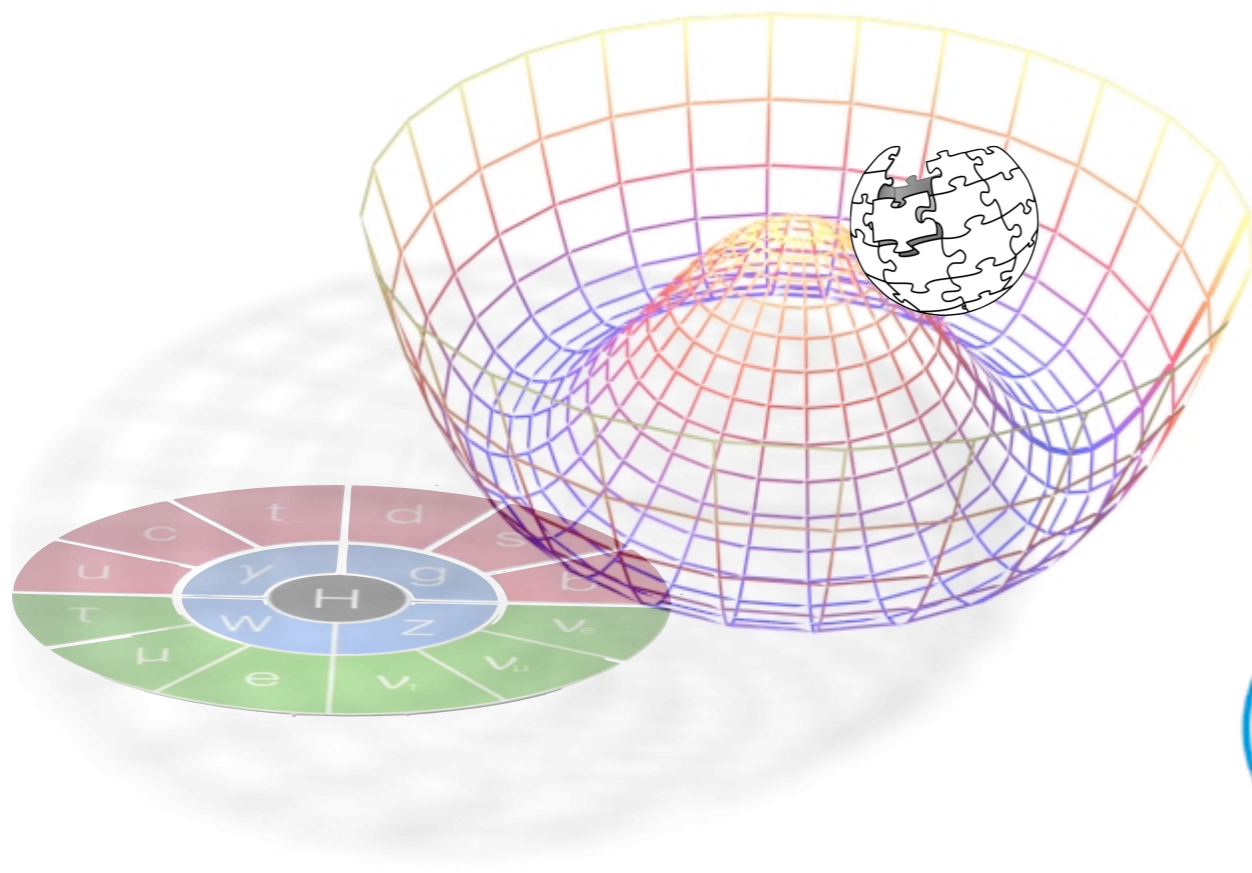


# The Standard Model of particle physics

*CERN summer student lectures 2022*

*Lecture 3/5*



*Christophe Grojean*

DESY (Hamburg)

Humboldt University (Berlin)

( [christophe.grojean@desy.de](mailto:christophe.grojean@desy.de) )



# Outline

## □ Monday

- Lagrangians
- Lorentz symmetry - scalars, fermions, gauge bosons
- Dimensional analysis: cross-sections and life-time.

## □ Tuesday

- Dimensional analysis: cross-sections and life-time
- Nuclear decay, Fermi theory

## □ Wednesday

- Gauge interactions: U(1) electromagnetism, SU(2) weak interactions, SU(3) QCD
- Chirality of weak interactions
- Pion decay

## □ Thursday

- Spontaneous symmetry breaking and Higgs mechanism
- Quark and lepton masses
- Neutrino masses

## □ Friday

- Running couplings
- Asymptotic freedom of QCD
- Anomalies cancelation

# Higgs Lifetime “Computation”

exercice:


Using similar dimensional analysis arguments, compute the Higgs boson lifetime (or its inverse aka as the Higgs decay width)

— Hints —

Higgs couplings proportional are proportional to the mass of the particles it couples to. It will therefore decay predominantly decay into the heaviest particle that is lighter than  $m_H/2$

$$\Gamma \sim \frac{1}{8\pi} \left(\frac{m_b}{v}\right)^2 m_h \sim \frac{1}{10} \left(\frac{4 \text{ GeV}}{246 \text{ GeV}}\right)^2 125 \text{ GeV} \sim 1 \text{ MeV}$$

phase space  $\nearrow$   $\left(\frac{m_b}{v}\right)^2$   $\nwarrow$  dimensionally  $[\Gamma]_{m=1}$   
 couplings to b-quark



<b>E</b>	<b>T</b>	<b>L</b>
1eV	$10^{-16}\text{s}$	$10^{-7}\text{m}$

Putting all factors and considering the other decay modes, Higgs width = 4MeV in the SM

(for Z gauge boson:  $\Gamma_Z = \frac{7}{48\pi} g^2 m_Z \sim 2 \text{ GeV}$  i.e.  $\tau_Z \sim 10^{-25} \text{ s}$ )

# Fermi Theory

Recap  
from yesterday

By analogy with electromagnetism, one can see the Fermi force as a current-current interaction

$$\mathcal{L} = G_F J_\mu^* J^\mu \quad \text{with} \quad J^\mu = (\bar{n}\gamma^\mu p) + (\bar{e}\gamma^\mu \nu_e) + (\bar{\mu}\gamma^\mu \nu_\mu) + \dots$$

The cross-terms generate both neutron decay and muon decay.

The life-times of the neutron and muon tell us that the relative factor between the electron and the muon is the current is of order one, i.e., the weak force has the same strength for electron and muon.

# Pathology at High Energy

What about weak scattering process, e.g.  $e\nu_e \rightarrow e\nu_e$ ?

$$\mathcal{L} = G_F J_\mu^* J^\mu \quad \text{with} \quad J^\mu = (\bar{n}\gamma^\mu p) + (\bar{e}\gamma^\mu \nu_e) + (\bar{\mu}\gamma^\mu \nu_\mu) + \dots$$

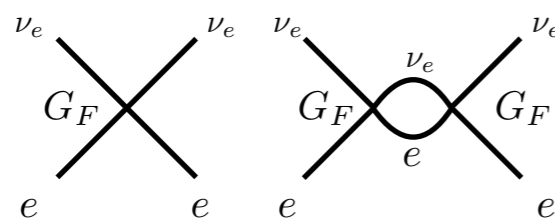
The same Fermi Lagrangian will thus also contain a term

$$G_F (\bar{e}\gamma^\mu \nu_e)(\bar{\nu}_e\gamma^\mu e)$$

that will generate e- $\nu_e$  scattering whose cross-section can be guessed by dimensional arguments

$$\begin{array}{ccc} \nearrow & \sigma \propto G_F^2 E^2 & \nwarrow \\ [\text{mass}]^{-2} & [\text{mass}]^{-2 \times 2} & [\text{mass}]^2 \end{array} \quad \rightarrow \quad \begin{array}{l} \text{non conservation of probability} \\ \text{(non-unitary theory)} \\ \text{inconsistent at high energy} \end{array}$$

It means that at high-energy the quantum corrections to the classical contribution can be sizeable:



$$\sigma \propto G_F^2 E^2 + \frac{1}{16\pi^2} G_F^4 E^6 + \dots$$

The theory becomes non-perturbative at an energy  $E_{\text{max}} = \frac{2\sqrt{\pi}}{\sqrt{G_F}} \sim 100 \text{ GeV} - 1 \text{ TeV}$

unless new degrees of freedom appear before to change the behaviour of the scattering

# U(1) Gauge Symmetry

**QED:** the phase of an electron is not physical and can be rotated away

$$\boxed{\phi \rightarrow e^{i\theta} \phi}$$

The phase transformation is **local**, i.e., depends on space-time coordinate, then

$$\partial_\mu \phi \rightarrow e^{i\theta} (\partial_\mu \phi + i(\partial_\mu \theta)\phi)$$

and the kinetic term is no-longer invariant due to the presence of the non-homogenous piece

To make the theory invariant under local transformation, one needs to introduce a **gauge field** that keeps track/memory of how the phase of the electron changes from one point to another.

For that, we build a **covariant derivative** that has nice homogeneous transformations

$$D_\mu \phi = \partial_\mu \phi + ieA_\mu \phi \rightarrow e^{i\theta} D_\mu \phi \quad \text{iff} \quad A_\mu \rightarrow A_\mu - \frac{1}{e} \partial_\mu \theta$$

Note that  $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \rightarrow F_{\mu\nu}$  and the full Lagrangian is invariant under local transformation

**Gauge invariance** is a dynamical principle: it predicts some interactions among particles

It also explains why the QED interactions are universal (an electron interacts with a photon in the same way on Earth, on the Moon and at the outskirts of the Universe)

# SU(N) non-Abelian Gauge Symmetry

We generalise the QED construction by considering general transformation of a N-vector

$$\phi \rightarrow U\phi$$

We build a **covariant derivative** that again has nice homogeneous transformations

$$D_\mu\phi = \partial_\mu\phi + igA_\mu\phi \rightarrow UD_\mu\phi \quad \text{iff} \quad A_\mu \rightarrow UA_\mu U^{-1} + \frac{i}{g}(\partial_\mu U)U^{-1}$$

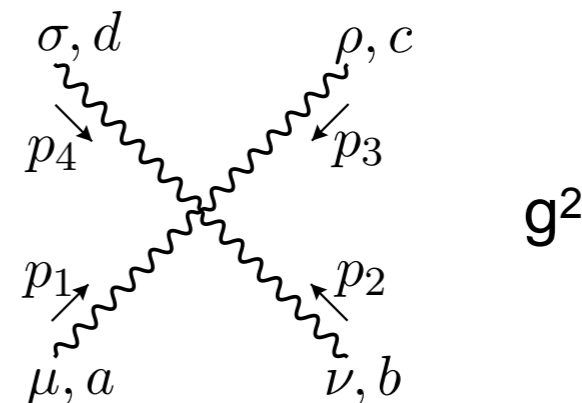
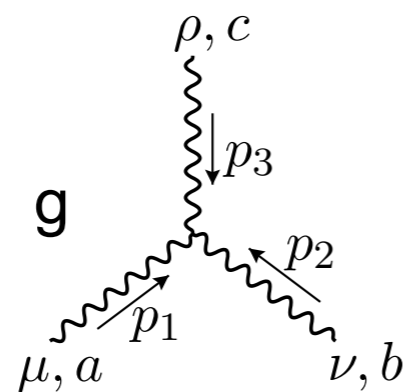
$g$  is the gauge coupling and defines the strength of the interactions

For the field strength to transform homogeneously, one needs to add a non-Abelian piece

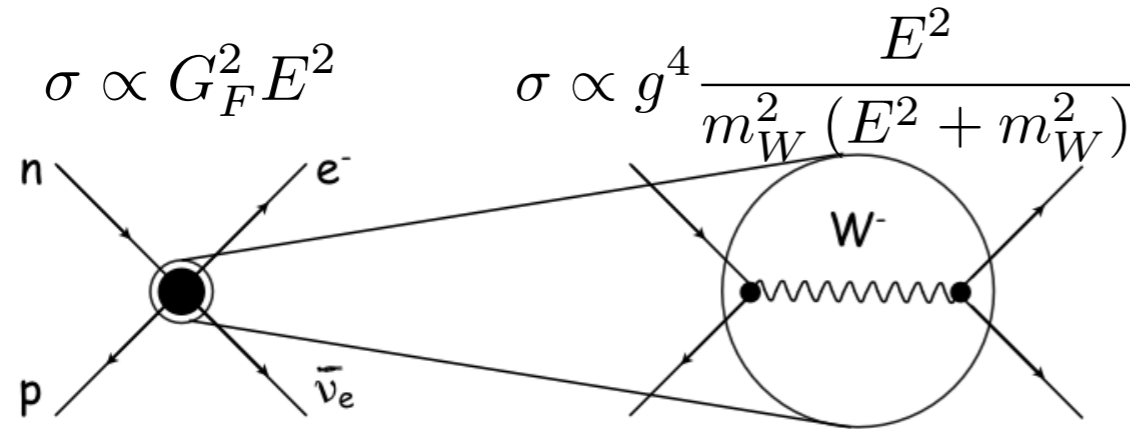
$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + ig[A_\mu, A_\nu] \rightarrow UF_{\mu\nu}U^{-1}$$

Contrary to the Abelian case, the gauge fields are now charged and interact with themselves

$$\mathcal{L} \propto F_{\mu\nu}F^{\mu\nu} \supset g\partial AAA + g^2 A AAA$$

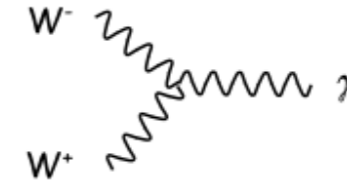


# Electroweak Interactions



$$G_F \propto \frac{g^2}{m_W^2}$$

charged W  $\Rightarrow$  must couple to photon:



$\Rightarrow$  non-abelian gauge symmetry  $[Q, T^\pm] = \pm T^\pm$

## 1. No additional “force” (Georgi, Glashow '72) $\Rightarrow$ extra matter

**SU(2)**

$$[T^a, T^b] = i\epsilon^{abc} T^c$$

$$[T^+, T^-] = Q \quad [Q, T^\pm] = +\pm T^\pm$$

$$T^\pm = \frac{1}{\sqrt{2}}(T^1 \pm iT^2)$$

$$\text{Tr}_{\text{irrep}} T^3 = 0 \Rightarrow \text{extra matter} \begin{pmatrix} X_L \\ \nu_L \\ e_L \end{pmatrix} \begin{pmatrix} X_R \\ \nu_R \\ e_R \end{pmatrix}$$

**SU(1, 1)**

$$[T^+, T^-] = -Q$$

$$[Q, T^\pm] = +\pm T^\pm$$

non-compact  
unitary rep. has dim  $\infty$

**E<sub>2</sub>**

2D Euclidean group

$$[T^+, T^-] = 0$$

$$[Q, T^\pm] = +\pm T^\pm$$

only one unitary rep.  
of finite dim. = trivial rep.

## 2. No additional “matter” (Glashow '61, Weinberg '67, Salam '68): **SU(2)xU(1)**

$\Rightarrow$  extra force

$$Q = T^3?$$

as Georgi-Glashow

$\Rightarrow$  extra matter

$$Q = Y?$$

$$Q(e_L) = Q(\nu_L)$$

$$Q = T^3 + Y!$$

Gell-Mann '56, Nishijima-Nakano '53



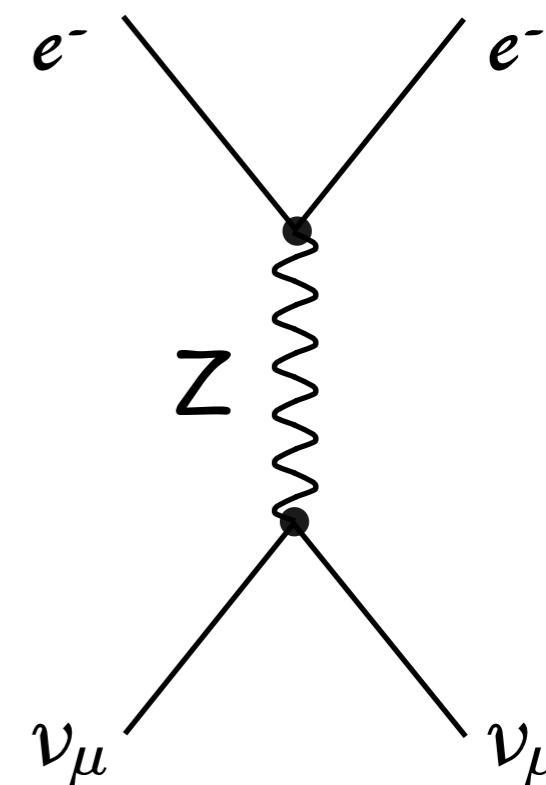
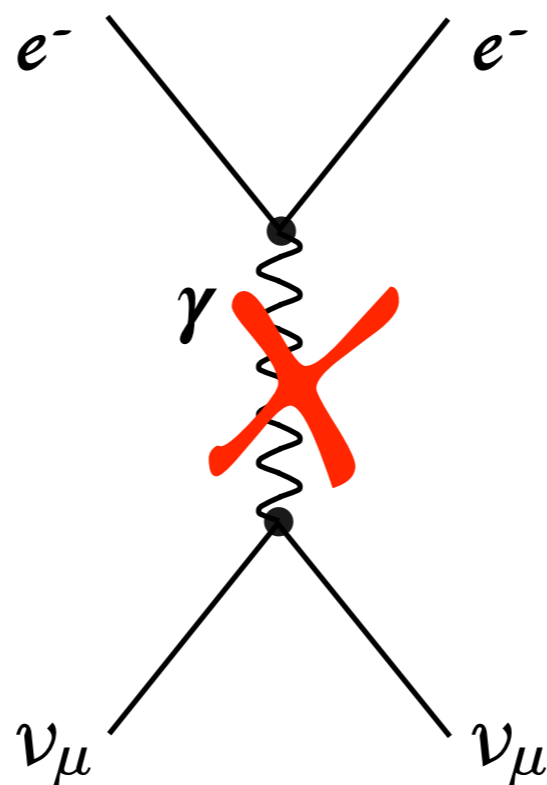
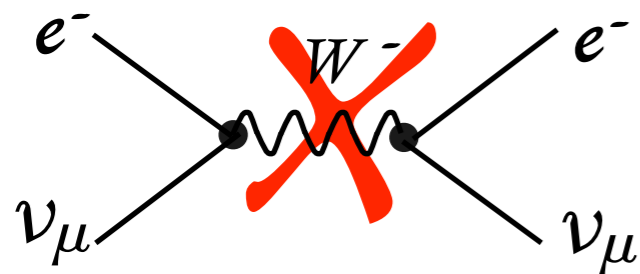
# Electroweak Interactions

**Gargamelle** experiment '73 first established the  $SU(2) \times U(1)$  structure

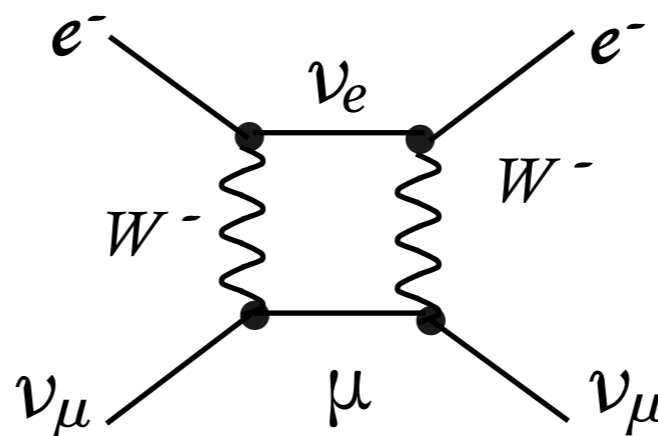
Idea:

rely on a particle that doesn't interact with photon to prove the existence a new neutral current process!

$$\nu_\mu e^- \rightarrow \nu_\mu e^-$$



loop-suppressed contribution from W:



# From Gauge Theory to Fermi Theory

We can derive the Fermi current-current contact interactions by “integrating out” the gauge bosons, i.e., by replacing in the Lagrangian the W’s by their equation of motion. Here is a simple derivation: (a better one should take into account the gauge kinetic term and the proper form of the fermionic current that we’ll figure out tomorrow, for the moment, take it as a heuristic derivation)

$$\mathcal{L} = -m_W^2 W_\mu^+ W_\nu^- \eta^{\mu\nu} + g W_\mu^+ J_\nu^- \eta^{\mu\nu} + g W_\nu^- J_\mu^+ \eta^{\mu\nu}$$

$$J^{+\mu} = \bar{n}\gamma^\mu p + \bar{e}\gamma^\mu \nu_e + \bar{\mu}\gamma^\mu \nu_\mu + \dots \quad \text{and} \quad J^{-\mu} = (J^{+\mu})^*$$

The equation of motion for the gauge fields:  $\frac{\partial \mathcal{L}}{\partial W_\mu^+} = 0 \Rightarrow W_\mu^- = \frac{g}{m_W^2} J_\mu^-$

Plugging back in the original Lagrangian, we obtain an *effective Lagrangian* (valid below the mass of the gauge bosons):

$$\mathcal{L} = \frac{g^2}{m_W^2} J_\mu^+ J_\nu^- \eta^{\mu\nu}$$

which is the Fermi current-current interaction. The Fermi constant is given by (the correct expression involves a different normalisation factor)

$$G_F = \frac{g^2}{m_W^2}$$

The next step is to relate  $m_W$  to  $v$ ... that’s the Higgs mechanism

$$G_F = \frac{1}{\sqrt{2}v^2}$$

# SU(3) QCD

Deep inelastic experiments in the 60's revealed the internal structure of the neutrons and protons  
Gell-Mann and others proposed that they are made of “**quarks**”

Up quark: spin-1/2, Q=2/3  
Down quark: spin-1/2, Q=-1/3

**SU(2)** weak symmetry that changes neutrino into electron also changes up-quark into down-quark

But **quarks** carry yet another quantum number: “**colour**”

There 3 possible colours and Nature is colour-blind, i.e, Lagrangian should remain the same when the colours of the quarks are changed, i.e., when we perform a rotation in the colour-space of quarks

$$Q^a \rightarrow U^a_b Q^b \quad \text{U: 3x3 matrix satisfying } U^\dagger U = 1_3 \quad \text{SU(3)}$$

such that the quark kinetic term is invariant

**hadrons** (spin-1/2, #hadronic=1):  $p = uud$      $n = udd$

**mesons** (spin-0, #hadronic=0):  $\pi^0 = \frac{u\bar{u} + d\bar{d}}{\sqrt{2}}$      $\pi^+ = u\bar{d}$      $\pi^- = d\bar{u}$

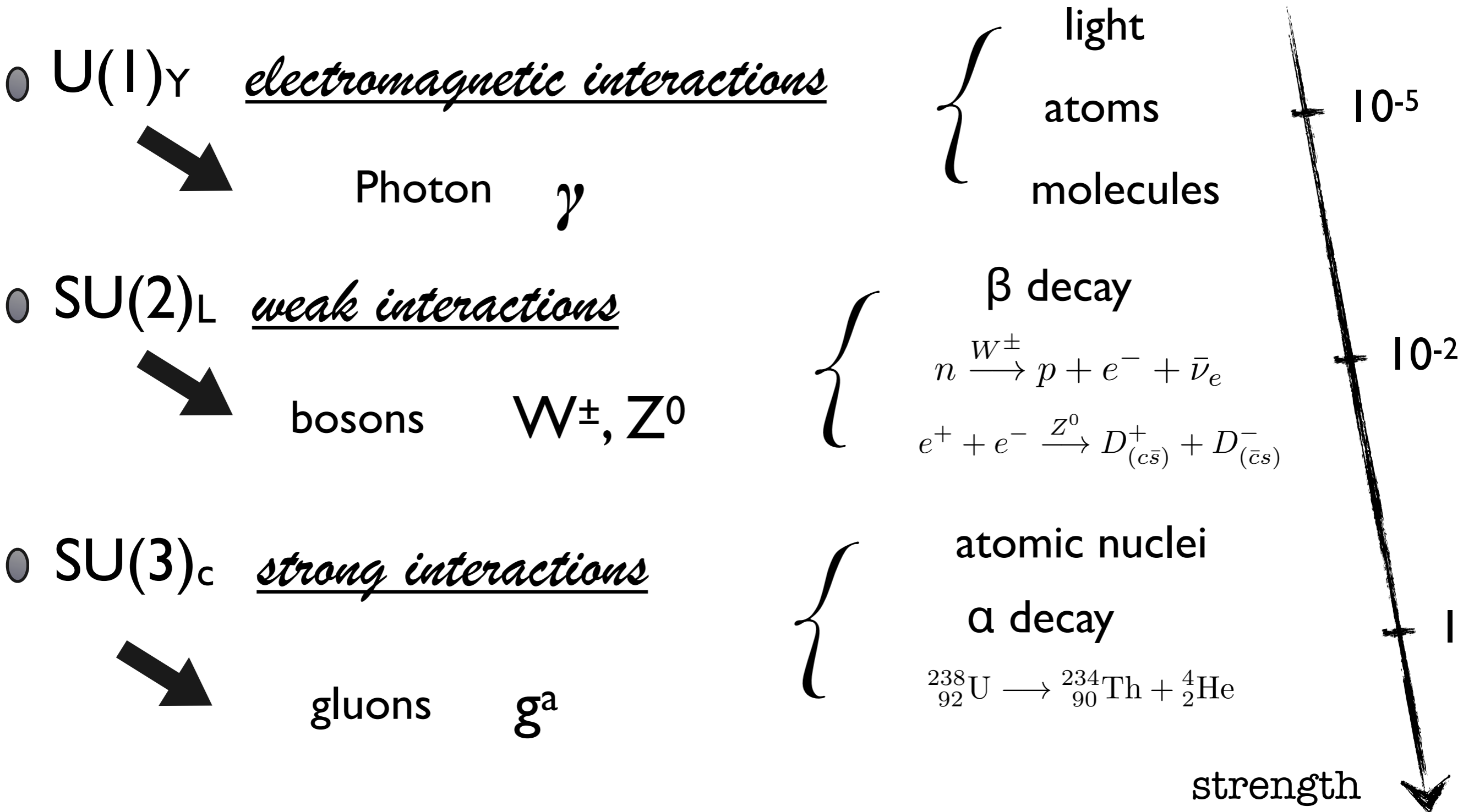
(Each quark carries a baryon number =1/3)

There are other (heavier) quarks and hence other baryons and mesons

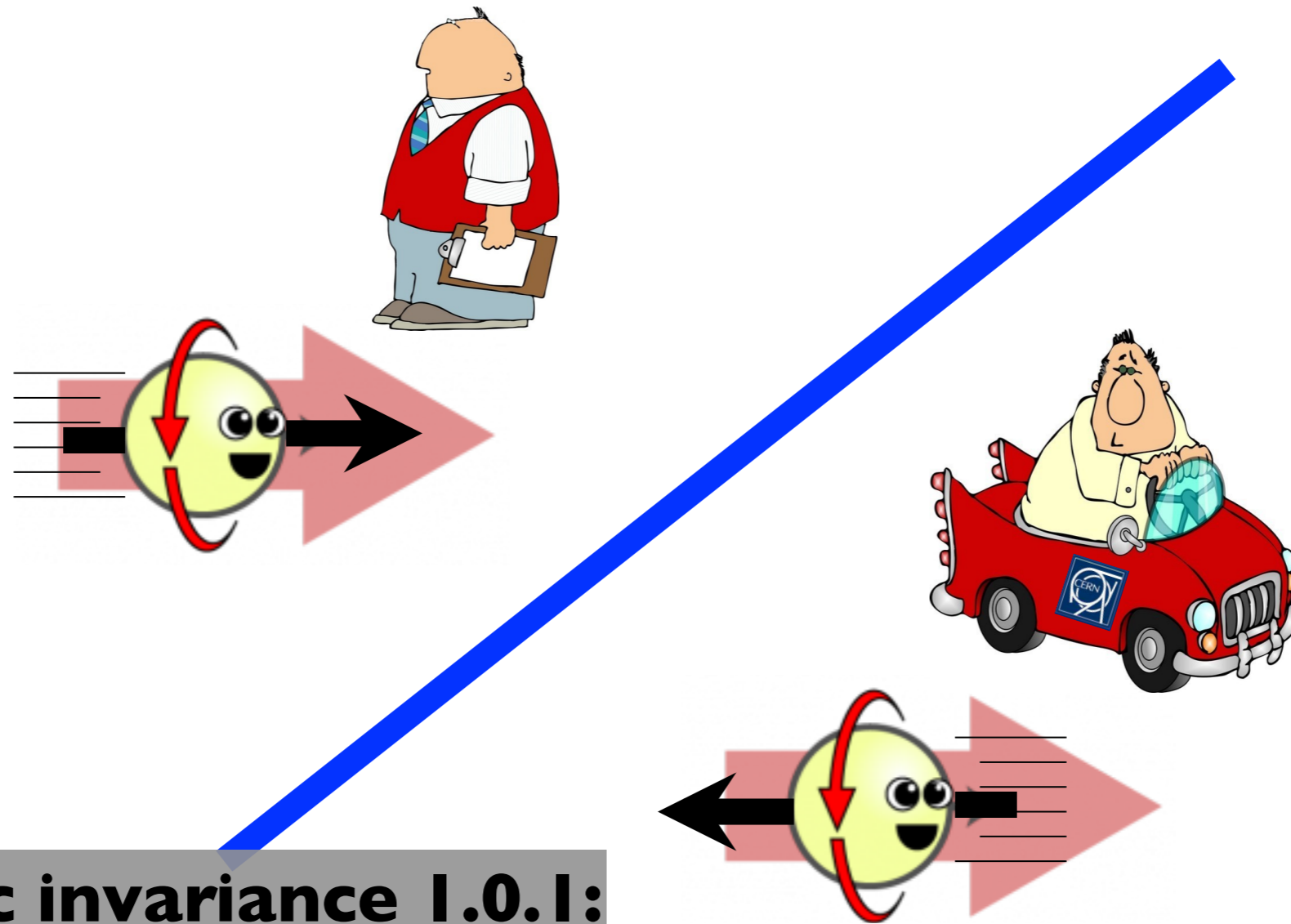
**All the interactions of the SM preserve baryon and lepton numbers**

$$\mu \rightarrow e \nu_\mu \bar{\nu}_e \quad n \rightarrow p e \bar{\nu}_e \quad \pi^- \rightarrow \mu^- \bar{\nu}_\mu \quad \pi^0 \rightarrow \gamma\gamma \quad p \not\rightarrow \pi^0 \bar{e}$$

# The Standard Model: Interactions



# Chirality & Masslessness



## Relativistic invariance 1.0.1:

there must be no distinction for massive particles between particles spinning clockwise or anti-clockwise

[chirality operator doesn't commute with the Hamiltonian]

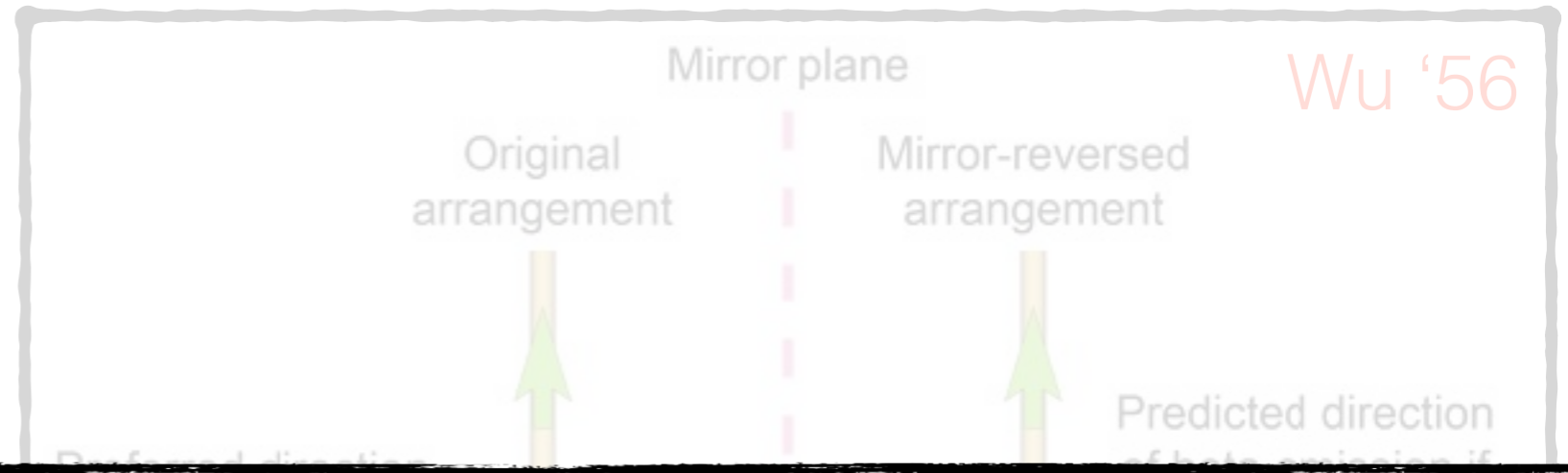
**If your theory sees a difference between  $e_L$  and  $e_R$ , either your theory is wrong or  $m_e=0$**

Theorem

# Chirality of SM & Mass problem

**Weak** interaction  
(force responsible for  
neutron decay)  
is chiral!

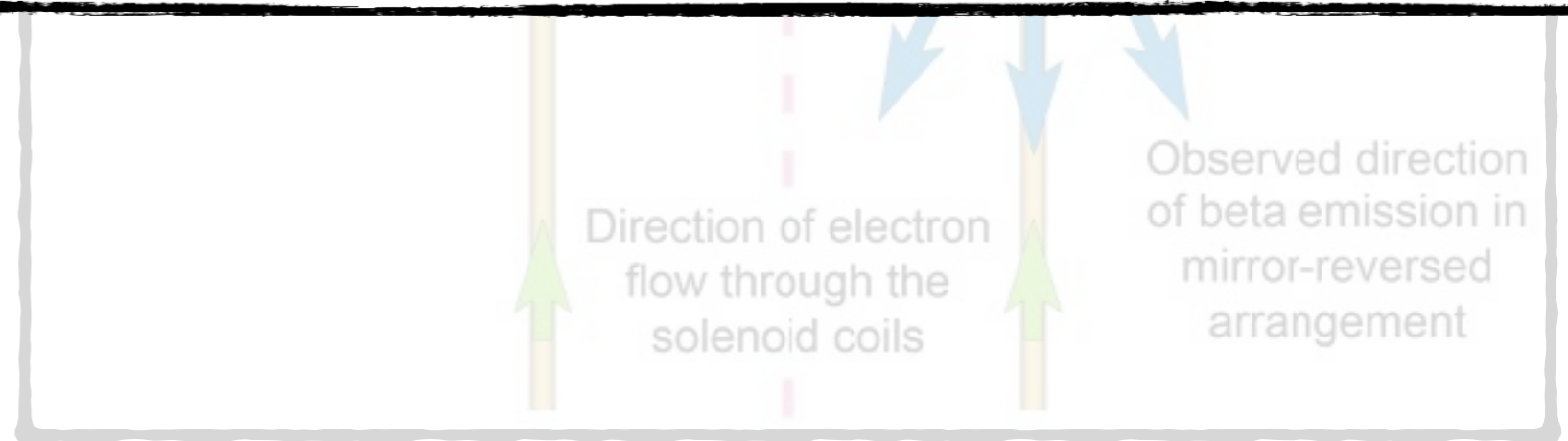
[ $e_L$  and  $e_R$  are fundamentally



Dextrorotation and Levorotation are essential for life to develop.  
To the best of our knowledge,  
in **molecular biology**, chirality seems an **emergent** property.  
At least, there is no clear evidence that it follows from chirality of the weak interactions.  
Are the chiral nature of the **weak** interactions **emergent** too?  
Some models of grand unification predict it. But we still don't know for sure.

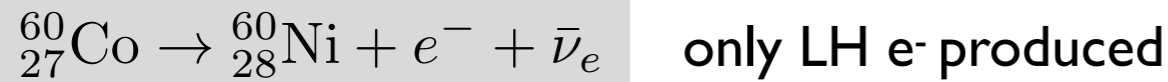
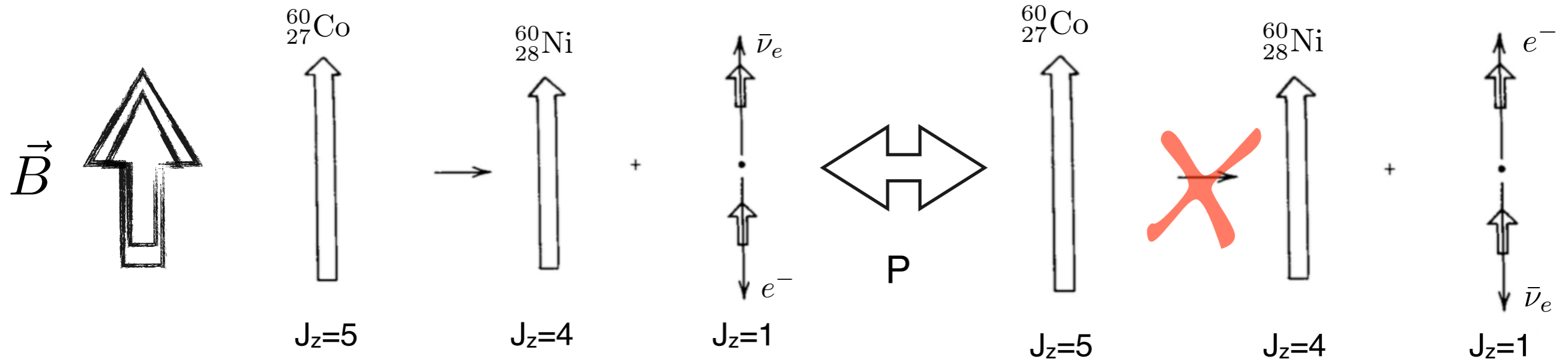
but since we know it is not true, we

**need a new  
phenomena to  
generate mass:  
Higgs mechanism**

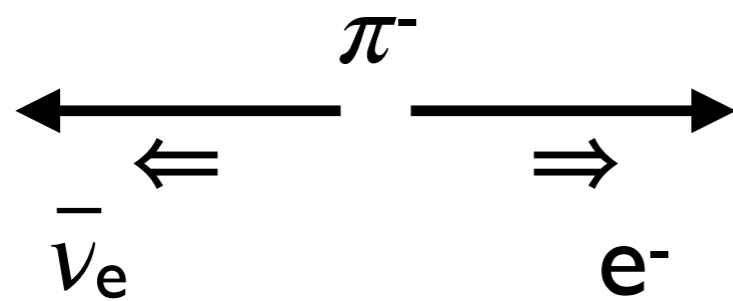


# SM is a Chiral Theory

Weak interactions maximally violates P



TH: Yang&Lee '56. EXP: Wu '57



Conservation of momentum and spin imposes to have a RH  $e^-$

Weak decays proceed only w/ LH  $e^-$   
So the amplitude is prop. to  $m_e$

$$\mathcal{L}_{\text{Dirac}} = \bar{\psi}_L \gamma^\mu \partial_\mu \psi_L + \bar{\psi}_R \gamma^\mu \partial_\mu \psi_R + m (\bar{\psi}_L \psi_R + \bar{\psi}_R \psi_L)$$

$$\frac{\Gamma(\pi^- \rightarrow e^- \bar{\nu}_e)}{\Gamma(\pi^- \rightarrow \mu^- \bar{\nu}_\mu)} \propto \frac{m_e^2}{m_\mu^2} \sim 2 \times 10^{-5} \sim 10_{\text{obs}}^{-4}$$

↑  
Extra phase-space factor

# Technical Details for Advanced Students



# Chirality

## Chirality matrix

$$\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$$

A few remarkable properties

$$(\gamma^5)^2 = 1_4$$

$$\{\gamma^5, \gamma^\mu\} = 0$$

$$\gamma^{5\dagger} = \gamma^5 = -\gamma^0\gamma^5\gamma^0$$

## Chiral/Weyl spinor

A **chiral/Weyl** spinor is an eigenvector of the chirality matrix  $\psi_{L,R} = \pm\gamma^5\psi_{L,R}$

From the Lorentz-transformation law of a spinor, it is obvious that the chirality condition is frame-independent

A Dirac spinor can also be written as a sum of two chiral spinors

$$\psi = \frac{1}{2} (1_4 + \gamma^5) \psi + \frac{1}{2} (1_4 - \gamma^5) \psi \equiv \psi_L + \psi_R$$

# Charge conjugation

In general,  $\psi$  and  $\psi^*$  do not transform in the same way under Lorentz transformations and the naive reality condition  $\psi = \psi^*$  is frame dependent

But it is possible to find a matrix  $C$ , called charge conjugation matrix, such that

$$\psi \quad \text{and} \quad \psi_C = C \psi^*$$

transform in the same way under Lorentz transformations

$$\text{The matrix } C \text{ needs to satisfy } C\gamma^\mu = -\gamma^\mu C$$

In the Dirac and Weyl representations,  $C = i\gamma^2$

In the Majorana representation,  $C = 1_4$

Basic properties of the charge conjugation matrix:  $C^2 = 1_4$ ,  $C^\dagger = C$ ,  $C^* = C$

The charge conjugated spinor,  $\psi_C$ , satisfies the same Dirac equation as  $\psi$ , with the same mass but opposite electric charge (when the spinor is minimally coupled to a U(1) gauge field)

A **Majorana** spinor satisfies the (Lorentz invariant!) condition  $\psi = \psi_C$

Note that in 4D, a spinor cannot be simultaneously chiral and Majorana

# Dirac and Majorana Masses

By construction, the following two mass terms in the Lagrangian are Lorentz-invariant

**Dirac mass:**  $\mathcal{L}_{\text{Dirac}} = m\bar{\psi}\psi$  (conserves fermion number)

**Majorana mass:**  $\mathcal{L}_{\text{Majorana}} = m\bar{\psi}_C\psi$  (changes fermion number by 2)

These two mass terms have different a chirality structure

$$\mathcal{L}_{\text{Dirac}} = m (\bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L)$$

$$\mathcal{L}_{\text{Majorana}} = m (\bar{\psi}_{LC}\psi_L + \bar{\psi}_{RC}\psi_R)$$

A chiral fermion can have a Majorana mass

A Dirac mass requires spinors of opposite chirality

Whether or not a Dirac or a Majorana mass can be included in the Lagrangian depends on transformation laws of the spinors under the gauge transformations

Within the SM (with the Higgs field), a Dirac mass can be written for the charged leptons and the quarks while a Majorana mass can be written for the neutrinos.