2. EFT/NATURALNESS

Last lecture

- SM is successful but arbitrary and incomplete.
- BSM is constrained not only by experiment/observation, but also by theoretical consistency.

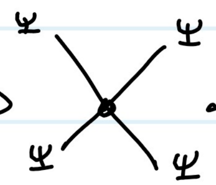
This Lecture

- Fermi theory (EFT of weak interactions)
- LHC no-lose theorem from finiteness of amplitudes
- SM is an EFT
- Naturalness expectation for EFT cut-off scale
- Q&A: universality of spin 2 interactions (gravity)

Fermi theory

At low energies $E \ll \Lambda$, where $\Lambda \sim M_W$ is the scale of the weak bosons, the weak interactions can be parametrised by a four-fermion operator in the Lagrangian:

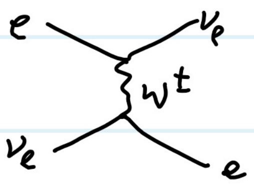
$$\mathcal{L}_{4\text{-fermion}} = \frac{c}{\Lambda^2}(\bar{\Psi}\gamma^\mu\Psi)(\bar{\Psi}\gamma_\mu\Psi) \quad \Rightarrow$$



$$\propto c/\Lambda^2$$

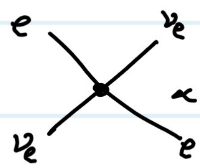
We now know the theory at higher energies is the SM: $W^\pm/Z$ bosons as the mediators with Higgs playing a crucial role.

SM reproduces Fermi theory at lower energies:



$$\propto \frac{g^2}{E^2 - M_W^2} = -\frac{g^2}{M_W^2}\frac{1}{\left(1 - E^2/M_W^2\right)}$$

$$\simeq -\frac{g^2}{M_W^2} \quad \text{for } E \ll M_W^2 \quad \Rightarrow$$

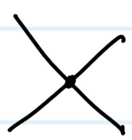


$$\propto \frac{g^2}{M_W^2}$$

LHC no-lose theorem

As another example of the power of theoretical consistency, we can start with Fermi theory and reproduce the structure of Yang-Mills theory and the Higgs mechanism just by requiring finiteness of amplitudes.

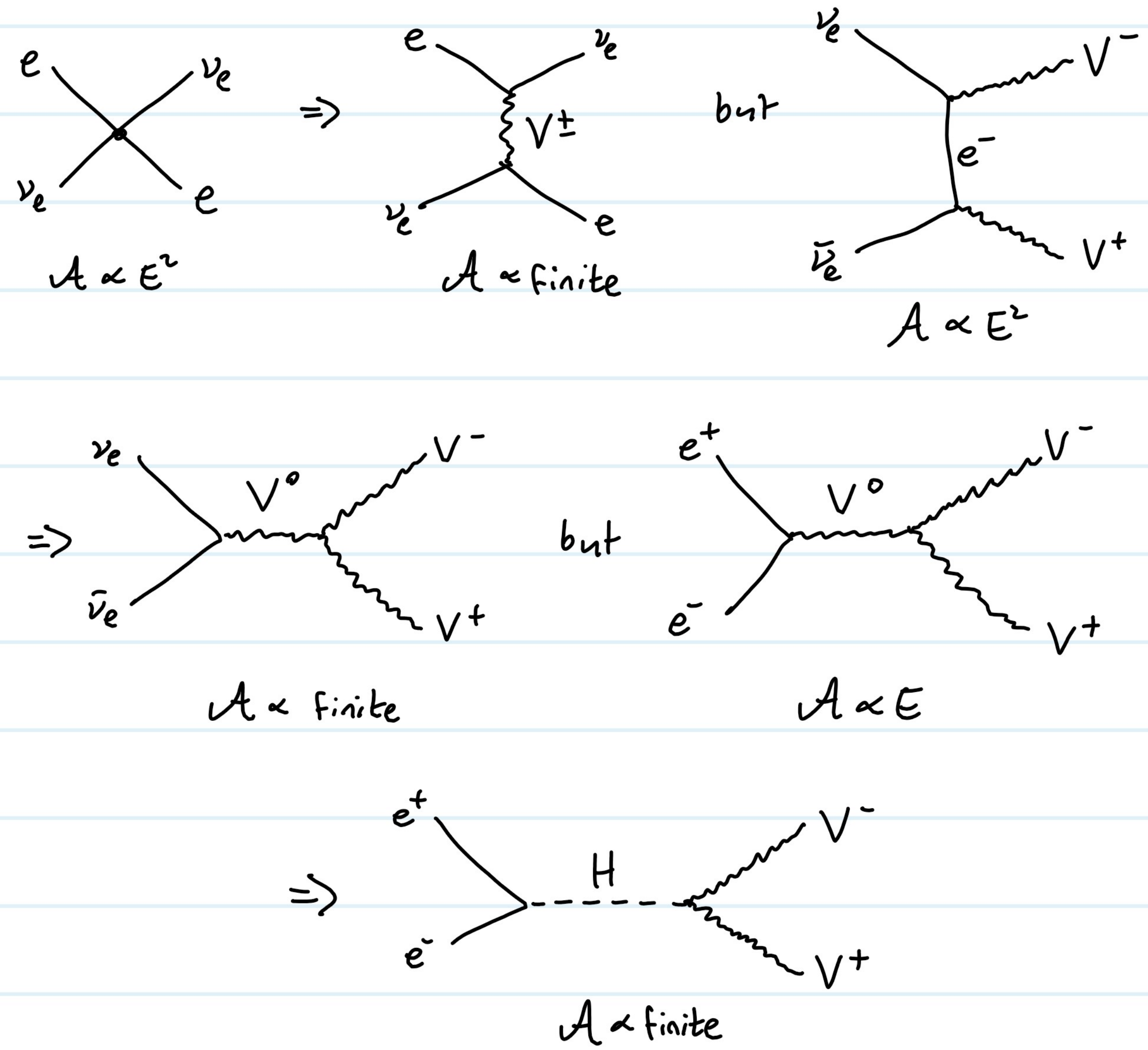
First note that $2 \to 2$ amplitudes must be dimensionless (follows from dimension of cross-section expression). Since the interaction coupling has dimension $1/\Lambda^2$, the amplitude must dimensionally be of the form



$$\sim \frac{c}{\Lambda^2} E^2$$

For $E \gtrsim \Lambda/c$ this divergent amplitude will violate unitarity.

we can add an intermediate vector boson to make this amplitude finite. However, this makes another process divergent which requires another boson for finiteness. Another amplitude is then divergent which requires a scalar to be added. The couplings of all these vector bosons and scalar reproduces Yang-Mills + Higgs.



Without the Higgs, amplitudes diverge and unitarity is violated at LHC energies. Therefore either the Higgs or something <u>must</u> show up!

the SM is an EFT

Old understanding of QFT: "Renormalisable" Lagrangian i.e. Operators up to mass-dimension 4 are more fundamental. e.g. QED:

$$\mathcal{L}_{QED} = \underbrace{\bar{\Psi} i \gamma^{\mu}(\partial_{\mu} - ig A_{\mu}) \Psi}_{\text{dim-4}} - m \underbrace{\bar{\Psi} \Psi}_{\text{dim-3}} - \frac{1}{4} \underbrace{F_{\mu\nu} F^{\mu\nu}}_{\text{dim-4}}$$

[Recall $S = \int d^4x \, \mathcal{L}$ is dimensionless, $e^{\frac{i}{\hbar} S}$, $[S] = 0 \Rightarrow [\mathcal{L}] = M^4$ since $[\underbrace{d^4x}_{dt\,dx\,dy\,dz}] = M^{-4}$ in natural units $\hbar = c = 1$.]

Modern understanding of QFT (Wilson, Weinberg '70s): Most general Lagrangian allowed by gauge and Lorentz symmetries. Our QFT's are not fundamental, they are <u>effective field theories</u> (EFT).

$$\mathcal{L}^{EFT}_{QED} = \mathcal{L}_{QED} + \frac{c_6}{\Lambda^2} \underbrace{\bar{\Psi} \gamma^{\mu} \Psi \bar{\Psi} \gamma_{\mu} \Psi}_{\text{dim-6}} + \frac{c_8}{\Lambda^4} \underbrace{(F_{\mu\nu} F^{\mu\nu})^2}_{\text{dim-8}} + \ldots$$

There is nothing special about $\mathcal{L}_{QED}$ restricted to operator dim ≤ 4. Indeed we saw that the higher-dimensional operators (dim > 4)

are generated by $\mathcal{L}_{SM}$ as a low-energy approximation.

Just like QED and Fermi theory, the SM is also an EFT.
We fix the experimentally known particle content and their quantum numbers, which defines the EFT and fixes the Lagrangian:

$$\{Q_L, L_L, u_R, d_R, l_R\} \times 3 \text{ generations } + H$$

$$(3,2,\tfrac{1}{6}),\ (1,2,-\tfrac{1}{2}),\ (3,1,\tfrac{2}{3}),\ (3,1,-\tfrac{1}{3}),\ (1,1,-1),\ (1,2,\tfrac{1}{2})$$

quantum numbers under $SU(3)_c \times SU(2)_L \times U(1)_Y$

Upto mass-dimension 4, the operators we can write down are

$$\mathcal{L}_{SM} = \mathcal{L}_M + \mathcal{L}_G + \mathcal{L}_H + \mathcal{L}_Y$$

$$\mathcal{L}_M = \bar{Q}_L i\gamma^\mu D^L_\mu Q_L + \bar{q}_R i\gamma^\mu D^R_\mu q_R + \bar{L}_L i\gamma^\mu D^L_\mu L_L + \bar{l}_R i\gamma^\mu D^R_\mu l_R$$

$$\mathcal{L}_G = -\frac{1}{4} B_{\mu\nu}B^{\mu\nu} - \frac{1}{4} W^a_{\mu\nu}W^{a\mu\nu} - \frac{1}{4} G_{\mu\nu}G^{\mu\nu} + \frac{\Theta}{16\pi^2} G_{\mu\nu}\tilde{G}^{\mu\nu}$$

$$\mathcal{L}_H = (D^L_\mu H)^\dagger (D^L_\mu H) + \mu^2 |H|^2 - \lambda (H^\dagger H)^2$$

$$\mathcal{L}_Y = y_d \bar{Q}_L H u_R + y_u \bar{Q}_L H^c u_R + y_L \bar{L}_L H \ell_R + h.c.$$

$$\left[D^L_\mu \equiv \partial_\mu - igW^a_\mu T^a - iYg'B_\mu \qquad D^R_\mu \equiv \partial_\mu - iYg'B_\mu \right]$$

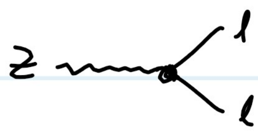
Note that there are global symmetries of $\mathcal{L}_{SM}$ (they are spacetime independent, unlike the local symmetries $SU(3)_c \times SU(2)_L \times U(1)_Y$). For example we can assign a lepton number L to the leptons and a baryon number B to the baryons. None of the operators in $\mathcal{L}_{SM}$ violate this. We did not impose this global symmetry; it is an accidental consequence of imposing the gauge and Lorentz symmetries. B and L are said to be accidental symmetries.

This is only an accident that holds for $\mathcal{L}_{SM}$, up to dim 4. At dim-5 we can write down the Weinberg operator that violates lepton number L:

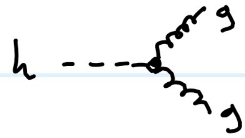
$$\mathcal{L}^{\text{dim-5}}_{SMEFT} = \frac{c_5}{\Lambda} (\bar{L}_L H^c)(L^c_L H^c)$$

At dim-6, we can write down many more operators.

e.g. $\frac{c_R^{\ell}}{\Lambda^2}\, iH^\dagger \overleftrightarrow{D}_\mu H\, \bar{\ell}_R \gamma^\mu \ell_R \supset Z_\mu \bar{\ell}_R \gamma^\mu \ell_R$

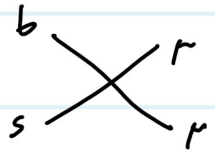


$\frac{c_R^{\ell}}{\Lambda^2}\, iH^\dagger \overleftrightarrow{D}_\mu H\, \bar{\ell}_R \gamma^\mu \ell_R \supset h\, G_{\mu\nu} G^{\mu\nu}$



$\frac{c_p}{\Lambda^2}\, qqq\ell$ violates B and L ($B-L$ conserved)

$\frac{c}{\Lambda^2}\, qq\ell\ell$



Flavour anomalies

Naturalness

Different notions of naturalness:

- Arbitrariness in structure e.g. yukawas, $m_{inertial} = m_{gravity}$
- Strange patterns or numbers (Dirac: $\mathcal{O}(1)$ natural)
- Fine-tuned cancellations

Fine-tuned cancellations are a sharp quantitative statement. How we interpret them is subjective.

We've seen that an EFT comes with a cut-off scale Λ.

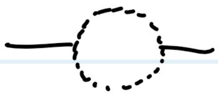
This gives a natural expectation for the sizes of parameters according to the symmetries of the theory. For example B- and L-violating effects are naturally small due to being (accidentally) conserved at the dim ≤ 4 level, while higher-dimensional (dim > 4) operators are suppressed by some inverse power of a mass scale.

Schematically, the EFT Lagrangian has a cut-off scale that sets the natural scale where one is needed dimensionally for operators $\mathcal{O}^{(n)}$ of dim-n:

$$\mathcal{L}_{EFT} = \Lambda^4 + \Lambda^2 \mathcal{O}^{(2)} + \Lambda \mathcal{O}^{(3)} + \mathcal{O}^{(4)} + \frac{1}{\Lambda^2}\mathcal{O}^{(6)} + \frac{1}{\Lambda^4}\mathcal{O}^{(8)} + \dots$$

Other dimensionful parameters smaller than Λ, such as masses or dimensionful couplings, can set the scale instead e.g. $m\mathcal{O}^{(3)}$ if $\mathcal{O}^{(3)} = \bar{\Psi}\Psi$ is a fermion mass term. However, quantum corrections will correct any dimensionful parameter and dominate, setting Λ again as the relevant dimensionful quantity, unless the dimensionful parameter is protected by some symmetry from quantum corrections.

For fermion masses, this is indeed the case. Quantum corrections δm to m is proportional to m itself:

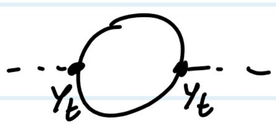
$$\delta m \sim$$  $$\sim m \log\left(\frac{\Lambda^2}{\mu^2}\right)$$

So if $m \ll \Lambda$ to begin with, it remains small under quantum corrections.

The reason is that the $m\underbrace{\bar{\Psi}\Psi}_{\bar{\Psi}_L\Psi_R + \bar{\Psi}_R\Psi_L}$ term breaks a symmetry:

without it, i.e. if $m = 0$, a chiral symmetry is restored where the left and right degrees of freedom can transform independently under separately conserved global symmetries. If m is the only parameter in the theory that breaks this chiral symmetry, then it follows that any quantum contributions to the $\mathcal{O}^{(3)} = \bar{\Psi}\Psi$ operator must be proportional to m itself, since in the limit $m \to 0$ the theory respects the chiral symmetry; quantum corrections calculated in a chirally symmetric theory can't violate chiral symmetry!

For scalar masses, $m^2\phi^2$, the same is true if there is no symmetry that is restored as $m \to 0$. The Higgs receives quantum corrections:

$$\delta m_H^2 \sim$$ 

$$= \frac{3y_t^2}{4\pi^2}\Lambda^2$$

$$\Delta_{\text{fine-tuning}} \sim \frac{\delta m_H^2}{m_H^2} \sim \frac{3y_t^2}{4\pi^2}\left(\frac{\Lambda}{m_H}\right)^2 \simeq \left(\frac{\Lambda}{450\ \text{GeV}}\right)^2$$

Note that this is not an artefact of using a momentum cut-off. For those familiar with loop integrals. e.g. using dim. reg., with heavy fermion Ψ at cut-off of EFT, $\Lambda \sim M_\Psi$, coupling to ϕ through $\mathcal{L} \supset -y\phi\bar{\Psi}\Psi$:

$$\delta m_\phi^2 = \frac{y^2}{4\pi^2}\left(1 - 3\log\frac{M^2}{\mu^2}\right)M_\Psi^2$$

correction is still proportional to Λ^2.

Note also that in the SM the Higgs mass is simply an input parameter and not calculable. There are no quantum corrections to its mass since it's just a number that we measure. However we expect the SM and the Higgs sector to emerge from some deeper underlying theory in which electroweak symmetry breaking can be understood more satisfactorily and the microscopic origins of the Higgs will lead to a calculable Higgs mass and potential. In this case there will be a very real fine tuning between different contributions to this calculation of the Higgs.

This reasoning is not new – it works! The divergent self-energy of the electron's electromagnetic field is the first example. Classically, we have the contribution to the electron's rest mass energy:

$$m_e^{exp} c^2 = m_e^0 c^2 + \frac{e^2}{4\pi\varepsilon_0 r_e^2}$$

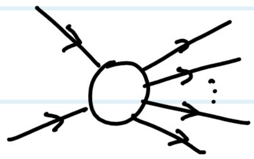
The Coulomb field energy contribution grows as the size of the electron becomes smaller, $r_e \to 0$. We can obtain the experimentally measured value $m_e^{exp} \simeq 0.511$ GeV by fixing the parameter m_e^0 appropriately. If $r_e \lesssim 10^{-15}$ m then we have a rather strange cancellation that is necessary to get the experimental value. As it happens, new physics kicks in at a distance scale of 10^{-13} m : Quantum mechanics! The contribution of electron-positron pairs fluctuating in and out of the quantum vacuum contributes to the calculation of the self-energy in such a way as to cancel the Coulomb contribution. The fine-tuning in this case is therefore avoided by requiring the existence of an antiparticle for every particle.

Another example is the pion. This is a scalar triplet consisting of a neutral π^0 and charged $\pi^\pm$. The quantum correction to the charged pion masses can be calculated as for the Higgs. To avoid fine-tuning, something should modify the calculation around $\Lambda \lesssim 750$ MeV. Indeed, this is where other QCD resonances such as the ρ and a_1 appear. The cut-off of the low-energy pion theory at which the underlying QCD theory takes over is set by considerations of naturalness.

The same arguments applied to Kaons led to the prediction of the charm quark. Clearly there is something to this reasoning of avoiding fine-tuned cancellations. The Higgs being a scalar, it was therefore widely expected to also come with a natural cut-off at which some new physics would enter to control its quadratically divergent mass correction.

Weinberg's soft emission amplitudes (Weinberg QFT vol. I Sec. 13.1)

Consider the matrix element for an arbitrary process $\alpha \to \beta$ with amplitude $\mathcal{A}_{\alpha\to\beta}$:

$\mathcal{A}_{\alpha\to\beta}$ = 

Attach a soft photon emission with outgoing momentum q to an outgoing charged-particle line with momentum p. We must then add an additional charged-particle propagator with momentum $p+q$. For spin-0 charged particles with mass m, charge $+e$, this propagator and the charged-particle-photon vertex gives the additional factor

$$i(2\pi)^4 e\,(2p^\mu + q^\mu)\left[\frac{-i}{(2\pi)^4}\frac{1}{(p+q)^2+m^2-i\varepsilon}\right]$$

$$\xrightarrow[\lim q\to 0]{} \frac{e\,p^\mu}{p\cdot q - i\varepsilon} \qquad \text{(redefining positive infinitesimal } \varepsilon\text{)}$$

(Can show that this is actually true for any spin in limit $q \to 0$).

A soft photon can be emitted from any incoming or outgoing charged-particle line. The amplitude $\mathcal{A}^{\mu}_{\alpha\to\beta+\gamma}(q)$ for the process $\alpha\to\beta$ plus a soft photon emission of momentum q and polarisation index μ can therefore be written as

$$\mathcal{A}^{\mu}_{\alpha\to\beta+\gamma}(q) \xrightarrow[q\to 0]{} \mathcal{A}_{\alpha\to\beta} \sum_n \frac{\eta_n e_n p_n^{\mu}}{p_n\cdot q - i\eta_n\varepsilon}$$

where p_n, e_n are the 4-momentum and charge of the n^{th} particle and $\eta_n = +1\ (-1)$ for final (initial) state.

The amplitude must be contracted with photon polarisation vector $e_{\mu}(\vec{q},\pm)$. But $e_{\mu}(\vec{q},\pm)$ is not a 4-vector; Lorentz transforms as $\Lambda^{\mu}{}_{\nu}e^{\nu}$ plus a term $\propto q^{\mu}$. $\therefore$ to not spoil Lorentz invariance, the amplitude contracted with q_{μ} should vanish:

$$q_{\mu}\mathcal{A}^{\mu}_{\alpha\to\beta+\gamma}(q) \xrightarrow[q\to 0]{} \mathcal{A}_{\alpha\to\beta} \sum_n \eta_n e_n \ := 0$$

This condition is <u>charge conservation</u>! Relativity (Lorentz invariance) + quantum mechanical amplitudes require charge to be conserved. We derived this independently of any assumptions about gauge invariance, using only consistency of soft emission amplitudes with relativity.

Consider now the amplitude for emitting a soft graviton of 4-momentum q and tensor indices μ, ν:

$$\mathcal{A}^{\mu\nu}_{\alpha\to\beta+G}(q) \xrightarrow[q\to 0]{} \mathcal{A}_{\alpha\to\beta} \sum_n \frac{\eta_n f_n p_n^\mu p_n^\nu}{p_n \cdot q - i\eta_n \varepsilon}$$

where f_n is the coupling constant of the soft graviton to the particle n. Lorentz invariance requires this to vanish when contracted with q_μ:

$$q_\mu \mathcal{A}^{\mu\nu}_{\alpha\to\beta+G}(q) \xrightarrow[q\to 0]{} \mathcal{A}_{\alpha\to\beta} \sum_n \eta_n f_n p_n^\nu \quad := 0$$

Here the sum $\sum_n f_n p_n^\nu = 0$ is conserved. But the only linear combination of 4-momenta that can be conserved is the total 4-momentum $\sum_n p_n^\nu = 0$, otherwise scattering becomes trivial. Therefore for this sum to vanish we must have f_n = constant for all n $= \sqrt{8\pi G_N}$.

Lorentz invariance requires massless particles of spin 2 to couple in the same way to all energy-momentum! i.e. Einstein's principle of equivalence is a necessary consequence of Lorentz invariance applied to massless spin 2 particles.

What about spin $j \geq 3$?

$$\mathcal{A}^{\mu\nu\rho\cdots}_{\alpha\to\beta+x}(q) \xrightarrow[q\to 0]{} \mathcal{A}_{\alpha\to\beta} \sum_n \frac{\eta_n g_n p_n^\mu p_n^\nu p_n^\rho \cdots}{p_n\cdot q - i\eta_n\varepsilon} := 0$$

However, no sum $\sum_n g_n p_n^\nu p_n^\rho \ldots$ can be conserved without making scattering trivial $\Rightarrow g_n = 0$!

Massless spin $j \geq 3$ particles may exist but they cannot have couplings that survive in the limit of low energy ($\therefore$ no inverse square law forces).