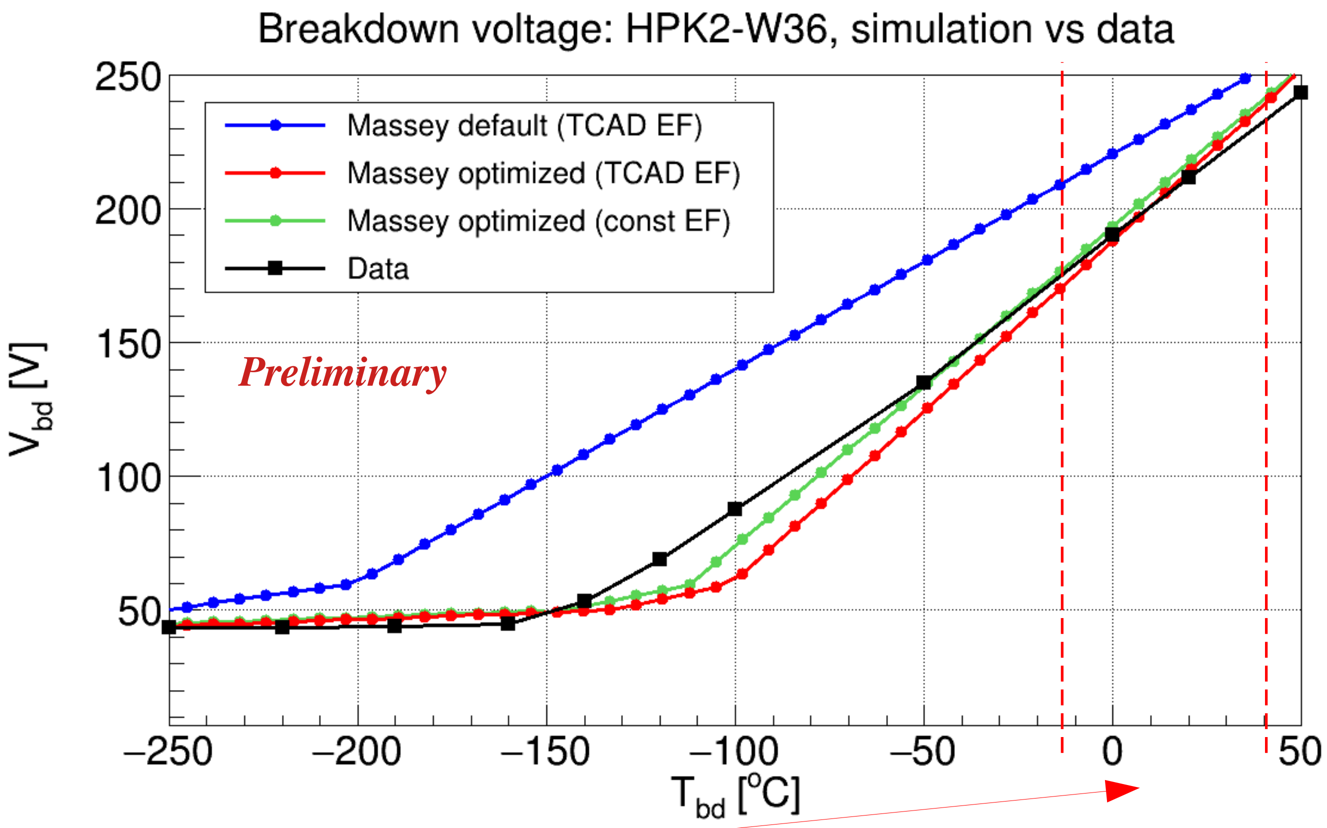


Breakdown voltage vs temperature (HPK2 LGADs)

E = const
Gain model = Massey opt.
Sensor = HPK2-LGAD-S3

In zone 3 we have the following slopes:

- Massey default using TCAD EF: …... **0.802 V/ºC**
- Massey optimized using TCAD EF: ... **1.274 V/ºC**
- Massey optimized using const EF: …. **1.194 V/ºC**
- From experimental data: ……...…. **1.031 V/ºC**



- Valid range in temperatures for our parametrization: **-15 ºC – 40 ºC.**
- The new parametrization gives better agreement in terms of the breakdown voltage than the default one.
- The const EF approximation agrees very well with the EF given by TCAD.
- To extend the parametrization to a wider range in temperatures, we need to include the breakdown conditions in the optimization process.

Summary and next steps

- TCAD simulations to extract the E profiles in the GL and simulate the gain with existing impact ionization models:
  - Massey, Van Overstraeten and Okuto-Crowell.
  - Parameters optimized in C++: same gain as in TCAD, but faster optimization enviroment.
- We presented a new parametrization that fits our data measured in TCT with an IR-laser :
  - The new parameters provide a good agreement with the experimental results.
  - Range in electric field: $2.8x10^5$ Vcm^{-1}– $4.5x10^5$ Vcm^{-1}.
  - Range in temperature: 258 K – 313 K.
- An error evaluation of the method was presented.
  - Method very sensitive to small variation in the input parameters.
- The new parametrization was used to simulate the breakdown voltage down to 20K.
  - Despite being outside of the T and E ranges of the new parametrization, it gives better results than the default ones.
  - Parameters need to be tuned including the breakdown conditions in the gain curves.
  - The model can be extended to a wider range in temperatures.
  - Breakdown voltage dependence as function of the temperature is now better understood.

For more details, Paper available from today:

"E.Curras and M.Moll, "Study of impact ionization coefficients in silicon with Low Gain Avalanche Diodes", arXiv

Thank you for your attention!