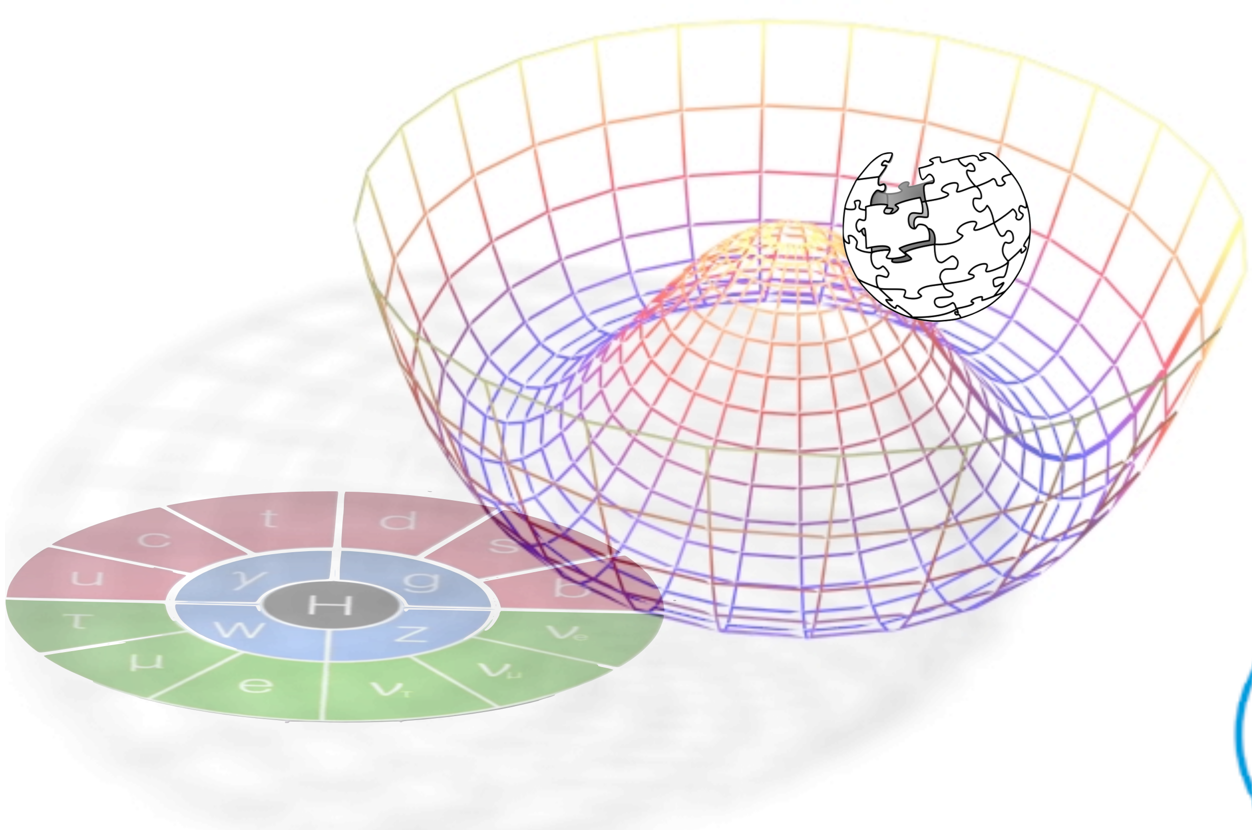
# The Standard Model
## of particle physics

*CERN summer student lectures 2022*

*Lecture 5/5*

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# Outline

## Monday
- Lagrangians
- Lorentz symmetry - scalars, fermions, gauge bosons
- Dimensional analysis: cross-sections and life-time.

## Tuesday
- Dimensional analysis: cross-sections and life-time
- Nuclear decay, Fermi theory

## Wednesday
- Breakdown of the Fermi theory
- Gauge interactions: U(1) electromagnetism, SU(2) weak interactions

## Thursday
- From SU(2) to the Fermi theory, SU(3) QCD
- Chirality of weak interactions, Pion decay
- Spontaneous symmetry breaking and Higgs mechanism
- Quark and lepton masses, Neutrino masses

## Friday
- Higgs mechanism and masses
- Running couplings: asymptotic freedom of QCD, Unification
- Hierarchy problem and how to solve it (maybe)

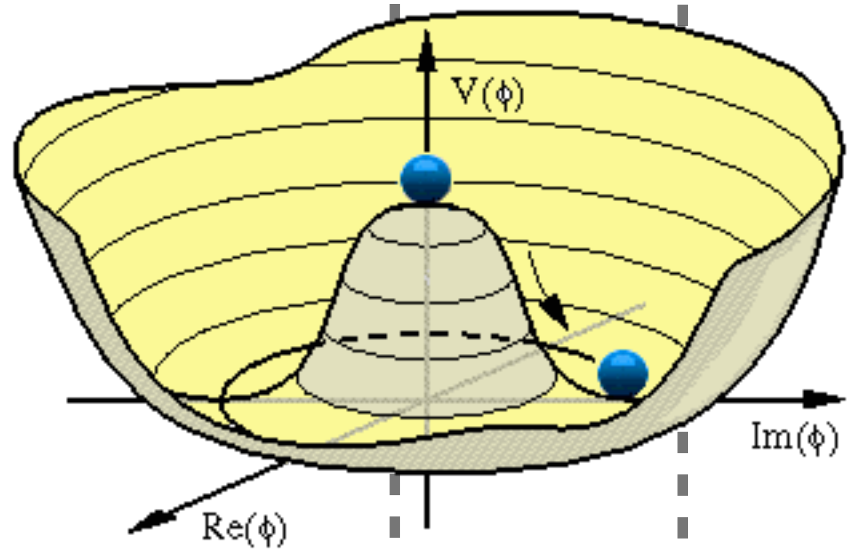
# Spontaneous Symmetry

## Symmetry of the Lagrangian

$$SU(2)_L \times U(1)_Y$$

Higgs Doublet

$$\underset{1/2}{H} = \begin{pmatrix} h^+ \\ h^0 \end{pmatrix}$$




## Symmetry of the Vacuum

$$U(1)_{e.m.}$$

Vacuum Expectation Value

$$\langle H \rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} \quad \text{with} \quad v \approx 246 \text{ GeV}$$

Most general Higgs (renormalisable) potential

$$V(H) = \lambda \left( |H|^2 - v^2/2 \right)^2$$

$v^2>0$ EW symmetry breaking, $v^2<0$ no breaking

Why Nature has decided that $v^2>0$? No dynamics explains it

$$\delta_{SU(2)} \langle H \rangle = \frac{i}{2} \left( \theta^1 \begin{pmatrix} & 1 \\ 1 & \end{pmatrix} + \theta^2 \begin{pmatrix} & -I \\ I & \end{pmatrix} + \theta^3 \begin{pmatrix} 1 & \\ & -1 \end{pmatrix} \right) \langle H \rangle \neq 0$$

$$\delta_Y \langle H \rangle = i\theta_Y \begin{pmatrix} 1/2 & \\ & 1/2 \end{pmatrix} \langle H \rangle \neq 0$$

$$\delta_Q \langle H \rangle = i\theta_{QED} \begin{pmatrix} 1 & \\ & 0 \end{pmatrix} \langle H \rangle = 0 \qquad \theta_{QED} = \theta_Y = \theta_3 \qquad Q = Y + T_{3L}$$

# Higgs Boson

## Before EW symmetry breaking

- 4 massless gauge bosons for SU(2)x(1): 4 x 2 = 8 dofs
- Complex scalar doublet: 4 dofs

## After EW symmetry breaking

- 1 massless gauge boson, photon: 2 dofs
- 3 massive gauge bosons, $W^{\pm}$ and Z: 3 x 3 = 9 dofs
- 1 real scalar: 1 dof

$$H = \begin{pmatrix} 0 \\ \frac{v+h(x)}{\sqrt{2}} \end{pmatrix}$$

h(x) describes the Higgs boson
(the fluctuation above the VEV).
The other components of the Higgs doublet H become
the longitudinal polarisations of the $W^{\pm}$ and Z

# Fermion Masses

SM is a **chiral** theory (≠ QED that is vector-like)

$$\underbrace{m_e \bar{e}_L}_{Y=1/2} \underbrace{e_R}_{Y=-1} + h.c.$$ is not gauge invariant

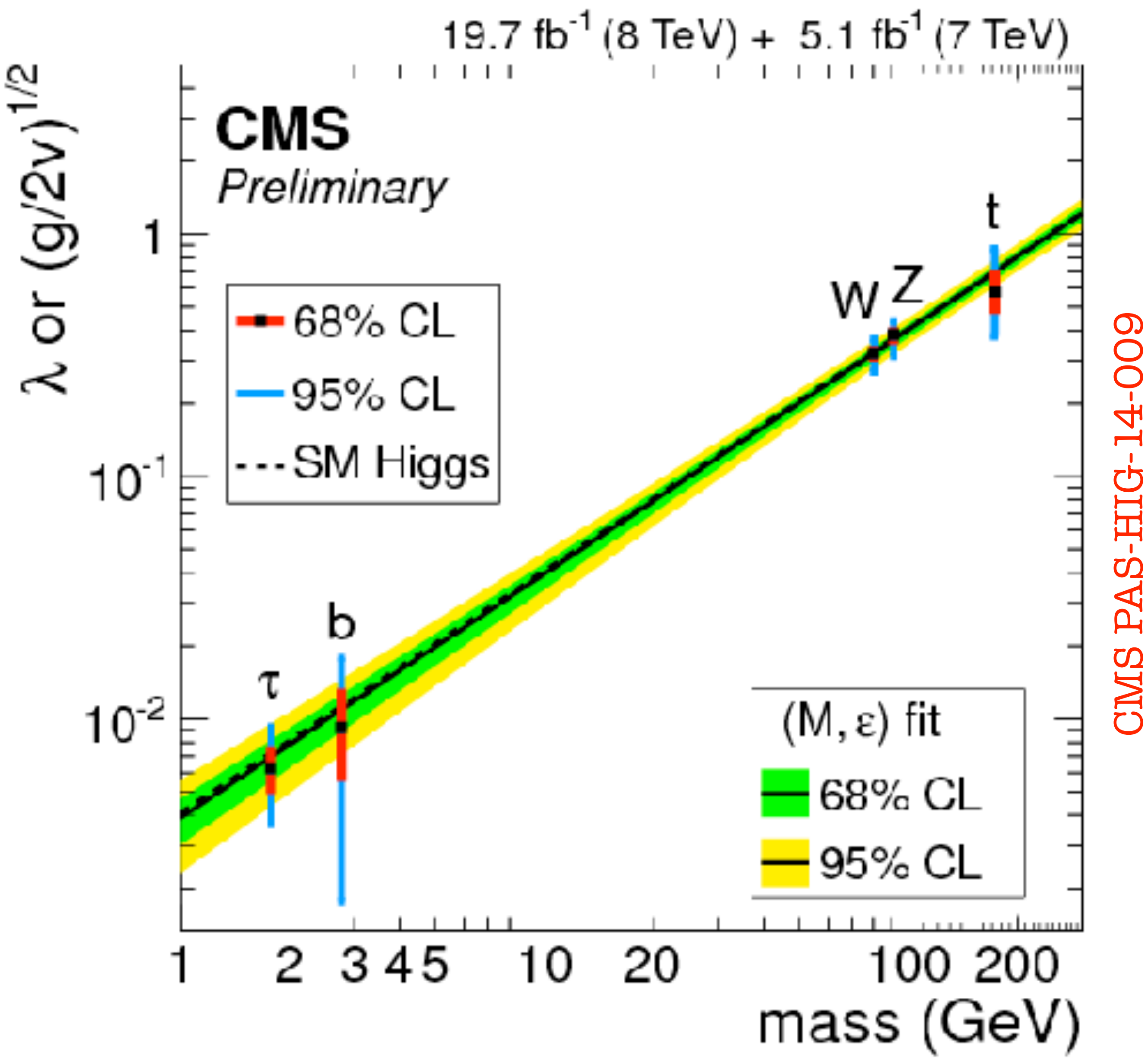
The SM Lagrangian cannot contain fermion mass term.
Fermion masses are **emergent** quantities
that originate from **interactions with Higgs VEV**

$$H = \begin{pmatrix} 0 \\ \frac{v+h}{\sqrt{2}} \end{pmatrix}$$

$$\mathcal{L} = y_e \underbrace{\begin{pmatrix} \bar{\nu}_L \\ \bar{e}_L \end{pmatrix}}_{Y=1/2} \cdot \underbrace{\begin{pmatrix} H^+ \\ H^0 \end{pmatrix}}_{Y=1/2} \underbrace{e_R}_{Y=-1} = \frac{y_e\, v}{\sqrt{2}} \left( \bar{e}_L e_R + \frac{1}{v} \bar{e}_L e_R \underbrace{\boxed{h}}_{\text{Higgs Boson}} \right)$$

Higgs couplings proportional to the mass of particles

# The Higgs PR plot




Higgs couplings
are proportional
to the masses of the particles

$$\lambda_\psi \propto \frac{m_\psi}{v}\,, \qquad \lambda_V^2 \equiv \frac{g_{VVh}}{2v} \propto \frac{m_V^2}{v^2}$$

# Fermion Masses

In SM, the Yukawa interactions are the only source of the fermion masses

$$y_{ij}\bar{f}_{L_i} H f_{R_j} = \frac{y_{ij}v}{\sqrt{2}}\bar{f}_{L_i} f_{R_j} + \frac{y_{ij}}{\sqrt{2}} h \bar{f}_{L_i} f_{R_j}$$

mass

Higgs-fermion interactions

both matrices are simultaneously diagonalisable

no tree-level Flavor Changing Current induced by the Higgs

Once the mass terms are diagonal, the Higgs interactions are diagonal too

Not true anymore if the SM fermions mix with vector-like partners or for non-SM Yukawa

$$y_{ij}\left(1 + c_{ij}\frac{|H|^2}{f^2}\right)\bar{f}_{L_i} H f_{R_j} = \frac{y_{ij}v}{\sqrt{2}}\left(1 + c_{ij}\frac{v^2}{2f^2}\right)\bar{f}_{L_i} f_{R_j} + \left(1 + 3c_{ij}\frac{v^2}{2f^2}\right)\frac{y_{ij}}{\sqrt{2}} h \bar{f}_{L_i} f_{R_j}$$

Look for SM forbidden Flavour Violating decays $h \to \mu\tau$ and $h \to e\tau$

(look also at $t \to hc$ )

- weak indirect constrained by flavour data ($\mu \to e\gamma$): BR<10%
- ATLAS and CMS have the sensitivity to set bounds O(1%)
- ILC/CLIC/FCC-ee can certainly do much better

# Fermion Masses: Quark Mixings

In SM, the Yukawa interactions are the only source of the fermion masses

$$\mathcal{L}_{\rm Yuk} = y^U_{ij}\bar{Q}^i_L H^\star u^i_R + y^D_{ij}\bar{Q}^i_L H d^i_R$$

$$\mathcal{U}_L^\dagger \left(\frac{v}{\sqrt{2}} y^U_{ij}\right)\mathcal{U}_R = \begin{pmatrix} m_u & & \\ & m_c & \\ & & m_t \end{pmatrix} \qquad \mathcal{D}_L^\dagger \left(\frac{v}{\sqrt{2}} y^D_{ij}\right)\mathcal{D}_R = \begin{pmatrix} m_d & & \\ & m_s & \\ & & m_b \end{pmatrix}$$

$$\Downarrow \qquad \Downarrow \qquad \Downarrow$$

$$\mathcal{L}_{\rm Yuk} = (\bar{u}_L \bar{c}_L \bar{t}_L)\begin{pmatrix} m_u & & \\ & m_c & \\ & & m_t \end{pmatrix}\begin{pmatrix} u_R \\ c_R \\ t_R \end{pmatrix} + (\bar{d}_L \bar{s}_L \bar{b}_L)\begin{pmatrix} m_d & & \\ & m_s & \\ & & m_b \end{pmatrix}\begin{pmatrix} d_R \\ s_R \\ b_R \end{pmatrix}$$

$$\mathcal{L}_{\rm gauge} = \frac{e}{\sqrt{2}\sin\theta_w}\left[W^+_\mu \bar{u} V \gamma^\mu \left(\frac{1-\gamma_5}{2}\right) d + W^-_\mu \bar{d} V^\dagger \gamma^\mu \left(\frac{1-\gamma_5}{2}\right) u\right] \qquad V = \mathcal{D}_L^\dagger \mathcal{U}_L$$

$$V_{\rm CKM} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}\begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix}\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Note: one complex phase → CP violation

# Neutrino Masses

The same construction doesn't work for neutrinos
since in the SM there are only Left Handed neutrinos

For an uncharged particle, it is possible to write a Majorana mass
another Lorentz-invariant quadratic term in the Lagrangian
(it involves the charge-conjugate spinor, see lecture #3-technical slides)

$$\mathcal{L}_{\text{Majorana}} = m\bar{\psi}_C\,\psi = m\left(\bar{\psi}_{L_C}\psi_L + \bar{\psi}_{R_C}\psi_R\right)$$

can build such a term with LH field only!

In SM, such neutrino Majorana mass can be obtained from dim-5 operator:

$$\mathcal{L} = \frac{y_\nu}{\Lambda}\underbrace{\begin{pmatrix} \nu_L \\ e_L \end{pmatrix}_C}_{\text{mass}^{3/2}} \cdot \underbrace{\begin{pmatrix} H^+ \\ H^0 \end{pmatrix}}_{\text{mass}} \underbrace{\begin{pmatrix} \nu_L \\ e_L \end{pmatrix}}_{\text{mass}^{3/2}} \cdot \underbrace{\begin{pmatrix} H^+ \\ H^0 \end{pmatrix}}_{\text{mass}} = \frac{y_\nu\, v^2}{\Lambda}\nu_{LC}\nu_L$$

Seesaw: $m_\nu = \frac{y_\nu v^2}{\Lambda}$

Order eV
for $y_\nu \sim 1$ and $\Lambda \sim 10^{14}$GeV

Note that such an operator breaks Lepton Number by 2 units

# SM Summary

| | | | color | chirality | hypercharge | weak isospin | electric charge | effective coupling to Z boson | |
|---|---|---|---|---|---|---|---|---|---|
| | SPIN | PARTICLES | $SU(3)_C$ × | $SU(2)_L$ × | $U(1)_Y$ | $T_{3_L}$ | $Q = T_{3_L} + Y$ | $g_{eff}$ | MEANING |
| LEPTONS | 1/2 | $L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L$ | 1 | 2 | $\begin{pmatrix} -1/2 \\ -1/2 \end{pmatrix}$ | $\begin{pmatrix} 1/2 \\ -1/2 \end{pmatrix}$ | $\begin{pmatrix} 0 \\ -1 \end{pmatrix}$ | $\begin{pmatrix} 1/2 \\ -1/2 + \sin^2\theta_W \end{pmatrix}$ | doublet under SU(2), singlet under SU(3) |
| | | $e_R$ | 1 | 1 | $-1$ | 0 | $-1$ | $\sin^2\theta_W$ | singlet under SU(2) and SU(3) |
| QUARKS | | $Q = \begin{pmatrix} u \\ d \end{pmatrix}_L$ | 3 | 2 | $\begin{pmatrix} 1/6 \\ 1/6 \end{pmatrix}$ | $\begin{pmatrix} 1/2 \\ -1/2 \end{pmatrix}$ | $\begin{pmatrix} 2/3 \\ -1/3 \end{pmatrix}$ | $\begin{pmatrix} 1/2 - \frac{2}{3}\sin^2\theta_W \\ -1/2 + \frac{1}{3}\sin^2\theta_W \end{pmatrix}$ | doublet under SU(2), triplet under SU(3) |
| | | $u_R$ | 3 | 1 | 2/3 | 0 | 2/3 | $-\frac{1}{3}\sin^2\theta_W$ | singlet under SU(2), triplet under SU(3) |
| | | $d_R$ | 3 | 1 | $-1/3$ | 0 | $-1/3$ | $\frac{1}{3}\sin^2\theta_W$ | singlet under SU(2), triplet under SU(3) |
| HIGGS | 0 | $H = \begin{pmatrix} h^+ \\ h^0 \end{pmatrix}$ | 1 | 2 | $\begin{pmatrix} 1/2 \\ 1/2 \end{pmatrix}$ | $\begin{pmatrix} 1/2 \\ -1/2 \end{pmatrix}$ | $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ | × | doublet under SU(2), singlet under SU(3) |

# Evolution of coupling constants

*Classical physics:* the forces depend on distances

*Quantum physics :* the charges depend on distances

QED virtual particles screen the electric charge: $\alpha \searrow$ when d $\nearrow$

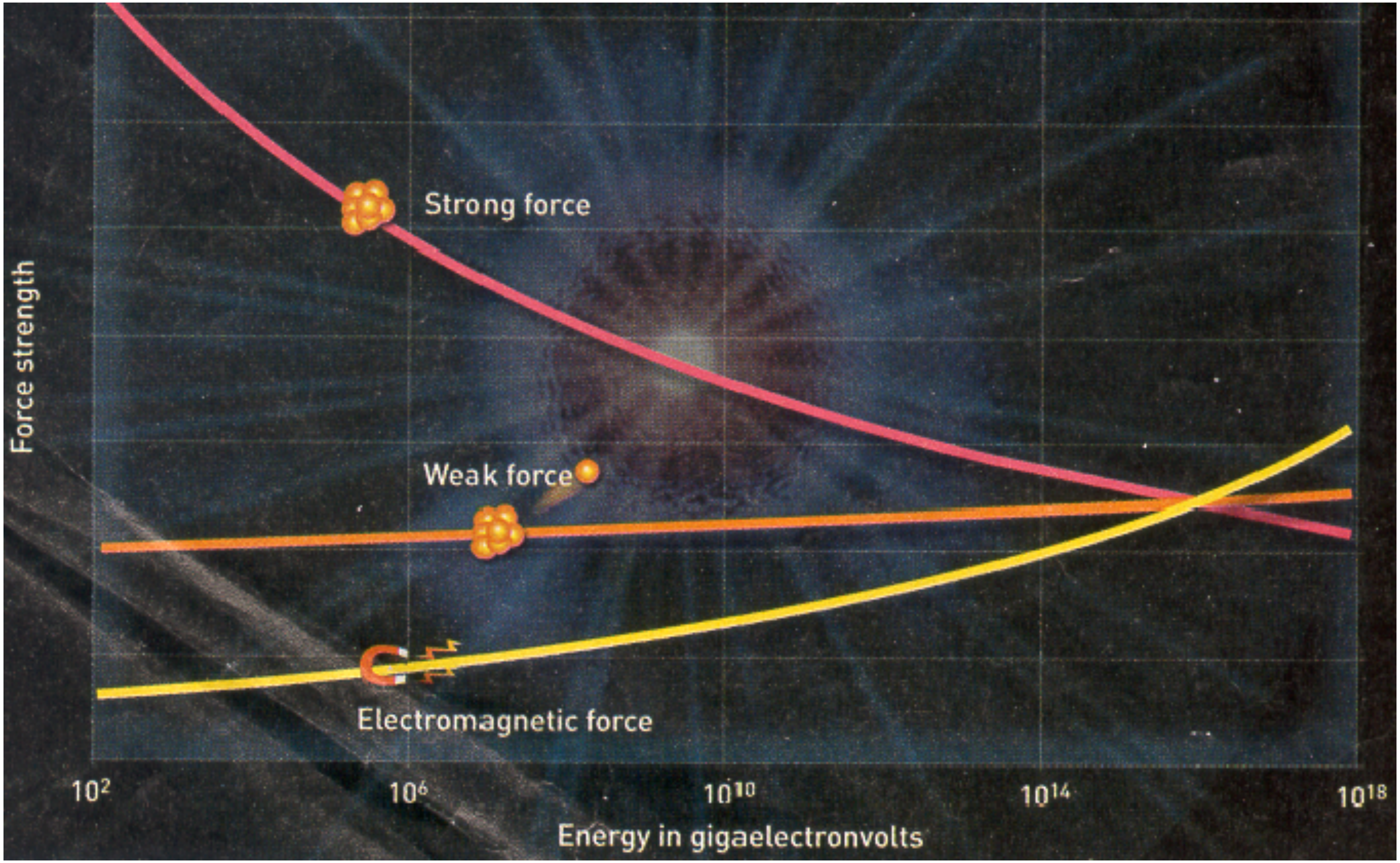
QCD virtual particles (quarks and *gluons*) screen the strong charge: $\alpha_s \nearrow$ when d $\nearrow$

'asymptotic freedom'

$$\frac{\partial \alpha_s}{\partial \log E} = \beta(\alpha_s) = \frac{\alpha_s^2}{\pi}\left(-\frac{11N_c}{6} + \frac{N_f}{3}\right)$$

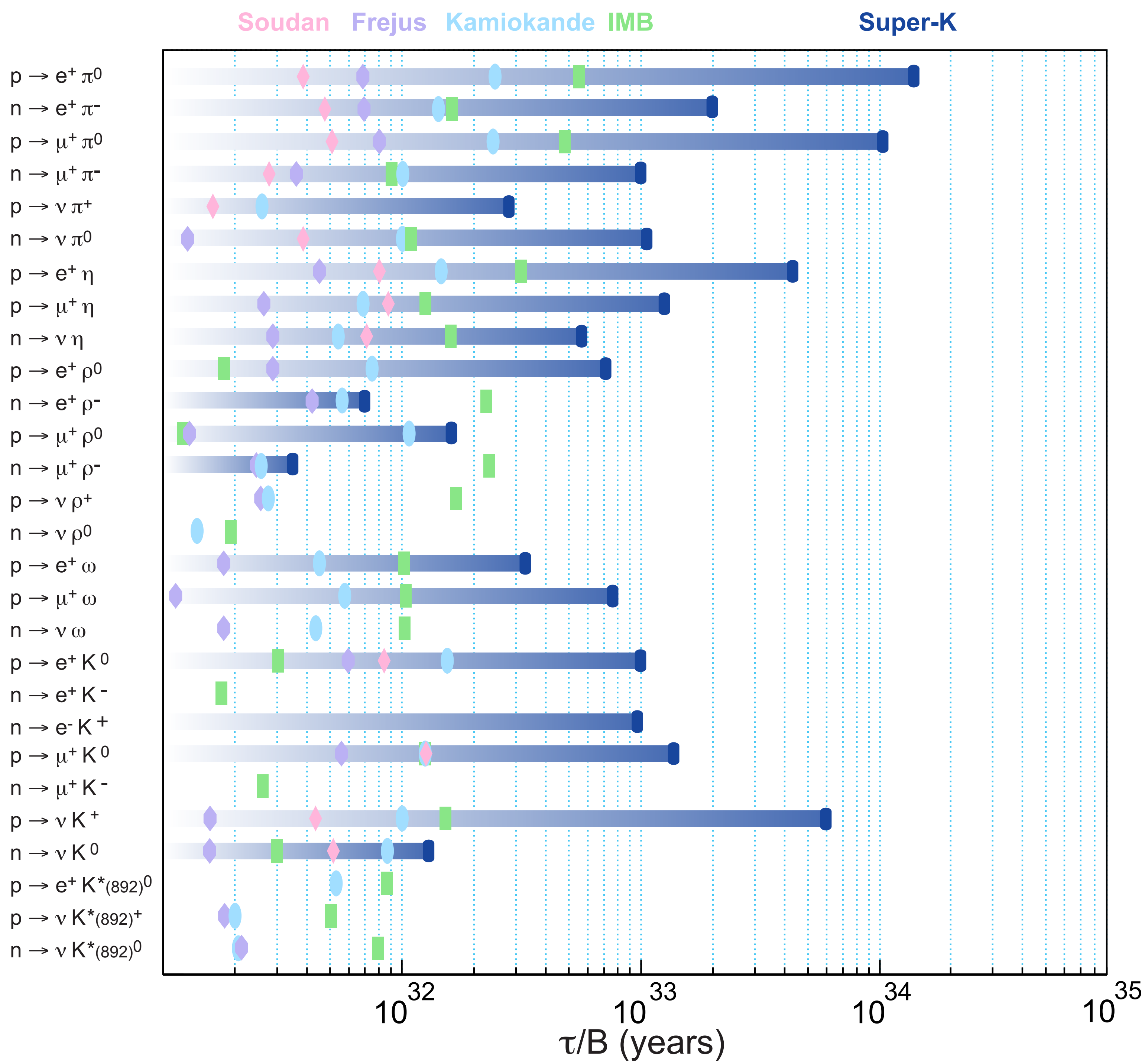
$\alpha_s$ becomes infinite at long distance: the quarks cannot escape →"confinement"

# Grand Unified Theories




A single form of matter
A single fundamental interaction

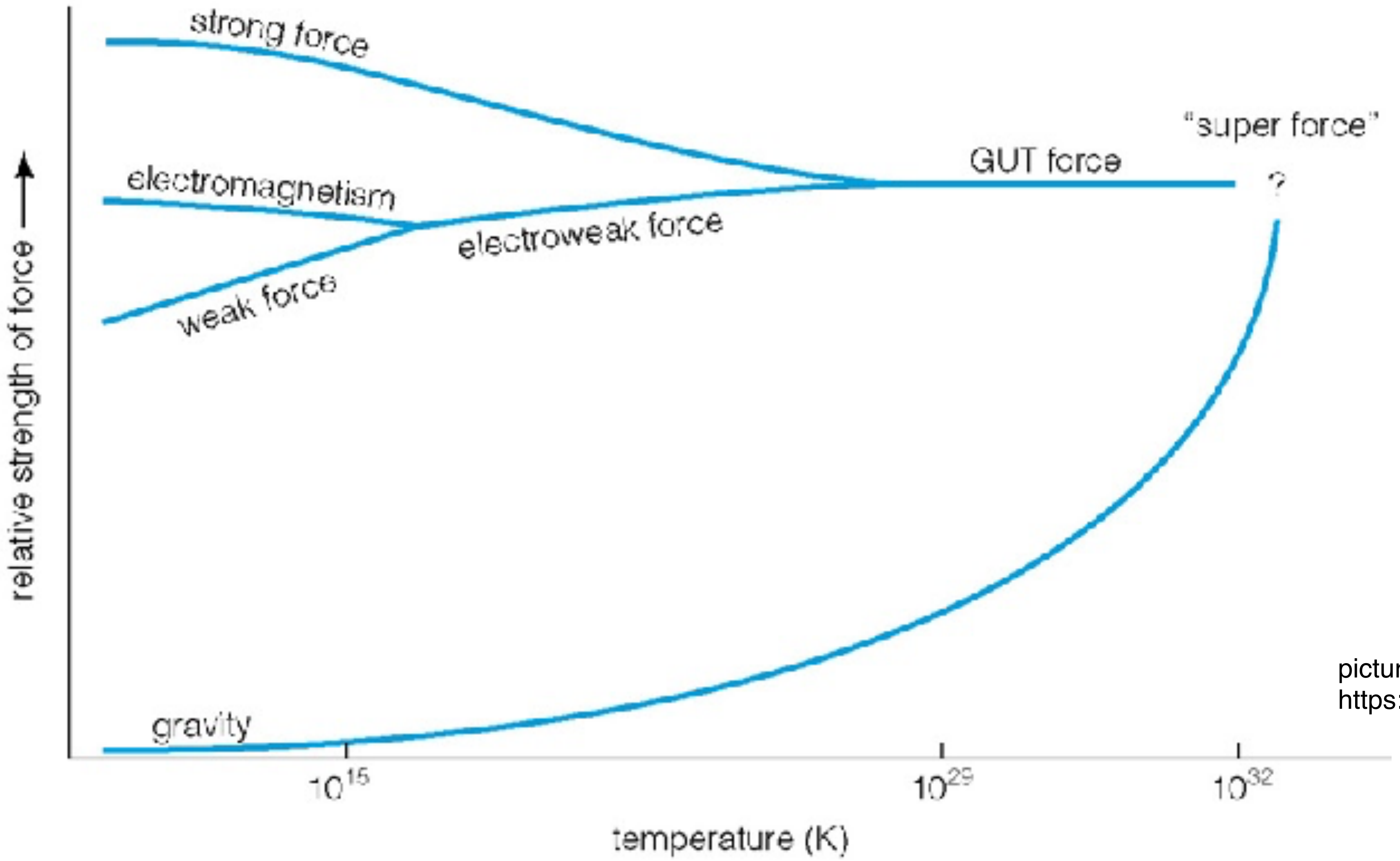
# Proton Decay




Babu et al '13

# SM & Gravity

It is actually possible to couple the SM to gravity and to quantise the graviton. The issue is that gravity is not renormalisable and to get ride of infinities in loop computation, one needs to add more and more counter-terms that are not present originally in the classical GR Lagrangian. At most gravity can be treated as an effective field theory and there are arguments that show that its UV completion is unlikely to be a quantum field theory but rather a theory of more complicated objects like matrices or strings. There is an important difference between gauge (spin-1) interactions and gravity: the gauge couplings of the former exhibit a logarithmic evolution with the energy of the process, while the strength of gravity grows like $E^2$. An important question is to figure out the scale of quantum gravity: is it $M_{Planck}$~$10^{19}$GeV? it could be lower down to few TeVs if there are (large or highly curved) extra dimensions. In that case, totally new phenomena could be observed at colliders… see the BSM lectures



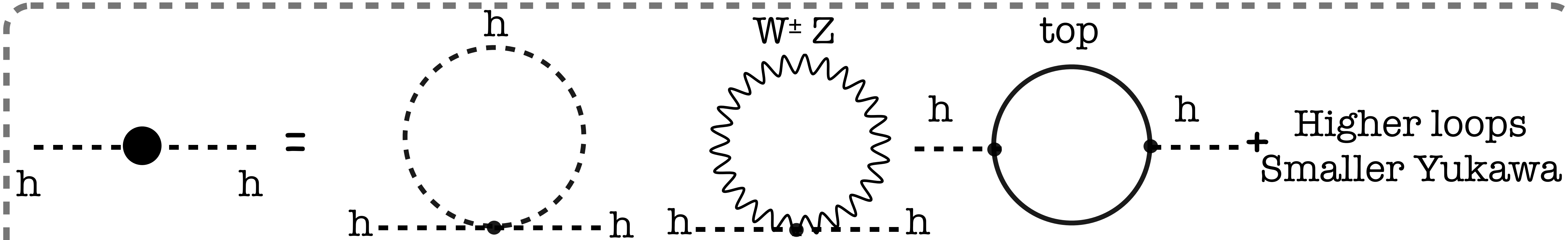

picture from:
https://is.gd/eoMCgC

# Quantum Instability of the Higgs Mass

The running of gauge couplings and of the Higgs quartic coupling is logarithmic:

$$\alpha_i^{-1}(\mu) = \alpha_i^{-1}(\mu_0) - \frac{b_i}{4\pi} \ln \mu^2/\mu_0^2$$

The Higgs mass has a totally different behaviour: it is highly dependent on the UV physics, which leads to the so called hierarchy problem.




$$\int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 - m^2} \propto \Lambda^2 \qquad \int \frac{d^4k}{(2\pi)^4} \frac{k^2}{(k^2 - m^2)^2} \propto \Lambda^2$$

Weisskopf '39
't hooft '79

$$\delta m_H^2 = \left(2m_W^2 + m_Z^2 + m_H^2 - 4m_t^2\right) \frac{3G_F\Lambda^2}{8\sqrt{2}\pi^2}$$

$$m_H^2 \sim m_0^2 - (115 \text{ GeV})^2 \left(\frac{\Lambda}{700 \text{ GeV}}\right)^2$$

# The hierarchy problem made easy

only a few electrons are enough to lift your hair (~ $10^{25}$ mass of $e^-$)

the electric force between 2 $e^-$ is $10^{43}$ times larger than their gravitational interaction

we don’t know why gravity is so weak?

we don’t know why the masses of particles are so small?

Several theoretical hypothesis

new dynamics? new symmetries? new space-time structure?

modification of special relativity? of quantum mechanics?

# How to Stabilise the Higgs Scale

## The spin trick

2s+1 polarisation states

a particle of spin s:

...with the only exception of a particle moving at the speed of light

... fewer polarisation states

Spin 1 Gauge invariance $\longrightarrow$ no longitudinal polarisation

Spin 1/2 Chiral symmetry $\longrightarrow$ only one helicity

$\searrow\nearrow$ m=0

If the symmetries are broken, the radiative mass will be set by the scale of symmetry breaking, not the UV/Planck scale

... but the Higgs is a spin 0 particle

# Symmetries to Stabilise a Scalar Potential

Supersymmetry

fermion ~ boson

Higher Dimensional
Lorentz invariance

$\Lleftarrow$ gauge-Higgs
unification models

$$A_\mu \sim A_5$$

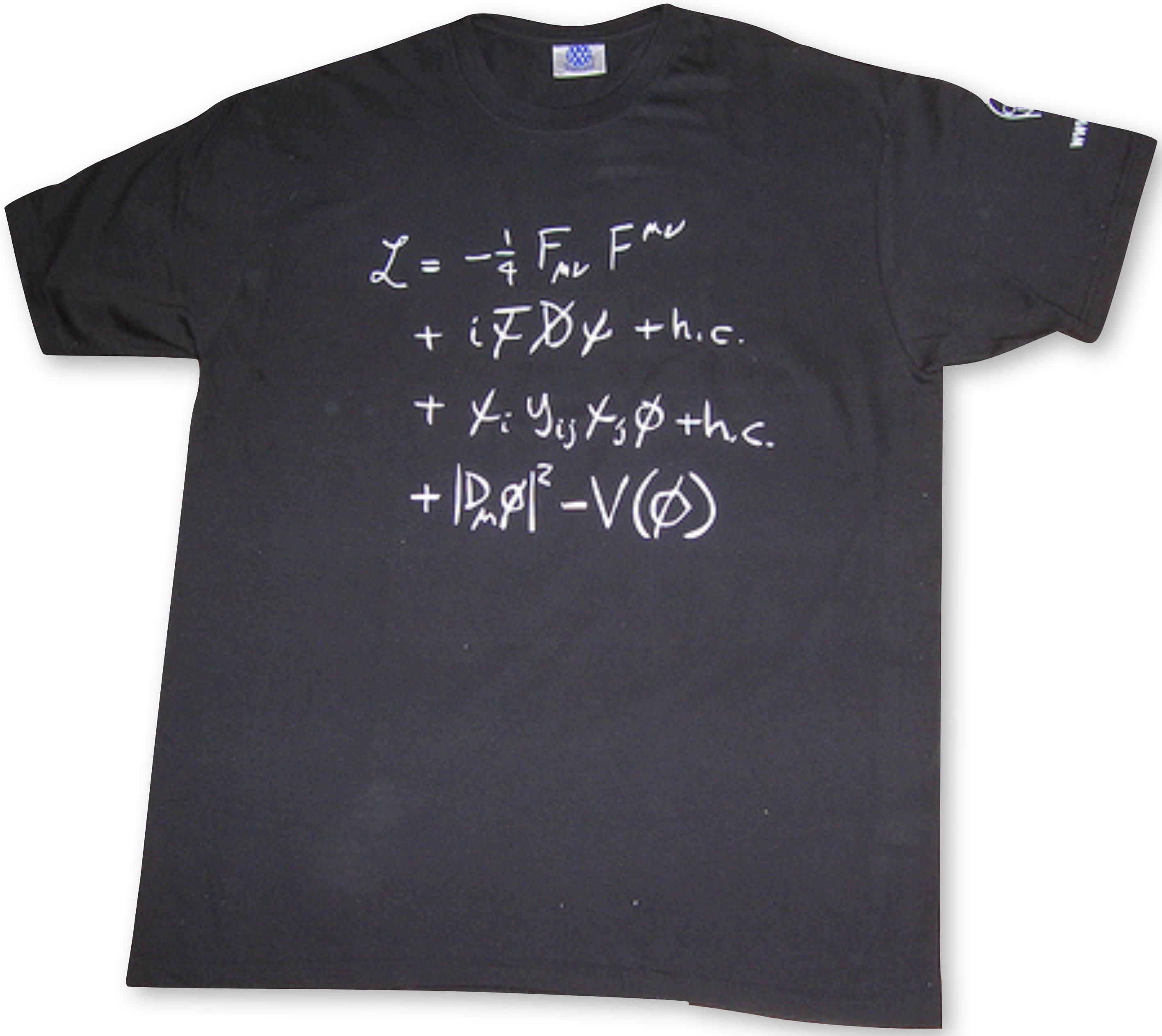
4D spin 1

4D spin 0

These symmetries cannot be exact symmetry of the Nature.
They have to be broken. We want to look for a soft breaking in
order to preserve the stabilisation of the weak scale.

# Conclusions

Hopefully you now understand all what is written on the CERN T-shirt




and you can safely go to the beach with it without fearing any question

# One day, one of you might take his job…

B. Clinton, Davos 2011
https://www.youtube.com/watch?v=p2dT7xVS6-s
(around 54'20")

Hopefully, that day you'll remember
what you have learnt during your stay at CERN

Thank you for your attention.
Good luck for your studies!

if you have question/want to know more

do not hesitate to send me an email

christophe.grojean@desy.de

# Technical Details for Advanced Students

# Dimensionality of π

In HEP natural units, we set c=ħ=1, such that [length]=[time]=[mass]$^{-1}$=[energy]$^{-1}$
But these fundamental constants are dimensionful. And it might be useful to keep track of the ħ-dimensions in addition to the mass dimension of any physical quantity

| | | $M^n$ | $\hbar^n$ |
|---|---|---|---|
| scalar field | $\phi$ | $1$ | $1/2$ |
| fermion field | $\psi$ | $3/2$ | $1/2$ |
| vector field | $A_\mu$ | $1$ | $1/2$ |
| mass | $m$ | $1$ | $0$ |
| gauge coupling | $g$ | $0$ | $-1/2$ |
| quartic coupling | $\lambda$ | $0$ | $-1$ |
| Yukawa coupling | $y_f$ | $0$ | $-1/2$ |

$$\mathcal{S} = \int d^4x \left(\mathcal{L}_0 + \hbar\mathcal{L}_1 + \hbar^2\mathcal{L}_2 + \ldots\right)$$

$[\mathcal{L}_0]_\hbar = 1$ $\quad [\mathcal{L}_1]_\hbar = 0$ $\quad [\mathcal{L}_2]_\hbar = -1$

$[\mathcal{L}_0]_M = 4$ $\quad [\mathcal{L}_1]_M = 4$ $\quad [\mathcal{L}_2]_M = 4$

example:
tree-level generated operator

$[\cdot]_\hbar = -1 \quad [\cdot]_\hbar = 2$

$$\frac{1}{M^2} g_*^2 \left(\partial^\mu |H|^2\right)^2$$

example:
one-loop generated operator

$$\frac{1}{M^2}\frac{g^2}{16\pi^2} g'^2 |H|^2 B_{\mu\nu}B^{\mu\nu}$$

The factors of π are very often associated to loop factors which are counting the ħ-dimension
Remember the normalisation of the states in QFT: $d^4k/(2\pi)^4$

# SU(5) GUT: Gauge Group Structure

## SU(3)$_C$xSU(2)$_L$xU(1)$_Y$: SM Matter Content

$$Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix} = (3,2)_{1/6}, \quad u_R^c = (\bar{3},1)_{-2/3}, \quad d_R^c = (\bar{3},1)_{1/3}, \quad L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} = (1,2)_{-1/2}, \quad e_R^c = (1,1)_1$$

How can you ever remember all these numbers?

## SU(3)$_C$xSU(2)$_L$xU(1)$_Y$ ⊂ SU(5)

SU(5)
Adjoint rep.

$$\left(\begin{array}{c|c} SU(2) & \\ \hline & SU(3) \end{array}\right)$$

$$\mathrm{Tr}(T^a T^b) = \frac{1}{2}\delta^{ab}$$

additional U(1) factor that commutes with SU(3)xSU(2)

$$T^{12} = \sqrt{\frac{3}{5}} \left(\begin{array}{cc|ccc} 1/2 & & & & \\ & 1/2 & & & \\ \hline & & -1/3 & & \\ & & & -1/3 & \\ & & & & -1/3 \end{array}\right)$$

$$\bar{5} = (1,2)_{-\frac{1}{2}\sqrt{\frac{3}{5}}} + (\bar{3},1)_{\frac{1}{3}\sqrt{\frac{3}{5}}}$$

$$\bar{5} = L + d_R^c$$

$$10 = (5\times 5)_A = (\bar{3},1)_{-\frac{2}{3}\sqrt{\frac{3}{5}}} + (3,2)_{\frac{1}{6}\sqrt{\frac{3}{5}}} + (1,1)_{\sqrt{\frac{3}{5}}}$$

$$10 = u_R^c + Q_L + e_R^c$$

$$T^{12} = \sqrt{\frac{3}{5}}Y \qquad g_5\sqrt{\frac{3}{5}} = g' \qquad g_5 = g = g_s$$

$$g_5 T^{12} = g' Y$$

$\sin^2\theta_W = \frac{3}{8}$ @ M$_{GUT}$

# SU(5) GUT: low energy consistency condition

$$\frac{1}{\alpha_i(M_Z)} = \frac{1}{\alpha_{GUT}} - \frac{b_i}{4\pi} \ln \frac{M_{GUT}^2}{M_Z^2} \qquad i = SU(3), SU(2), U(1)$$

$\alpha_3(M_Z), \alpha_2(M_Z), \alpha_1(M_Z)$ ⟵ experimental inputs

$b_3, b_2, b_1$ ⟵ predicted by the matter content

3 equations & 2 unknowns $(\alpha_{GUT}, M_{GUT})$

one consistency relation on low energy parameters

$$\epsilon_{ijk} \frac{b_j - b_k}{\alpha_i(M_Z)} = 0 \quad \Rightarrow \quad \boxed{\sin^2\theta_W = \frac{3(b_3-b_2)}{8b_3-3b_2-5b_1} + \frac{5(b_2-b_1)}{8b_3-3b_2-5b_1}\frac{\alpha_{em}(M_Z)}{\alpha_s(M_Z)}}$$

$$\alpha_{em}(M_Z) \approx \frac{1}{128} \qquad \alpha_s(M_Z) \approx 0.1184 \pm 0.0007$$

$$\Downarrow$$

$\sin^2\theta_W \approx 0.207$ not bad… (observed value: 0.23)
Even better in MSSM

# SU(5) GUT: low energy consistency condition

$$\frac{1}{\alpha_i(M_Z)} = \frac{1}{\alpha_{GUT}} - \frac{b_i}{4\pi} \ln \frac{M_{GUT}^2}{M_Z^2} \qquad i = SU(3), SU(2), U(1)$$

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3 equations & 2 unknowns $(\alpha_{GUT}, M_{GUT})$

one consistency relation on low energy parameters

$$M_{GUT} = M_Z \exp\left(2\pi \frac{3\alpha_s(M_Z) - 8\alpha_{em}(M_Z)}{(8b_3 - 3b_2 - 5b_1)\alpha_s(M_Z)\alpha_{em}(M_Z)}\right) \approx 7 \times 10^{14} \text{ GeV}$$

$$\alpha_{GUT}^{-1} = \frac{3b_3\alpha_s(M_Z) - (5b_1 + 3b_2)\alpha_{em}(M_Z)}{(8b_3 - 3b_2 - 5b_1)\alpha_s(M_Z)\alpha_{em}(M_Z)} \approx 41.5$$

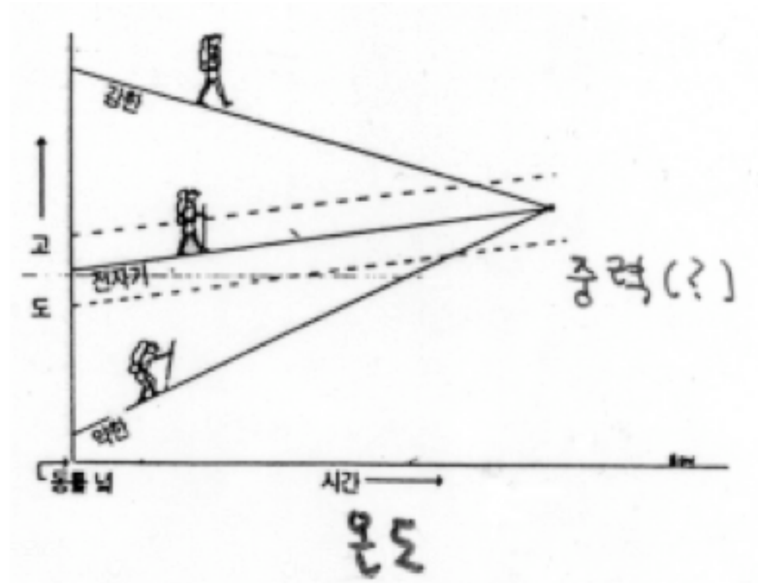
self-consistent computation:

- $M_{GUT} << M_{Pl}$ safe to neglect quantum gravity effects
- $\alpha_{GUT} << 1$ perturbative computation valid

# SU(5) GUT: SM $\beta$ fcts

g, g' and $g_s$ are different but this is a low energy artefact!

$$\boxed{\beta = \frac{dg}{d\log\mu} = -\frac{1}{16\pi^2} b g^3 + \dots}$$




$$\boxed{\frac{1}{g^2(Q)} = \frac{1}{g^2(Q_0)} + \frac{b}{16\pi^2}\ln\frac{Q^2}{Q_0^2}}$$

$$b = \frac{11}{3}T_2(\texttt{spin-1}) - \frac{2}{3}T_2(\texttt{chiral spin-1/2}) - \frac{1}{3}T_2(\texttt{complex spin-0})$$

$$\mathrm{Tr}\left(T^a(R)T^b(R)\right) = T_2(R)\delta^{ab} \qquad T_2(\texttt{fund}) = \frac{1}{2} \qquad T_2(\texttt{adj}) = N$$

$$b_{SU(3)} = \underbrace{\frac{11}{3}\times 3}_{g} - \frac{2}{3}\left(\underbrace{\frac{1}{2}\times 2\times 3}_{Q_L} + \underbrace{\frac{1}{2}\times 1\times 3}_{u_R^c} + \underbrace{\frac{1}{2}\times 1\times 3}_{d_R^c}\right) = 7$$

$$b_{SU(2)} = \underbrace{\frac{11}{3}\times 2}_{W^\pm, Z} - \frac{2}{3}\left(\underbrace{\frac{1}{2}\times 3\times 3}_{Q_L} + \underbrace{\frac{1}{2}\times 1\times 3}_{L}\right) - \underbrace{\frac{1}{3}\times\frac{1}{2}}_{H} = \frac{19}{6}$$

$$b_Y = -\frac{2}{3}\left(\underbrace{\left(\frac{1}{6}\right)^2 3\times 2\times 3}_{Q_L} + \underbrace{\left(-\frac{2}{3}\right)^2 3\times 3}_{u_R^c} + \underbrace{\left(\frac{1}{3}\right)^2 3\times 3}_{d_R^c} + \underbrace{\left(-\frac{1}{2}\right)^2 2\times 3}_{L} + \underbrace{(1)^2\times 3}_{e_R^c}\right) - \underbrace{\frac{1}{3}\left(\frac{1}{2}\right)^2\times 2}_{H} = -\frac{41}{6} \Rightarrow b_{T^{12}} = -\frac{41}{10}$$

# SU(5) GUT: SM vs MSSM $\beta$ fcts

**chiral superfield**

complex spin-0
Weyl spin-1/2
in same representation of gauge group

**vector superfield**

Weyl spin-1/2
real spin-1
in same representation of gauge group

$$b = \frac{11}{3} T_2(\texttt{vector}) - \frac{2}{3} T_2(\texttt{vector}) - \frac{2}{3} T_2(\texttt{chiral}) - \frac{1}{3} T_2(\texttt{chiral}) = 3\, T_2(\texttt{vector}) - T_2(\texttt{chiral})$$

## MSSM Chiral Content

$$Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix} = (3,2)_{1/6}, \quad U = (\bar{3},1)_{-2/3}, \quad D = (\bar{3},1)_{1/3}, \quad L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} = (1,2)_{-1/2}, \quad E = (1,1)_1, \; H_u = (1,2)_{1/2}, \quad H_d = (1,2)_{-1/2}$$

$$b_{SU(3)} = \overset{g}{3 \times 3} - \left( \overset{Q_L}{\frac{1}{2} \times 2 \times 3} + \overset{U}{\frac{1}{2} \times 1 \times 3} + \overset{D}{\frac{1}{2} \times 1 \times 3} \right) = 3$$

$$b_{SU(2)} = \overset{W^\pm, Z}{3 \times 2} - \left( \overset{Q_L}{\frac{1}{2} \times 3 \times 3} + \overset{L}{\frac{1}{2} \times 1 \times 3} \right) - \overset{H_u}{\frac{1}{2}} - \overset{H_d}{\frac{1}{2}} = -1$$

$$b_Y = -\left( \overset{Q_L}{\left(\frac{1}{6}\right)^2 3 \times 2 \times 3} + \overset{U}{\left(-\frac{2}{3}\right)^2 3 \times 3} + \overset{D}{\left(\frac{1}{3}\right)^2 3 \times 3} + \overset{L}{\left(-\frac{1}{2}\right)^2 2 \times 3} + \overset{E}{(1)^2 \times 3} \right) - \overset{H_u}{\left(\frac{1}{2}\right)^2 \times 2} - \overset{H_d}{\left(\frac{1}{2}\right)^2 \times 2} = -11 \;\Rightarrow\; b_{T^{12}} = -\frac{33}{5}$$

# SU(5) GUT: MSSM GUT

$$b_3 = 3, \quad b_2 = -1, \quad b_1 = -33/5$$

low-energy consistency relation for unification

$$\sin^2\theta_W = \frac{3(b_3 - b_2)}{8b_3 - 3b_2 - 5b_1} + \frac{5(b_2 - b_1)}{8b_3 - 3b_2 - 5b_1}\frac{\alpha_{em}(M_Z)}{\alpha_s(M_Z)} \approx 0.23$$

squarks and sleptons form complete SU(5) reps ➙ they don't improve unification!
gauginos and higgsinos are improving the unification of gauge couplings

GUT scale predictions

$$M_{GUT} = M_Z \exp\left(2\pi\frac{3\alpha_s(M_Z) - 8\alpha_{em}(M_Z)}{(8b_3 - 3b_2 - 5b_1)\alpha_s(M_Z)\alpha_{em}(M_Z)}\right) \approx 2 \times 10^{16}\ \text{GeV}$$

$$\alpha_{GUT}^{-1} = \frac{3b_3\alpha_s(M_Z) - (5b_1 + 3b_2)\alpha_{em}(M_Z)}{(8b_3 - 3b_2 - 5b_1)\alpha_s(M_Z)\alpha_{em}(M_Z)} \approx 24.3$$

# Proton Decay

| | Mode | Partial mean life ($10^{30}$ years) | Confidence level |
|---|---|---|---|
| | | **Antilepton + meson** | |
| $\tau_1$ | $N \to e^+\pi$ | $> 2000$ ($n$), $> 8200$ ($p$) | 90% |
| $\tau_2$ | $N \to \mu^+\pi$ | $> 1000$ ($n$), $> 6600$ ($p$) | 90% |
| $\tau_3$ | $N \to \nu\pi$ | $> 1100$ ($n$), $> 390$ ($p$) | 90% |
| $\tau_4$ | $p \to e^+\eta$ | $> 4200$ | 90% |
| $\tau_5$ | $p \to \mu^+\eta$ | $> 1300$ | 90% |
| $\tau_6$ | $n \to \nu\eta$ | $> 158$ | 90% |
| $\tau_7$ | $N \to e^+\rho$ | $> 217$ ($n$), $> 710$ ($p$) | 90% |
| $\tau_8$ | $N \to \mu^+\rho$ | $> 228$ ($n$), $> 160$ ($p$) | 90% |
| $\tau_9$ | $N \to \nu\rho$ | $> 19$ ($n$), $> 162$ ($p$) | 90% |
| $\tau_{10}$ | $p \to e^+\omega$ | $> 320$ | 90% |
| $\tau_{11}$ | $p \to \mu^+\omega$ | $> 780$ | 90% |
| $\tau_{12}$ | $n \to \nu\omega$ | $> 108$ | 90% |
| $\tau_{13}$ | $N \to e^+K$ | $> 17$ ($n$), $> 1000$ ($p$) | 90% |
| $\tau_{14}$ | $p \to e^+K^0_S$ | | |
| $\tau_{15}$ | $p \to e^+K^0_L$ | | |
| $\tau_{16}$ | $N \to \mu^+K$ | $> 26$ ($n$), $> 1600$ ($p$) | 90% |
| $\tau_{17}$ | $p \to \mu^+K^0_S$ | | |
| $\tau_{18}$ | $p \to \mu^+K^0_L$ | | |
| $\tau_{19}$ | $N \to \nu K$ | $> 86$ ($n$), $> 5900$ ($p$) | 90% |
| $\tau_{20}$ | $n \to \nu K^0_S$ | $> 260$ | 90% |
| $\tau_{21}$ | $p \to e^+K^*(892)^0$ | $> 84$ | 90% |
| $\tau_{22}$ | $N \to \nu K^*(892)$ | $> 78$ ($n$), $> 51$ ($p$) | 90% |
| | | **Antilepton + mesons** | |
| $\tau_{23}$ | $p \to e^+\pi^+\pi^-$ | $> 82$ | 90% |
| $\tau_{24}$ | $p \to e^+\pi^0\pi^0$ | $> 147$ | 90% |
| $\tau_{25}$ | $n \to e^+\pi^-\pi^0$ | $> 52$ | 90% |
| $\tau_{26}$ | $p \to \mu^+\pi^+\pi^-$ | $> 133$ | 90% |
| $\tau_{27}$ | $p \to \mu^+\pi^0\pi^0$ | $> 101$ | 90% |
| $\tau_{28}$ | $n \to \mu^+\pi^-\pi^0$ | $> 74$ | 90% |
| $\tau_{29}$ | $n \to e^+K^0\pi^-$ | $> 18$ | 90% |

**ΔB=ΔL=1 decay bounds**

| | Mode | Partial mean life ($10^{30}$ years) | Confidence level |
|---|---|---|---|
| | | **Lepton + meson** | |
| $\tau_{30}$ | $n \to e^-\pi^+$ | $> 65$ | 90% |
| $\tau_{31}$ | $n \to \mu^-\pi^+$ | $> 49$ | 90% |
| $\tau_{32}$ | $n \to e^-\rho^+$ | $> 62$ | 90% |
| $\tau_{33}$ | $n \to \mu^-\rho^+$ | $> 7$ | 90% |
| $\tau_{34}$ | $n \to e^-K^+$ | $> 32$ | 90% |
| $\tau_{35}$ | $n \to \mu^-K^+$ | $> 57$ | 90% |
| | | **Lepton + mesons** | |
| $\tau_{36}$ | $p \to e^-\pi^+\pi^+$ | $> 30$ | 90% |
| $\tau_{37}$ | $n \to e^-\pi^+\pi^0$ | $> 29$ | 90% |
| $\tau_{38}$ | $p \to \mu^-\pi^+\pi^+$ | $> 17$ | 90% |
| $\tau_{39}$ | $n \to \mu^-\pi^+\pi^0$ | $> 34$ | 90% |
| $\tau_{40}$ | $p \to e^-\pi^+K^+$ | $> 75$ | 90% |
| $\tau_{41}$ | $p \to \mu^-\pi^+K^+$ | $> 245$ | 90% |

**ΔB=-ΔL=1 decay bounds**

# Naturalness principle @ work

Following the arguments of Wilson, ‘t Hooft (and others):
only small numbers associated to the breaking of a symmetry survive quantum corrections

**Introduce new degrees of freedom to regulate the high-energy behavior**

Beautiful examples of naturalness to understand the need of “new” physics

see for instance Giudice ’13 (and refs. therein) for an account

- the need of the **positron** to screen the electron self-energy: $\Lambda < m_e/\alpha_{\text{em}}$
- the **rho meson** to cutoff the EM contribution to the charged pion mass: $\Lambda < \delta m_\pi^2/\alpha_{\text{em}}$
- the kaon mass difference regulated by the **charm** quark: $\Lambda^2 < \frac{\delta m_K}{m_K} \frac{6\pi^2}{G_F^2 f_K^2 \sin^2\theta_C}$
- the light **Higgs** boson to screen the EW corrections to gauge bosons self-energies
- ...
- **new physics** at the weak scale to cancel the UV sensitivity of the Higgs mass?