Summer Student Lectures 2022

Particle Accelerators and Beam Dynamics
Part 3

by

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01.07.2022

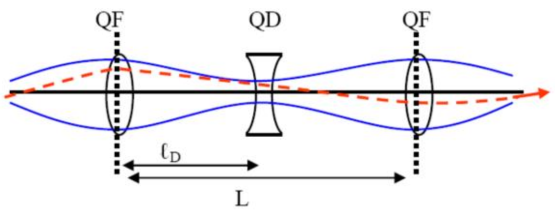
A Little recap from yesterday …

Relation between particle *momentum*, *magnetic field* and trajectory *radius*: **Beam rigidity.**

$$F_L = F_{centr} \quad \Rightarrow \quad \frac{p}{q} = B\,\rho$$

Particle beam focusing is equivalent to focusing of light with lenses.

Typical alternating sequence of focusing elements.



F = focusing
0 = nothing (dipole, RF, …)
D = defocusing
0

Particle motion through this lattice is described by as an harmonic oscillator.

Particles perform ***betatron oscillations*** around the design orbit.
Number of oscillation in one turn is called ***tune***.

$$x'' + Kx = 0$$
$$y'' - ky = 0$$

The particle's trajectory through the accelerator can easily be calculated by transfer matrices.

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{s_1} = M \begin{pmatrix} x \\ x' \end{pmatrix}_{s_0}$$

$$M_{total} = M_{QF} \cdot M_D \cdot M_{Bend} \cdot M_D \cdot M_{QD} \cdot \ldots$$

A Little recap from yesterday …

Phase space

A space that represents all possible states of a system.

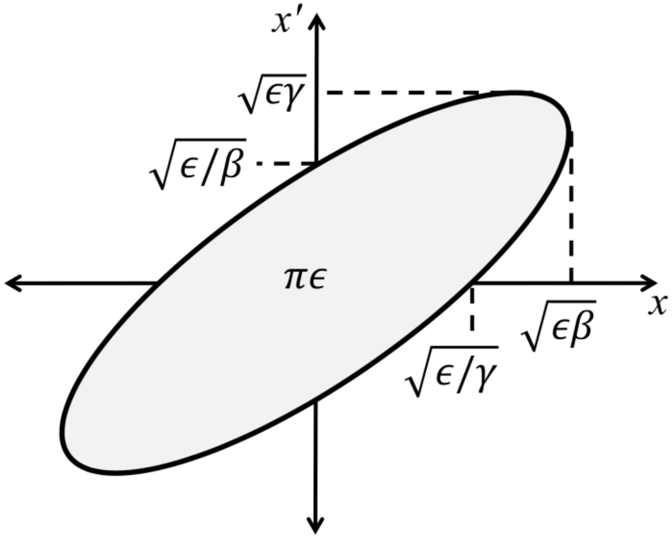
A particle's trajectory points or coordinates at a given element draw an ***ellipse in phase space***.

The orientation and shape of that ellipse is described by the optical (Courant-Snyder) parameters. → β-function

The area of that ellipse is $\propto$ ***emittance***.

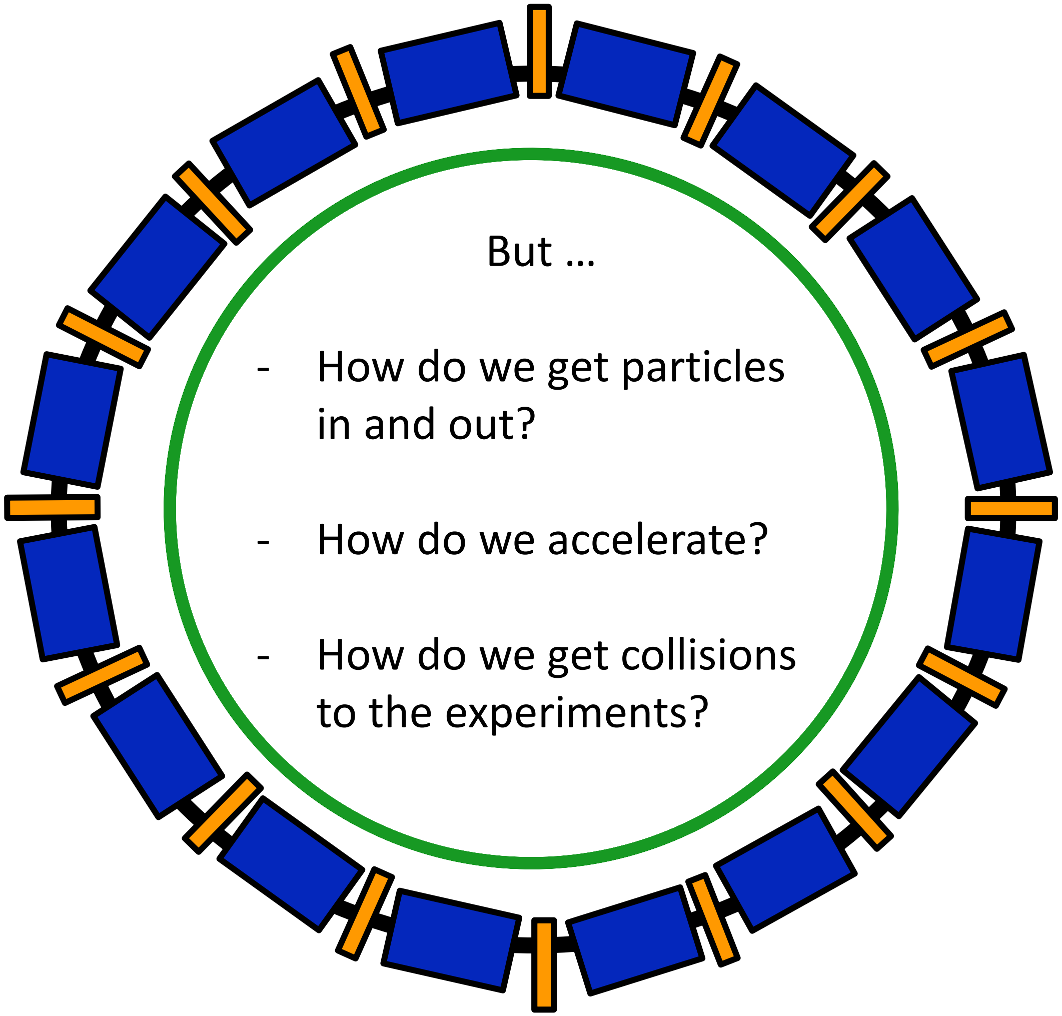
Emittance is a beam property that cannot be changed by focusing.

The ***beam size*** of a particle ensemble is defined by $\sigma = \sqrt{\epsilon\beta}$.



What have we learned so far?

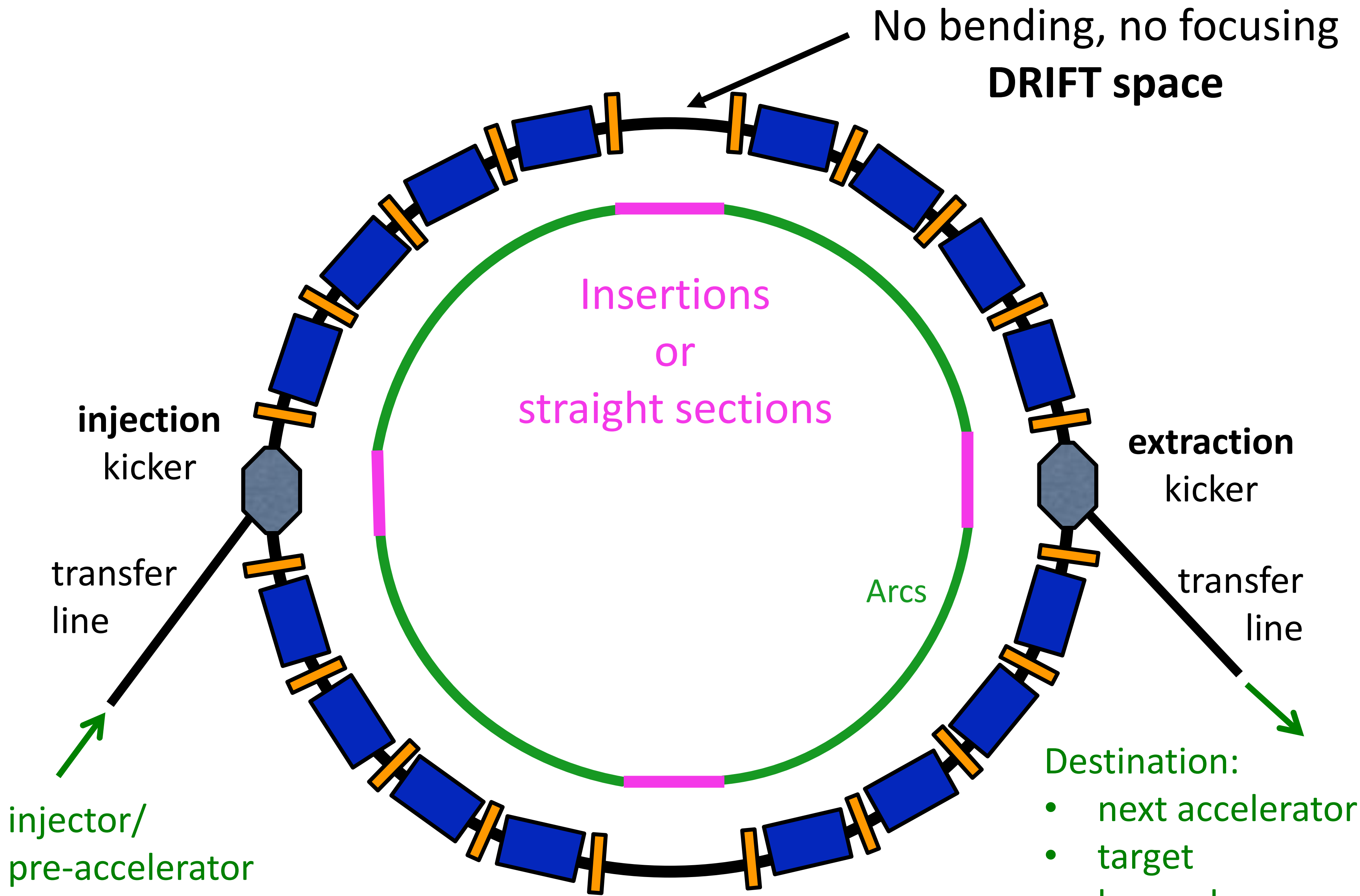
We know, how particles behave along the magnetic lattice of an accelerator.



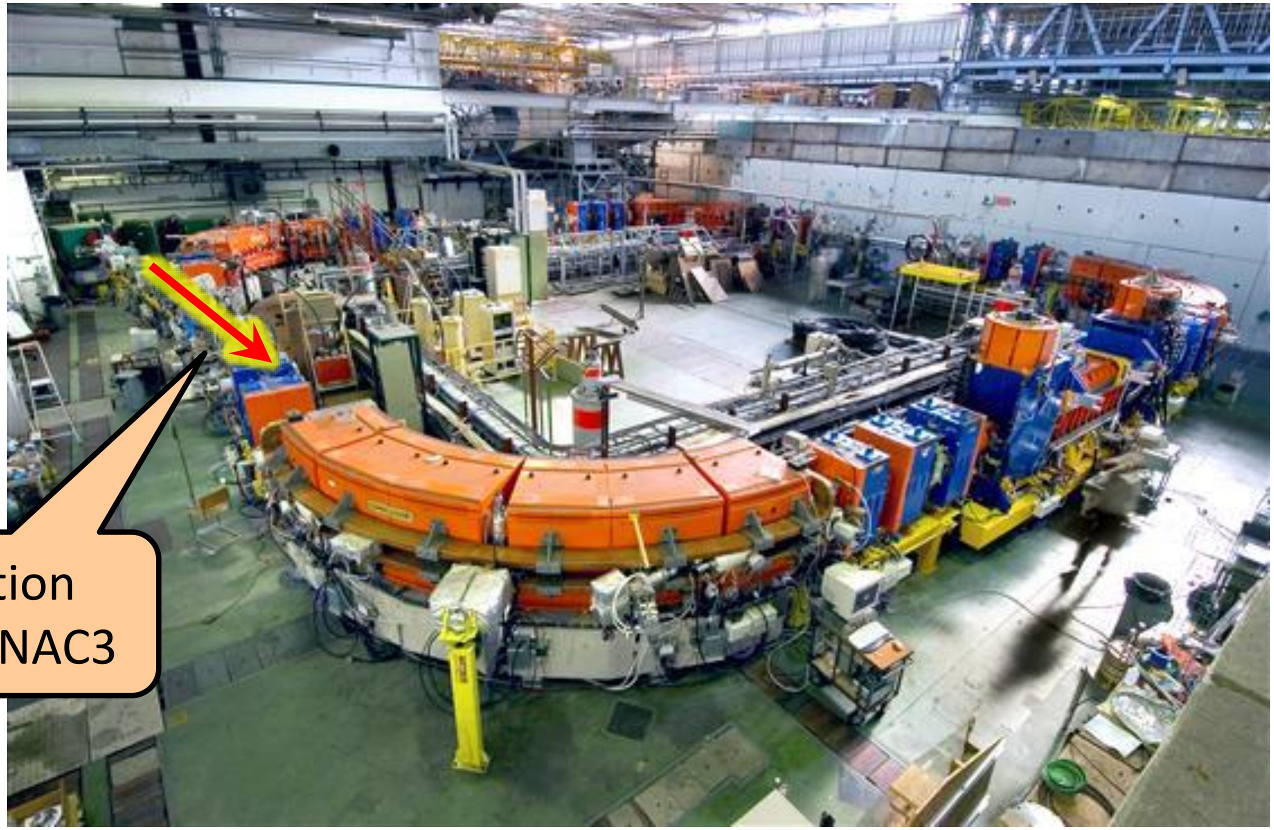
But …

- How do we get particles in and out?
- How do we accelerate?
- How do we get collisions to the experiments?

Straight Sections and Insertions



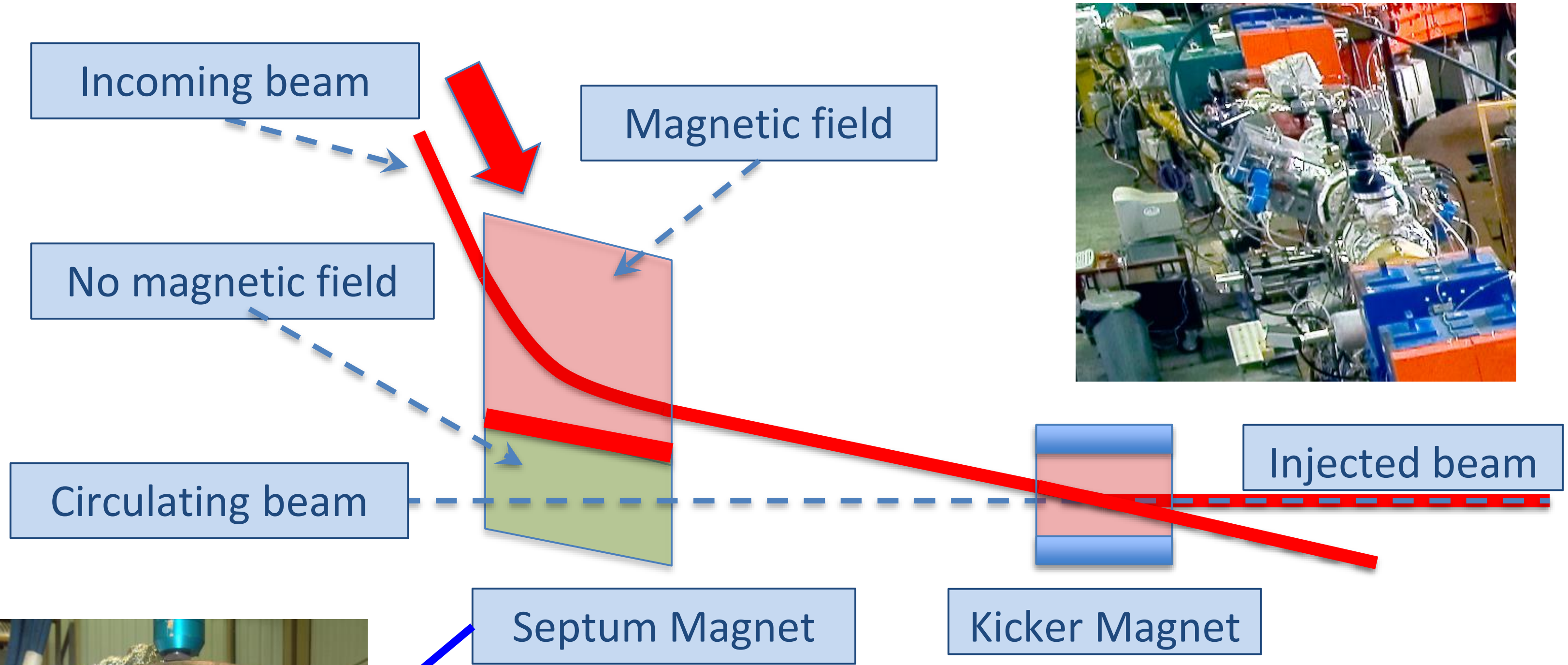
Injection and Extraction

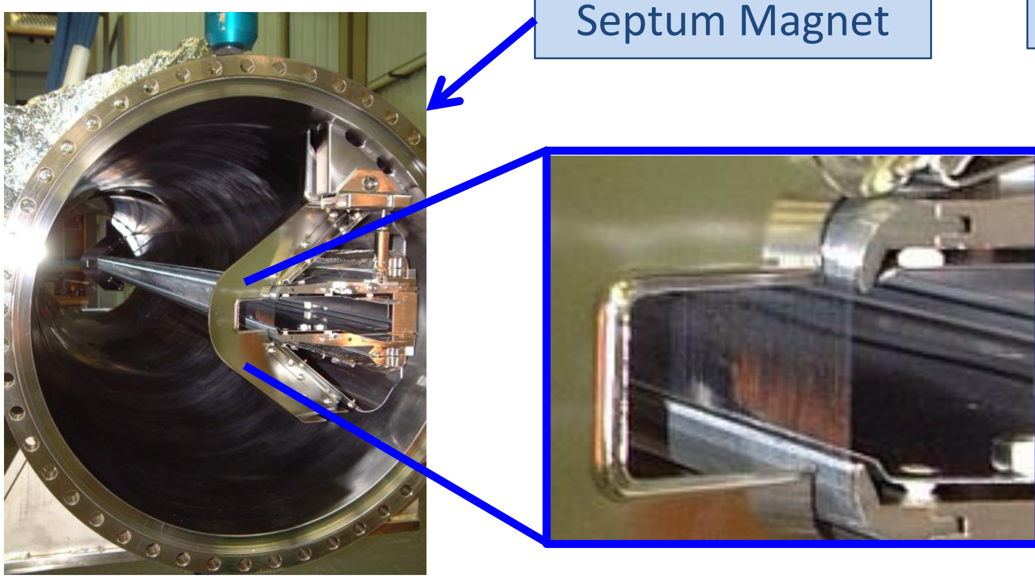


Injection from LINAC3

LEIR – first circular accelerator in for CERN‘s heavy-ions on the way to LHC

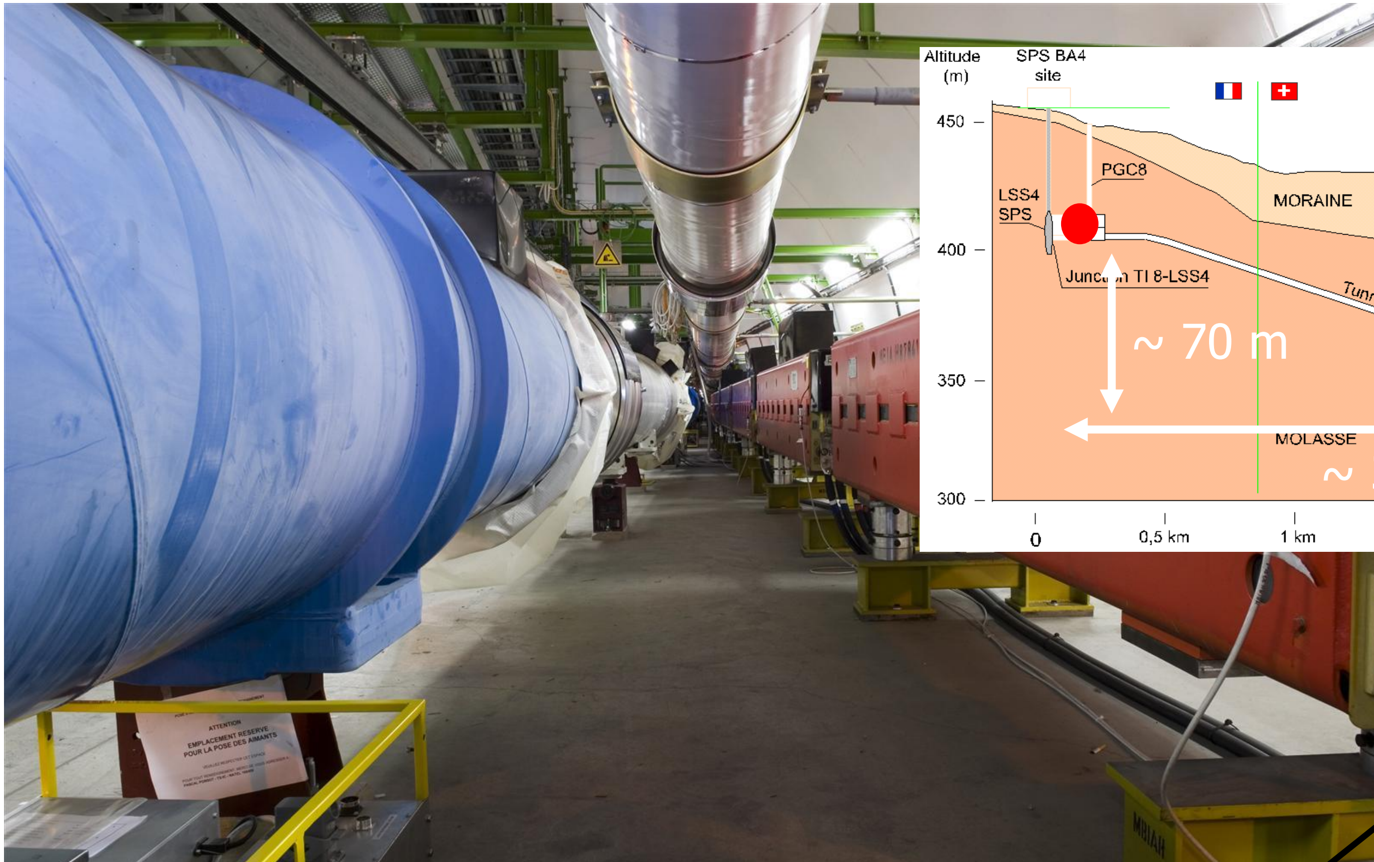
Injection and Extraction

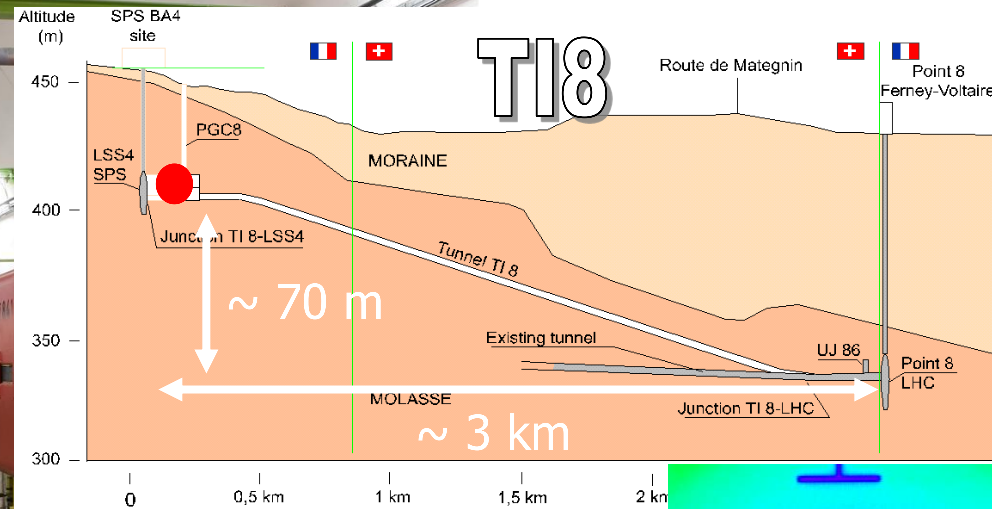


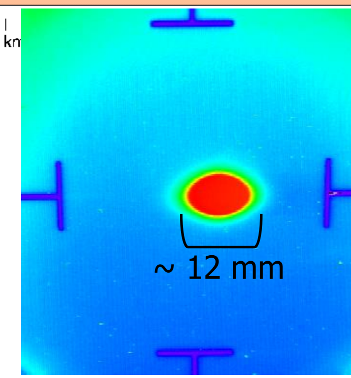


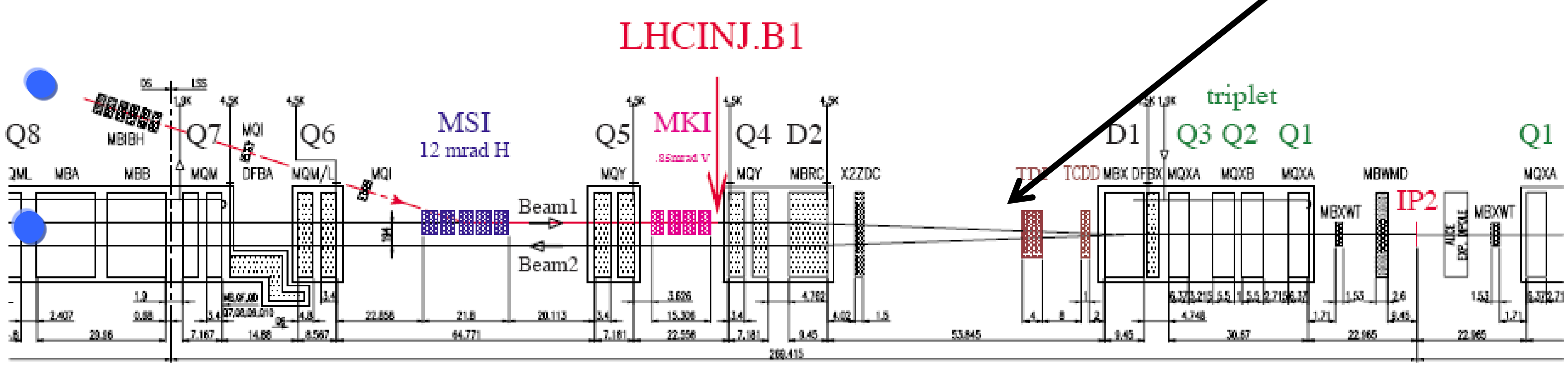
Extraction follows the same principle, but the beam travels in the **opposite direction**.

Injection of Beam 2 into LHC



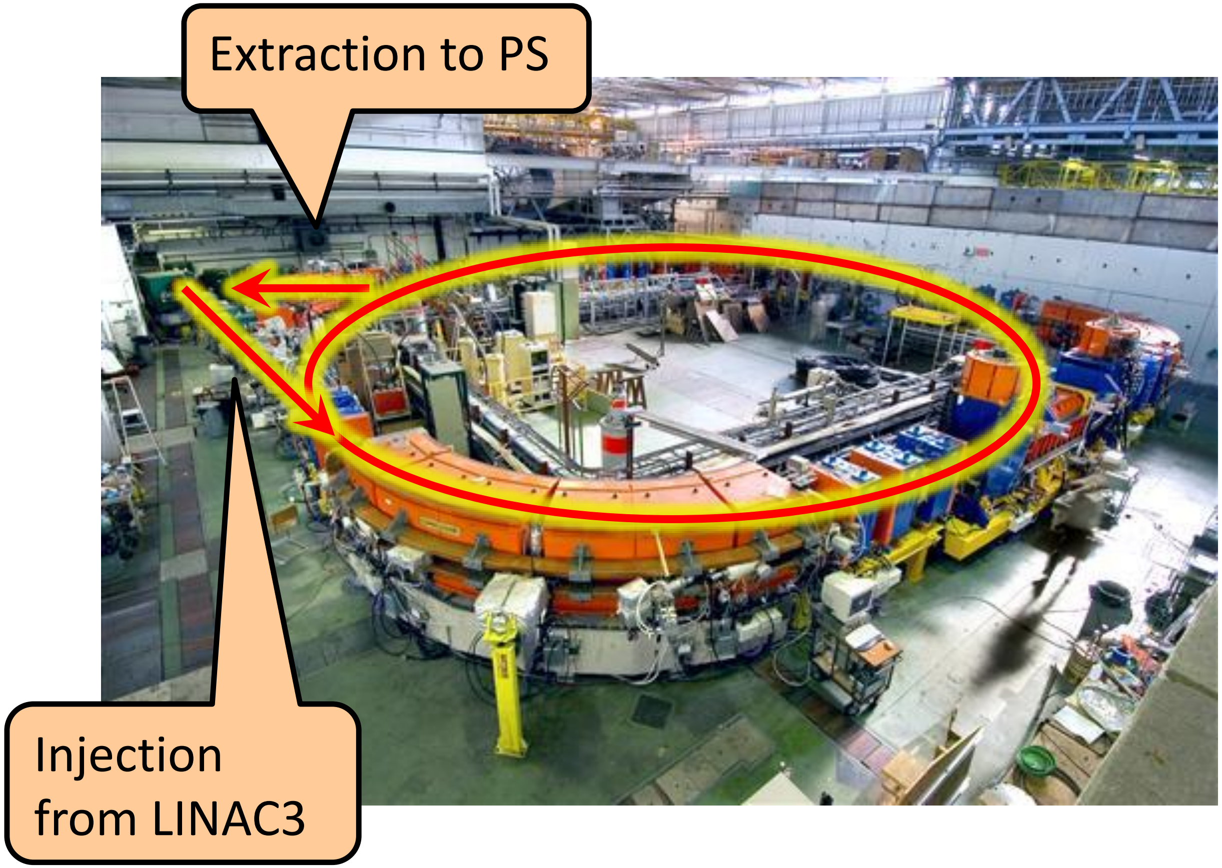






court.
R. Alemany

Filling and Circulating Beam

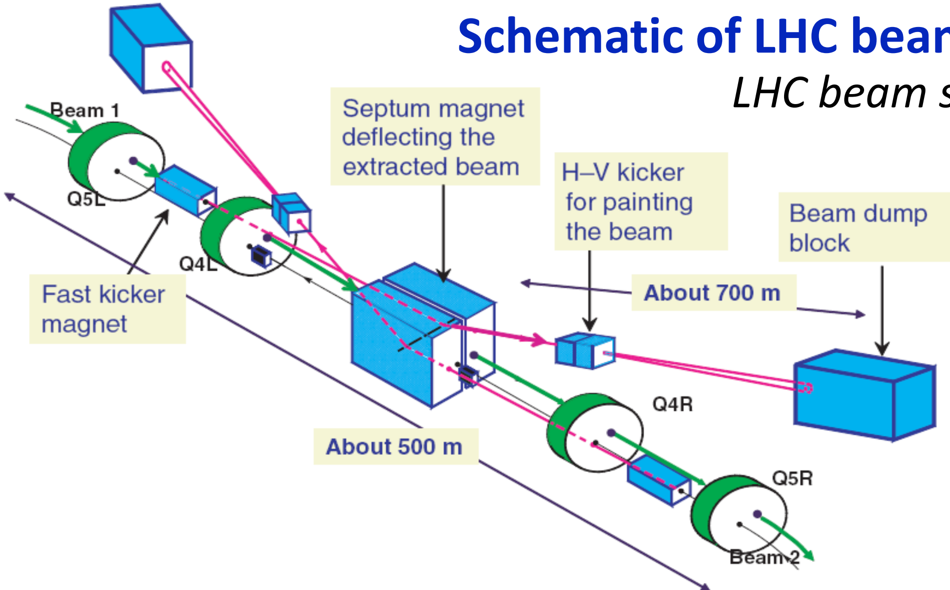


- Several injections are accumulated,
- while the already injected particles circulate/wait.
- Only once the ring is fully filled, acceleration starts.

LEIR – first circular accelerator in for CERN‘s heavy-ions on the way to LHC

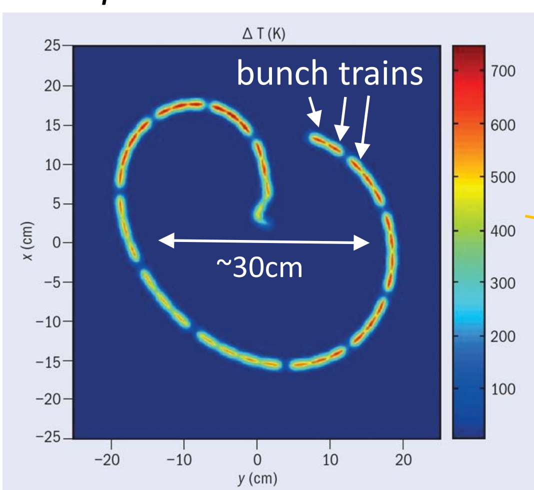
Beam Dump – How to safely kill the LHC beam

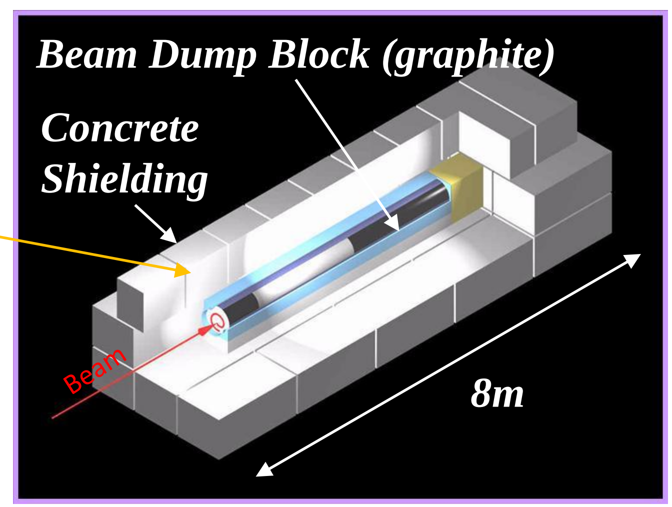
Schematic of LHC beam dump system



LHC beam stores ~360MJ energy.

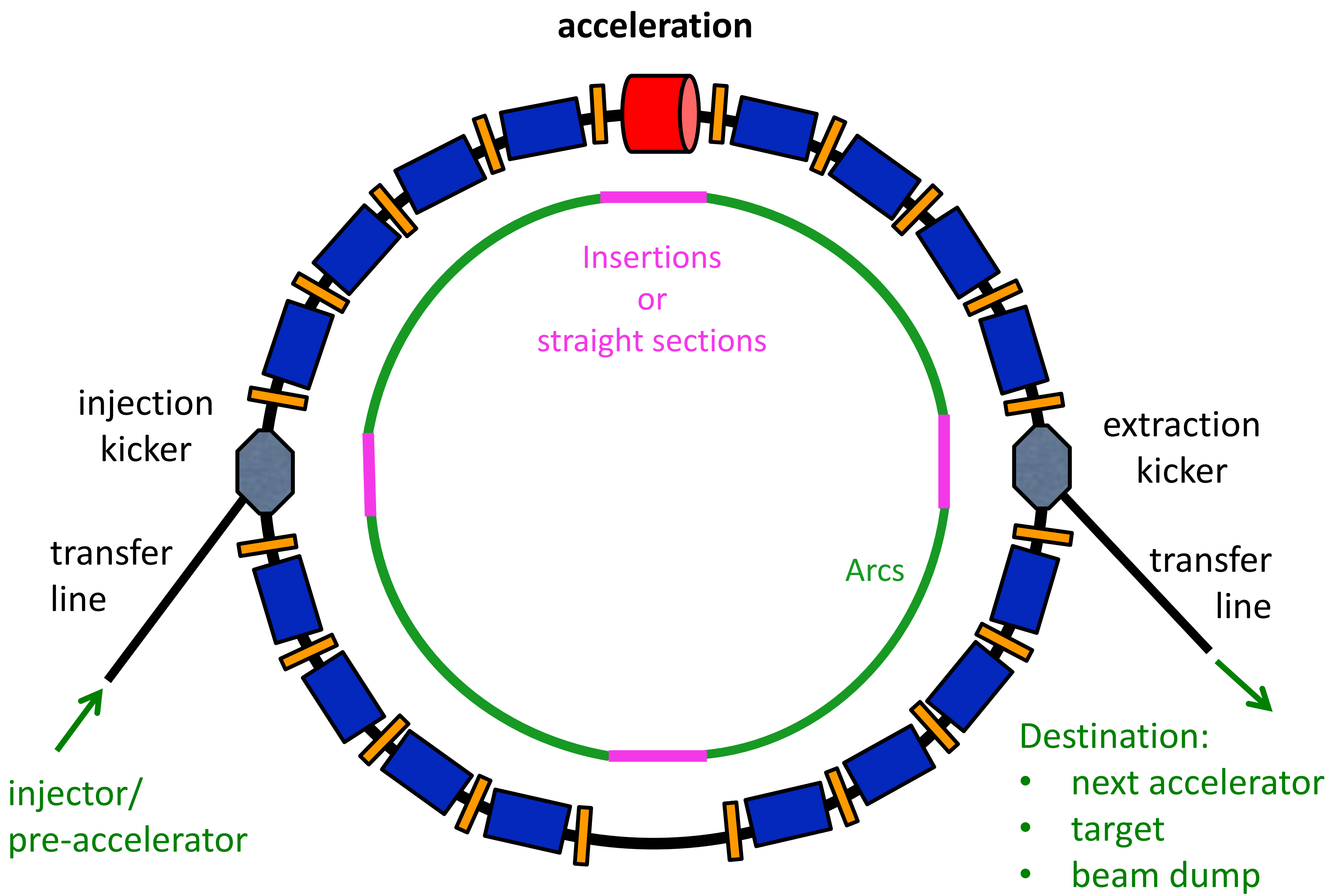
Sweep of beam on beam dump window





Source graphics: http://clipart-library.com

Acceleration



How can we increase the energy of a particle?

A *charged* particles that travels through an electro-magnetic field feels the **Lorentz force**:

$$\vec{F} = q(\vec{v} \times \vec{B} + \vec{E})$$

Magnetic field *B*:

Force acts perpendicular to path.

→ **Can change direction of particle**

→ cannot accelerate

Electric field E:

Force acts parallel to path.

→ **Can accelerate**

→ not optimal for deflection

The ***energy gain ΔE*** of the particle is defined by the integral of the force *F* over the travelled path *dr*:

$$\Delta E = q\int_{r_1}^{r_2}(\vec{v} \times \vec{B} + \vec{E})d\vec{r}$$

$$= q\int_{r_1}^{r_2} \vec{E}d\vec{r} = qU.$$

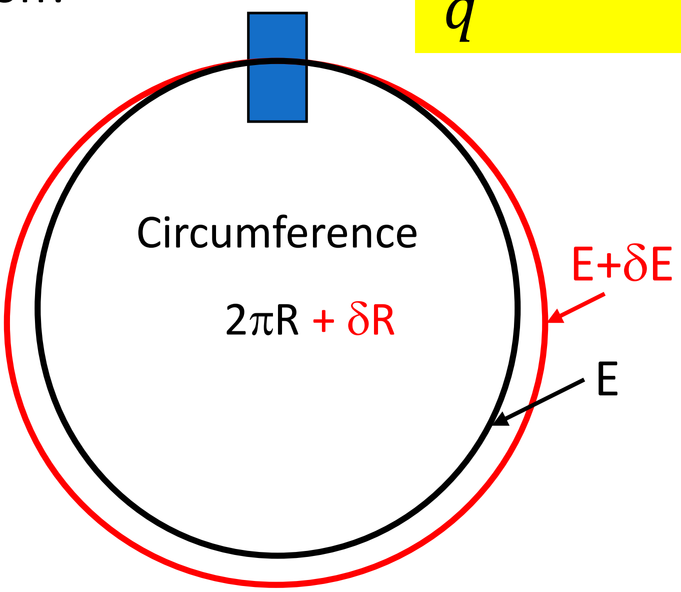
↑

$(\vec{v} \times \vec{B})d\vec{r}$ *= 0*

RF Acceleration and Magnetic Field Increase

$$\frac{p}{q} = B\,\rho$$

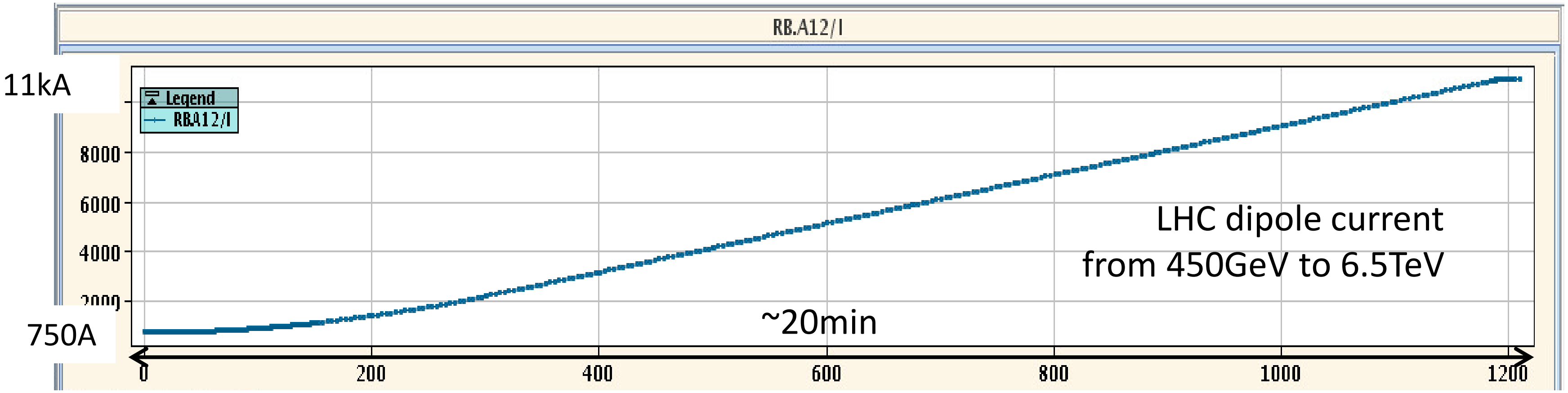
What about the magnetic field during acceleration?



Beam rigidity needs to be increased proportionally to increasing energy.

→ Machine radius is constant.

→ Need to increase dipole field accordingly!



Acceleration without Magnetic Field Increase

LHC magnetic dipole field at 450 GeV:

$$B = \frac{p}{q\rho} = \frac{450\,\mathrm{GeV}/c}{e \times 2803\,\mathrm{m}} = 0.535\,\mathrm{T}$$

Required bending radius at 7 TeV with B_{inj}=0.5T:

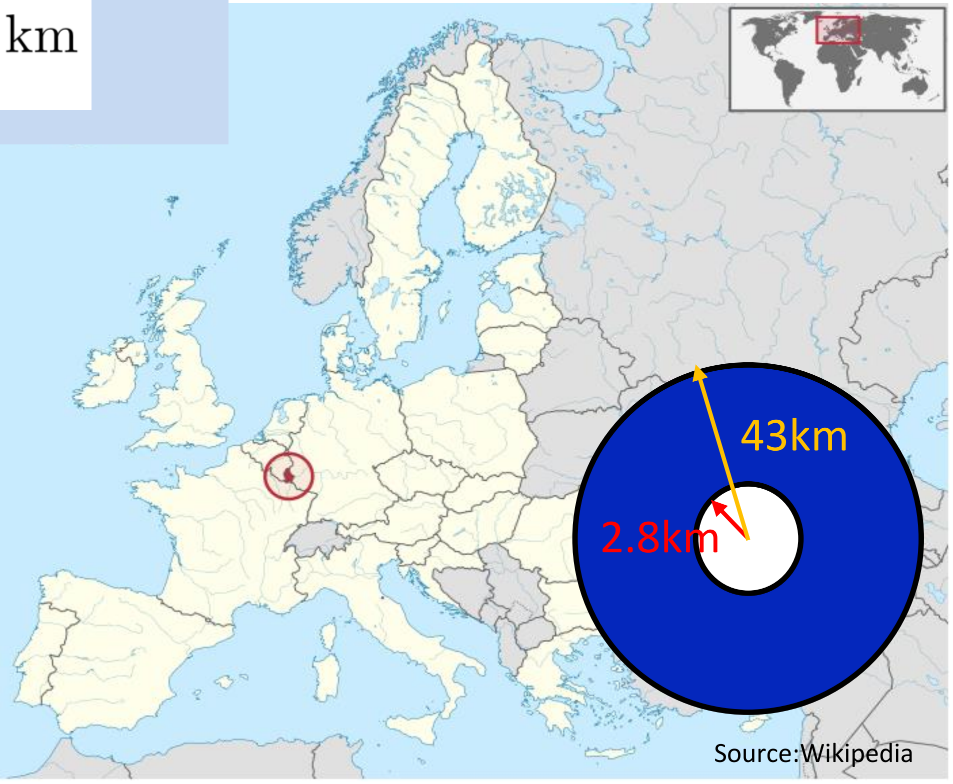
$$\rho = \frac{p}{qB} = \frac{7\,\mathrm{TeV}/c}{e \times 0.535\,\mathrm{T}} = 43.6\,\mathrm{km}$$

How does the bending radius changes, when accelerating without adjusting the magnetic field?

$$\frac{p}{q} = B\,\rho$$

Equivalent to 270km circumference (pure dipole field! without any insertions or quadrupoles)

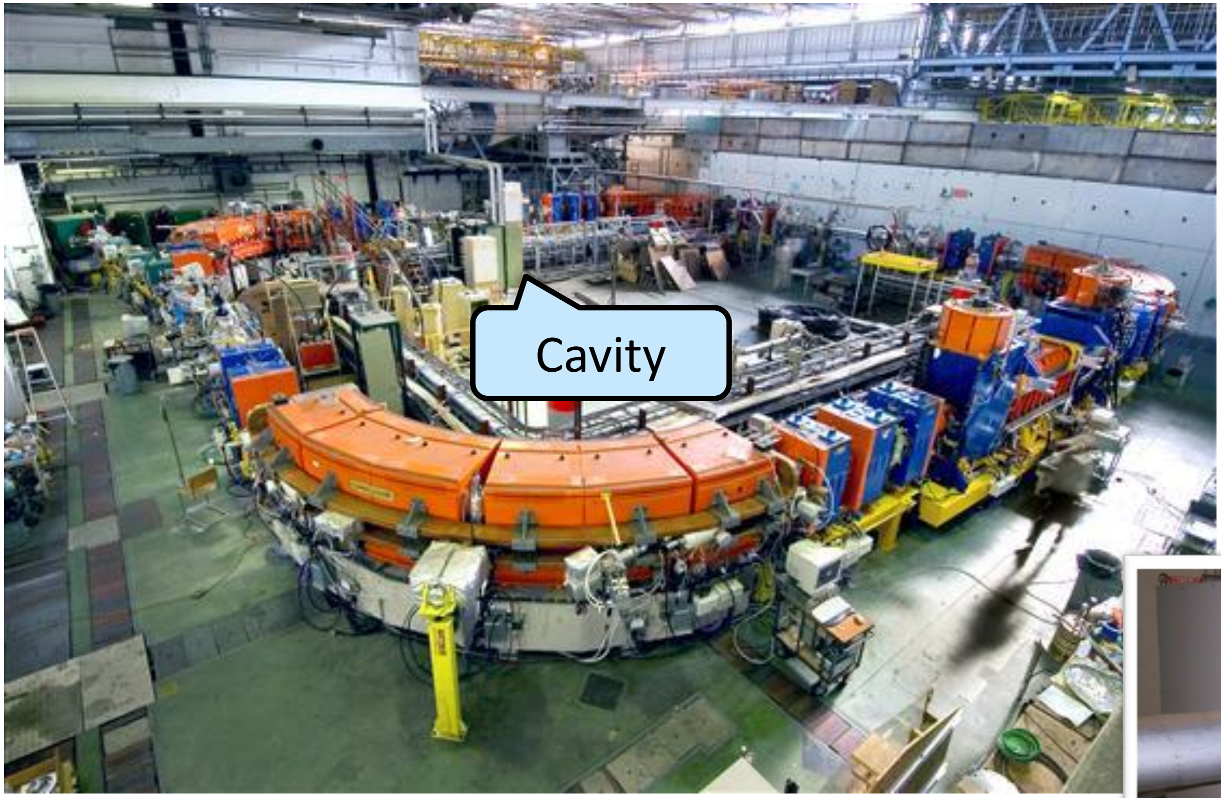
Magnet surface = 5800km^2
→Area of Brunei (South-Eastern Asia)
→ Area of 2x Luxemburg

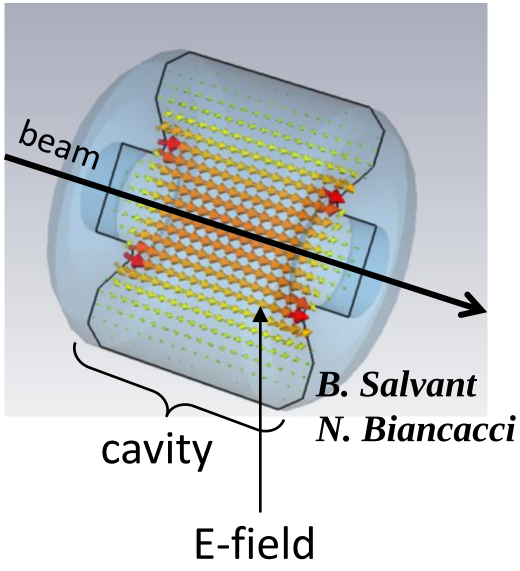


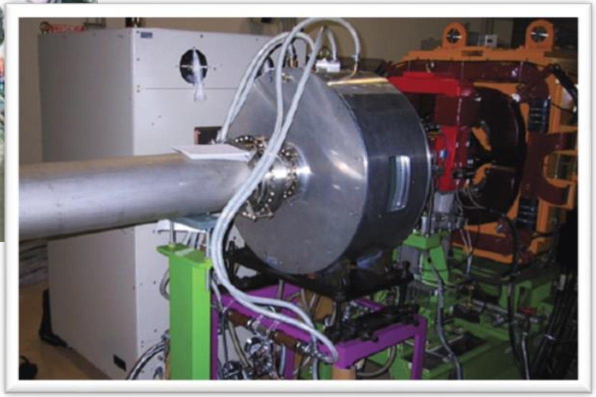
Source:Wikipedia

Where do we accelerate?

Use **RF cavities** to apply the same accelerating voltage on each passage. → Gradually increase total energy by **gaining a small amount each turn**.

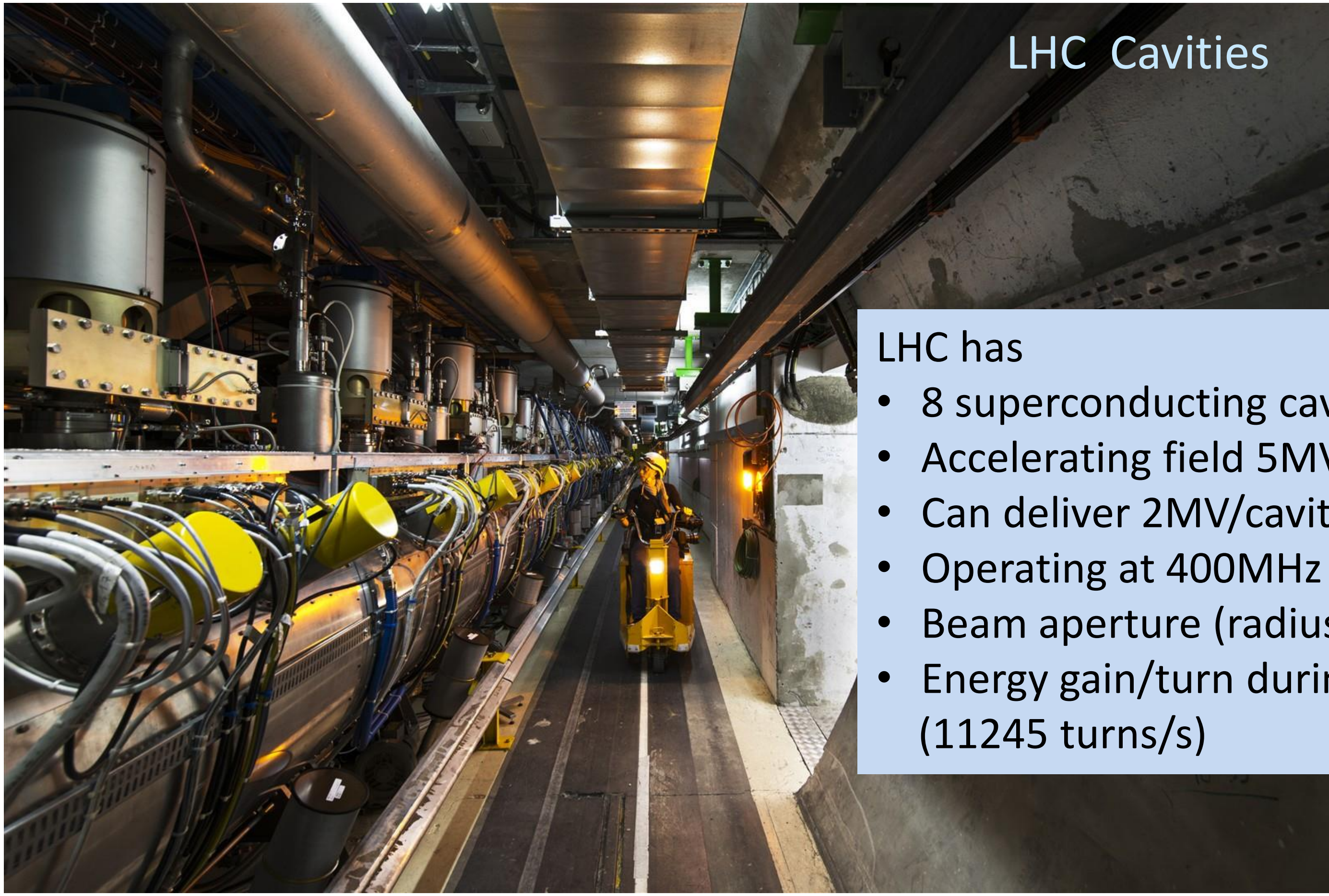






Example of an accelerating cavity in a synchrotron

Example: LHC Accelerating System



LHC has

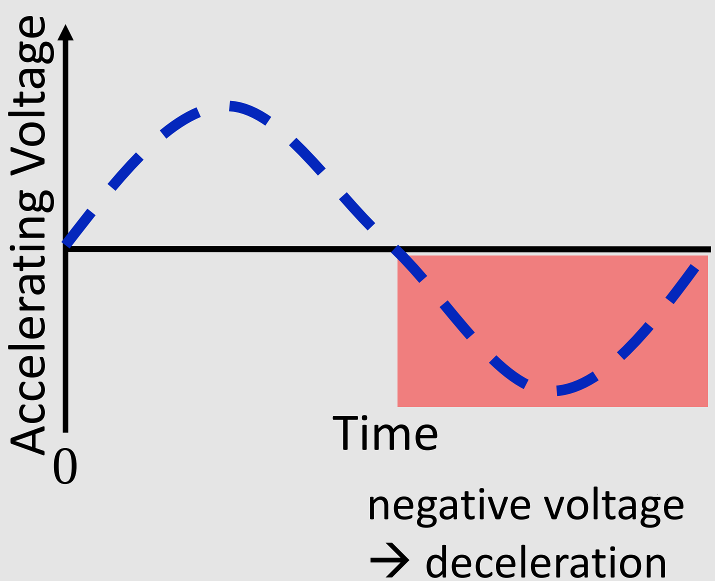
- 8 superconducting cavities per beam
- Accelerating field 5MV/m
- Can deliver 2MV/cavity
- Operating at 400MHz
- Beam aperture (radius) ~30cm
- Energy gain/turn during ramp 485 keV (11245 turns/s)

RF Acceleration

Accelerating voltage is changing with time. That has two consequences:

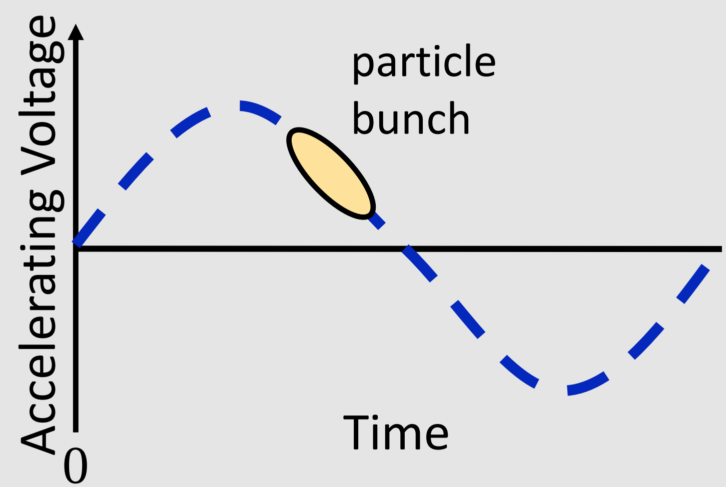
Need ***synchronization*** between beam and RF phase to gain energy.

There is a ***synchronous RF phase*** for which the energy gain fits the increase of the magnetic field.



Not all particles see the same voltage, because they arrive at different times.

Not all particles gain the same energy.

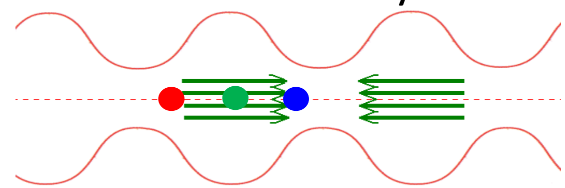


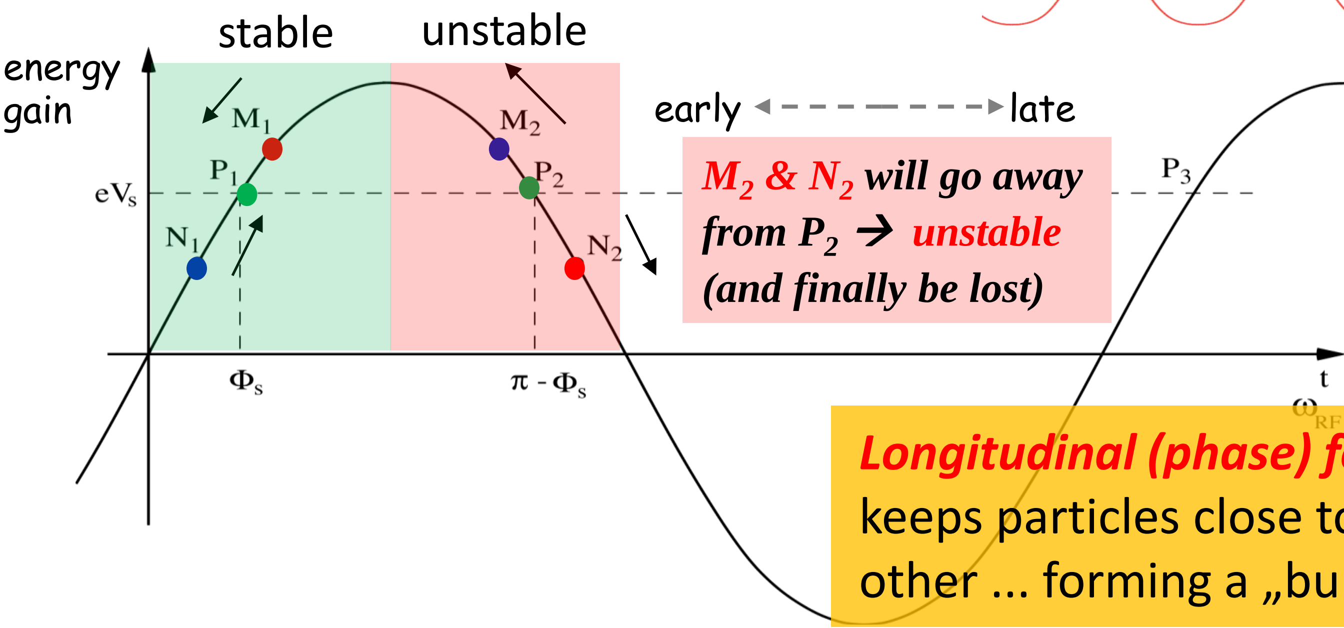
Phase Stability (non-relativistic regime)

Assume the situation where ***energy increase is transferred into a velocity increase*** (non-relativistic regime).

Particles P_1, P_2 have the synchronous phase.

field in a cavity





M_2 & N_2 will go away from P_2 → unstable (and finally be lost)

Longitudinal (phase) focusing keeps particles close to each other ... forming a „bunch“

- Ideal particle
- Particle with Δt < 0 (early) →lower energy gain → gets slower
- Particle with Δt > 0 (late) →higher energy gain → gets faster

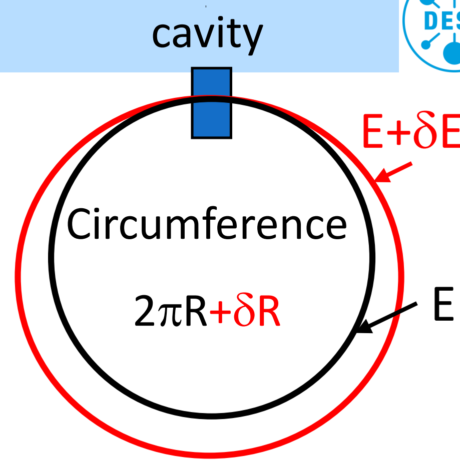
→ M_1 & N_1 will move towards P_1 → stable

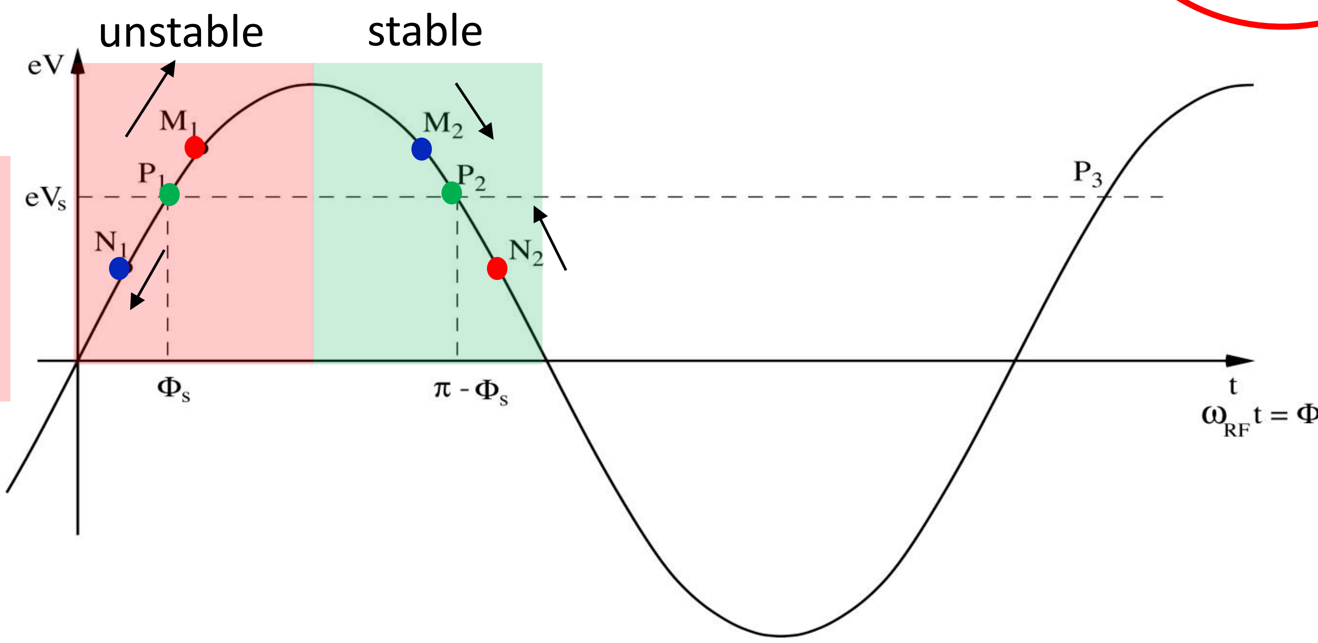
Courtesy F. Tecker for drawings

Phase Stability (relativistic regime)

Now assume relativistic energies ($v \approx c$):
An ***increase in momentum*** transforms into a ***longer orbit and thus a longer revolution time.***

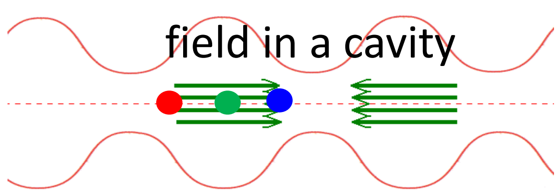
$$\frac{p}{q} = B\,\rho$$





M_1 & N_1 will go away from P_1 → unstable (and finally be lost)

- Ideal particle
- Particle with Δt < 0 → higher energy gain → gets longer orbit
- Particle with Δt > 0 → lower energy gain → gets shorter orbit
- ***M_2 & N_2 will move towards P_2 → stable***

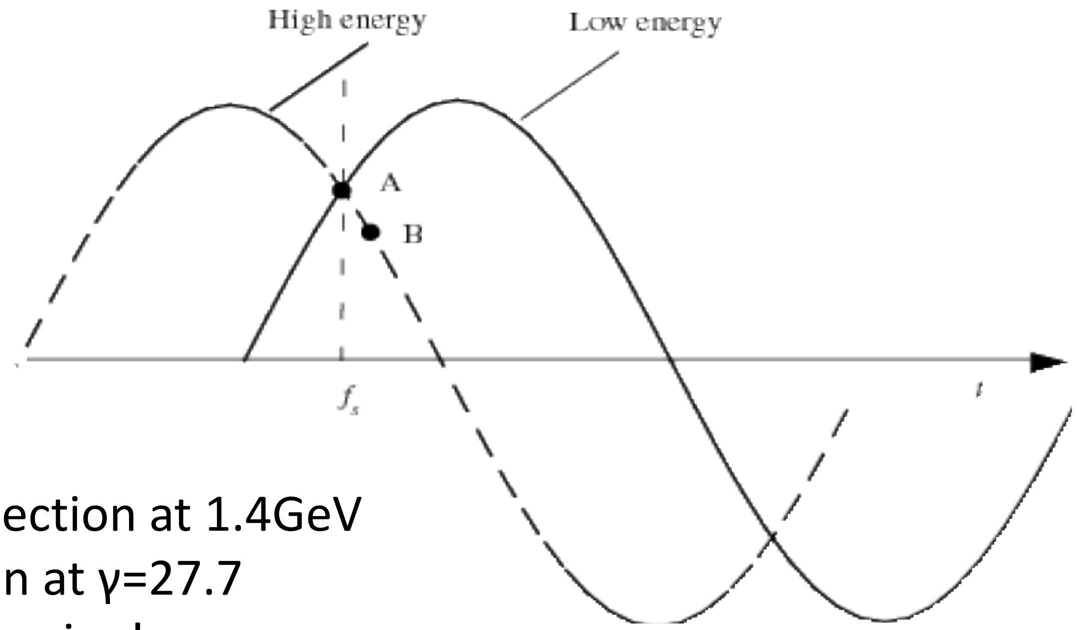


Courtesy F. Tecker for drawings

Crossing Transition

The previously stable synchronous phase becomes unstable when v => c and the gain in path length overtakes the gain in velocity → ***Transition***

Transition from one slope to the other during acceleration → ***Crossing Transition.*** The RF system needs to make a rapid change of the RF phase, a 'phase jump'.



In the PS: γ_t is at ~6 GeV, injection at 1.4GeV
In the SPS: γ_t= 22.8, injection at γ=27.7
=> no transition crossing!
In the LHC: γ_t is at ~55 GeV, also far below injection energy

Transition crossing not needed in leptons machines, why?

Synchrotron Oscillation

Like in the transverse plane the particles are oscillating in longitudinal space.

Particles keep *oscillating around the stable synchronous particle* varying phase and dp/p.

Typically one synchrotron oscillation takes many turns (much slower than betatron oscillation)

Phase-space ellipse defines ***longitudinal emittance***.

Separatrix is the trajectory separating stable and unstable motion.

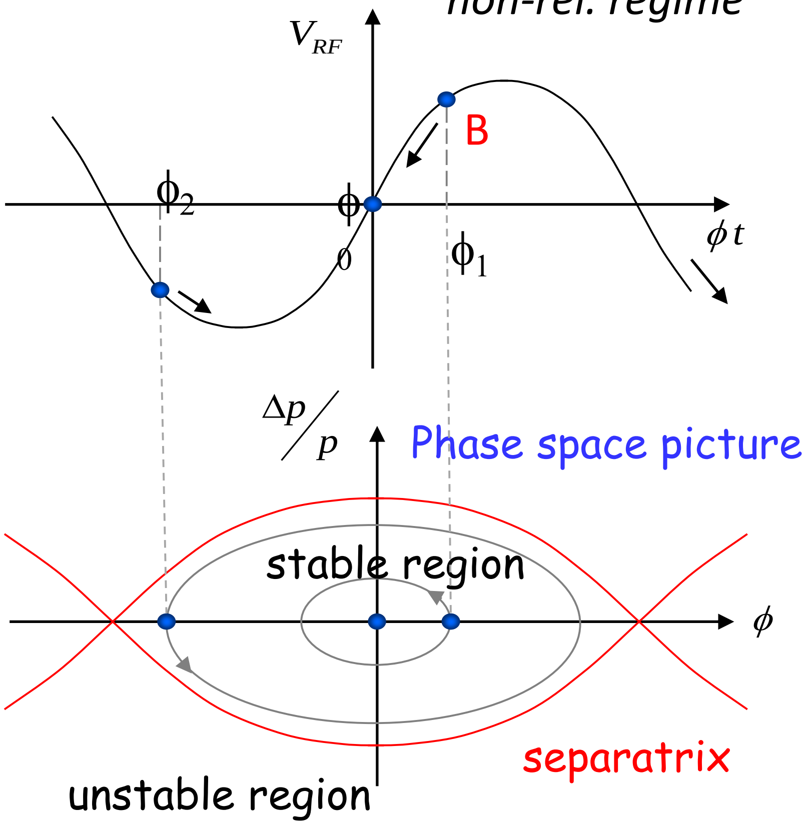
Stable region is also called ***bucket***.
→ Harmonic number h = number of buckets:

$$f_{RF} = h\, f_{rev}$$

Simple case (no accel.): B = const.

- Stable phase: $\phi_0 = 0$
- Particle B oscillates around ϕ_0.

non-rel. regime



Courtesy F. Tecker for drawings

Emittance during Acceleration

What happens to the emittance if the reference momentum P_0 changes?

Can write down transfer matrix for reference momentum change:

$$M_x = \begin{pmatrix} 1 & 0 \\ 0 & P_0/P_1 \end{pmatrix} \rightarrow \quad \epsilon_{x1} = \frac{P_0}{P_1}\epsilon_{x0}$$

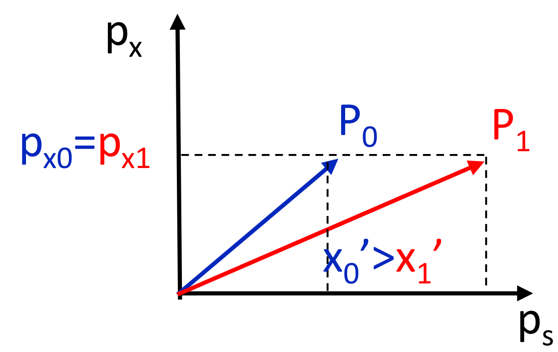
The emittance shrinks with acceleration!

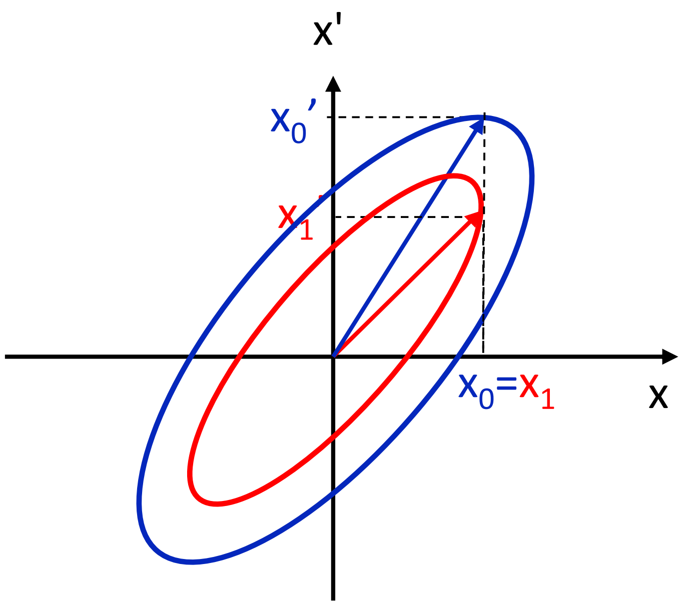
With $P = \beta\gamma mc$ where γ, β are the relativistic parameters.

The conserved quantity is

$$\beta_1\gamma_1\epsilon_{x1} = \beta_0\gamma_0\epsilon_{x0}$$

It is called ***normalized emittance***.





How big are the beams in the LHC?

Normalized emittance at LHC : $\varepsilon_n = 3.5\ \mu m$

→ ε_n preserved during acceleration.

$$\varepsilon_{7TeV} = \varepsilon_{450GeV} \frac{\gamma_{450GeV}}{\gamma_{7TeV}}$$

The **geometric emittance:**

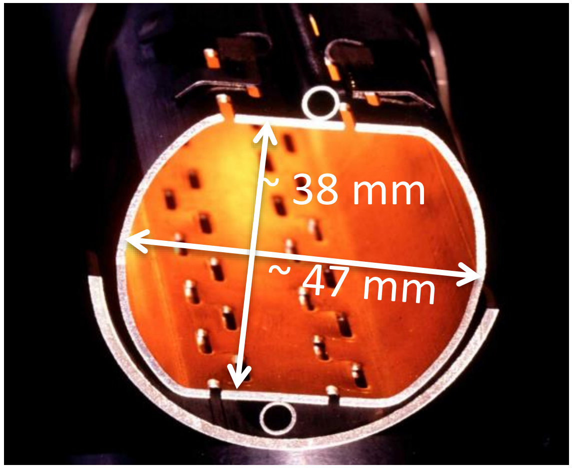
- Injection energy of 450 GeV: $\varepsilon = 7.3$ nm
- Top energy of 7 TeV: $\varepsilon = 0.5$ nm

The corresponding max. **beam sizes** in the arc,
at the location with the maximum beta function ($\beta_{max} = 180$ m):

– $\sigma_{450GeV} = 1.1$ mm
– $\sigma_{7TeV} = 300\ \mu m$

Aperture requirement: $a > 10\ \sigma$
LHC beam pipe radius:

- Vertical plane: 19 mm ~ 17 σ @ 450 GeV
- Horizontal plane: 23 mm ~ 20 σ @ 450 GeV

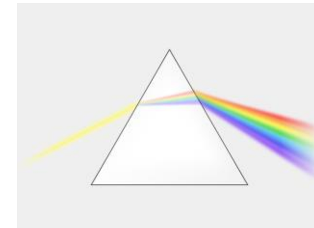


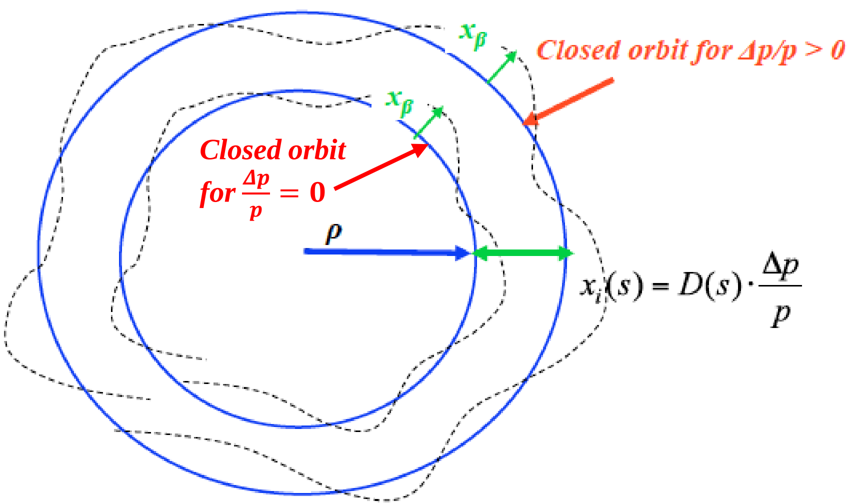
Transverse-Longitudinal Coupling: Dispersion

Dipole magnets generate dispersion:

→ Particles with different momentum are bent differently.

Due the momentum spread in the beam $\frac{\Delta p}{p}$, this has to be taken into account for the particle trajectory.





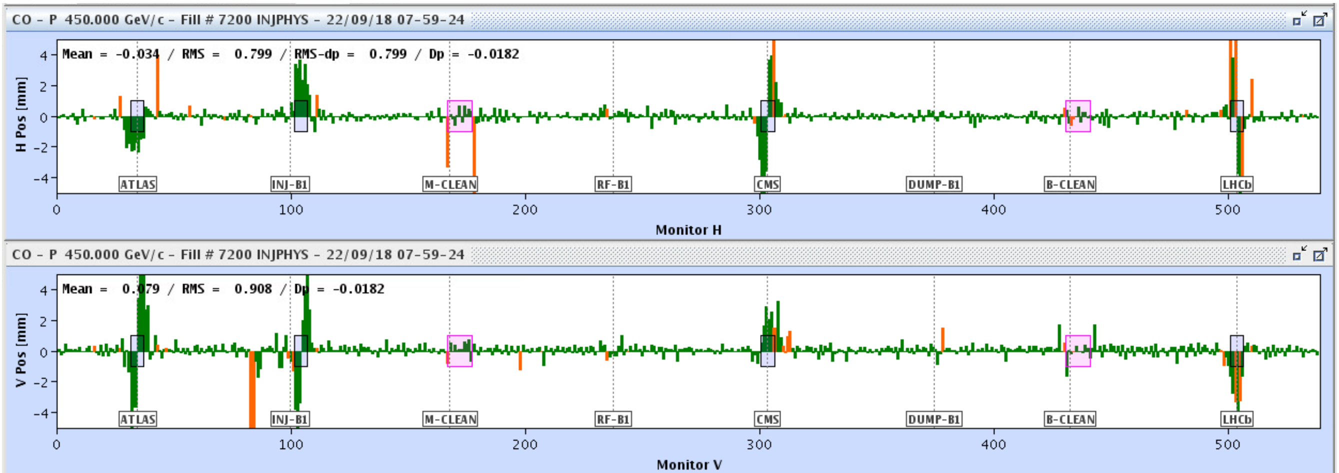
$$x(s) = x_\beta(s) + D(s)\frac{\Delta p}{p}$$

Dispersion function *D(s)* corresponds to the trajectory of a particle with momentum offset $\frac{\Delta p}{p} = 1$.

This also has an effect on the beam size:

$$\sigma = \sqrt{\beta\varepsilon} \quad \rightarrow \quad \sigma = \sqrt{\beta\varepsilon + D^2\left(\frac{\Delta p}{p}\right)^2}$$

Dispersive Orbit



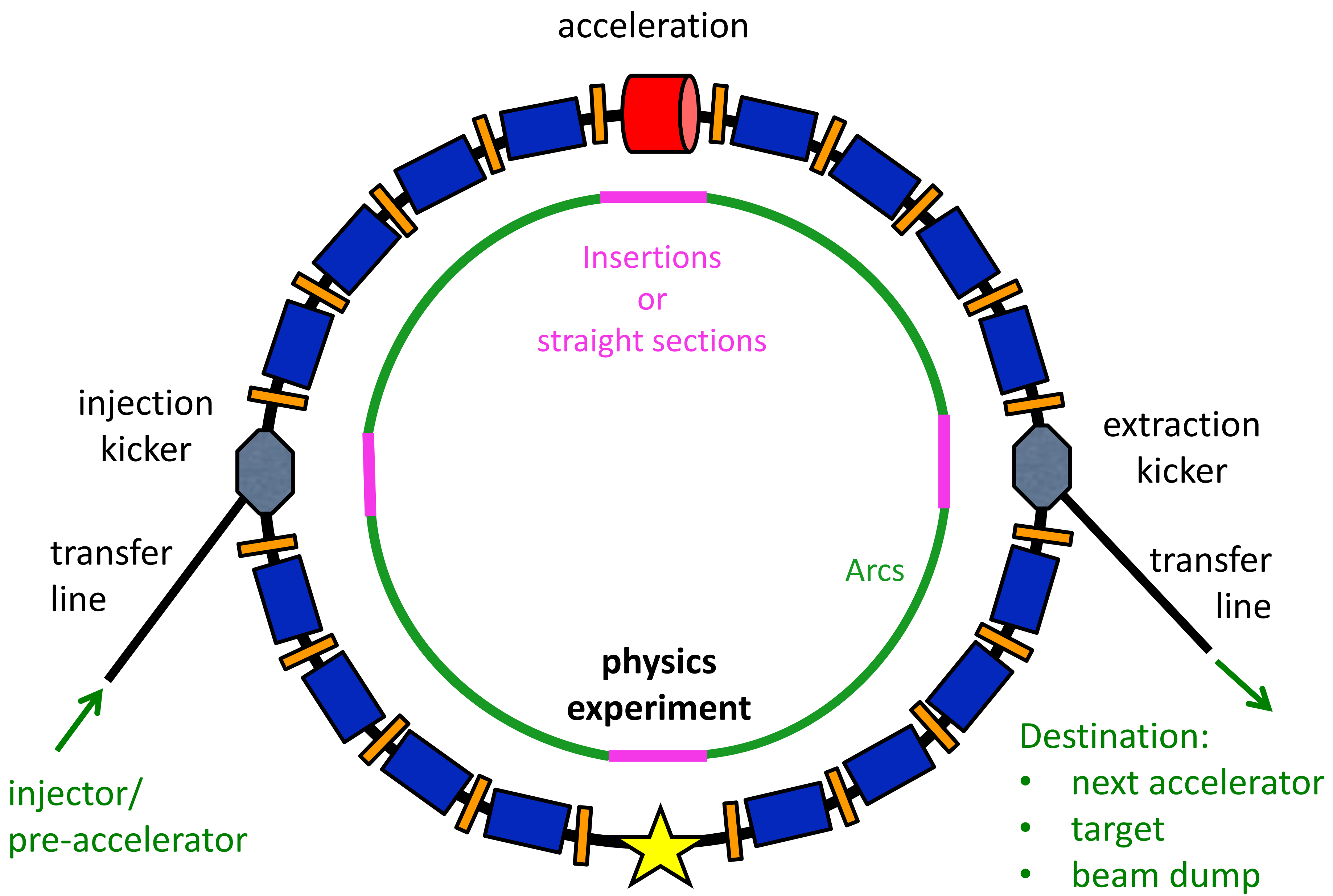
On-momentum orbit in LHC

Off-momentum orbit

Dedicated energy (i.e. f_{RF}) change of the stored beam.

- Horizontal orbit is moved to a dispersions trajectory.
- Vertical orbit unchanged (no vertical dispersion)

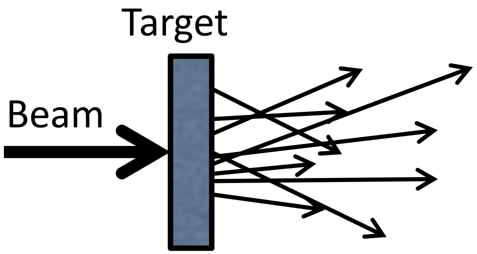
Experiments and Luminosity



Experimental Application

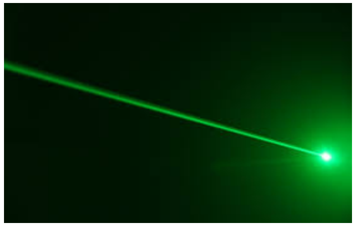
Each accelerator and experiment requires specific beam properties.
Fundamentally different are:

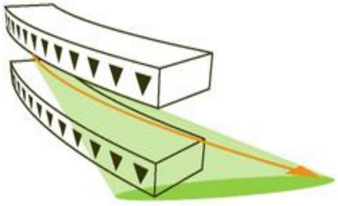
Fixed Target:



Secondary particles

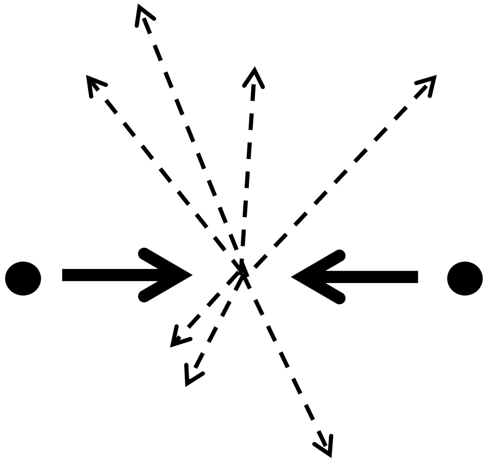
Light Sources:





Particles that are bent to a circular orbit emit energy/light.

Collider:



‘Smashing’ Modes and Center-of-Mass Energy

The ***center-of-mass energy*** defines
the upper limit of the newly created particle’s mass.

Fixed Target

$$E \propto \sqrt{E_{beam}}$$

Most of the Energy is lost in the target, only a fraction is transformed into useful secondary particles.

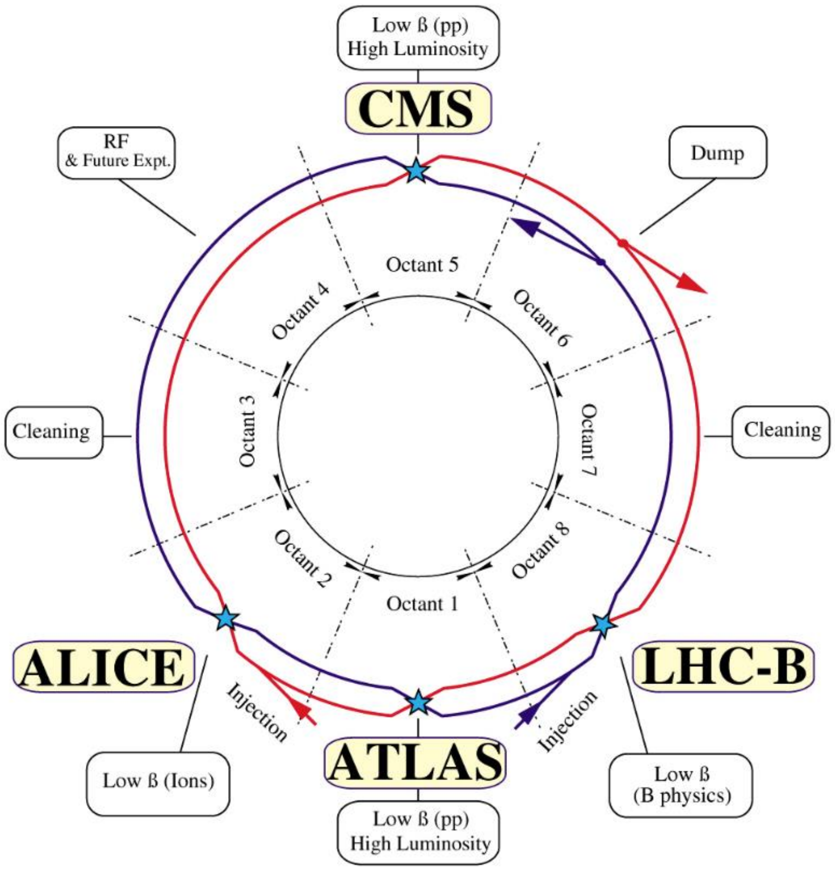
Collider

$$E = E_{beam1} + E_{beam2}$$

All energy is available for the production of new particles.

Exercise 2: Derive center-of-mass energy in fixed target and collider experiment.

LHC and its Experiments



In LHC has **4 interaction points (IPs)** hosting particle physics experiments:
→ ATLAS, ALICE, CMS, LHCb

Therefore the two counterrotating **beams collide 4 times per turn**

When they collide the outer beam **cross over** to the inner circle and vise versa.

Particle Collisions

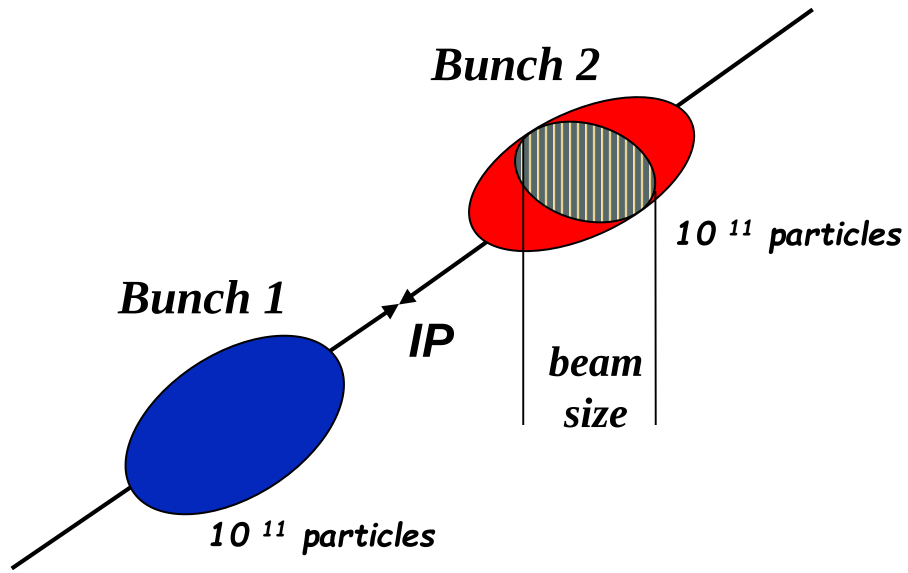
Experiments are interested in maximum number of interactions per second. The event rate in an experiment is proportional to the luminosity.

Interaction Cross-section
(= constant probability for the given interaction to take place)

$$\frac{dR}{dt} = L \times \sigma_p$$

Luminosity

Event rate
(= number of collisions per time)



"Quality Factor" of a Collider

The most important measure to describe the potential of a collider is the Luminosity.

$$L = \frac{k N^2 f \gamma}{4\pi \beta^* \varepsilon} \cdot F$$

N..... No. particles per bunch
k...... No. bunches
f....... revolution freq.
g...... rel. gamma
β^*.... beta-function at IPs
ε...... norm. trans. emit

Defined by the injectors

Overall Goal of an Collider: Maximizing Luminosity!

→ Many particles (N, k)

→ In a small transverse cross-section (ε, β)

Limitation:
"Collective effects" cause beam instabilities for too high bunch intensities, too small bunch spacing, too "bright" beams.

Performance depends on the injectors:

→ Production of large N and small ε

→ Preservation of these parameters until collisions.

Optimizing Luminosity

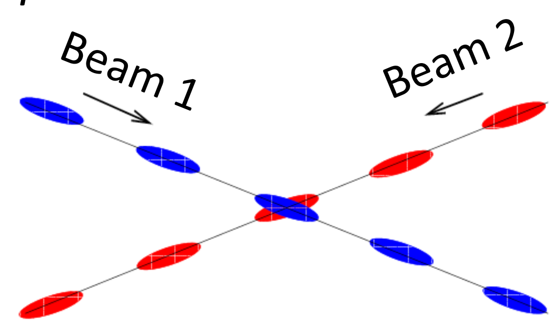
Bunch properties (N & ϵ) are defined in the injectors.
But what can we do in the Collider?

$$L = \frac{kN^2 f\gamma}{4\pi\beta^*\varepsilon}\cdot F$$

N….. No. particles per bunch
k…… No. bunches
f……. revolution freq.
g…… rel. gamma
β^*…. beta-function at IPs
ε…... norm. trans. emit

f_{rev}, γ: defined by the design of the accelerator

F [0,1]: When colliding with many bunches, a **crossing angle is needed** to avoid unwanted collisions. However this ***reduces the beam overlap*** and therefore the luminosity. Keep as *small as possible*!
→Limited by beam-beam effects.



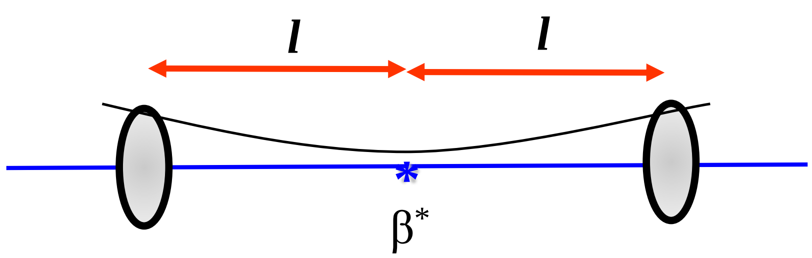
k: Optimize filling scheme and bunch spacing.

β^*: Can be optimized by focusing!

Mini-Beta Insertions

Mini-beta insertion is a ***symmetric drift space*** with a ***waist of the β-function*** in the center of the insertion.

On each side of the symmetry point a quadrupole **doublet** or **triplet** is used to generate the waist.
They are not part of the regular lattice.



$$\beta(s) = \beta^* + \frac{s^2}{\beta^*}$$

Collider experiments are located in mini-beta insertions: **smallest beam size possible** for the colliding beam to increase probability of collisions.

There is a price to pay: **The smaller β*, the larger β at the triplet.**

Example: Mini-Beta Insertion at LHC

Example of the LHC
(design report values):

At the interaction point:

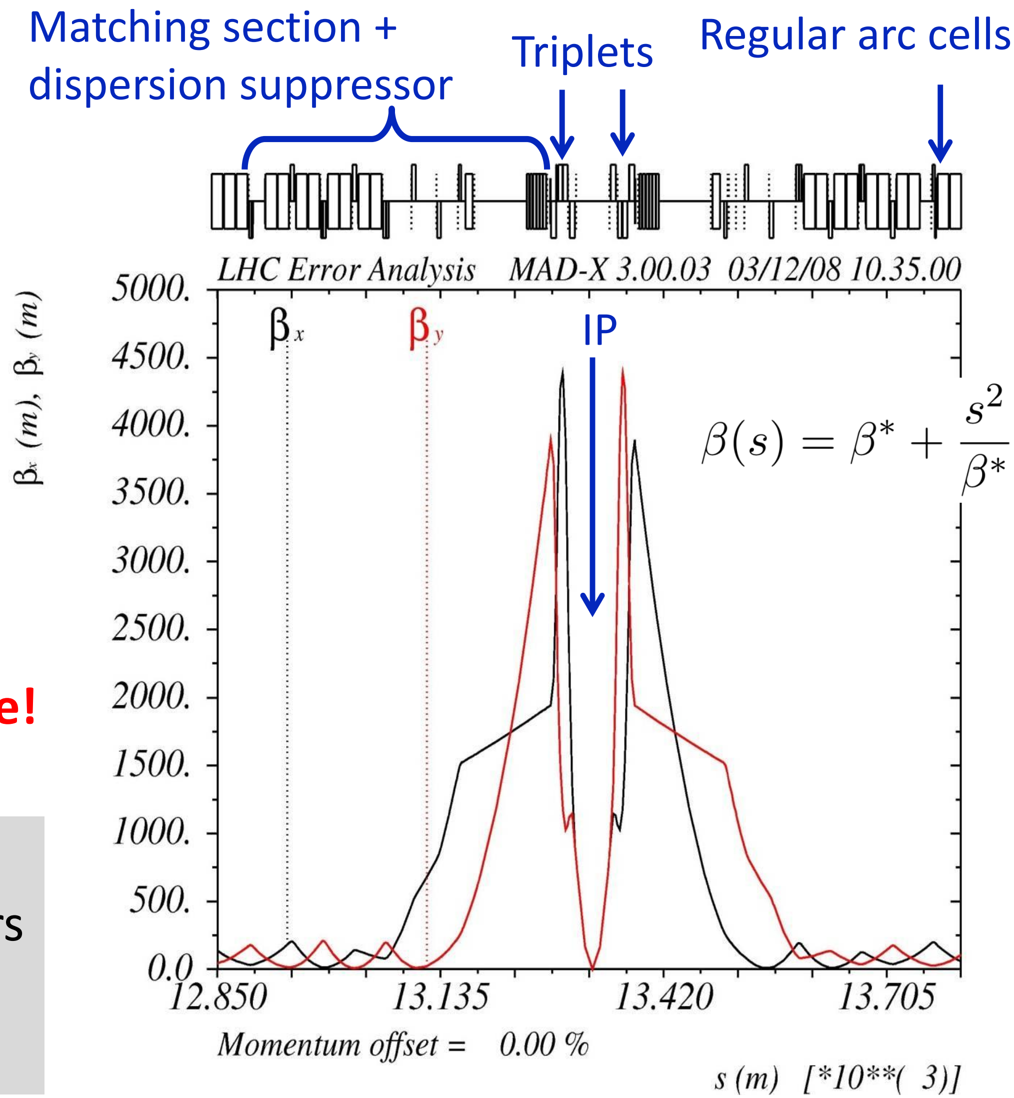
$\beta^* = 0.55$ m
$\sigma^* = 16\ \mu$m
That's smaller than a hair's diameter!

At the triplet:

$\beta = 4500$ m
$\sigma = 1.5$ mm $= 1500\ \mu$m
Largest beams size in the lattice!

Limitations:

- Tighter tolerances on field errors
- Triplet aperture limits β^* together with crossing angle.

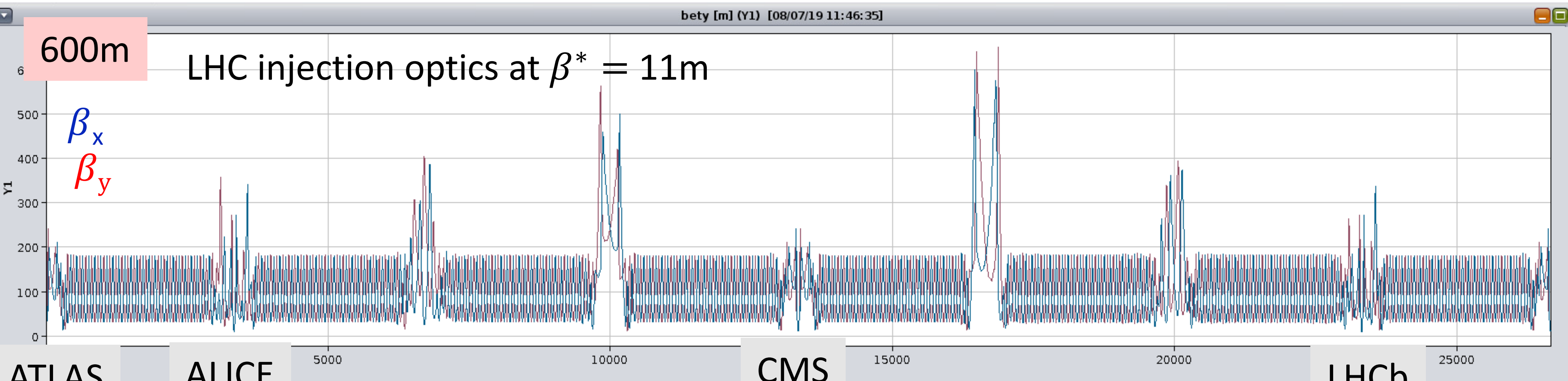


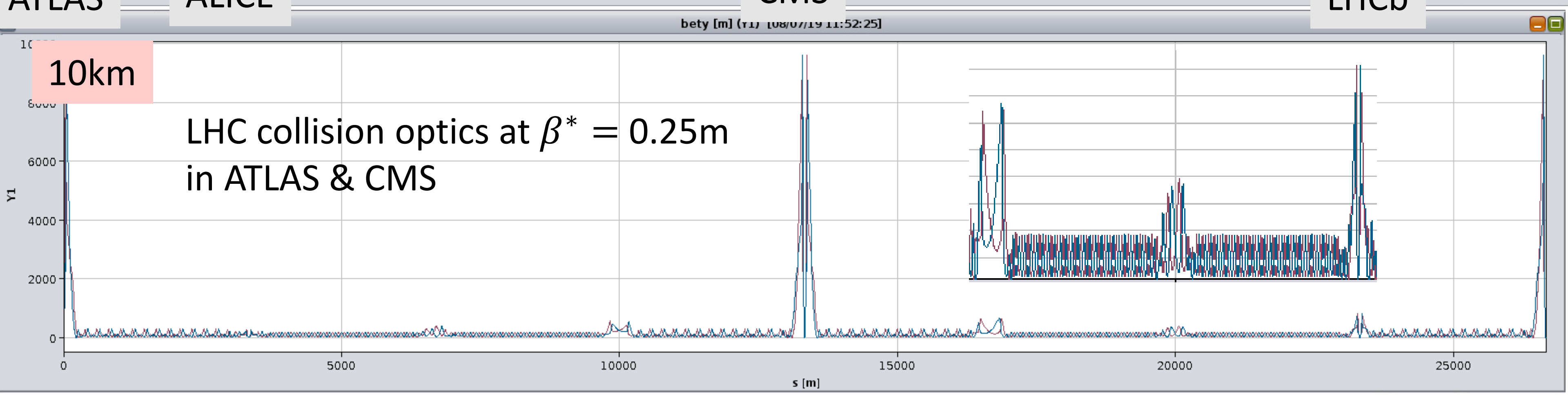
Squeeze

The β-functions, and thus beam sizes, in the triplet for small β^* is too large at injection energy → aperture problems.

→ beam size shrinks with energy $\sigma \propto \sqrt{1/\gamma}$

→ Mini-beta **squeeze** done at top energy when beam size is smaller.





Integrated Luminosity

What counts for the experiments is not peak performance, but total accumulated number of events:

$$\underbrace{\sigma_p}\cdot\underbrace{\mathcal{L}_{int}} = \sigma_p \cdot \int_0^T \mathcal{L}dt$$

unit "barn" = 10^{-24} cm^2

unit " inverse barn" = 10^{24} cm^{-2}

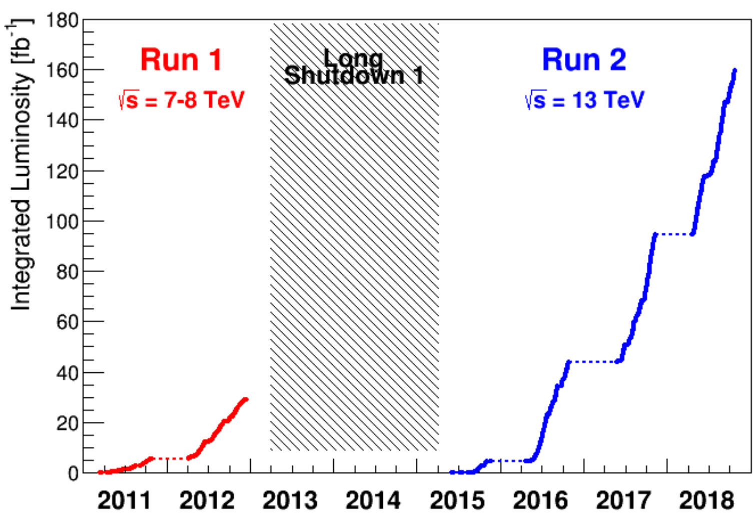
Common order of magnitude: 1 fb^{-1} = 10^{39} cm^{-2}

For example:
To integrate 1 fb^{-1} it requires 10^7 s at

$$\mathcal{L} = 10^{32}\mathrm{cm}^{-2}\mathrm{s}^{-1}$$

For comparison: a year has about $\pi \times 10^7$ s.

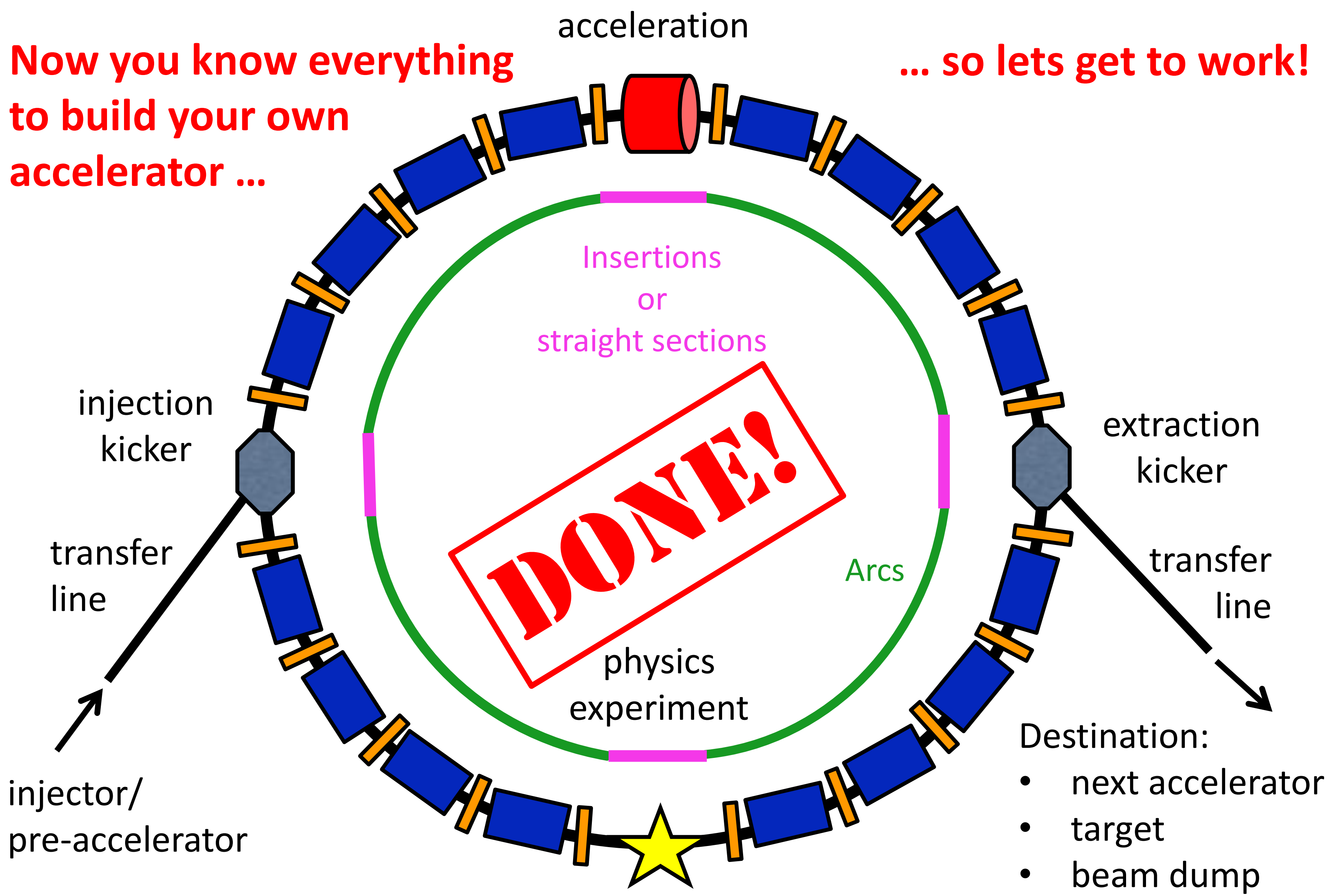
LHC has delivered so far ~190 fb^{-1} to ATLAS/CMS in proton-proton collisions over 7 production years.

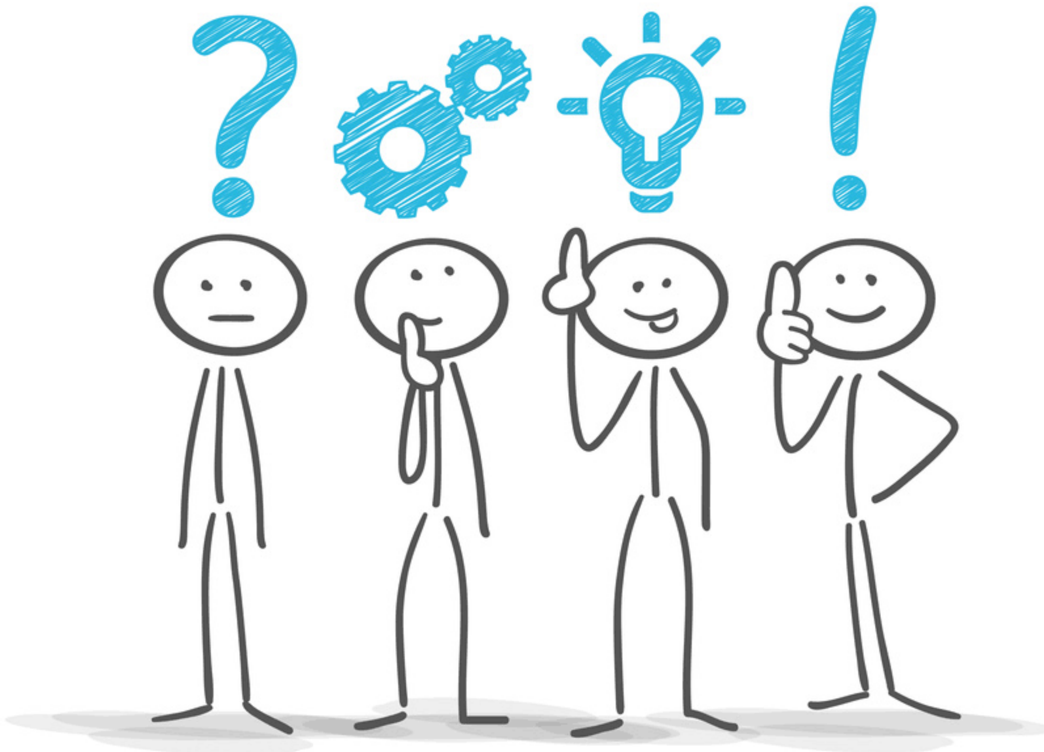


“Complete” Collider

Now you know everything to build your own accelerator …

… so lets get to work!





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Everything clear! Hmm ….

Solutions to Exercises

Exercise 2: Center-of-Mass Energy

Exercise 2: Derive center-of-mass energy in fixed target and collider experiment.

Center-of-mass (CM) frame is defined where sum of all momenta is zero: $\sum \vec{p}_i = \vec{0}$

4-momentum

$$p^\mu = (E/c, \vec{p}) \quad \longrightarrow \quad p^\mu p_\mu = \frac{E^2}{c^2} - \vec{p}^2$$

can be transformed to center-of-mass frame by Lorentz transformation:

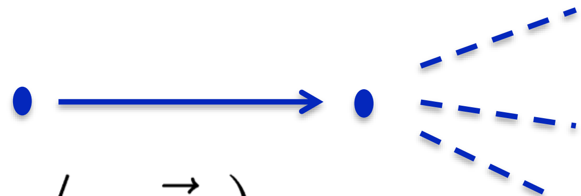
$$p'^\mu = L^\nu_\mu p^\nu$$ Lorentz Transformation

$$p^\mu p_\mu = p'^\mu p'_\mu$$ The norm: is Lorentz invariant

Energy conversation between both frames:

$$\frac{E^2_{CM}}{c^2} - \vec{0}^2 = \frac{E^2_{tot}}{c^2} - \vec{p}^2_{tot} \quad \longrightarrow \quad \frac{E^2_{CM}}{c^2} = \frac{E^2_{tot}}{c^2} - \vec{p}^2_{tot}$$

Exercise 2 : E_{CM} in Fixed Target Experiment



$$p_2 = (m_2 c, \vec{0})$$

$$p_1 = (E_1/c, \vec{p}_1)$$

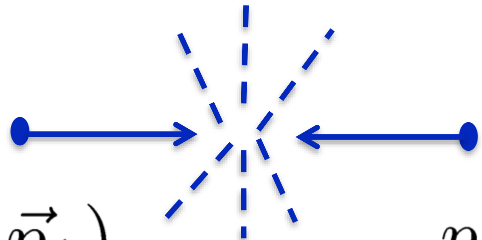
$$p_{tot} = (E_1/c + m_2 c, \vec{p}_1)$$

$$E_{CM}^2 = (m_1^2 + m_2^2)c^4 + 2E_1 m_2 c^2$$

$$E_{CM} \propto \sqrt{E_1}$$

Exercise 2: E_{CM} in Collider Experiment

Laboratory Frame = CM Frame



$$p_1 = (E_1/c, \vec{p}_1) \qquad p_2 = (E_2/c, -\vec{p}_1)$$

$$E_{CM} = E_1 + E_2$$

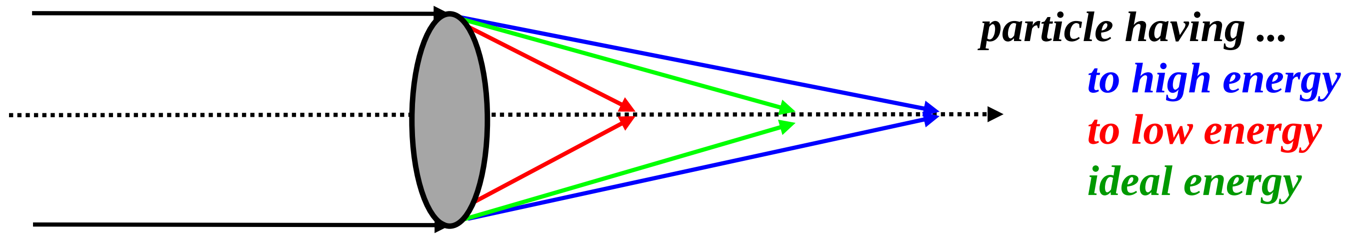
Back-up Slides

Chromaticity

Dipole magnets generate dispersion, which is then focused by quadrupoles.

focusing strength

$$k = \frac{g}{p/q} [m^{-2}]$$

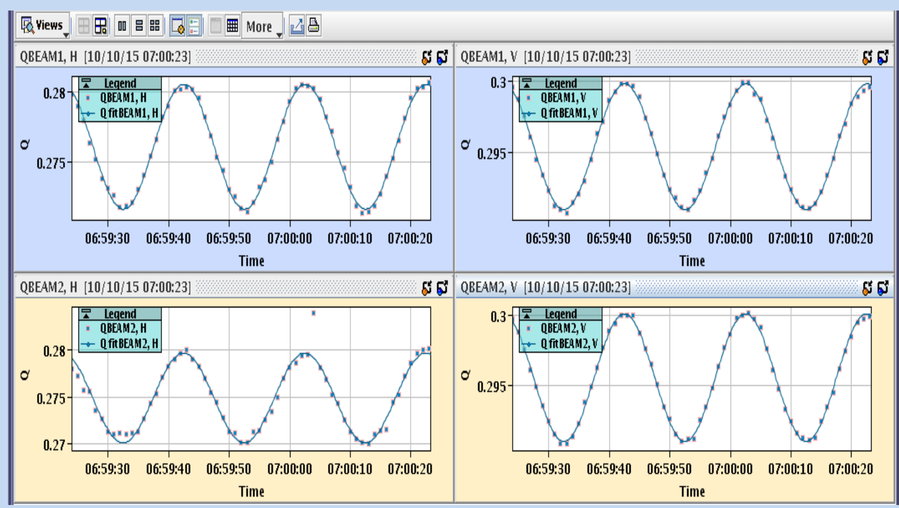


Chromaticity Q' acts like a quadrupole error and leads to a tune spread.
Definition of Chromaticity:

$$\Delta Q = Q' \frac{\Delta p}{p} \qquad Q' = -\frac{1}{4\pi} \oint k(s)\beta(s)ds$$

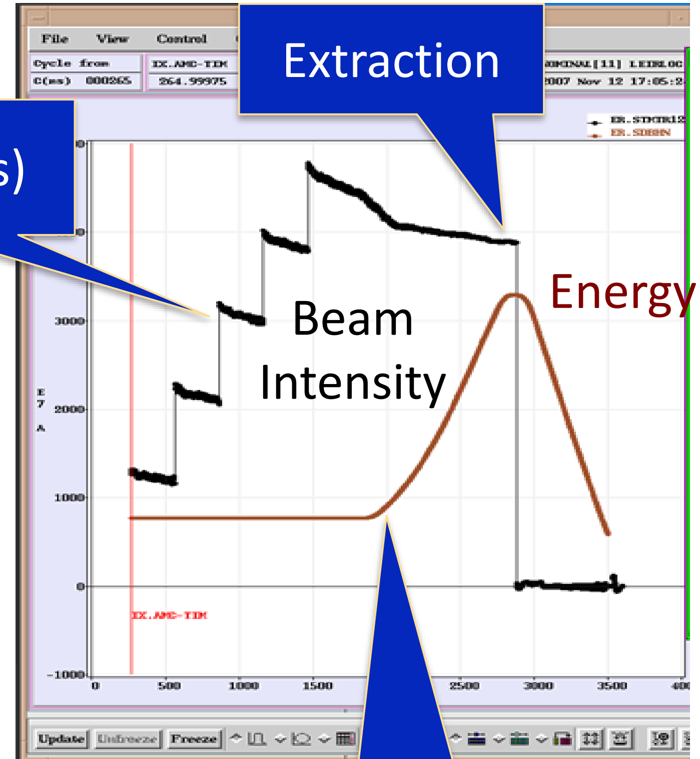
Q' measurement at LHC

The chromaticity is measured by changing / **modulating the energy** offset dp/p **through the RF frequency** while recording the tune change ΔQ.



Accelerator Cycle

LEIR Operational Cycle



Extraction

Injection(s)

Acceleration & increase of magnetic field

Storage/Collisions

LHC Operational Cycle