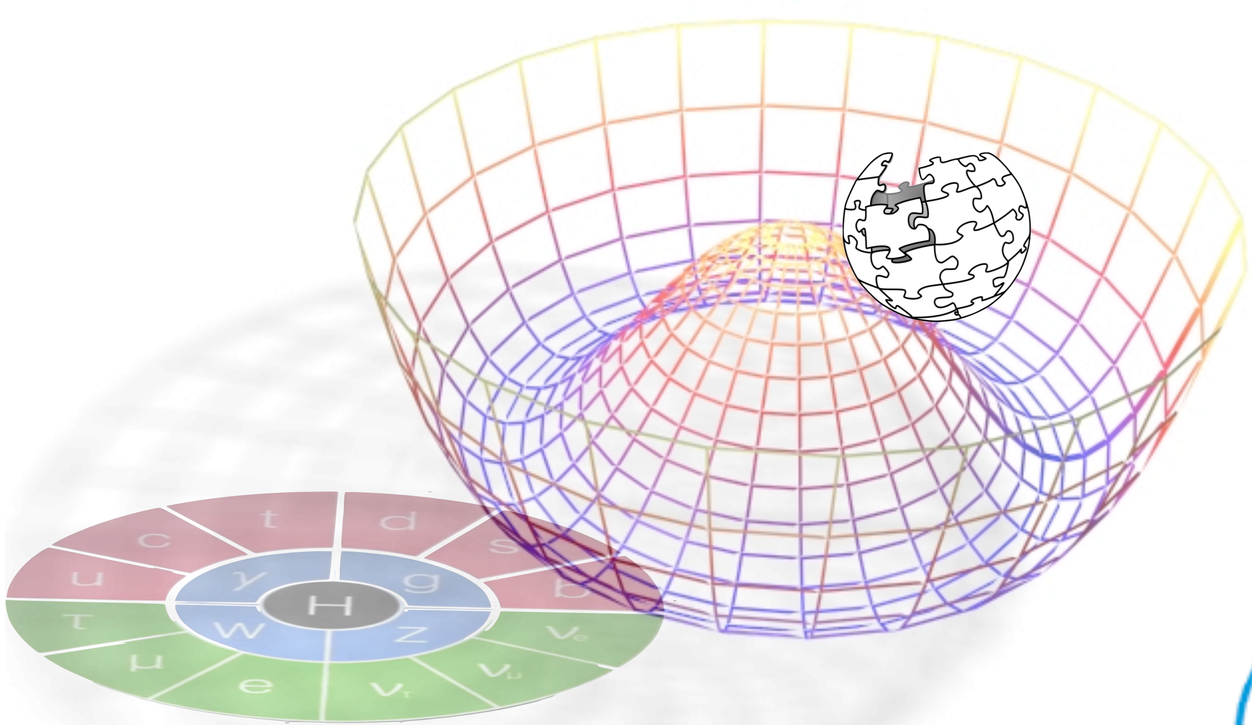
The Standard Model
of particle physics

CERN summer student lectures 2022

Lecture 4/5



Christophe Grojean
DESY (Hamburg)
Humboldt University (Berlin)
(christophe.grojean@desy.de)

Outline

Monday
- Lagrangians
- Lorentz symmetry - scalars, fermions, gauge bosons
- Dimensional analysis: cross-sections and life-time.

Tuesday
- Dimensional analysis: cross-sections and life-time
- Nuclear decay, Fermi theory

Wednesday
- Breakdown of the Fermi theory
- Gauge interactions: U(1) electromagnetism, SU(2) weak interactions

Thursday
- From SU(2) to the Fermi theory, SU(3) QCD
- Chirality of weak interactions, Pion decay
- Spontaneous symmetry breaking and Higgs mechanism
- Quark and lepton masses, Neutrino masses

Friday
- Running couplings
- Asymptotic freedom of QCD
- Anomalies cancelation

From Gauge Theory to Fermi Theory

We can derive the Fermi current-current contact interactions by "integrating out" the gauge bosons, i.e., by replacing in the Lagrangian the W's by their equation of motion. Here is a simple derivation:
(a better one should take taking into account the gauge kinetic term and the proper form of the fermionic current that we'll figure out tomorrow, for the moment, take it as a heuristic derivation)

$$\mathcal{L} = -m_W^2 W_\mu^+ W_\nu^- \eta^{\mu\nu} + g W_\mu^+ J_\nu^- \eta^{\mu\nu} + g W_\nu^- J_\nu^+ \eta^{\mu\nu}$$

$$J^{+\mu} = \bar{n}\gamma^\mu p + \bar{e}\gamma^\mu \nu_e + \bar{\mu}\gamma^\mu \nu_\mu + \dots \qquad \text{and} \qquad J^{-\mu} = \left(J^{+\mu}\right)^*$$

The equation of motion for the gauge fields: $\dfrac{\partial \mathcal{L}}{\partial W_\mu^+} = 0 \qquad \Rightarrow \qquad W_\mu^- = \dfrac{g}{m_W^2} J_\mu^-$

Plugging back in the original Lagrangian, we obtain an *effective Lagrangian* (valid below the mass of the gauge bosons):

$$\boxed{\mathcal{L} = \frac{g^2}{m_W^2} J_\mu^+ J_\nu^- \eta^{\mu\nu}}$$

which is the Fermi current-current interaction. The Fermi constant is given by
(the correct expression involves a different normalisation factor)

$$\boxed{G_F = \frac{g^2}{m_W^2}}$$

The next step is to relate m_W to v… that's the Higgs mechanism

$$\boxed{G_F = \frac{1}{\sqrt{2}v^2}}$$

SU(3) QCD

Deep inelastic experiments in the 60's revealed the internal structure of the neutrons and protons
Gell-Mann and others proposed that they are made of "**quarks**"

Up quark: spin-1/2, Q=2/3
Down quark: spin-1/2, Q=-1/3

SU(2) weak symmetry that changes neutrino into electron also changes up-quark into down-quark

But **quarks** carry yet another quantum number: "**colour**"
There 3 possible colours and Nature is colour-blind, i.e, Lagrangian should remain the same when the colours of the quarks are changed, i.e., when we perform a rotation in the colour-space of quarks

$$Q^a \to U^a{}_b Q^b$$

U: 3x3 matrix satisfying $U^\dagger U = 1_3$ such that the quark kinetic term is invariant

SU(3)

hadrons (spin-1/2, #hadronic=1):	$p = uud$	$n = udd$
mesons (spin-0, #hadronic=0):	$\pi^0 = \frac{u\bar{u} + d\bar{d}}{\sqrt{2}}$	$\pi^+ = u\bar{d}$ $\quad \pi^- = d\bar{u}$

(Each quark carries a baryon number =1/3)

There are other (heavier) quarks and hence other baryons and mesons

All the interactions of the SM preserve baryon and lepton numbers

$\mu \to e\nu_\mu\bar{\nu}_e \qquad n \to p\, e\, \bar{\nu}_e \qquad \pi^- \to \mu^- \bar{\nu}_\mu \qquad \pi^0 \to \gamma\gamma \qquad p \not\to \pi^0 \bar{e}$

The Standard Model: Interactions

- $U(1)_Y$ ***electromagnetic interactions***

  Photon γ

  light, atoms, molecules — 10^{-5}

- $SU(2)_L$ ***weak interactions***

  bosons $W^{\pm}, Z^0$

  β decay

  $$n \xrightarrow{W^{\pm}} p + e^- + \bar{\nu}_e$$

  $$e^+ + e^- \xrightarrow{Z^0} D^+_{(c\bar{s})} + D^-_{(\bar{c}s)}$$

  10^{-2}

- $SU(3)_c$ ***strong interactions***

  gluons g^a

  atomic nuclei

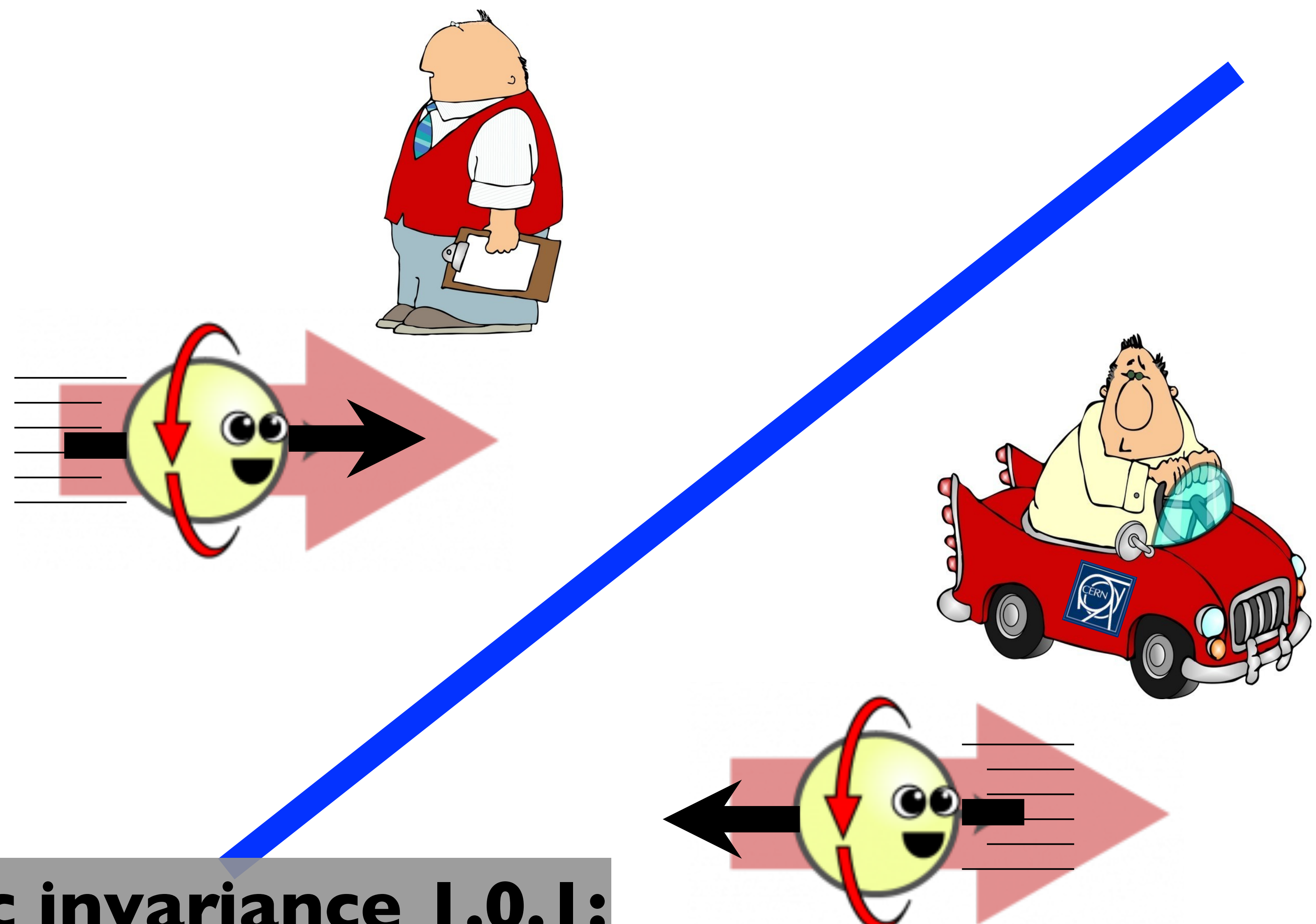
  α decay

  $$^{238}_{92}\mathrm{U} \longrightarrow {}^{234}_{90}\mathrm{Th} + {}^{4}_{2}\mathrm{He}$$

  1

strength

Chirality & Masslessness



Picture courtesy to G. Giudice

Relativistic invariance 1.0.1:

there must be no distinction for massive particles between particles spinning clockwise or anti-clockwise

[chirality operator doesn't commute with the Hamiltonian]

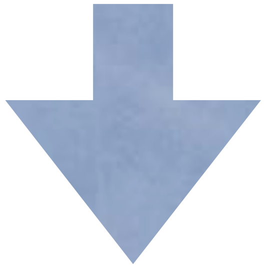
Theorem

If your theory sees a difference between e_L and e_R, either your theory is wrong or $m_e=0$

Chirality of SM & Mass problem

Weak interaction
(force responsible for
neutron decay)
is chiral!

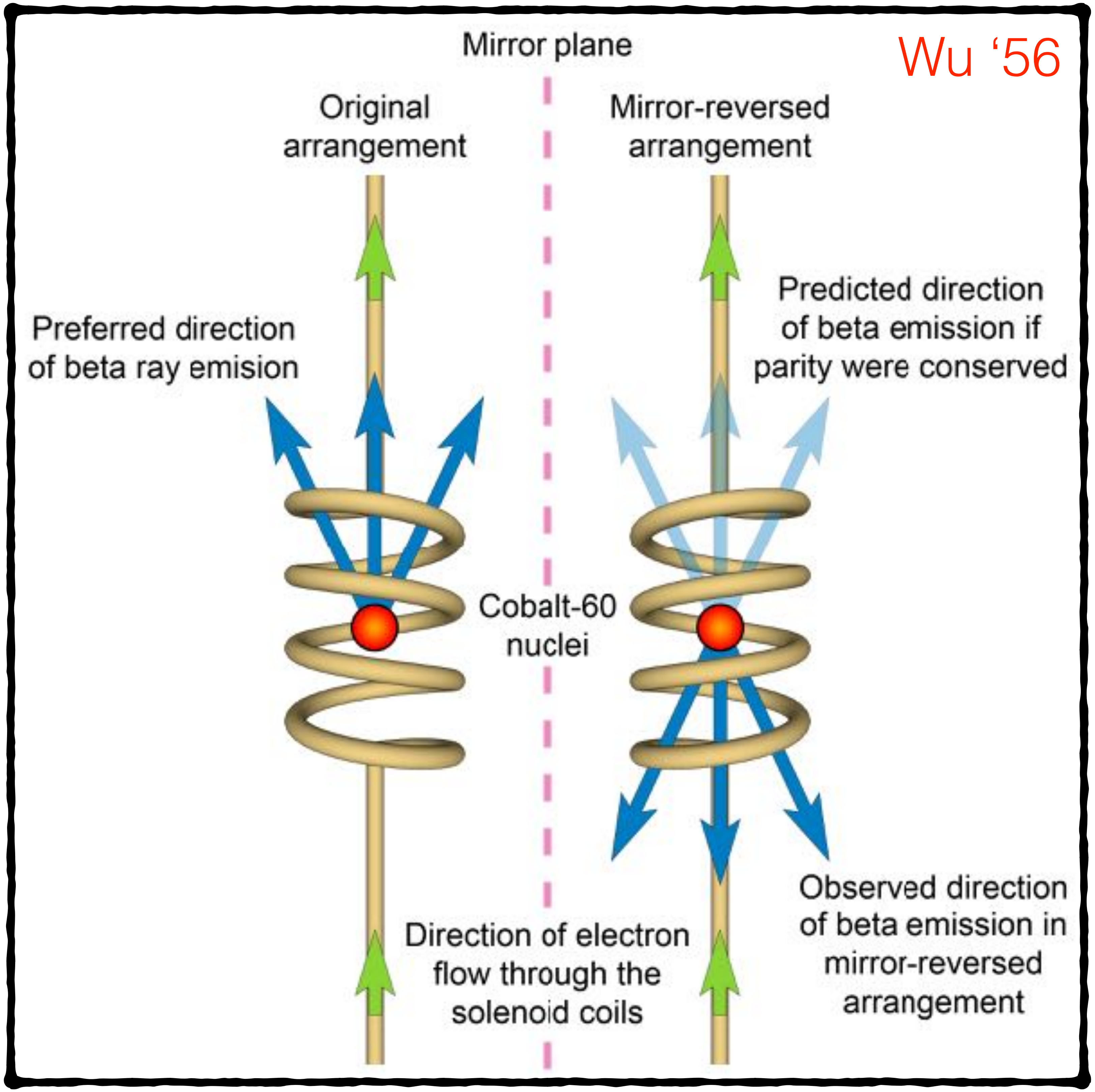
[e_L and e_R are fundamentally
two different particles
Only an accident of the history of
physics that they are both called
electron]



$m_e=0$

but since we know it is not true, we

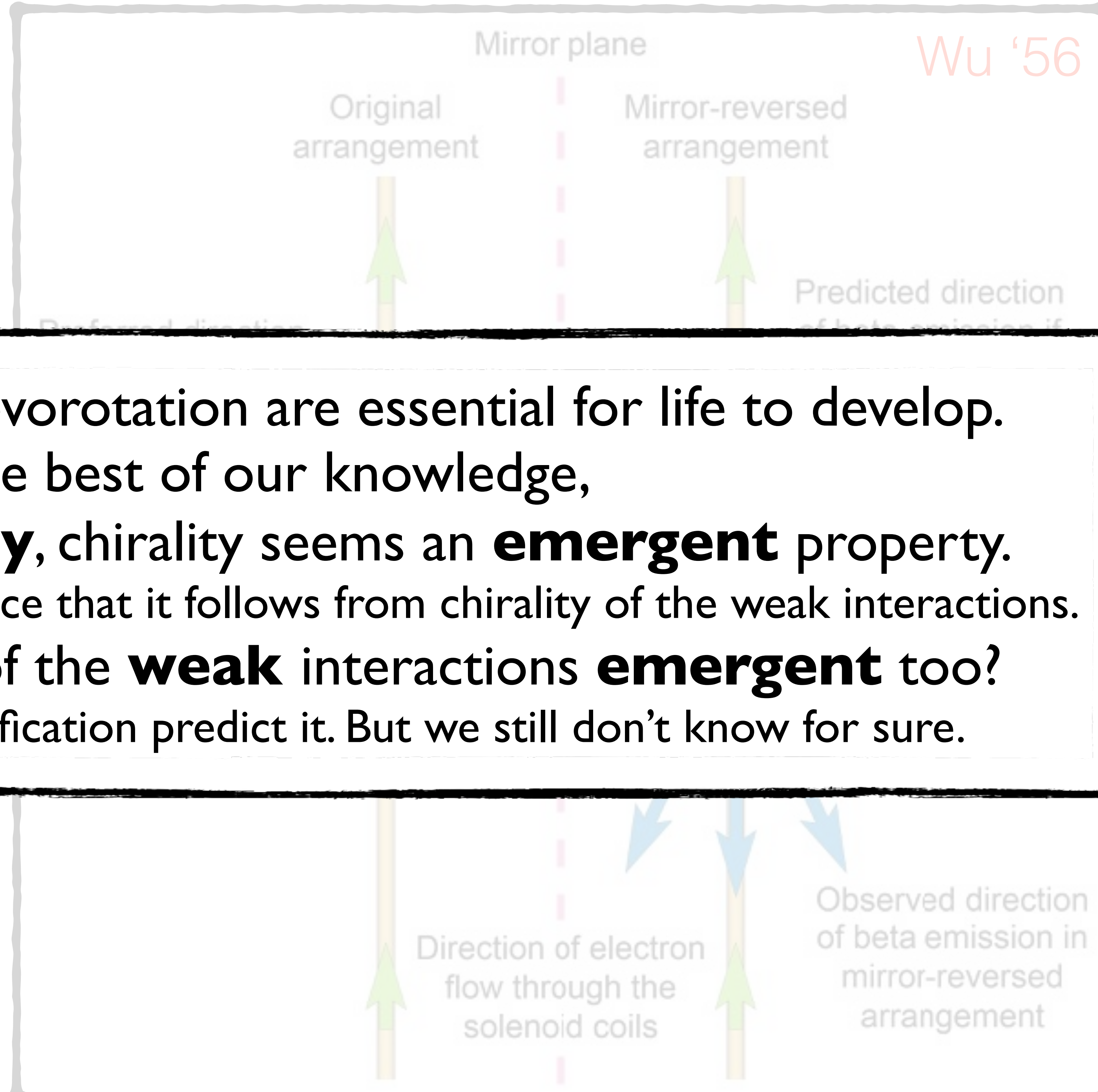
need a new phenomena to generate mass: Higgs mechanism



Chirality of SM & Mass problem

Weak interaction
(force responsible for
neutron decay)
is chiral!

[e_L and e_R are fundamentally



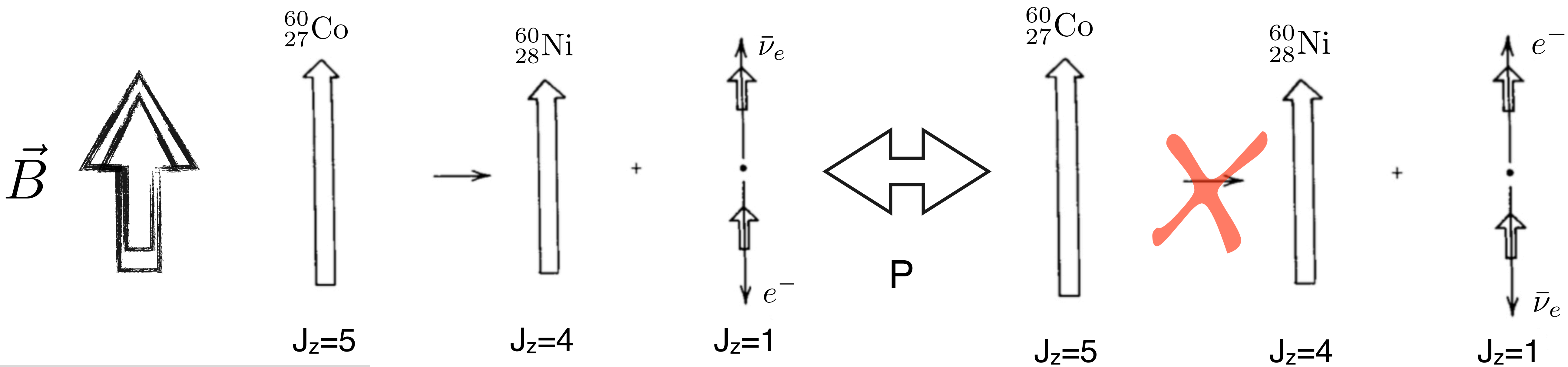
Dextrorotation and Levorotation are essential for life to develop.
To the best of our knowledge,
in **molecular biology**, chirality seems an **emergent** property.
At least, there is no clear evidence that it follows from chirality of the weak interactions.
Are the chiral nature of the **weak** interactions **emergent** too?
Some models of grand unification predict it. But we still don't know for sure.

But since we know it is not true, we

need a new phenomena to generate mass: Higgs mechanism

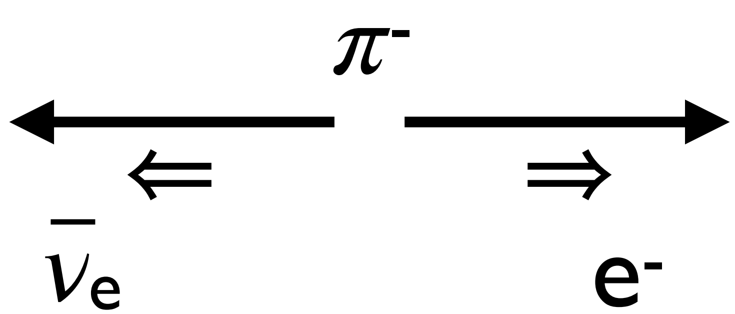
SM is a Chiral Theory

Weak interactions maximally violates P



$^{60}_{27}\mathrm{Co} \to {}^{60}_{28}\mathrm{Ni} + e^- + \bar{\nu}_e$ only LH e$^-$ produced

TH: Yang&Lee '56. EXP: Wu '57



Conservation of momentum and spin imposes to have a RH e$^-$

Weak decays proceed only w/ LH e$^-$
So the amplitude is prop. to m_e

$$\mathcal{L}_{\text{Dirac}} = \bar{\psi}_L \gamma^\mu \partial_\mu \psi_L + \bar{\psi}_R \gamma^\mu \partial_\mu \psi_R + m\left(\bar{\psi}_L \psi_R + \bar{\psi}_R \psi_L\right)$$

$$\frac{\Gamma(\pi^- \to e^- \bar{\nu}_e)}{\Gamma(\pi^- \to \mu^- \bar{\nu}_\mu)} \propto \frac{m_e^2}{m_\mu^2} \sim 2 \times 10^{-5} \sim 10^{-4}_{\text{obs}}$$

Extra phase-space factor

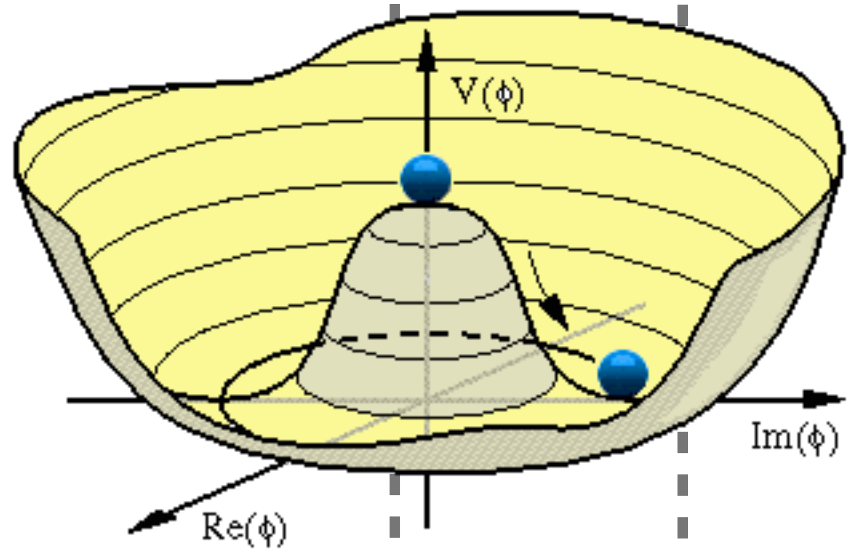
Higgs Mechanism

Symmetry of the Lagrangian

$$SU(2)_L \times U(1)_Y$$

Higgs Doublet

$$H = \begin{pmatrix} h^+ \\ h^0 \end{pmatrix}$$



Symmetry of the Vacuum

$$U(1)_{e.m.}$$

Vacuum Expectation Value

$$\langle H \rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} \quad \text{with} \quad v \approx 246 \text{ GeV}$$

$$D_\mu H = \partial_\mu H - \frac{i}{2} \begin{pmatrix} g W^3_\mu + g' B_\mu & \sqrt{2} g W^+_\mu \\ \sqrt{2} g W^-_\mu & -g W^3_\mu + g' B_\mu \end{pmatrix} H \quad \text{with} \quad \mathrm{W}^\pm_\mu = \frac{1}{\sqrt{2}} (\mathrm{W}^1_\mu \mp \mathrm{W}^2_\mu)$$

$$|D_\mu H|^2 = \frac{1}{4} g^2 v^2 \, W^+_\mu W^{-\,\mu} + \frac{1}{8} (W^3_\mu \; B_\mu) \begin{pmatrix} g^2 v^2 & -g g' v^2 \\ -g g' v^2 & g'^2 v^2 \end{pmatrix} \begin{pmatrix} W^{3\,\mu} \\ B^\mu \end{pmatrix}$$

- Gauge boson spectrum
  - electrically charged bosons $M_W^2 = \frac{1}{4} g^2 v^2$
  - electrically neutral bosons

$$Z_\mu = c W^3_\mu - s B_\mu$$
$$\gamma_\mu = s W^3_\mu + c B_\mu$$

Weak mixing angle

$$c = \frac{g}{\sqrt{g^2 + g'^2}} \qquad s = \frac{g'}{\sqrt{g^2 + g'^2}}$$

$$M_Z^2 = \frac{1}{4} (g^2 + g'^2) v^2$$

$$M_\gamma = 0$$

Fermion Masses

SM is a **chiral** theory (≠ QED that is vector-like)

$$\underbrace{m_e \bar{e}_L}_{Y=1/2} \underbrace{e_R}_{Y=-1} + h.c. \qquad \text{is not gauge invariant}$$

The SM Lagrangian cannot contain fermion mass term.
Fermion masses are **emergent** quantities
that originate from **interactions with Higgs VEV**

$$H = \begin{pmatrix} 0 \\ \frac{v+h}{\sqrt{2}} \end{pmatrix}$$

$$\mathcal{L} = y_e \underbrace{\begin{pmatrix} \bar{\nu}_L \\ \bar{e}_L \end{pmatrix}}_{Y=1/2} \cdot \underbrace{\begin{pmatrix} H^+ \\ H^0 \end{pmatrix}}_{Y=1/2} \underbrace{e_R}_{Y=-1} = \frac{y_e\, v}{\sqrt{2}} \left(\bar{e}_L e_R + \frac{1}{v}\, \bar{e}_L e_R \underbrace{\boxed{h}}_{\text{Higgs Boson}} \right)$$

Higgs couplings proportional to the mass of particles

Fermion Masses

In SM, the Yukawa interactions are the only source of the fermion masses

$$y_{ij}\bar{f}_{L_i}Hf_{R_j} = \frac{y_{ij}v}{\sqrt{2}}\bar{f}_{L_i}f_{R_j} + \frac{y_{ij}}{\sqrt{2}}h\bar{f}_{L_i}f_{R_j}$$

mass

Higgs-fermion interactions

both matrices are simultaneously diagonalisable

no tree-level Flavor Changing Current induced by the Higgs

Once the mass terms are diagonal, the Higgs interactions are diagonal too

Not true anymore if the SM fermions mix with vector-like partners or for non-SM Yukawa

$$y_{ij}\left(1+c_{ij}\frac{|H|^2}{f^2}\right)\bar{f}_{L_i}Hf_{R_j} = \frac{y_{ij}v}{\sqrt{2}}\left(1+c_{ij}\frac{v^2}{2f^2}\right)\bar{f}_{L_i}f_{R_j} + \left(1+3c_{ij}\frac{v^2}{2f^2}\right)\frac{y_{ij}}{\sqrt{2}}h\bar{f}_{L_i}f_{R_j}$$

Look for SM forbidden Flavour Violating decays h → μτ and h → eτ

(look also at t→hc)

- weak indirect constrained by flavour data (μ→ eγ): BR<10%
- ATLAS and CMS have the sensitivity to set bounds O(1%)
- ILC/CLIC/FCC-ee can certainly do much better

Neutrino Masses

The same construction doesn't work for neutrinos
since in the SM there are only Left Handed neutrinos

For an uncharged particle, it is possible to write a Majorana mass
another Lorentz-invariant quadratic term in the Lagrangian
(it involves the charge-conjugate spinor, see lecture #3-technical slides)

$$\mathcal{L}_{\text{Majorana}} = m\bar{\psi}_C\,\psi = m\left(\bar{\psi}_{L_C}\psi_L + \bar{\psi}_{R_C}\psi_R\right)$$

can build such a term with LH field only!

In SM, such neutrino Majorana mass can be obtained from dim-5 operator:

$$\mathcal{L} = \frac{y_\nu}{\Lambda}\underbrace{\begin{pmatrix} \nu_L \\ e_L \end{pmatrix}_C}_{\text{mass}^{3/2}} \cdot \underbrace{\begin{pmatrix} H^+ \\ H^0 \end{pmatrix}}_{\text{mass}} \underbrace{\begin{pmatrix} \nu_L \\ e_L \end{pmatrix}}_{\text{mass}^{3/2}} \cdot \underbrace{\begin{pmatrix} H^+ \\ H^0 \end{pmatrix}}_{\text{mass}} = \frac{y_\nu\, v^2}{\Lambda}\nu_{LC}\nu_L$$

Seesaw: $m_\nu = \dfrac{y_\nu v^2}{\Lambda}$

Order eV
for $y_\nu \sim 1$ and $\Lambda \sim 10^{14}$GeV

Note that such an operator breaks Lepton Number by 2 units

SM Summary

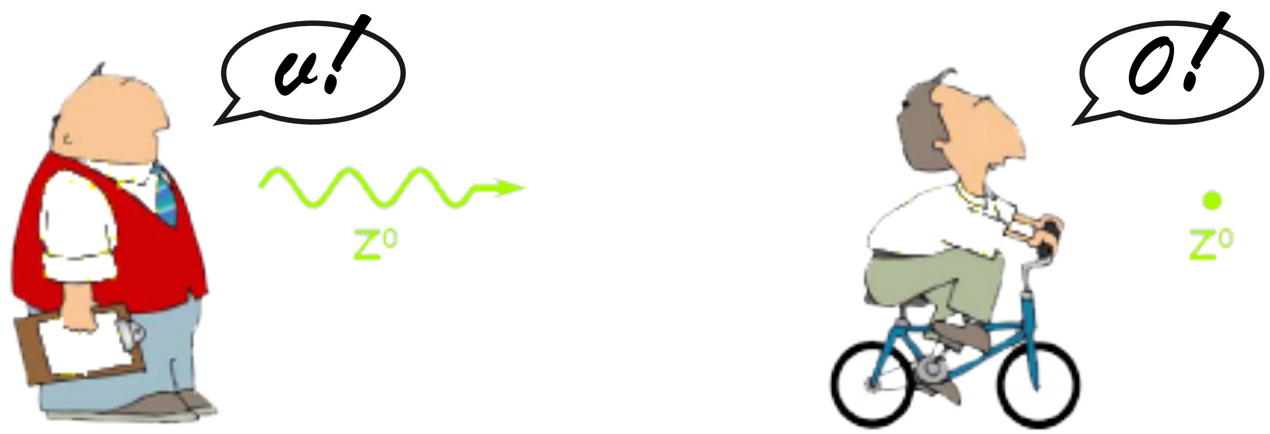
	SPIN	PARTICLES	color $SU(3)_C$ ×	chirality $SU(2)_L$ ×	hypercharge $U(1)_Y$	weak isospin T_{3_L}	electric charge $Q = T_{3_L} + Y$	effective coupling to Z boson g_{eff}	MEANING
LEPTONS	1/2	$L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L$	1	2	$\begin{pmatrix} -1/2 \\ -1/2 \end{pmatrix}$	$\begin{pmatrix} 1/2 \\ -1/2 \end{pmatrix}$	$\begin{pmatrix} 0 \\ -1 \end{pmatrix}$	$\begin{pmatrix} 1/2 \\ -1/2 + \sin^2\theta_W \end{pmatrix}$	doublet under SU(2), singlet under SU(3)
		e_R	1	1	-1	0	-1	$\sin^2\theta_W$	singlet under SU(2) and SU(3)
QUARKS		$Q = \begin{pmatrix} u \\ d \end{pmatrix}_L$	3	2	$\begin{pmatrix} 1/6 \\ 1/6 \end{pmatrix}$	$\begin{pmatrix} 1/2 \\ -1/2 \end{pmatrix}$	$\begin{pmatrix} 2/3 \\ -1/3 \end{pmatrix}$	$\begin{pmatrix} 1/2 - \frac{2}{3}\sin^2\theta_W \\ -1/2 + \frac{1}{3}\sin^2\theta_W \end{pmatrix}$	doublet under SU(2), triplet under SU(3)
		u_R	3	1	2/3	0	2/3	$-\frac{1}{3}\sin^2\theta_W$	singlet under SU(2), triplet under SU(3)
		d_R	3	1	$-1/3$	0	$-1/3$	$\frac{1}{3}\sin^2\theta_W$	singlet under SU(2), triplet under SU(3)
HIGGS	0	$H = \begin{pmatrix} h^+ \\ h^0 \end{pmatrix}$	1	2	$\begin{pmatrix} 1/2 \\ 1/2 \end{pmatrix}$	$\begin{pmatrix} 1/2 \\ -1/2 \end{pmatrix}$	$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$	×	doublet under SU(2), singlet under SU(3)

Technical Details
for Advanced Students

The longitudinal polarisation of massive W, Z

a massless particle is never at rest: always possible to distinguish (and eliminate!) the longitudinal polarisation

3=2+1
Guralnik et al '64



the longitudinal polarisation is physical for a massive spin-1 particle

(pictures: courtesy of G. Giudice)

symmetry breaking: new phase with more degrees of freedom

$$\epsilon_{\|} = \left(\frac{|\vec{p}|}{M}, \frac{E}{M}\frac{\vec{p}}{|\vec{p}|}\right)$$ polarization vector grows with the energy

The longitudinal polarisation of massive W, Z

Indeed a massive spin 1 particle has 3 physical polarizations:

$$A_\mu = \epsilon_\mu \ e^{ik_\mu x^\mu}$$

$$\epsilon^\mu \epsilon_\mu = -1 \qquad k^\mu \epsilon_\mu = 0$$

$$k^\mu = (E, 0, 0, k)$$

with $k_\mu k^\mu = E^2 - k^2 = M^2$

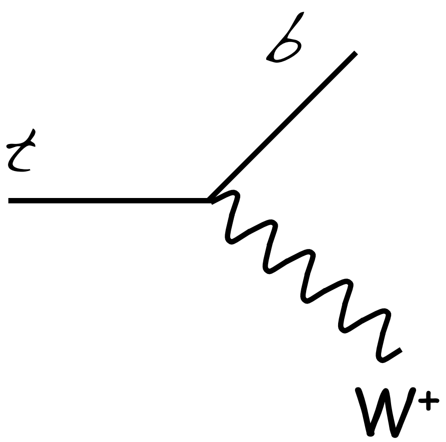
- 2 transverse: $\begin{cases} \epsilon_1^\mu = (0,1,0,0) \\ \epsilon_2^\mu = (0,0,1,0) \end{cases}$
- 1 longitudinal: $\epsilon_\parallel^\mu = (\frac{k}{M}, 0, 0, \frac{E}{M}) \approx \frac{k^\mu}{M} + \mathcal{O}(\frac{E}{M})$

(in the R-ξ gauge, the time-like polarization ($\epsilon^\mu \epsilon_\mu = 1 \quad k^\mu \epsilon_\mu = M$) is arbitrarily massive and decouple)

in the particle rest-frame, no distinction between L and T polarisations
in a frame where the particle carries a lot of kinetic energy,
the L polarisation "dominates"

The BEH mechanism: “V_L=Goldstone bosons”

At high energy, the physics of the gauge bosons becomes simple



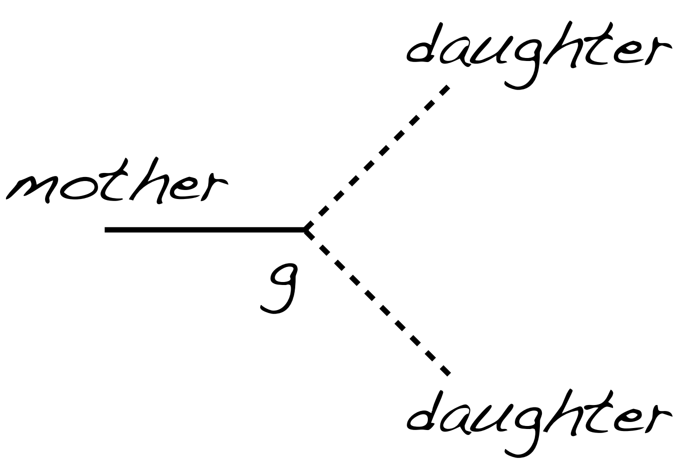
$$\Gamma(t \to bW_L) = \frac{g^2}{64\pi} \frac{m_t^2}{m_W^2} \frac{(m_t^2 - m_W^2)^2}{m_t^3}$$

$$\Gamma(t \to bW_T) = \frac{g^2}{64\pi} \frac{2(m_t^2 - m_W^2)^2}{m_t^3}$$

- at threshold ($m_t \sim m_W$) democratic decay
- at high energy ($m_t >> m_W$) W_L dominates the decay

At high energy, the dominant degrees of freedom are W_L

~~ why you should be stunned by this result: ~~



we expect: (dimensional analysis) $\Gamma \sim g^2\, m_{\mathrm{mother}}$

instead $\Gamma \propto m^3_{\mathrm{mother}}$ means $g \propto m$ like the Higgs couplings!

very efficient way to get energy from the mother particle $\tau \ll \tau_{\mathrm{naive}}$

Goldstone equivalence theorem

$W^\pm_L, Z_L \approx SO(4)/SO(3)$

This is the physics that was understood at LEP
The pending question was then: is there something else?
That was the job of the LHC

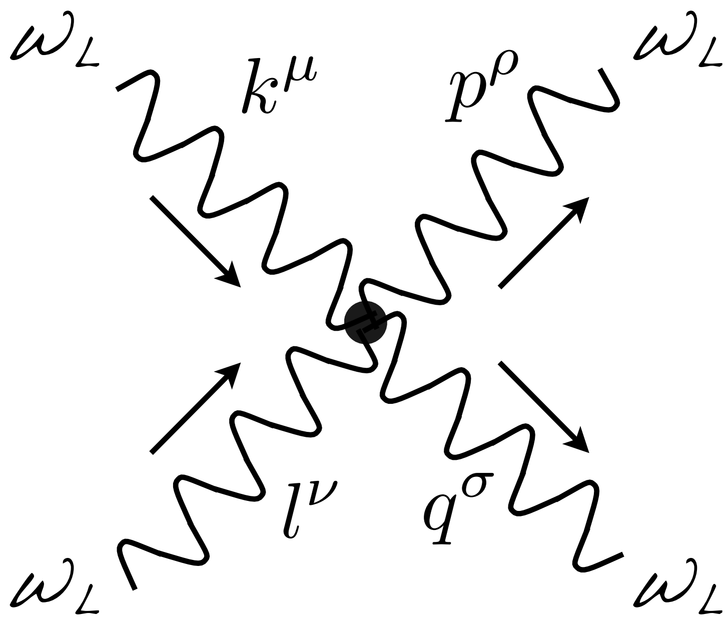
Call for extra degrees of freedom

NO LOSE THEOREM

Bad high-energy behaviour for
the scattering of the longitudinal
polarisations

$$\mathcal{A} = \epsilon_{\parallel}^{\mu}(k)\epsilon_{\parallel}^{\nu}(l)g^2\left(2\eta_{\mu\rho}\eta_{\nu\sigma} - \eta_{\mu\nu}\eta_{\rho\sigma} - \eta_{\mu\sigma}\eta_{\nu\rho}\right)\epsilon_{\parallel}^{\rho}(p)\epsilon_{\parallel}^{\sigma}(q)$$

$$\mathcal{A} = g^2 \frac{E^4}{4M_W^4}$$

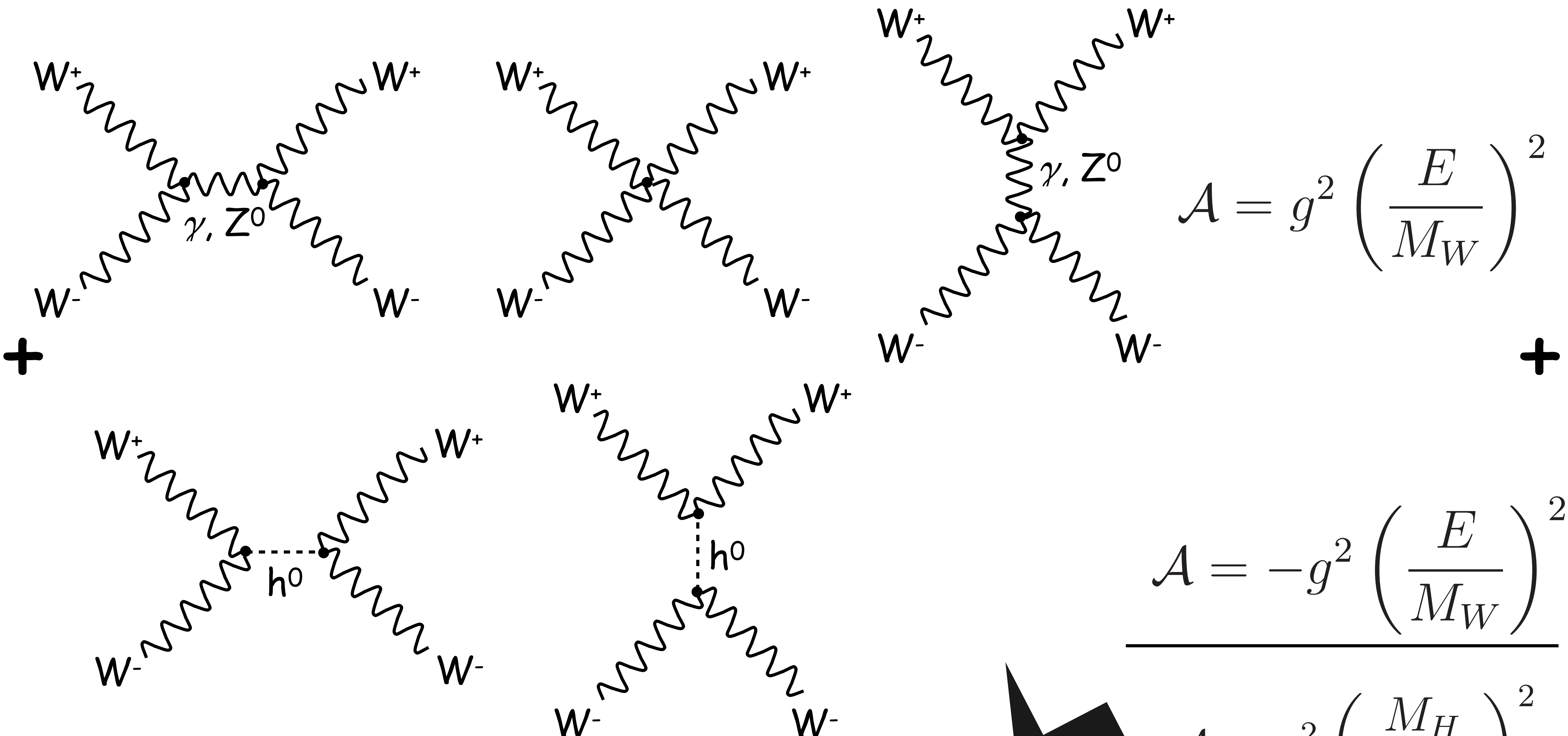


violations of perturbative unitarity around E ~ M/√g (actually M/g)

Extra degrees of freedom are needed to have a good description
of the W and Z masses at higher energies

numerically: E ~ 3 TeV ⇦ the LHC was sure to discover something!

Call for extra degrees of freedom



$$\mathcal{A} = g^2 \left(\frac{E}{M_W} \right)^2$$

$$\mathcal{A} = -g^2 \left(\frac{E}{M_W} \right)^2$$

$$\mathcal{A} = g^2 \left(\frac{M_H}{2M_W} \right)^2$$

The Higgs boson unitarizes the W scattering
(if its mass is below ~ 1 TeV)

W_L scattering = pion scattering
Goldstone equivalence theorem

Lewellyn Smith '73
Dicus, Mathur '73
Cornwall, Levin, Tiktopoulos '73
Lee, Quigg, Thacker '77

What is the SM Higgs?

A single scalar degree of freedom that couples to the mass of the particles

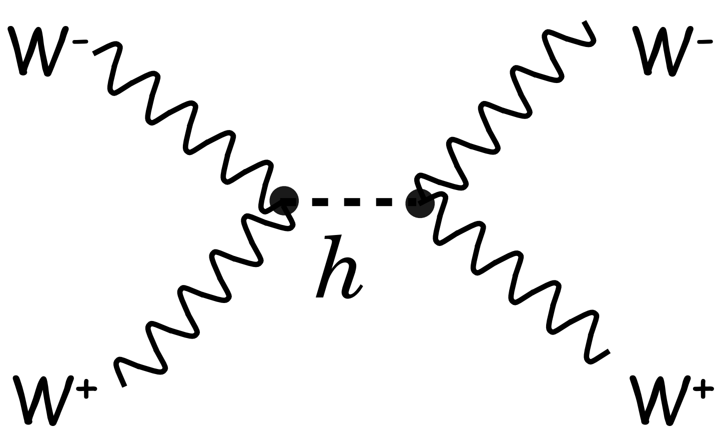
$$\Sigma = e^{i\pi^a \sigma^2 / v}$$ parametrises the coset SO(4)/SO(3)

$$\mathcal{L} = \frac{v^2}{4} \mathrm{Tr}\, D_\mu \Sigma^\dagger D^\mu \Sigma \xrightarrow{\Sigma = \mathbb{1}} m_W^2 W_\mu^+ W_\mu^+ + \frac{1}{2} m_Z Z_\mu Z^\mu$$

$$\mathcal{L} = \frac{v^2}{4} \mathrm{Tr}\, D_\mu \Sigma^\dagger D^\mu \Sigma \xrightarrow[g = g' = 0]{} \frac{1}{2}(\partial\pi)^2 + \frac{1}{v^2}\partial^2\pi^4 + \dots$$

$$\mathcal{L}_{\mathrm{EWSB}} = m_W^2 W_\mu^+ W_\mu^+ \left(1 + 2a\frac{h}{v} + b\frac{h^2}{v^2}\right) - m_\psi \bar{\psi}_L \psi_R \left(1 + c\frac{h}{v}\right)$$

'a', 'b' and 'c' are arbitrary free couplings



$$\mathcal{A} = \frac{1}{v^2}\left(s - \frac{a^2 s^2}{s - m_h^2}\right)$$

growth cancelled for
a = 1
restoration of perturbative
unitarity

What is the Higgs the name of?

A single scalar degree of freedom that couples to the mass of the particles

$$\mathcal{L}_{\text{EWSB}} = m_W^2 W_\mu^+ W_\mu^+ \left(1 + 2a\frac{h}{v} + b\frac{h^2}{v^2}\right) - m_\psi \bar{\psi}_L \psi_R \left(1 + c\frac{h}{v}\right)$$

'a', 'b' and 'c' are arbitrary free couplings

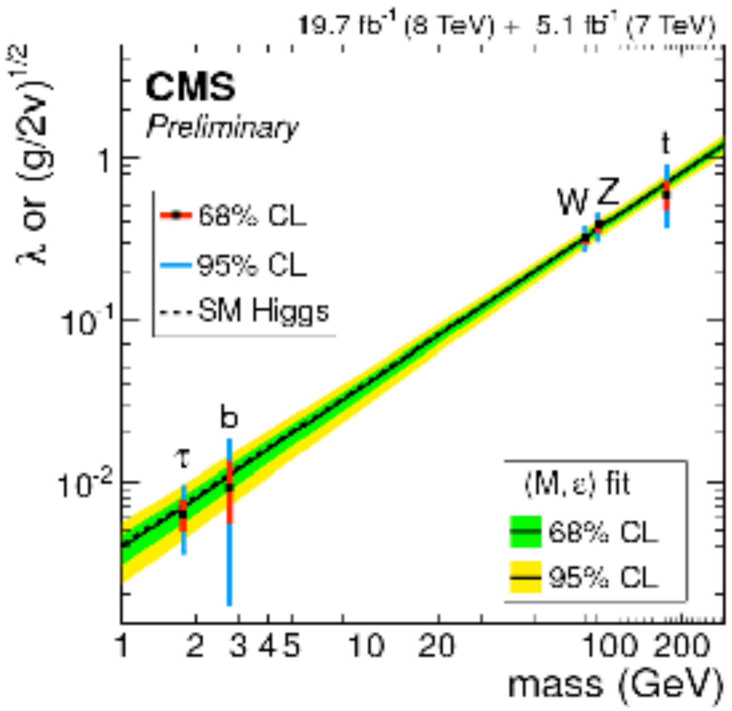
For a=1: perturbative unitarity in elastic channels WW → WW

For b = a^2: perturbative unitarity in inelastic channels WW → hh

For ac=1: perturbative unitarity in inelastic WW → $\psi\,\psi$

Cornwall, Levin, Tiktopoulos '73

Contino, Grojean, Moretti, Piccinini, Rattazzi '10



CMS PAS-HIG-14-009

Higgs couplings
are proportional
to the masses of the particles

$$\lambda_\psi \propto \frac{m_\psi}{v}, \qquad \lambda_V^2 \equiv \frac{g_{VVh}}{2v} \propto \frac{m_V^2}{v^2}$$