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Exercice 1: Two-body decays

Using energy and momentum conservation, show that in a 2-body decay, $A \to B + C$, the energy and momentum of the daughter particles in the rest frame of the mother particle are given by

$$E_B = \frac{m_A^2 + m_B^2 - m_C^2}{2m_A} c^2, \quad E_C = \frac{m_A^2 + m_C^2 - m_B^2}{2m_A} c^2$$

$$p = \frac{\sqrt{\lambda(m_A, m_B, m_C)}}{2m_A} c$$

with

$$\lambda(m_A, m_B, m_C) = (m_A + m_B + m_C)(m_A + m_B - m_C)(m_A - m_B + m_C)(m_A - m_B - m_C).$$

Exercice 2: Colliders vs fixed-target experiments

Consider collisions between two particles A and B.

a) Calculate the center-of-mass energy in the rest frame of the B particle. What is the beam energy of the particles A needed to reach an energy of 2 TeV in the center-of-mass (we will assume that A and B are protons). This is a fixed-target experiment. For instance, CERN is thinking of building an experiment called SHiP that will use the SPS 400 GeV proton beam sent to a fixed target. What is the center-of-mass energy available?

b) CERN is also considering a Future Circular Collider with a 100 km tunnel accelerating protons at an energy of 50 TeV that would collide electrons accelerated to 60 GeV in a different tunnel. Compute the center-of-mass energy avaialble?

c) Calculate the center-of-mass energy in the frame where the two beams are colliding head-on like at the LHC.

Exercice 3: EM action for photons

The photon field strength is defined to be

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu\,,$$

where the 4-vector A^μ is defined from the scalar and vector potential $(\phi, \vec{A})$.

a) From the classical EM definition of the electric and magnetic fields from the scalar and vector potential

$$\vec{E} = -\vec{\nabla}\phi - \frac{1}{c}\frac{\partial \vec{A}}{\partial t}, \quad \vec{B} = \vec{\nabla} \wedge \vec{A}$$

show that

$$\vec{E}_i = -F_{i0} \quad \vec{B}_i = -\frac{1}{2}\epsilon_{ijk} F_{jk}$$

b) Derive the expression of the gauge field Lagrangian density, $-\frac{1}{4\pi}F_{\mu\nu}F^{\mu\nu}$, in terms of the electric and magnetic fields and recognise the usual expression of the energy density stored in the electromagnetic fields.
c) There is another Lorentz-invariant Lagrangian density that one can construct from the electromagnetic field:

$$\epsilon_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma}.$$

Compute this Lagrangian density in terms of $\vec{E}$ and $\vec{B}$.