Perey-Buck's nonlocality in nucleon scattering: sixty years after

Guillaume Blanchon[1] **& Hugo F. Arellano**[2]

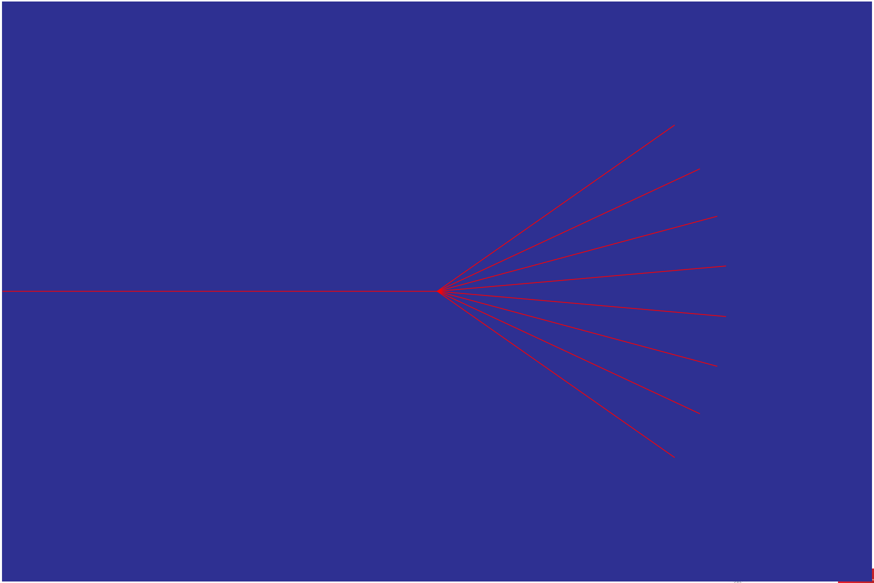
[1] CEA,DAM,DIF, F-91297 Arpajon, France

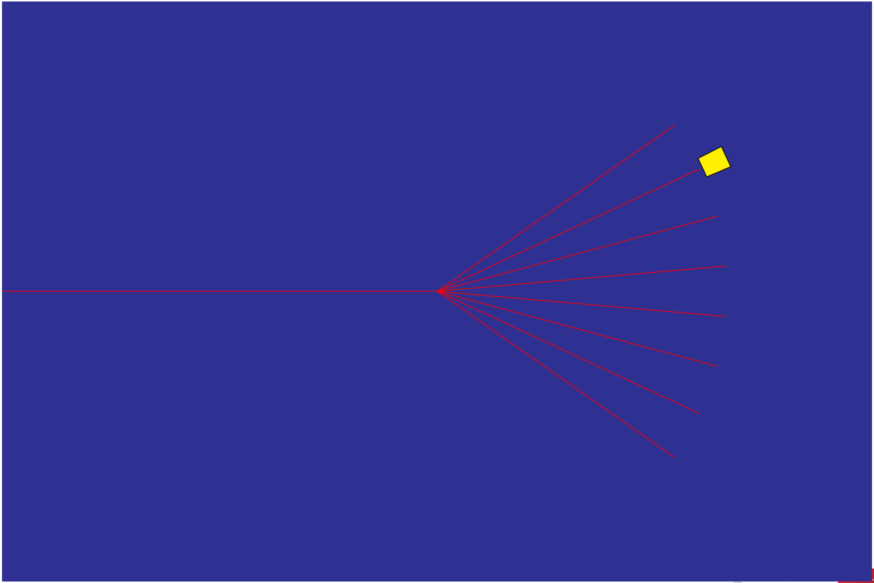
[2] Department of Physics - FCFM, U. Chile

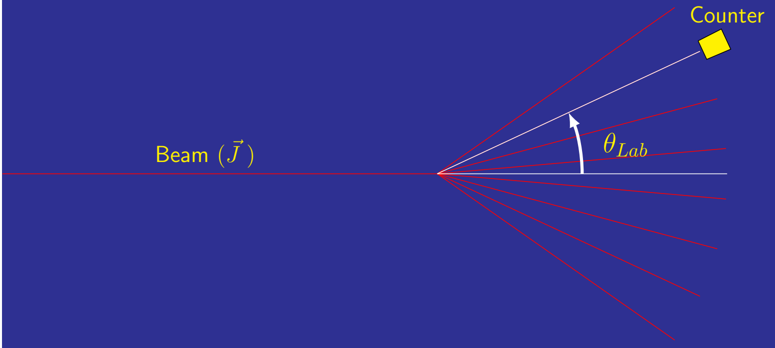
16th Varenna Conference on Nuclear Reaction Mechanisms Jun 11 - 16, 2023

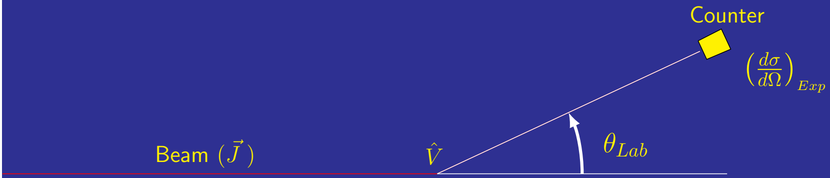
Content

Outline

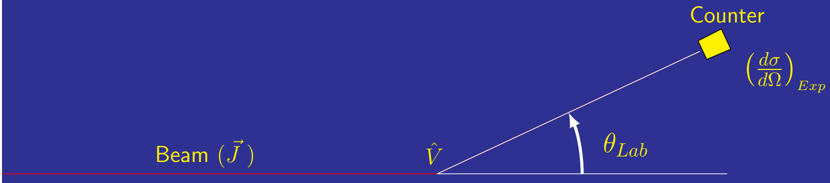





Counter
θ_{Lab}
Beam ($\vec{J}$)



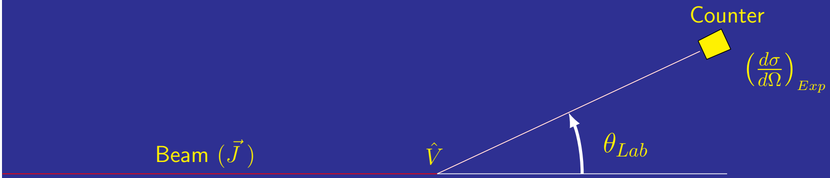
$$(\hat{K} + \hat{V})|\Psi\rangle = E|\Psi\rangle$$

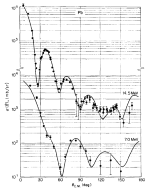


$$(\hat{K} + \hat{V})|\Psi\rangle = E|\Psi\rangle$$

Assuming $\hat{V}$ local: $\langle \vec{r}\,'|\hat{V}|\vec{r}\rangle = v(\vec{r})\,\delta(\vec{r}\,' - \vec{r})$

Then: $-\frac{\hbar^2}{2m}\nabla^2\Psi(\vec{r}) + v(\vec{r})\,\Psi(\vec{r}) = E\Psi(\vec{r})$



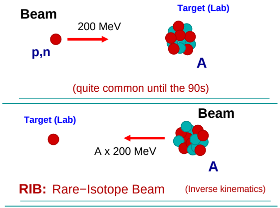


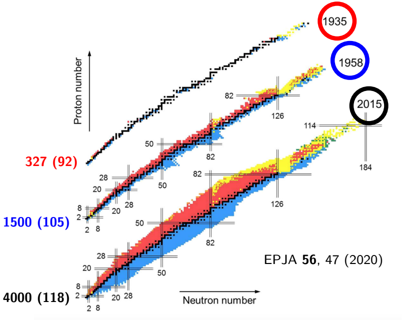
$$(\hat{K} + \hat{V})|\Psi\rangle = E|\Psi\rangle$$

Assuming $\hat{V}$ local: $\langle \vec{r}\,'|\hat{V}|\vec{r}\rangle = v(\vec{r})\,\delta(\vec{r}\,' - \vec{r})$

Then: $-\frac{\hbar^2}{2m}\nabla^2\Psi(\vec{r}) + v(\vec{r})\,\Psi(\vec{r}) = E\Psi(\vec{r})$ → $\left(\frac{d\sigma}{d\Omega}\right)_{Th}$

Probes and targets





- **Motivations for studying optical potential:**
  - Key element for evaluations
  - Interpretation of experiments
  - Interesting by itself

- **Motivations for studying optical potential:**
  - Key element for evaluations
  - Interpretation of experiments
  - Interesting by itself

- **Different strategies:**
  - Microscopy: build the potential from NN interaction and many-body theory
  - Phenomenology: postulate a shape of potential and calibrate on experiment
  - Dialogue microscopy/phenomenology

Origin of nonlocality

- **Antisymmetrization**

$$V_{HF}(\mathbf{r},\mathbf{r'}) = \int d\mathbf{r}_1 \rho(\mathbf{r}_1) v(\mathbf{r},\mathbf{r}_1) - \rho(\mathbf{r},\mathbf{r'}) v(\mathbf{r},\mathbf{r'})$$

- **Antisymmetrization**

$$V_{HF}(\mathbf{r},\mathbf{r'}) = \int d\mathbf{r}_1 \rho(\mathbf{r}_1) v(\mathbf{r},\mathbf{r}_1) - \rho(\mathbf{r},\mathbf{r'}) v(\mathbf{r},\mathbf{r'})$$

- **Polarization**
  Surface term...

$$\Delta U(\mathbf{r},\mathbf{r}';E) = \sum_i V_{0i}(\mathbf{r}) G_{ii}(\mathbf{r},\mathbf{r'};E) V_{i0}(\mathbf{r'}),$$

where G_{ii} is a propagator

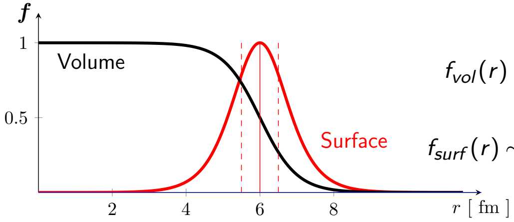
$$V_{i0}(\mathbf{r}) = \beta_i r \frac{dU(r)}{dr} Y_\lambda^\mu(\widehat{\mathbf{r}}),$$

transition potential in the Bohr collective model.

A. Lev, W. P. Beres, and M. Divadeenam. PRC 9 :2416–2434, Jun 1974.

Woods and Saxon (phenomenological)

$$V(r) = -[V_0 + iW_0] f_{vol}(r) - iW_D f_{surf}(r) - (U_{so} + iW_{so}) f_{surf}(r) \boldsymbol{\ell} \cdot \boldsymbol{\sigma}$$



$$f_{vol}(r) \sim \frac{1}{1 + e^{(r - R_0)/a}}$$

$$f_{surf}(r) \sim \frac{4\, e^{(r - R_0)/a}}{[1 + e^{(r - R_0)/a}]^2}$$

Integro-différential scattering equation,

$$-\frac{\hbar^2}{2\mu}\Delta\psi(\mathbf{r})+\int V_{NL}(\mathbf{r},\mathbf{r'})\psi(\mathbf{r'})\mathrm{d}\mathbf{r'}=E\psi(\mathbf{r}),$$

When potential is local

$$V_{NL}(\mathbf{r},\mathbf{r'})=V_L(\mathbf{r})\delta(\mathbf{r},\mathbf{r'}).$$

Scattering equation reduces to **differential**,

$$-\frac{\hbar^2}{2\mu}\Delta\psi(\mathbf{r})+V_L(\mathbf{r})\psi(\mathbf{r})=E\psi(\mathbf{r}).$$

$\rightarrow$ **Need for numerical tools**

Outline

2.E

Nuclear Physics **32** (1962) 353—380; © *North-Holland Publishing Co., Amsterdam*

A NON-LOCAL POTENTIAL MODEL FOR THE SCATTERING OF NEUTRONS BY NUCLEI

F. PEREY and B. BUCK

Oak Ridge National Laboratory † *Oak Ridge, Tennessee*

Received 25 September 1961

Abstract: An energy independent non-local optical potential for the elastic scattering of neutrons from nuclei is proposed and the wave-equation solved numerically in its full integro-differential form. The non-local kernel is assumed separable into a potential form factor times a Gaussian non-locality. The potential form factor, of argument $\frac{1}{2}(\mathbf{r}+\mathbf{r}')$, is that of a real Saxon form plus an imaginary term having the shape of the derivative of a Saxon form. A real local spin-orbit potential of the usual Thomas form is included. The parameters of the potential obtained solely from the fitting of the differential cross sections for lead at 7 MeV and 14.5 MeV are used unchanged to calculate the elastic differential cross sections, total and reaction cross sections and polarizations on some elements ranging from Al to Pb at various energies from 0.4 MeV to 24 MeV. The S-wave strength functions and the effective scattering radius R' are also calculated with the same parameters. The parameters in the usual notations are: real potential $V = 71$ MeV, $r = 1.22$ fm, $a = 0.65$ fm; surface imaginary potential $W = 15$ MeV, $a = 0.47$ fm; non-locality $\beta = 0.85$ fm; spin-orbit potential, using the nucleon mass in the Thomas form, $U_{so} = 1300$ MeV. The energy independence of the

F. PEREY AND B. BUCK

linate representation, a non-local potential operating on a
the form

$$V\Psi(\mathbf{r}) = \int V(\mathbf{r}, \mathbf{r}')\Psi(\mathbf{r}')\mathrm{d}\mathbf{r}'.$$

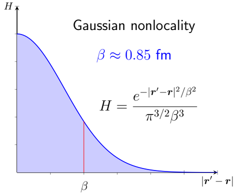
faced with an integro-differential Schrödinger equation whic
by numerical integration and iteration.
al grounds it is necessary that the kernel function $V(\mathbf{r},$
, i.e.,

$$V(\mathbf{r}, \mathbf{r}') = V(\mathbf{r}', \mathbf{r}).$$

te the numerical calculations a separable form was chosen fo
rnel function:

$$V(\mathbf{r}, \mathbf{r}') = U\left(\frac{\mathbf{r}+\mathbf{r}'}{2}\right) H\left(\frac{\mathbf{r}-\mathbf{r}'}{\beta}\right).$$

nonlocality



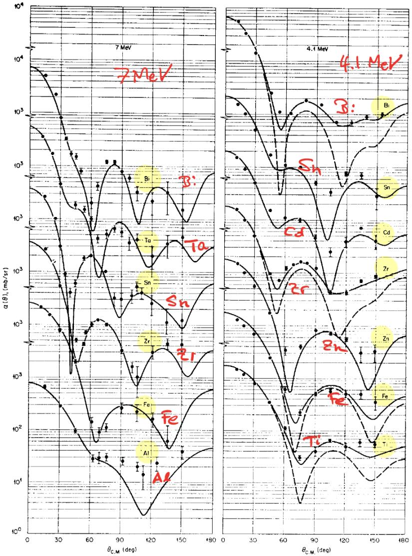


Fig. 2. Comparison of predictions of the energy independent non-local optical model with experimental elastic differential cross-sections of neutrons at 4.1 MeV and 7 MeV. The parameters are

Perey-Buck's assumptions:

- Separability
- Gaussian nonlocality
- Low incident energy $E < 24$ MeV
- Energy-independent
- Local spin-orbit

Perey-Buck's assumptions:

- Separability
- Gaussian nonlocality
- Low incident energy $E < 24$ MeV
- Energy-independent
- Local spin-orbit

$\rightarrow$ Shape used in most of nowadays phenomenology

Perey-Buck's assumptions:

- Separability
- Gaussian nonlocality
- Low incident energy $E < 24$ MeV
- Energy-independent
- Local spin-orbit

$\rightarrow$ Shape used in most of nowadays phenomenology

$\rightarrow$ Is it validated by microscopy? (at least by a given microscopic model)

Outline

Representations

$$\langle \text{post} | \hat{U} | \text{prior} \rangle$$

relative coordinates

$r', r, \hat{\boldsymbol{r}}\cdot\hat{\boldsymbol{r}}'$

$U(\boldsymbol{r}', \boldsymbol{r})$ - - - - - - $\tilde{U}(\boldsymbol{k}', \boldsymbol{k})$

$k', k, \hat{\boldsymbol{k}}\cdot\hat{\boldsymbol{k}}'$

relative momenta

$\updownarrow$ $\updownarrow$

$$\boldsymbol{R} = \frac{1}{2}(\boldsymbol{r}' + \boldsymbol{r})$$

'nonlocality' $\boldsymbol{s} = \boldsymbol{r}' - \boldsymbol{r}$

$$\boldsymbol{q} = \boldsymbol{k}' - \boldsymbol{k}$$ momentum transfer

$$\boldsymbol{K} = \frac{1}{2}(\boldsymbol{k}' + \boldsymbol{k})$$

$s, R, \hat{\boldsymbol{R}}\cdot\hat{\boldsymbol{s}}$

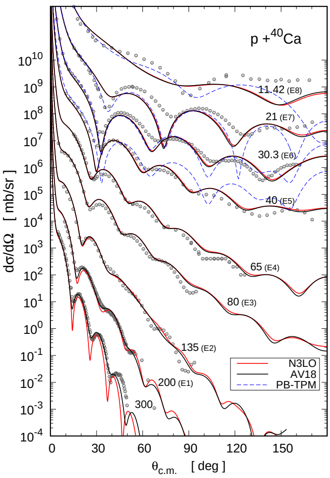
$K, q, \hat{\boldsymbol{K}}\cdot\hat{\boldsymbol{q}}$

Microscopic optical model

Bare $NN \rightarrow N+A$ connection

$$\underbrace{\boldsymbol{NN} \rightarrow \left\{ \begin{array}{c} \left(\frac{d\sigma}{d\Omega}\right)_{NN} \\ \downarrow\uparrow \\ {\color{red}\delta_L(E)} \end{array} \right\}}_{\text{2-Body}} \rightleftharpoons v_{NN} \underbrace{\begin{array}{c} \nearrow \quad \mathscr{G}_{NN} \\ \\ \searrow \quad t_{NN} \end{array} \Bigg\}}_{\text{Effective interaction}} \underbrace{\rightarrow \underbrace{U_{opt}(\mathbf{k}',\mathbf{k};E)}_{\uparrow\, |\Psi_{g.s.}\rangle} \rightarrow \left\{ \begin{array}{c} |\psi\rangle \\ \frac{d\sigma}{d\Omega} \\ \sigma \end{array} \right.}_{\text{N+A}}$$

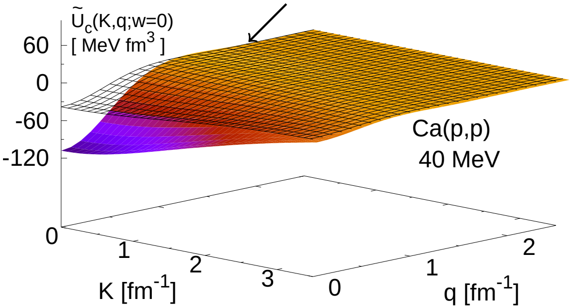
$$U(\mathbf{k}',\mathbf{k};E) = \int d\mathbf{p}\, d\mathbf{p}' \quad \underbrace{{\color{red}\rho(\mathbf{p}',\mathbf{p})}}_{\sim\sum \phi_\alpha(p')\phi_\alpha^\dagger(p)} \quad \underbrace{\langle \mathbf{k}'\,\mathbf{p}' | G(E,\rho) | \mathbf{k}\,\mathbf{p}\rangle}_{V_{NN}}$$



- Tian-Pang-Ma
- N3LO / Density from Gogny HFB
- AV18 / Density from Gogny HFB

$\tilde{U}$ in the K-q plane

Weak angular dependence ($w = \hat{K} \cdot \hat{q}$)



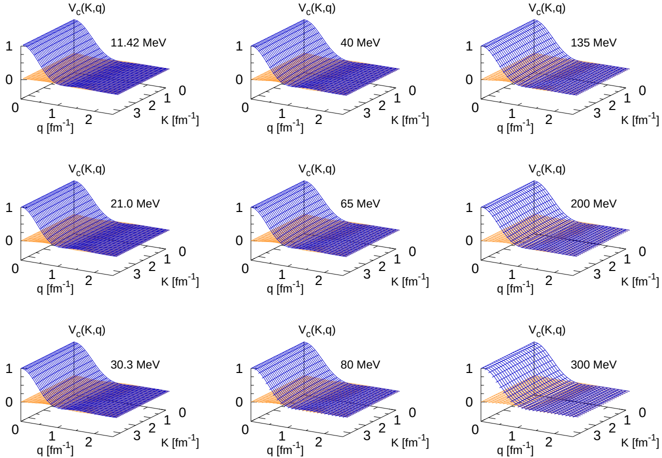
$$\tilde{U}(K,q) = \frac{\tilde{U}(K,q)}{\tilde{U}(K,0)} \times \frac{\tilde{U}(K,0)}{\tilde{U}(0,0)} \times \tilde{U}(0,0) \equiv \tilde{V}(K,q) \times \tilde{H}(K) \times W$$

Nonlocality

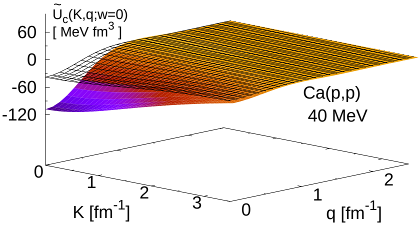
Strength

Weak K-dependence!

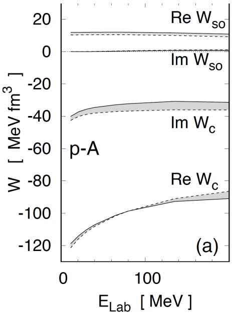
$\tilde{V}_c(K,q) = \tilde{U}(K,q)/\tilde{U}(K,0) \sim \tilde{v}(q)$ in the range 10–300 MeV

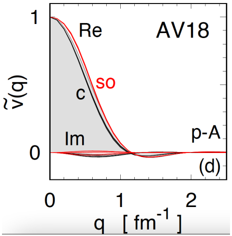


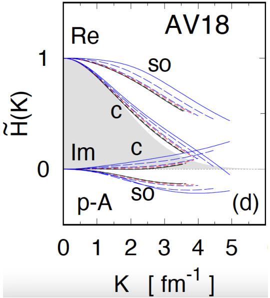
JvH factorization

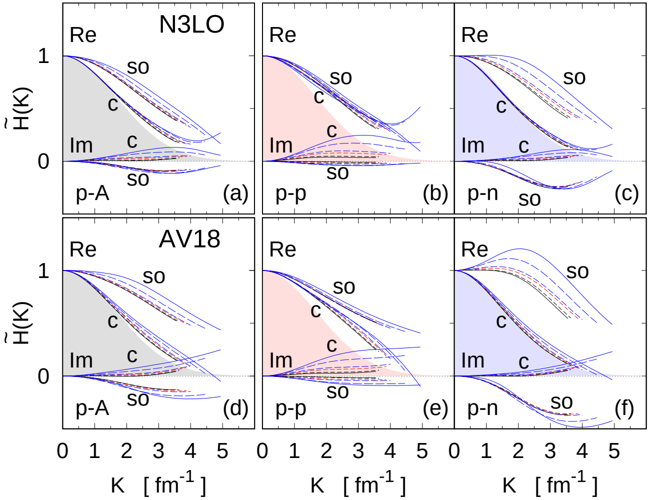


$$\tilde{U}(K,q) = W\,\tilde{v}(q)\,\tilde{H}(K)$$





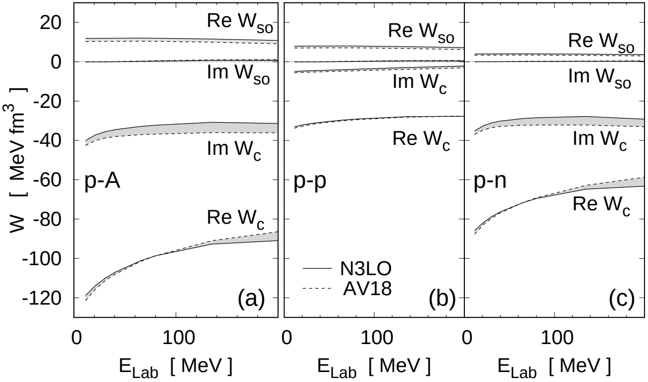




$$H_{PB}(s) = \frac{1}{\pi^{3/2}\beta^3} e^{-s^2/\beta^2}$$

$$\tilde{H}_{PB}(K) = e^{-\beta^2 K^2/4}$$

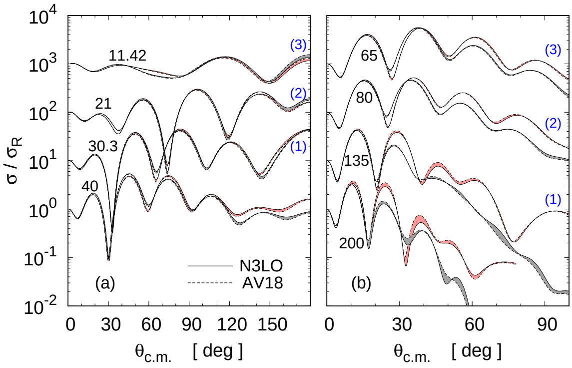
		N3LO			AV18		
	Energy (MeV)	β_{pA} (fm)	β_{pp} (fm)	β_{pn} (fm)	β_{pA} (fm)	β_{pp} (fm)	β_{pn} (fm)
Central	11.42	0.89	0.72	0.94	0.89	0.73	0.95
	21.0	0.88	0.72	0.94	0.89	0.72	0.94
	30.3	0.88	0.71	0.93	0.88	0.71	0.94
	40.0	0.88	0.71	0.93	0.88	0.71	0.93
	61.4	0.87	0.72	0.93	0.86	0.70	0.92
	80.0	0.87	0.72	0.93	0.86	0.71	0.92
	135.0	0.87	0.75	0.92	0.83	0.71	0.89
	200.0	0.86	0.78	0.90	0.80	0.72	0.85
Spin-orbit	11.42	0.57	0.61	0.51	0.47	0.58	0.03
	21.0	0.58	0.61	0.50	0.46	0.59	—
	30.3	0.58	0.62	0.49	0.48	0.59	—
	40.0	0.58	0.63	0.48	0.48	0.60	—
	61.4	0.57	0.63	0.43	0.46	0.60	—
	80.0	0.55	0.62	0.37	0.37	0.59	—
	135.0	0.48	0.59	0.13	0.32	0.56	—
	200.0	0.44	0.56	—	0.22	0.51	—



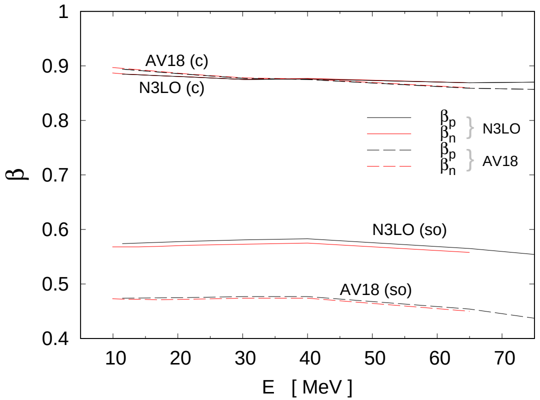
Volume integral

$$W = \frac{J}{(2\pi)^3}$$

Microscopic potential *vs.* JvH



Nonlocalities for n+A & p+A



Outline

Schrödinger Integro-Differential Equation Solver

G. Blanchon, M. Dupuis, H.F. Arellano, R.N. Bernard, B. Morillon, CPC 254 (2020) 107340

$$-\frac{\hbar^2}{2\mu}\left[\frac{d^2}{dr^2}-\frac{l(l+1)}{r^2}\right]f_{lj}(k,r)+r\int_0^\infty v_{lj}(r,r';E)f_{lj}(k,r')r'\mathrm{d}r'=Ef_{lj}(k,r)$$

- Language: Fortran
- Method: modified Numerov method
- Refinement procedure of the solution
- Extension for potentials with first derivative

  (G. Blanchon, M. Dupuis, H. F. Arellano, R. N. Bernard, B. Morillon, P. Romain, EPJA (2021) 57:13)
- Available with pairing *(not published)*

Scattering WAves off NonLocal Optical Potentials

H. F. Arellano, G. Blanchon, CPC 259 (2021) 107543

- Lippmann-Schwinger equation
- Momentum and coordinate space potentials

H. F. Arellano, G. Blanchon, PLB 789 (2019) 256-261

Potentials included

- Koning-Delaroche global local potential *(de 1 keV à 200 MeV)*

  A. J. Koning and J.-P. Delaroche NPA 713(3-4) 231 - 310, 2003.

- Morillon-Romain global dispersive local potential *(de 1 keV à 200 MeV)*

  B. Morillon and P. Romain. PRC, 70 014601 (2004) and PRC, 76(4) 044601 (2007).

- Morillon global dispersive nonlocal potential *(Talk)*

- Perey-Buck global nonlocal potential *(below 30 MeV)*

  F. Perey and B. Buck. Nucl. Phys., 32 353 – 380, 1962.

- Tian-Pang-Ma global nonlocal potential *(below 30 MeV)*

  Y. Tian, D.-Y. Pang, and Z.-Y. Ma. IJMP E, 24(01) 1550006, 2015.

- Mahzoon nonlocal dispersive potential

  M. H. Mahzoon, R. J. Charity, W. H. Dickhoff, H. Dussan, and S. J. Waldecker. PRL 112 162503, 2014.

+ Potentials given on a radial mesh

- To the lower order in the angular expansion $\widehat{\mathbf{q},\mathbf{K}}$ the microscopic potential leads to *JvH* separable structure of the central and the spin-orbit.
- *JvH* validates Gaussian nonlocality.
- For $E < 65\ MeV$, the range of the nonlocality β is 0.86-0.89 fm for the central part and 0.46-0.58 for spin-orbit part.
- *JvH* offers a new link between theory and phenomenology. It will be interesting to explore higher orders.