Summer Student Lectures 2022

Particle Accelerators and Beam Dynamics

Part 3

by

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Relation between particle momentum, magnetic field and trajectory radius: **Beam rigidity.**

\[
\frac{p}{q} = B \rho
\]

Particle beam focusing is equivalent to focusing of light with lenses.

Typical alternating sequence of focusing elements.

Particle motion through this lattice is described by as an **harmonic oscillator.**

Particles perform **betatron oscillations** around the design orbit.

Number of oscillation in one turn is called **tune.**

The particle’s trajectory through the accelerator can easily be calculated by transfer matrices.

\[
\begin{pmatrix}
  x \\
  x'
\end{pmatrix}
_{s_1} =
M
\begin{pmatrix}
  x \\
  x'
\end{pmatrix}
_{s_0}
\]

\[
M_{total} = M_{QF} \cdot M_D \cdot M_{Bend} \cdot M_D \cdot M_{QD} \cdot \ldots
\]
Phase space
A space that represents all possible states of a system.

A particle’s trajectory points or coordinates at a given element draw an ellipse in phase space.

The orientation and shape of that ellipse is described by the optical (Courant-Snyder) parameters. $\beta$-function

The area of that ellipse is $\propto$ emittance.

Emittance is a beam property that cannot be changed by focusing.

The beam size of a particle ensemble is defined by $\sigma = \sqrt{\epsilon \beta}$. 
What have we learned so far?

We know, how particles behave along the magnetic lattice of an accelerator.

But ...

- How do we get particles in and out?
- How do we accelerate?
- How do we get collisions to the experiments?
Insertions or straight sections

No bending, no focusing
DRIFT space

Destination:
- next accelerator
- target
- beam dump

Injection kicker
Transfer line
Injector/pre-accelerator
Extraction kicker
Transfer line

Arcs
LEIR – first circular accelerator in for CERN’s heavy-ions on the way to LHC
Injection and Extraction

Extraction follows the same principle, but the beam travels in the opposite direction.
Injection of Beam 2 into LHC

~ 70 m
~ 3 km
~ 12 mm

cour. R. Alemany
Filling and Circulating Beam

LEIR – first circular accelerator in for CERN’s heavy-ions on the way to LHC

- Several injections are accumulated,
- while the already injected particles circulate/wait.
- Only once the ring is fully filled, acceleration starts.
Schematic of LHC beam dump system

LHC beam stores ~360MJ energy.

Sweep of beam on beam dump window

Beam Dump Block (graphite)

Concrete Shielding

Source graphics: http://clipart-library.com
Acceleration

Insertions or straight sections

Destinations:
- next accelerator
- target
- beam dump
How can we increase the energy of a particle?

A charged particles that travels through an electro-magnetic field feels the Lorentz force:

\[ \vec{F} = q(\vec{v} \times \vec{B} + \vec{E}) \]

**Magnetic field** \(B\):
- Force acts perpendicular to path.
- Can change direction of particle
- cannot accelerate

**Electric field** \(E\):
- Force acts parallel to path.
- Can accelerate
- not optimal for deflection

The **energy gain** \(\Delta E\) of the particle is defined by the integral of the force \(F\) over the travelled path \(dr\):

\[
\Delta E = q \int_{r_1}^{r_2} (\vec{v} \times \vec{B} + \vec{E}) \, dr
\]

\[
= q \int_{r_1}^{r_2} \vec{E} \, d\vec{r} = qU.
\]

\[
(\vec{v} \times \vec{B}) \, d\vec{r} = 0
\]
What about the magnetic field during acceleration?

Beam rigidity needs to be increased proportionally to increasing energy.
→ Machine radius is constant.
→ Need to increase dipole field accordingly!

\[ \frac{p}{q} = B \rho \]

Circumference
\[ 2\pi R + \delta R \]

LHC dipole current from 450GeV to 6.5TeV

~20min
LHC magnetic dipole field at 450 GeV:

\[ B = \frac{p}{q\rho} = \frac{450 \text{ GeV}/c}{e \times 2803 \text{ m}} = 0.535 \text{ T} \]

Required bending radius at 7 TeV with \( B_{\text{inj}} = 0.5 \text{T} \):

\[ \rho = \frac{p}{qB} = \frac{7 \text{ TeV}/c}{e \times 0.535 \text{ T}} = 43.6 \text{ km} \]

Equivalent to **270km circumference** (pure dipole field! without any insertions or quadrupoles)

Magnet surface = 5800km²
- Area of Brunei (South-Eastern Asia)
- Area of 2x Luxemburg
Use **RF cavities** to apply the same accelerating voltage on each passage. → Gradually increase total energy by **gaining a small amount each turn**.
LHC has
- 8 superconducting cavities per beam
- Accelerating field 5MV/m
- Can deliver 2MV/cavity
- Operating at 400MHz
- Beam aperture (radius) ~30cm
- Energy gain/turn during ramp 485 keV (11245 turns/s)
RF Acceleration

Accelerating voltage is changing with time. That has two consequences:

Need **synchronization** between beam and RF phase to gain energy.

There is a **synchronous RF phase** for which the energy gain fits the increase of the magnetic field.

Not all particles see the same voltage, because they arrive at different times.

Not all particles gain the same energy.
Phase Stability (non-relativistic regime)

Assume the situation where *energy increase is transferred into a velocity increase* (non-relativistic regime).

Particles $P_1, P_2$ have the synchronous phase.

- Ideal particle
- Particle with $\Delta t < 0$ (early) $\rightarrow$ lower energy gain $\rightarrow$ gets slower
- Particle with $\Delta t > 0$ (late) $\rightarrow$ higher energy gain $\rightarrow$ gets faster
  $\rightarrow M_1 & N_1 will move towards P_1 \rightarrow stable$

*M_2 & N_2 will go away from P_2 $\rightarrow$ unstable (and finally be lost)*

Longitudinal (phase) focusing keeps particles close to each other ... forming a „bunch“

Courtesy F. Tecker for drawings

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Now assume relativistic energies ($\nu \approx c$): An *increase in momentum* transforms into a *longer orbit and thus a longer revolution time*.

\[ \frac{p}{q} = B \rho \]

$M_1$ & $N_1$ will go away from $P_1$ → *unstable* (and finally be lost)

$M_2$ & $N_2$ will move towards $P_2$ → *stable*
The previously stable synchronous phase becomes unstable when \( v \rightarrow c \) and the gain in path length overtakes the gain in velocity \( \rightarrow \text{Transition} \)

Transition from one slope to the other during acceleration \( \rightarrow \text{Crossing Transition.} \)
The RF system needs to make a rapid change of the RF phase, a ‘phase jump’.

In the PS: \( \gamma_t \) is at \( \sim 6 \) GeV, injection at 1.4GeV
In the SPS: \( \gamma_t = 22.8 \), injection at \( \gamma = 27.7 \)
\( \Rightarrow \) no transition crossing!
In the LHC: \( \gamma_t \) is at \( \sim 55 \) GeV, also far below injection energy

*Transition crossing not needed in leptons machines, why?*
Synchrotron Oscillation

Like in the transverse plane the particles are oscillating in longitudinal space.

Particles keep *oscillating around the stable synchronous particle* varying phase and $\frac{dp}{p}$.

Typically one synchrotron oscillation takes many turns (much slower than betatron oscillation).

Phase-space ellipse defines *longitudinal emittance*.

*Separatrix* is the trajectory separating stable and unstable motion.

Stable region is also called *bucket*.

→ Harmonic number $h = \text{number of buckets}$:

$$f_{RF} = h f_{rev}$$

Simple case (no accel.): $B = \text{const.}$
- Stable phase: $\phi_0 = 0$
- Particle B oscillates around $\phi_0$.

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*Phase space picture*

*Courtesy F. Tecker for drawings*
What happens to the emittance if the reference momentum $P_0$ changes?

Can write down transfer matrix for reference momentum change:

$$M_x = \begin{pmatrix} 1 & 0 \\ 0 & P_0/P_1 \end{pmatrix} \quad \Rightarrow \quad \epsilon_{x1} = \frac{P_0}{P_1} \epsilon_{x0}$$

The emittance shrinks with acceleration!

With $P = \beta \gamma mc$ where $\gamma, \beta$ are the relativistic parameters.

The conserved quantity is

$$\beta_1 \gamma_1 \epsilon_{x1} = \beta_0 \gamma_0 \epsilon_{x0}$$

It is called *normalized emittance*. 
How big are the beams in the LHC?

**Normalized emittance** at LHC : $\varepsilon_n = 3.5$ $\mu$m

$\rightarrow \varepsilon_n$ preserved during acceleration.

The **geometric emittance**:

\[ \varepsilon_{7 TeV} = \varepsilon_{450 GeV} \frac{\gamma_{450 GeV}}{\gamma_{7 TeV}} \]

- Injection energy of 450 GeV: $\varepsilon = 7.3$ nm
- Top energy of 7 TeV: $\varepsilon = 0.5$ nm

The corresponding max. **beam sizes** in the arc, at the location with the maximum beta function ($\beta_{\text{max}} = 180$ m):

- $\sigma_{450 GeV} = 1.1$ mm
- $\sigma_{7 TeV} = 300$ $\mu$m

Aperture requirement: $a > 10 \sigma$

LHC beam pipe radius:

- Vertical plane: 19 mm $\sim 17 \sigma$ @ 450 GeV
- Horizontal plane: 23 mm $\sim 20 \sigma$ @ 450 GeV
Transverse-Longitudinal Coupling: Dispersion

Dipole magnets generate dispersion:
→ **Particles with different momentum are bent differently.**

Due the momentum spread in the beam $\frac{\Delta p}{p}$, this has to be taken into account for the particle trajectory.

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\[
x(s) = x_\beta(s) + D(s) \frac{\Delta p}{p}
\]

**Dispersion function $D(s)$**
corresponds to the trajectory of a particle with momentum offset $\frac{\Delta p}{p} = 1$.

This also has an effect on the beam size:

\[
\sigma = \sqrt{\beta \varepsilon} \quad \rightarrow \quad \sigma = \sqrt{\beta \varepsilon + D^2 \left(\frac{\Delta p}{p}\right)^2}
\]
Dispersive Orbit

Dedicated energy (i.e. $f_{RF}$) change of the stored beam.
- Horizontal orbit is moved to a dispersions trajectory.
- Vertical orbit unchanged (no vertical dispersion)
Experiments and Luminosity

- **Acceleration**
- **Insertions or straight sections**
- **Injection kicker**
- **Transfer line**
- **Injector/pre-accelerator**
- **Extraction kicker**
- **Transfer line**
- **Physics experiment**
- **Arrows**
- **Destination:**
  - next accelerator
  - target
  - beam dump
Each accelerator and experiment requires specific beam properties. Fundamentally different are:

- **Fixed Target:**
  - Beam
  - Target
  - Secondary particles
  - Particles that are bent to a circular orbit emit energy/light.

- **Light Sources:**
  - Green laser
  - Particles that are bent to a circular orbit emit energy/light.

- **Collider:**
  - Dartboard
  - Particles that are bent to a circular orbit emit energy/light.
‘Smashing’ Modes and Center-of-Mass Energy

The center-of-mass energy defines the upper limit of the newly created particle’s mass.

Fixed Target

Most of the Energy is lost in the target, only a fraction is transformed into useful secondary particles.

Collider

All energy is available for the production of new particles.

**Exercise 2:** Derive center-of-mass energy in fixed target and collider experiment.
In LHC has **4 interaction points (IPs)** hosting particle physics experiments: → ATLAS, ALICE, CMS, LHCb

Therefore the two counterrotating **beams collide 4 times per turn**

When they collide the outer beam **cross over** to the inner circle and vise versa.
Experiments are interested in maximum number of interactions per second. The event rate in an experiment is proportional to the luminosity.

\[ \frac{dR}{dt} = L \times \sigma_p \]

Interaction Cross-section
(= constant probability for the given interaction to take place)

Luminosity

Event rate
(= number of collisions per time)
“Quality Factor” of a Collider

The most important measure to describe the potential of a collider is the Luminosity.

\[
L = \frac{kN^2 f \gamma}{4\pi \beta^* \varepsilon} \cdot F
\]

Defined by the injectors

N..... No. particles per bunch
k..... No. bunches
f....... revolution freq.
g...... rel. gamma
\(\beta^*\) .... beta-function at IPs
\(\varepsilon\) ...... norm. trans. emit

Overall Goal of an Collider: Maximizing Luminosity!

→ Many particles (N, k)
→ In a small transverse cross-section (\(\varepsilon, \beta\))

Limitation:
“Collective effects” cause beam instabilities for too high bunch intensities, too small bunch spacing, too “bright” beams.

Performance depends on the injectors:
→ Production of large N and small \(\varepsilon\)
→ Preservation of these parameters until collisions.
Optimizing Luminosity

Bunch properties (N & ε) are defined in the injectors.

But what can we do in the Collider?

$$L = \frac{kN^2 f \gamma}{4\pi \beta^* \varepsilon} F$$

$N$: No. particles per bunch
$k$: No. bunches
$f$: revolution freq.
$\gamma$: rel. gamma
$\beta^*$: beta-function at IPs
$\varepsilon$: norm. trans. emit

$f_{rev} \gamma$: defined by the design of the accelerator

$F$: When colliding with many bunches, a crossing angle is needed to avoid unwanted collisions. However this reduces the beam overlap and therefore the luminosity. Keep as small as possible! → Limited by beam-beam effects.

$k$: Optimize filling scheme and bunch spacing.

$\beta^*$: Can be optimized by focusing!
Mini-Beta Insertions

Mini-beta insertion is a **symmetric drift space** with a **waist of the β-function** in the center of the insertion.

![Diagram of Mini-Beta Insertion](image)

On each side of the symmetry point a quadrupole **doublet** or **triplet** is used to generate the waist. *They are not part of the regular lattice.*

\[
\beta(s) = \beta^* + \frac{s^2}{\beta^*}
\]

Collider experiments are located in mini-beta insertions: **smallest beam size possible** for the colliding beam to increase probability of collisions.

There is a price to pay: **The smaller β*, the larger β at the triplet.**
Example: Mini-Beta Insertion at LHC

Example of the LHC (design report values):

At the interaction point:
\[ \beta^* = 0.55 \text{ m} \]
\[ \sigma^* = 16 \text{ \(\mu\)m} \]
That’s smaller than a hair’s diameter!

At the triplet:
\[ \beta = 4500 \text{ m} \]
\[ \sigma = 1.5 \text{ mm} = 1500 \text{ \(\mu\)m} \]
Largest beams size in the lattice!

Limitations:
• Tighter tolerances on field errors
• Triplet aperture limits \(\beta^*\) together with crossing angle.
The $\beta$-functions, and thus beam sizes, in the triplet for small $\beta^*$ is too large at injection energy $\rightarrow$ aperture problems.

$\rightarrow$ beam size shrinks with energy $\sigma \propto \sqrt{1/\gamma}$

$\rightarrow$ Mini-beta squeeze done at top energy when beam size is smaller.

**LHC injection optics at $\beta^* = 11m$**

**LHC collision optics at $\beta^* = 0.25m$ in ATLAS & CMS**
Integrated Luminosity

What counts for the experiments is not peak performance, but total accumulated number of events:

\[ \sigma_p \cdot \mathcal{L}_{int} = \sigma_p \cdot \int_0^T \mathcal{L} \, dt \]

unit “barn” = \(10^{-24} \text{ cm}^2\)

unit “inverse barn” = \(10^{24} \text{ cm}^{-2}\)

Common order of magnitude: \(1 \text{ fb}^{-1} = 10^{39} \text{ cm}^{-2}\)

For example:
To integrate 1 fb\(^{-1}\) it requires \(10^7\) s at

\[ \mathcal{L} = 10^{32} \text{ cm}^{-2} \text{s}^{-1} \]

For comparison: a year has about \(\pi \times 10^7\) s.

LHC has delivered so far \(~190 \text{ fb}^{-1}\) to ATLAS/CMS in proton-proton collisions over 7 production years.
Now you know everything to build your own accelerator …

… so let's get to work!

Destination:
• next accelerator
• target
• beam dump
Everything clear! Hmm ....
Solutions to Exercises
Exercise 2: Center-of-Mass Energy

Center-of-mass (CM) frame is defined where sum of all momenta is zero: \[ \vec{p}_i = 0 \]

4-momentum \[ p^\mu = (E/c, \vec{p}) \rightarrow p^\mu p_\mu = \frac{E^2}{c^2} - \vec{p}^2 \]

can be transformed to center-of-mass frame by Lorentz transformation:

\[ p'^\mu = L^\nu_\mu p^\nu \]

Lorentz Transformation

\[ p^\mu p_\mu = p'^\mu p'_\mu \]

The norm: is Lorentz invariant

Energy conversation between both frames:

\[ \frac{E_{CM}^2}{c^2} - \vec{0}^2 = \frac{E_{tot}^2}{c^2} - \vec{p}_{tot}^2 \rightarrow \frac{E_{CM}^2}{c^2} = \frac{E_{tot}^2}{c^2} - \vec{p}_{tot}^2 \]
Exercise 2: $E_{CM}$ in Fixed Target Experiment

\[ p_1 = \left( \frac{E_1}{c}, \vec{p}_1 \right) \]

\[ p_2 = (m_2c, \vec{0}) \]

\[ p_{tot} = \left( \frac{E_1}{c} + m_2c, \vec{p}_1 \right) \]

\[ E_{CM}^2 = (m_1^2 + m_2^2)c^4 + 2E_1m_2c^2 \]

\[ E_{CM} \propto \sqrt{E_1} \]
Exercise 2: $E_{\text{CM}}$ in Collider Experiment

Laboratory Frame = CM Frame

\[ p_1 = \left( \frac{E_1}{c}, \vec{p}_1 \right) \quad \quad p_2 = \left( \frac{E_2}{c}, -\vec{p}_1 \right) \]

\[ E_{CM} = E_1 + E_2 \]
Back-up Slides
Chromaticity

Dipole magnets generate dispersion, which is then focused by quadrupoles.

\[ k = \frac{q}{p/q}[m^{-2}] \]

Chromaticity \( Q' \) acts like a quadrupole error and leads to a tune spread. Definition of Chromaticity:

\[ \Delta Q = Q' \frac{\Delta p}{p} \]

\[ Q' = -\frac{1}{4\pi} \int k(s)\beta(s)ds \]

**Q’ measurement at LHC**

The chromaticity is measured by changing / modulating the energy offset \( dp/p \) through the RF frequency while recording the tune change \( \Delta Q \).
Accelerator Cycle

LEIR Operational Cycle

- Extraction
- Injection(s)
- Storage/Collisions
- Energy
- Beam Intensity
- Acceleration & increase of magnetic field

LHC Operational Cycle

- Ramp down
- Cycling
- Injection
- Prepare ramp
- Flattop
- Squeeze
- Adjust
- Stable beams
- Beam dump

Graph showing:
- $E_b$ and $N$ [arbitrary units]
- Time [h]
- Energy
- Intensity B1
- Intensity B2

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