



# Making Predictions for Hadron Colliders

Mike Seymour – University of Manchester

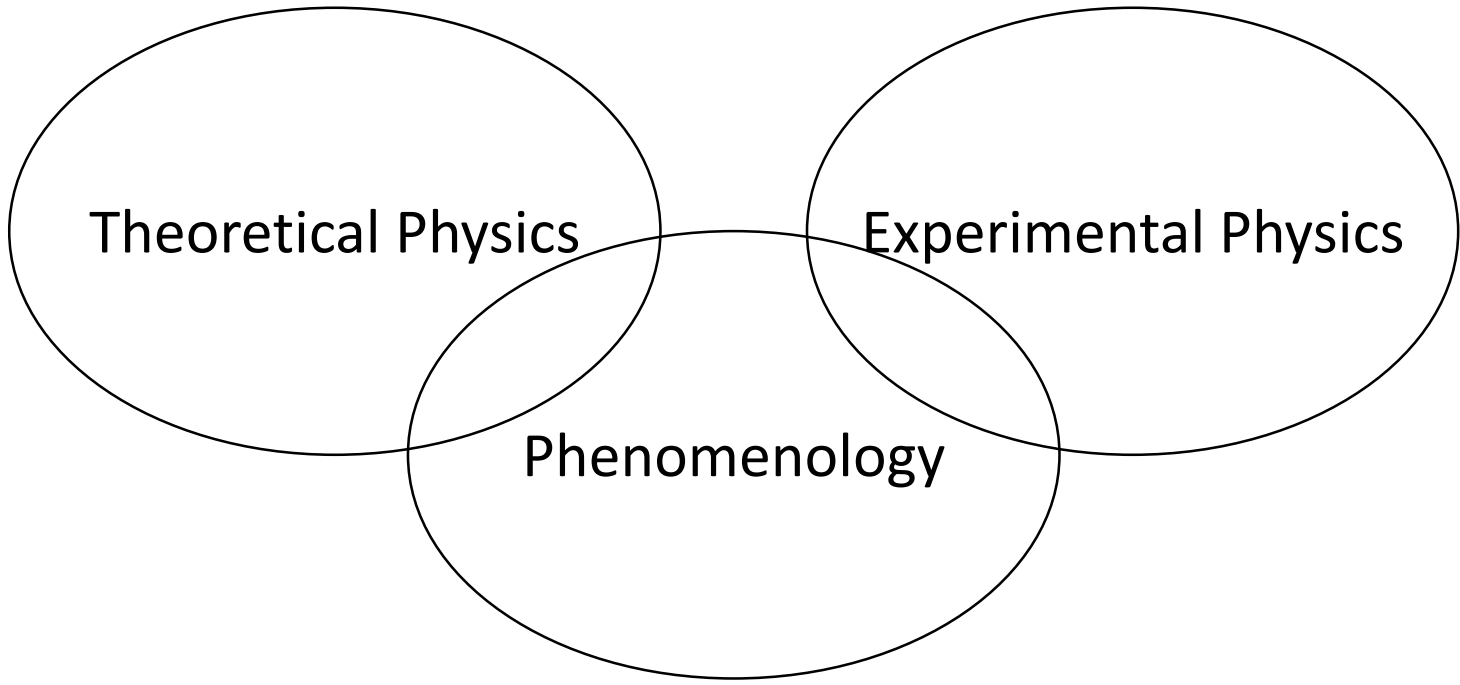


# Making Predictions for Hadron Colliders

## 1. From Feynman Diagrams to Cross Sections



# Phenomenology



# Calculating Event Rates

$$N = \mathcal{L} \sigma$$

Number of  
events

Integrated  
Luminosity

Cross  
section

# Calculating Cross Sections

$$d\sigma = \frac{1}{F} |\mathcal{M}|^2 dLIPS$$

Flux factor

$$= 2s = 4E_1E_2$$

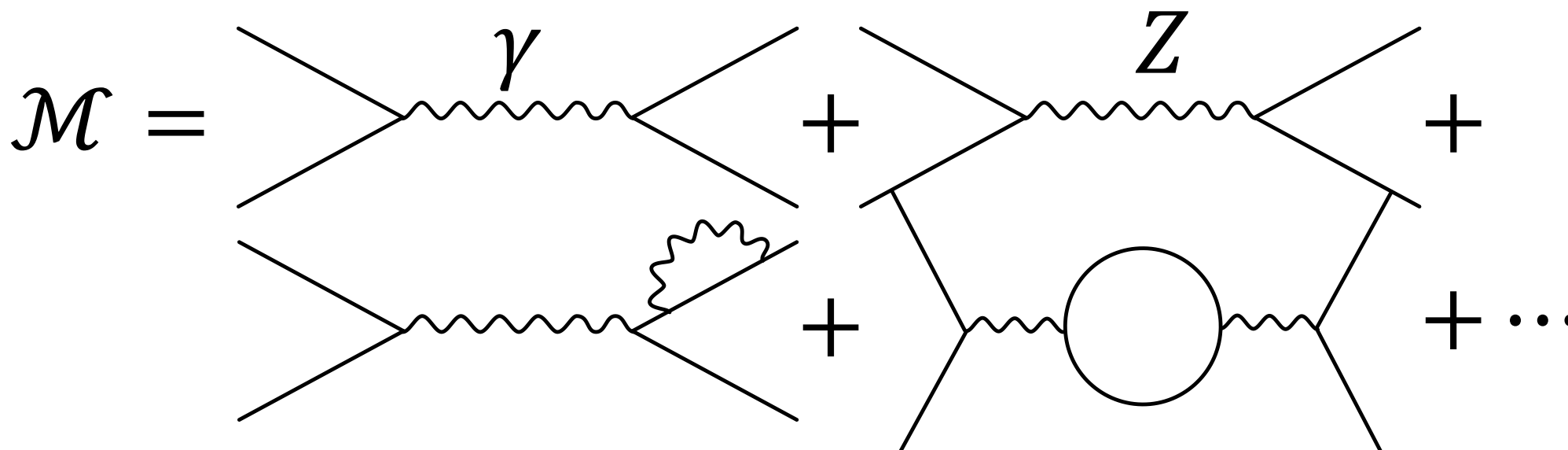
(Quantum mechanical) amplitude squared

Lorentz Invariant Phase Space

$$= \frac{d^4p_i}{(2\pi)^4} (2\pi)\delta(p_i^2 - m_i^2) \frac{d^4p_j}{(2\pi)^4} (2\pi)\delta(p_j^2 - m_j^2) \dots (2\pi)^4 \delta(p_1 + p_2 - p_i - p_j - \dots)$$

# Calculating Cross Sections

$$d\sigma = \frac{1}{F} |\mathcal{M}|^2 dLIPS$$



# Feynman Rules

$$\alpha \longrightarrow \beta \quad \rightarrow \quad \left( \frac{i}{\not{p} - m + i\epsilon} \right)_{\beta\alpha}$$

$$\mu \text{ (wavy) } \nu \quad \rightarrow \quad \frac{-i\eta_{\mu\nu}}{p^2 + i\epsilon}$$

$$\begin{array}{l} \beta \\ \nearrow \\ \alpha \end{array} \text{ (solid lines) } \text{---} \text{ (wavy line) } \mu \quad \rightarrow \quad -ie\gamma_{\beta\alpha}^{\mu} (2\pi)^4 \delta^{(4)}(p_1 + p_2 + p_3).$$

Elementary charge

$$e = \sqrt{4\pi\alpha} \quad \alpha \approx 1/137$$

# Tree Diagrams as Leading Order of Expansion in $\alpha$

$$\mathcal{M} = \begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array} + \begin{array}{c} \text{Diagram 3} \\ \text{Diagram 4} \end{array} + \dots + \mathcal{O}(\alpha^2)$$

The diagram shows the expansion of the amplitude  $\mathcal{M}$  in powers of the coupling constant  $\alpha$ . It consists of several terms:
 

- Two tree-level diagrams (order  $\alpha$ ):
  - Diagram 1: A wavy line labeled  $\gamma$  connects two vertices, each with two external lines.
  - Diagram 2: A wavy line labeled  $Z$  connects two vertices, each with two external lines.
- Two higher-order diagrams (order  $\alpha^2$ ):
  - Diagram 3: A tree-level diagram with a wavy line and a loop (represented by a circle) attached to it.
  - Diagram 4: A tree-level diagram with a wavy line and a loop (represented by a circle) attached to it.

$$\alpha \approx 1/137 \text{ but } \alpha_s \approx 0.1$$

$\Rightarrow$  QCD corrections important



Example: The Drell-Yan process ( $pp \rightarrow \mu^+ \mu^-$ )

$$\mathcal{M} = \begin{array}{c} q \\ \swarrow \\ \gamma \\ \nwarrow \\ \bar{q} \end{array} \begin{array}{c} \mu^- \\ \swarrow \\ \mu^+ \\ \nwarrow \end{array}$$

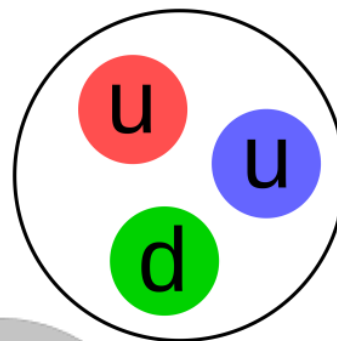
$$\Rightarrow |\mathcal{M}|^2 \propto e_q^2 \alpha^2 \frac{t^2 + u^2}{s^2} \propto e_q^2 \alpha^2 (1 + \cos^2 \theta)$$

$$\Rightarrow \sigma = \frac{4\pi\alpha^2}{9Q^2} e_q^2$$

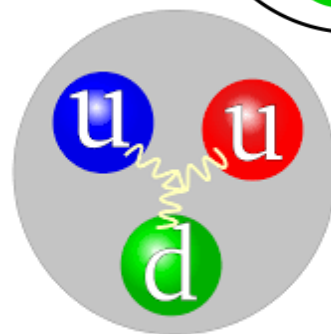
$$(s = Q^2 = (p_q + p_{\bar{q}})^2)$$

# Proton structure

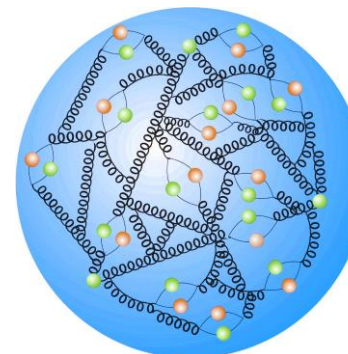
- Proton = uud ?



- Held together by gluons?



- Quantum Field Theory: gluons can create  $q\bar{q}$  pairs

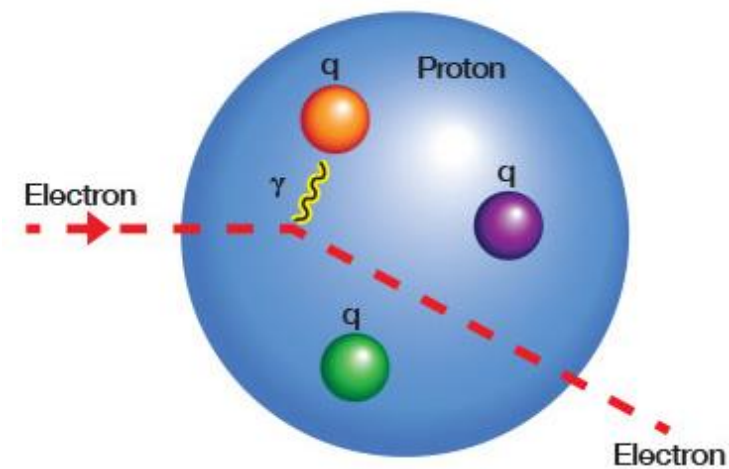


- Proton can interact through any of its partons

# Proton structure: parton distribution functions

- How is the proton's energy shared between its parton constituents?
- Measure in deep inelastic electron scattering
- Quantify by *parton distribution function*

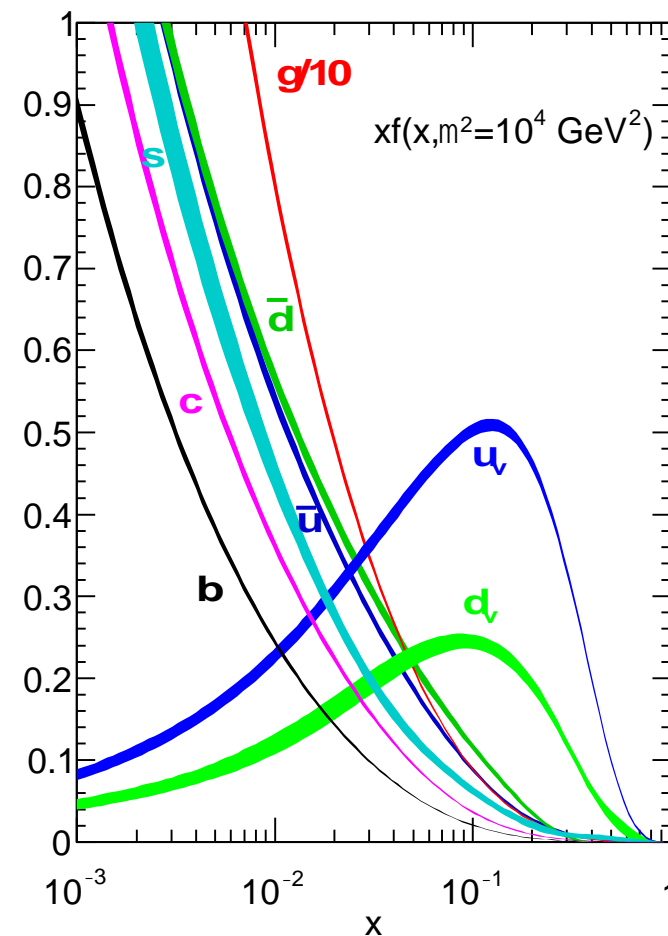
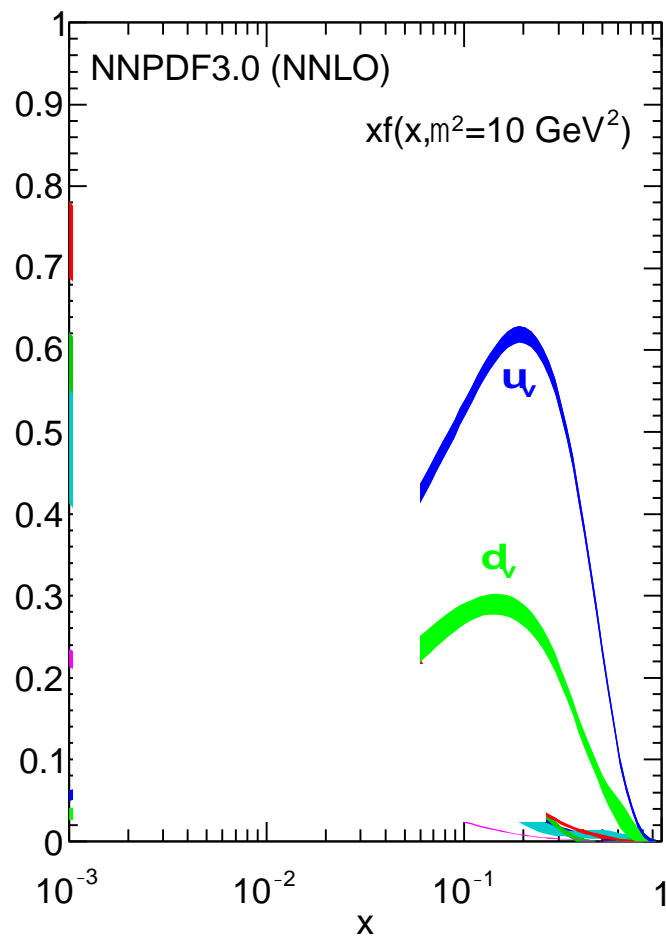
$f_i(x)dx$  = probability that parton of type  $i$  is found with fraction of proton's momentum between  $x$  and  $x + dx$



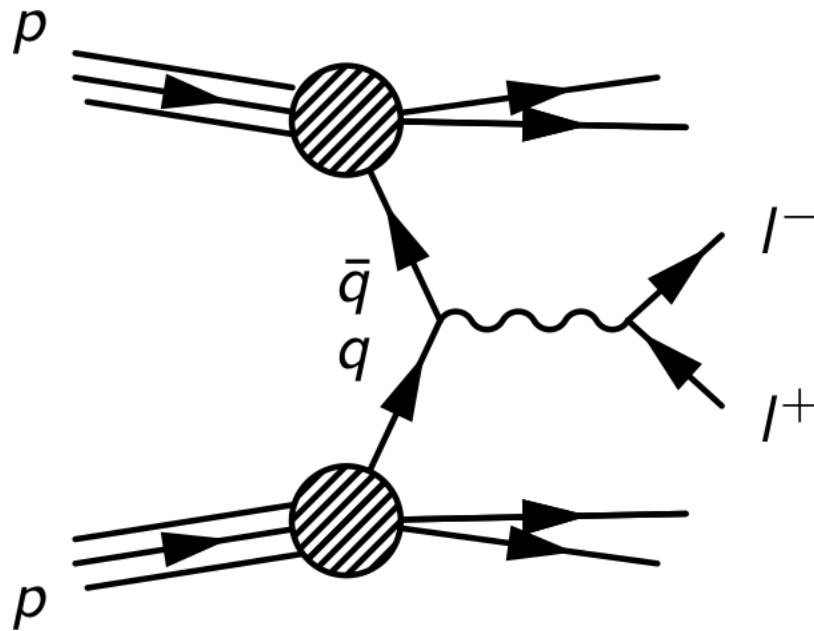
- But how long do those quantum fluctuations live?

⇒ PDFs depend on the momentum scale of the probe  $f_i(x, Q^2)dx$

# Proton structure: parton distribution functions



# The Drell-Yan process ( $pp \rightarrow \mu^+ \mu^-$ )



$$\frac{d\sigma}{dQ^2} = \sum_q \int dx_1 f_q(x_1, Q^2) dx_2 f_{\bar{q}}(x_2, Q^2) \frac{4\pi\alpha^2}{9Q^2} e_q^2 \delta(x_1 x_2 s - Q^2)$$

# Loop Diagrams as Higher Order Corrections

$$\mathcal{M} = \text{tree diagram } \mathcal{O}(\alpha) + \text{loop diagram } \mathcal{O}(\alpha\alpha_s) + \dots$$

$$|\mathcal{M}|^2 = |\mathcal{M}_0|^2 + 2\Re(\mathcal{M}_0^* \mathcal{M}_1) + |\mathcal{M}_1|^2 + \dots$$

$\mathcal{O}(\alpha^2)$                        $\mathcal{O}(\alpha^2\alpha_s)$

- Quantum mechanics: sum over unobserved quantum numbers = integrate over gluon momenta

# Loop Diagrams as Higher Order Corrections

$$\mathcal{M} = \text{tree diagram } \mathcal{O}(\alpha) + \text{loop diagram } \mathcal{O}(\alpha\alpha_s) + \dots$$

- Gluon momentum integral is divergent! (= *minus* infinity)
- Divergence comes from:
  - Momentum = 0
  - Momentum = parallel to quark or antiquark

# Gluon Emission as Higher Order Correction

$$\mathcal{M} = \text{[Diagram 1]} + \text{[Diagram 2]} \quad \mathcal{O}(\alpha\sqrt{\alpha_s})$$

The diagram shows the matrix element  $\mathcal{M}$  as the sum of two terms. The first term is a tree-level process where a quark and antiquark annihilate into a virtual photon, which then decays into a muon and antimuon. A gluon is emitted from the incoming quark line. The second term is similar, but the gluon is emitted from the incoming antiquark line. The order of the correction is  $\mathcal{O}(\alpha\sqrt{\alpha_s})$ .

- Gluon emission describes a different process ( $q\bar{q} \rightarrow \mu^+\mu^-g$ )
- But if we are only interested in the total cross section for Drell-Yan pairs, must integrate over gluon momenta
- Divergent from momentum = 0 or parallel to quark or antiquark
- Cancels loop divergence

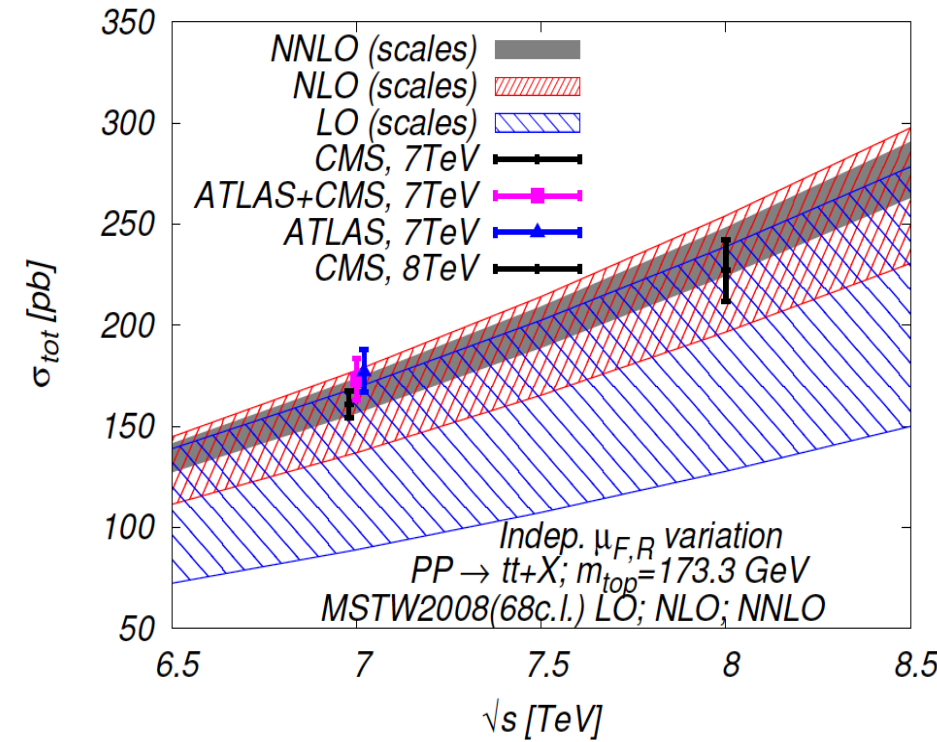
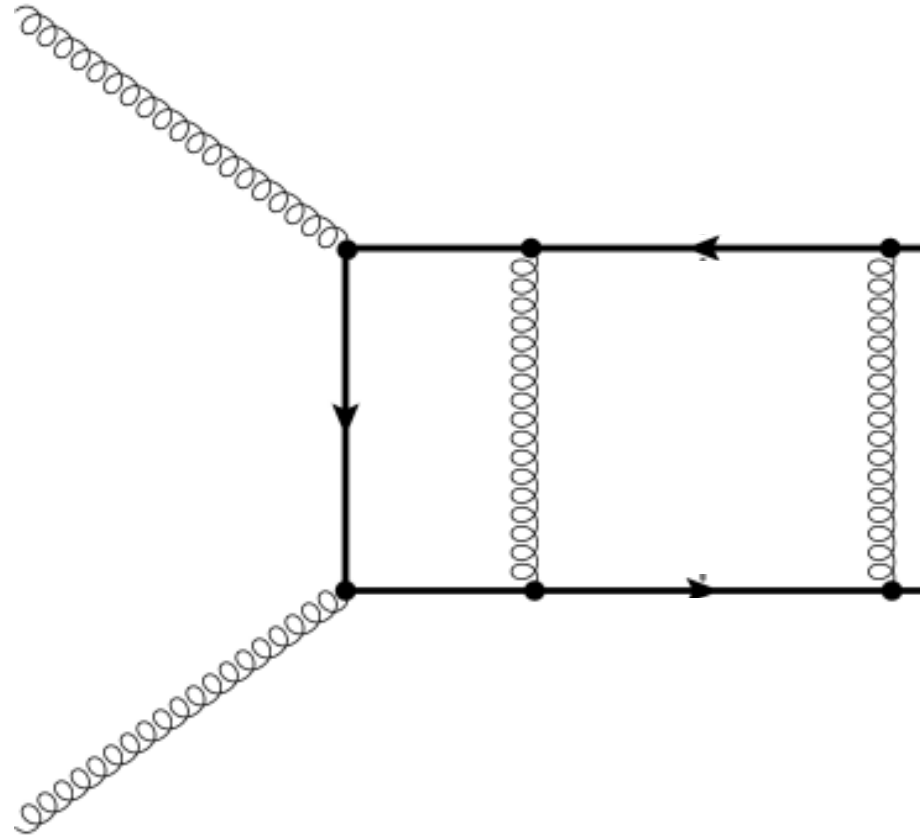


# Next-to-Leading Order (NLO) cross section

- $\sigma_{NLO} = \sigma_{tree} + \sigma_{loop} + \sigma_{emission}$
- $\sigma_{loop}$  and  $\sigma_{emission}$  each divergent
  - must regularize and expose singularities of each
  - *Subtraction algorithms*
- Fully automated,
  - e.g. in Madgraph/aMC@NLO, MCFM, Sherpa, Herwig ...

# State of the Art – NNLO Calculations

e.g.  $pp \rightarrow t\bar{t}$



# From Feynman Diagrams to Cross Sections

- Major part of phenomenology = calculating cross sections
- LO = write down all tree diagrams, integrate phase space numerically
- Convolute with parton distribution functions (fitted to data)
- NLO = one-loop diagrams, one-emission processes
  - Extract singularities from integrals, integrate analytically
  - Integrate remainders numerically
- NNLO = two-loop diagrams, one-emission at one-loop, and two emissions
- But LHC events contain *hundreds* of additional particles...