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Standard Model **Homework 1** **Date: 11.07.2022**

Feel free to send me (`christophe.grojean@desy.de`) your solutions and I'll give you feedback.

Exercice 1: Natural units

a) Show that $[\hbar] = M \cdot L^2 \cdot T^{-1}$ and $[c] = L \cdot T^{-1}$.
b) Check the consistency of the classical/quantum correspondence at the dimensional level:

$$E \to i\hbar\frac{\partial}{\partial t} \quad \& \quad p \to i\hbar\frac{\partial}{\partial x}$$

c) Show that

$$1\,\mathrm{s} = 1.52 \cdot 10^{27}\,\hbar/\mathrm{TeV}, \quad 1\,\mathrm{m} = 5.1 \cdot 10^{18}\,\hbar c/\mathrm{TeV}, \quad 1\,\mathrm{kg} = 5.61 \cdot 10^{23}\,\mathrm{TeV}/c^2$$

d) Using the Newton constant, $\hbar$ and c, construct a mass scale, a length scale and a time scale. They are defining the Planck scales. Compute the matter density of the universe today (10^{-29} g/cm^3) in Planck units.
e) The Schwarzschild radius of an object of mass m is the measure of its mass in Planck units. The Compton wavelength is defined as $\hbar/(mc)$. Compute the Schwarzschild radius of the Earth, the Sun, a neutron star, a stellar black-hole, a super-massive BH, a micro-BH (you'll check on Wikipedia the characteristic mass of these objects). What do you conclude? Compute the Schwarzschild radius of a micro-BH assuming that the Planck scale has been reduced to 1 TeV. What do you conclude?
f) Using e, m_e and c, construct a length scale. This is the classical radius of the electron.
Using e, m_e and $\hbar$, construct a length scale. This is the Bohr radius of the electron.
g)The pion Compton wavelength in natural units is $\lambda_\pi = \hbar/(M_\pi c) \to 1/M_\pi \simeq (140\ \mathrm{MeV})^{-1}$. Convert this to conventional units by multiplying with a combination of $\hbar$ and c to get a distance unit.
h) A typical hadronic cross section is of order $\sigma \simeq \lambda_\pi^2 \simeq 1/M_\pi^2 \simeq 1/(140)^2\mathrm{MeV}^{-2}$. Express this quantity in units of barns (1 barn = 10^{-28} m^2).

Exercice 2: Value of e in HEP units

The electromagnetic fine-structure constant was defined by A. Sommerfeld in 1916. It is given by

$$\alpha = \frac{e^2}{4\pi\epsilon_0\hbar c},$$

where $e = 1.6 \times 10^{-19}$ C is the unit electric charge, $\epsilon_0 = 8.8 \times 10^{-12}\,\mathrm{F}\cdot\mathrm{m}^{-1}$ is the vacuum permittivity.

a) Compute the value of α. Check that it is a dimensionless quantity (we remind that $1\,\mathrm{F} = 1\,\mathrm{C}^2.\mathrm{J}^{-1}$)
b) Deduce the value of the electric charge e in the HEP units ($\hbar = c = \epsilon_0 = 1$).

Exercice 3: Average temperatures on the planets of the Solar system

We'll assume that the sun and the solar system planets are perfect black-bodies, and we'll neglect any effects of the planet atmospheres.
a) Using the Stefan–Boltzmann law, compute the luminosity of the sun (we recall that the average surface temperature of the Sun in $\langle T_\odot^{\text{surface}} \rangle = 5778\,\mathrm{K}$).
b) Assuming that the planets radiate away all the energy received from the Sun, estimate the average temperature on the different planets. Compare with the data you can find on Wkipedia?

Exercice 4: Lorentz-invariance of space-time measure

In QFT, the action corresponds to integration of whole space-time of the Lagrangian

$$S = \int c\,dt\,d^3x\,\mathcal{L}$$

By construction, the Lagragian for scalars, fermions and gauge bosons are invariant under Lorentz transformations. Prove that the space-time integration measure, $c\,dt\,d^3x$, is also invariant under Lorentz transformations.