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Standard Model | **Homework 2** | **Date: 12.07.2022**

Exercice 1: Particle physics units

We recall that $[\hbar] = M \cdot L^2 \cdot T^{-1}$ and $[c] = L \cdot T^{-1}$.

a) Using the Newton constant, $\hbar$ and c, construct a mass scale, a length scale and a time scale. They are defining the Planck scales. Compute the matter density of the universe today (10^{-29} g/cm^3) in Planck units.

b) Using e, m_e and c, construct a length scale. This is the classical radius of the electron. Using e, m_e and $\hbar$, construct a length scale. This is the Bohr radius of the electron.

Exercice 2: Order of magnitude estimates

a) Estimate the energy of the cosmic rays given that the lifetime of a muon is about 1 μs.

b) A 1 cm^3 piece of ice melts in about 40 minutes under the sun. Compute the volume of oil to burn 1 cm-thick ice cap surrounding the sun at a distance of 150 million kilometres (the effects of the atmosphere will be neglected). Assuming that all the energy radiated by the Sun would be of chemical origin, what would be the maximal lifetime of the Sun? What do you conclude concerning the origin of the energy radiated by the Sun? We recall that burning 1 liter of oil produces about 30 MJ and that 333 kJ are needed to melt 1kg of ice.

Exercice 3: Hawking Black Hole radiation

a) S. Hawking understood that the laws of quantum mechanics imply that a BH is actually radiating particles, hence energy. Based on dimensional arguments, find the Hawking temperature of a BH of mass M. There is a priori a 1D infinite family of solution, you'll retain the solution that scales with a single power of $\hbar$. The exact formula derived by Hawking is smaller by a factor $1/(8\pi)$ compared to the naive estimate.

b) Assuming that a BH is a perfect black-body, use the Stefan–Boltzmann law ($P \propto T^4$) to derive the luminosity of a BH of mass M. Numerically, compute the power of a BH of solar mass (we recall that the Stefan–Boltzmann constant is equal to $\pi^2 k_B^4/(60\hbar^3 c^2)$).

c) Using the conservation of energy, derive the differential equation controlling the time evolution of the BH mass. Integrate this equation to obtain the BH life time.

d) What is the lower bound on the mass of a BH to be as old as the Universe?