JUSTUS-LIEBIG-UNIVERSITÄT GIESSEN

Institut für Theoretische Physik

Formal Theory of Heavy Ion Double Charge Exchange Reactions

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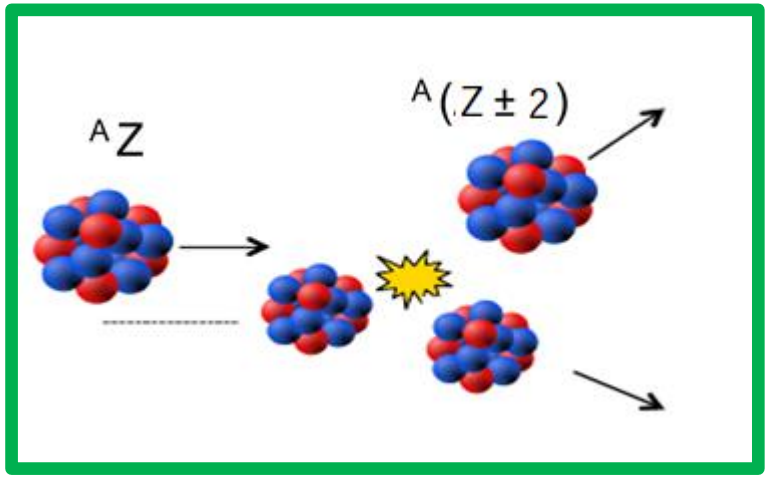
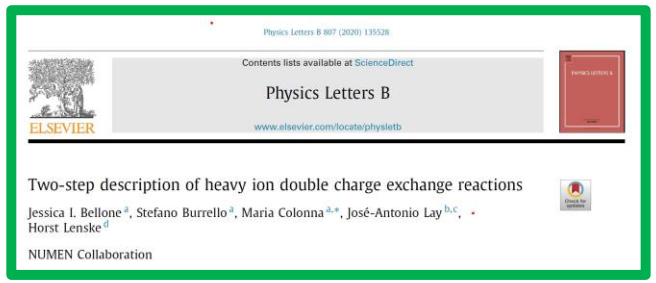
The Topic:
Double Charge Exchange Reactions



Agenda:

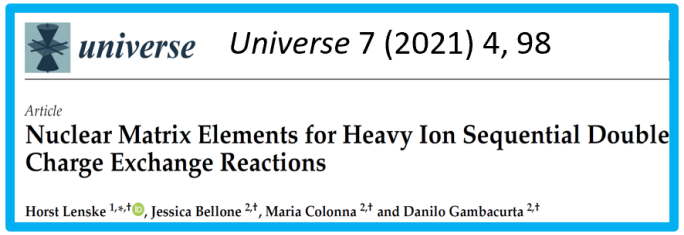
Charged current physics with LOW ENERGY heavy ion beams

- **DCE Theory Overview**
- **Direct DCE: Mesonic „Majorana“ (MDCE) Double Charge Exchange**
- **MDCE Pion-Nucleon Box Diagrams**
- **Exploratory Investigations**
- **Outlook**



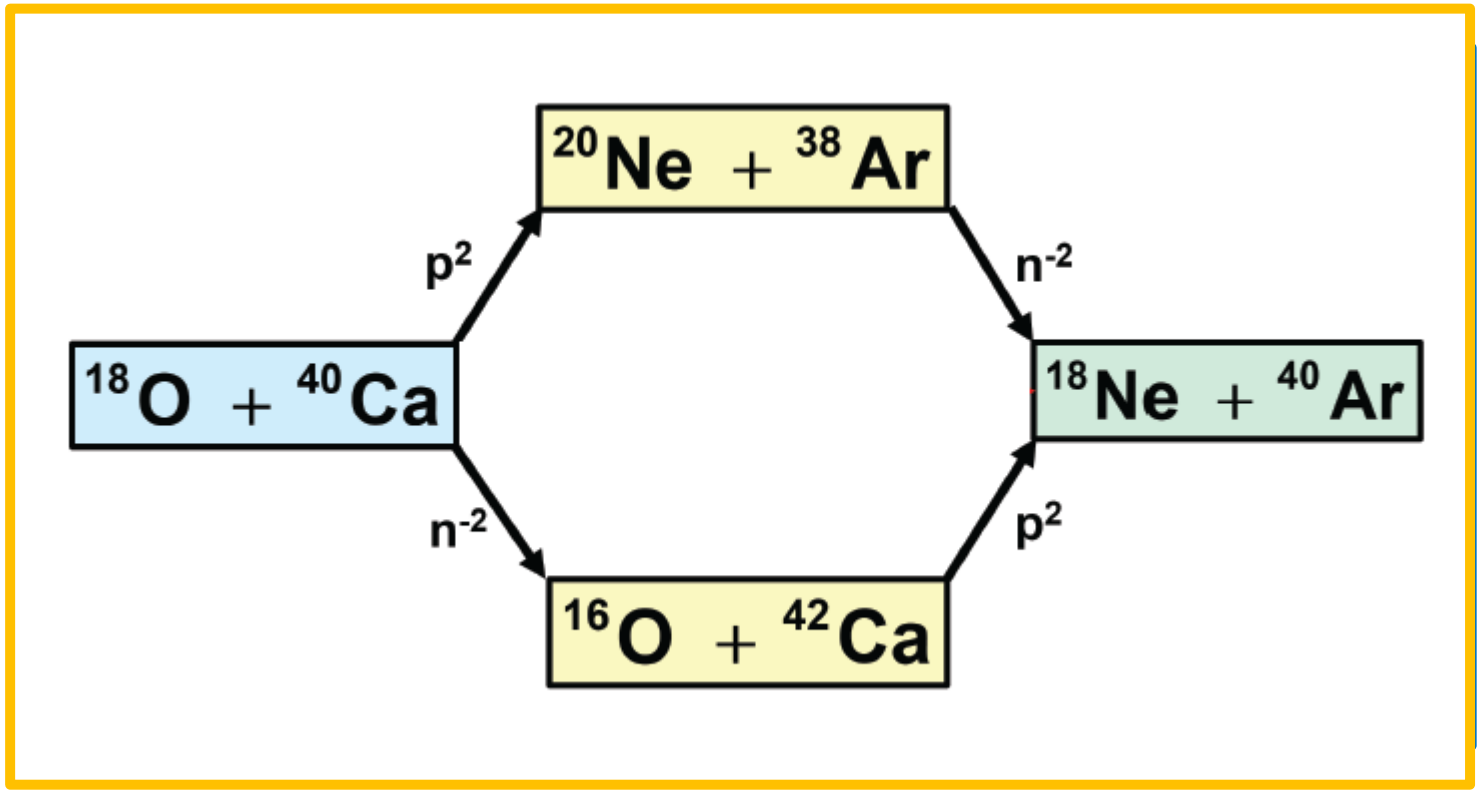
Progress in Particle and Nuclear Physics
Shedding light on nuclear aspects of neutrinoless double beta decay by heavy-ion double charge exchange reactions

Prog.Part.Nucl.Phys. **128 (2023) 103999**

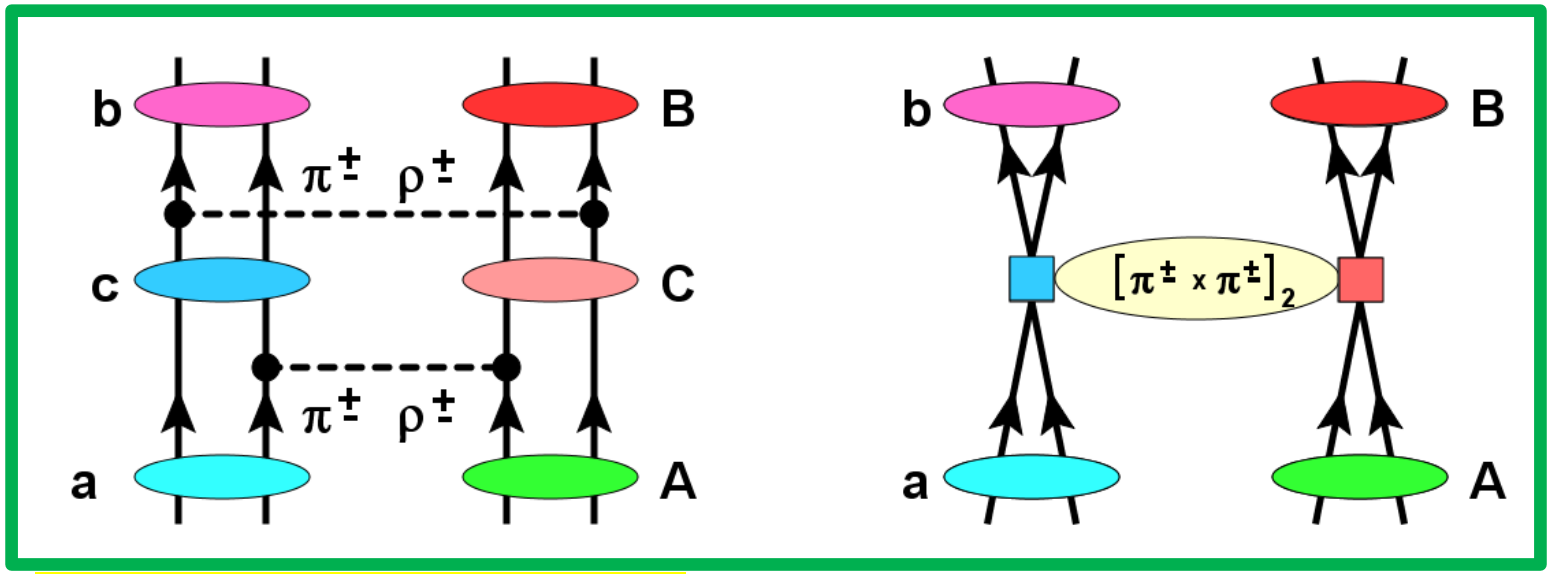


Heavy Ion DCE Reaction Mechanism

The Complete Picture: from Transfer DCE to Mesonic „Majorana“ DCE



Nuclear Double Charge Exchange (DCE) Reactions

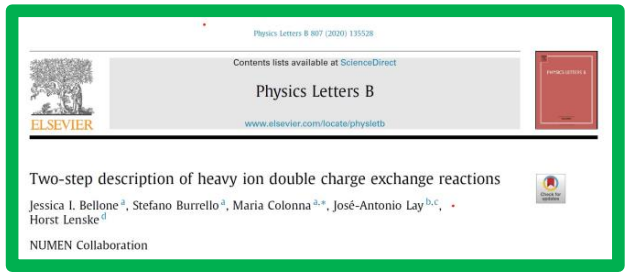


DSCE (talk by Maria Colonna)

→ 2ν2β

MDCE

→ 0ν2β



Physics Letters B 807 (2020) 135528

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ELSEVIER

Two-step description of heavy ion double charge exchange reactions

Jessica I. Bellone [a], Stefano Burrello [a], Maria Colonna [a,*], José-Antonio Lay [b,c], Horst Lenske [d]

NUMEN Collaboration

universe

Universe 7 (2021) 4, 98

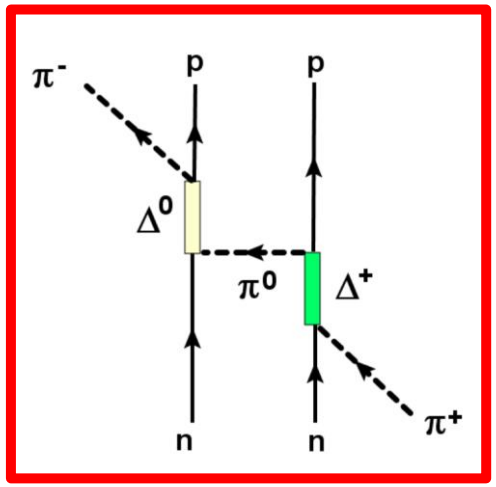
Article

Nuclear Matrix Elements for Heavy Ion Sequential Double Charge Exchange Reactions

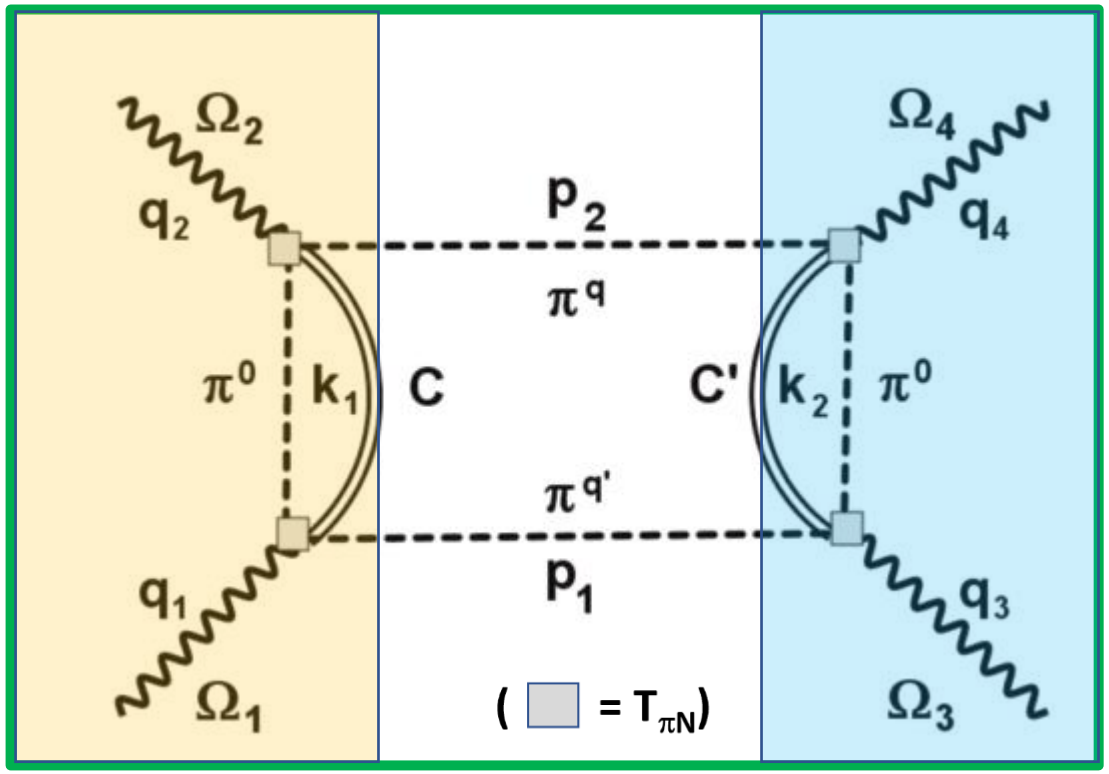
Horst Lenske [1,*,†], Jessica Bellone [2,†], Maria Colonna [2,†] and Danilo Gambacurta [2,†]

DCE Reactions via 2-Nucleon Processes

DCE by Double Meson Exchange and Pair-Correlation Interaction by πN T-matrix, not NN T-matrix



On-shell counterpart:
(π^+,π^-) and (π^-,π^+)
DCE reactions



MDCE
Box Diagrams

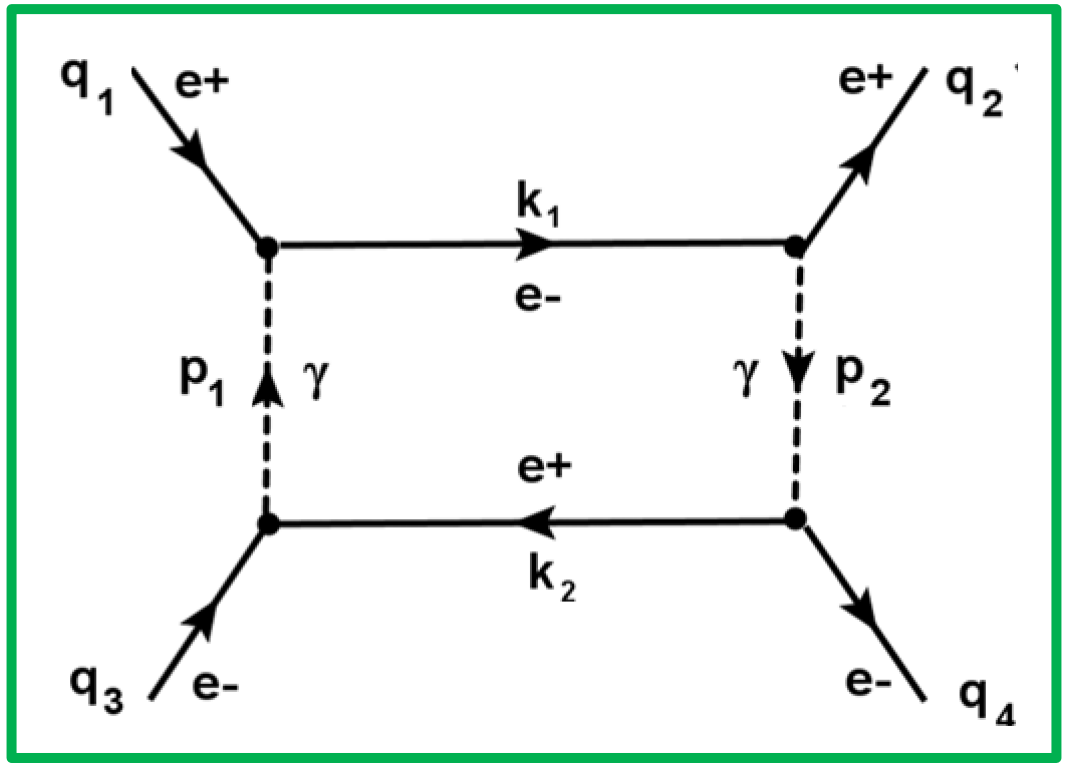
$\Omega_{1,2} \approx (pn^{-1}) \leftrightarrow \Omega_{3,4} \approx (np^{-1})$

Two-Nucleon nn ↔ pp Process

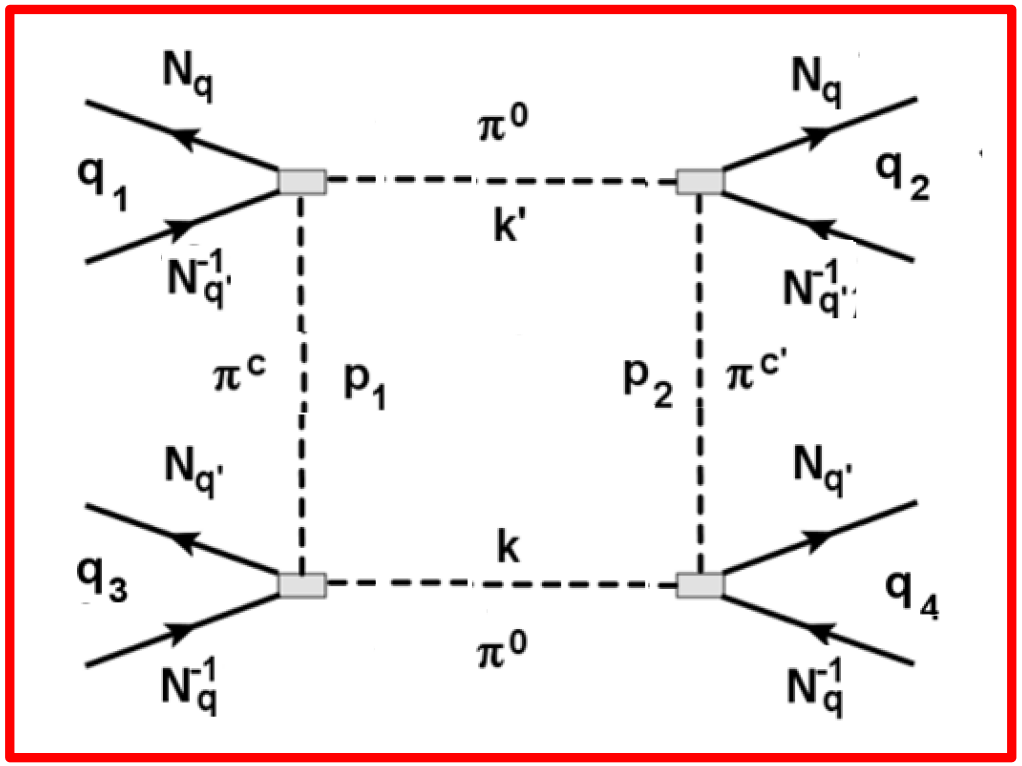
- Final states are **2p2h-configurations** w.r.t. the parent nuclei

DCE by dynamically generated rank-2 Isotensor Interaction!

The MDCE Box Diagrams

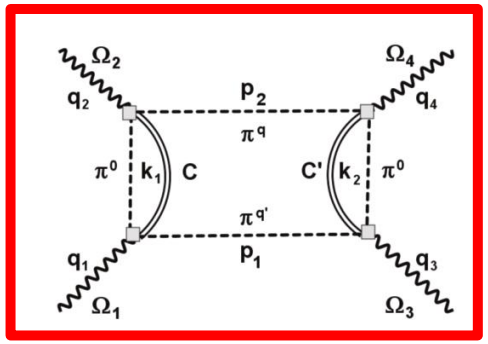


e^+e^- Bhabha Scattering QED Box
One independent momentum variable k or p
Integral over product of 2 lepton and 2 photon Propagators
Closed form by Mellin-Barnes Integrals



MDCE Isotensor Hadronic Box
$N^{-1}N$ momenta are fixed by k,k‘ and $p_{1,2}$
Meson Potentials and Nuclear Vertices
Numerical Evaluation

MDCE Isotensor Transition Kernel



$$\mathcal{U}_{\alpha\beta}(\mathbf{p}_1,\mathbf{p}_2) = \mathcal{W}_{12}(\mathbf{p}_1,\mathbf{p}_2) D_{\pi^q\pi^{q'}}(\mathbf{p}_1,\mathbf{p}_2) \mathcal{W}_{34}(\mathbf{p}_2,\mathbf{p}_1),$$

$$\mathcal{W}_{ij}(\mathbf{p},\mathbf{p}') = \sum_n \int \frac{d^3k}{(2\pi)^3} F_{jn}(\mathbf{p}',\mathbf{k}) D_{\pi^0}(k^2,\omega_n) F_{ni}(\mathbf{p},\mathbf{k}).$$

$$F_{dc}(\mathbf{p},\mathbf{k}) = \langle d | e^{\pm i\mathbf{q}_d\cdot\mathbf{r}} \tilde{T}_{\pi N}(\mathbf{p},\mathbf{k}) | c \rangle \qquad (\mathbf{q}_d = \mathbf{p} \pm \mathbf{k})$$

MDCE-NME in projectile and target: two SCE-vertices connected by π^0 exchange

2-Nucleon Mechanism fostered by 2-body correlations

MDCE Vertices in Closure Approximation

$$D_{\pi^0 C}(k^2) \sim -\frac{1}{k^2 + m_{\pi^0}^2}\left(1 + \frac{(\omega_A - \omega_C)^2}{m_{\pi^0}^2} + \ldots\right)$$

Pion Mass → NATURAL SEPARATION SCALE! → keep the 1st term only → Closure

MDCE Transition Potential in Closure Approximation

$$\mathcal{W}_{FD}^{(c)}(\mathbf{p}_1, \mathbf{p}_2) = -\int \frac{d^3k}{(2\pi)^3} \langle F | V_{\pi N}^{\pm}(\mathbf{k}, \mathbf{p}_2) \frac{e^{i\mathbf{k}\cdot\mathbf{x}_{12}}}{k^2 + m_{\pi^0}^2} V_{\pi N}^{\pm}(\mathbf{k}, \mathbf{p}_1) | D \rangle$$

The MDCE Nuclear Transition Amplitude

$$\mathcal{W}_{AB}(\mathbf{p}_1,\mathbf{p}_2) = -\langle B|e^{-i\mathbf{p}_2\cdot\mathbf{r}_2}\mathcal{U}_\pi(\mathbf{x})e^{i\mathbf{p}_1\cdot\mathbf{r}_1}\mathcal{T}_{2\pm2}|A\rangle,$$

Rank-2 Isotensor Operator

$$\mathcal{T}_{2\pm2} = [\boldsymbol{\tau}_1 \otimes \boldsymbol{\tau}_2]_{2\pm2}$$

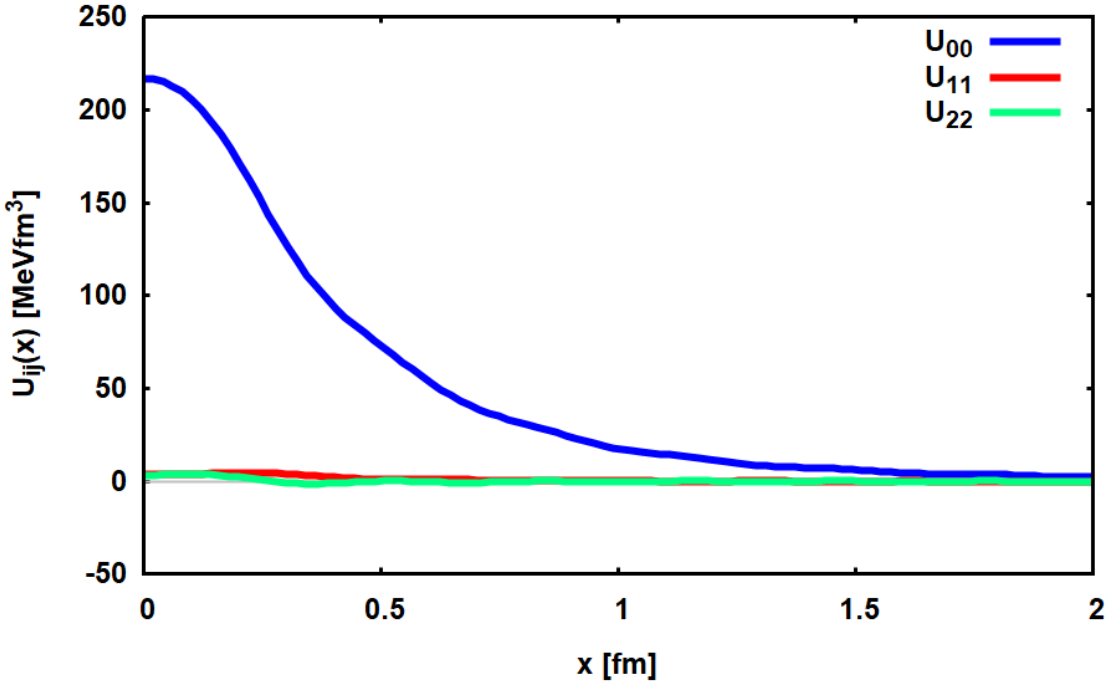
Pion-Potential

$$\mathcal{U}_\pi(\mathbf{x}) = \int \frac{d^3k}{(2\pi)^3} T_{\pi N}(\mathbf{p}_2,\mathbf{k}) \frac{e^{i\mathbf{k}\cdot\mathbf{x}}}{k^2+m_{\pi^0}^2} T_{\pi N}(\mathbf{p}_1,\mathbf{k}).$$

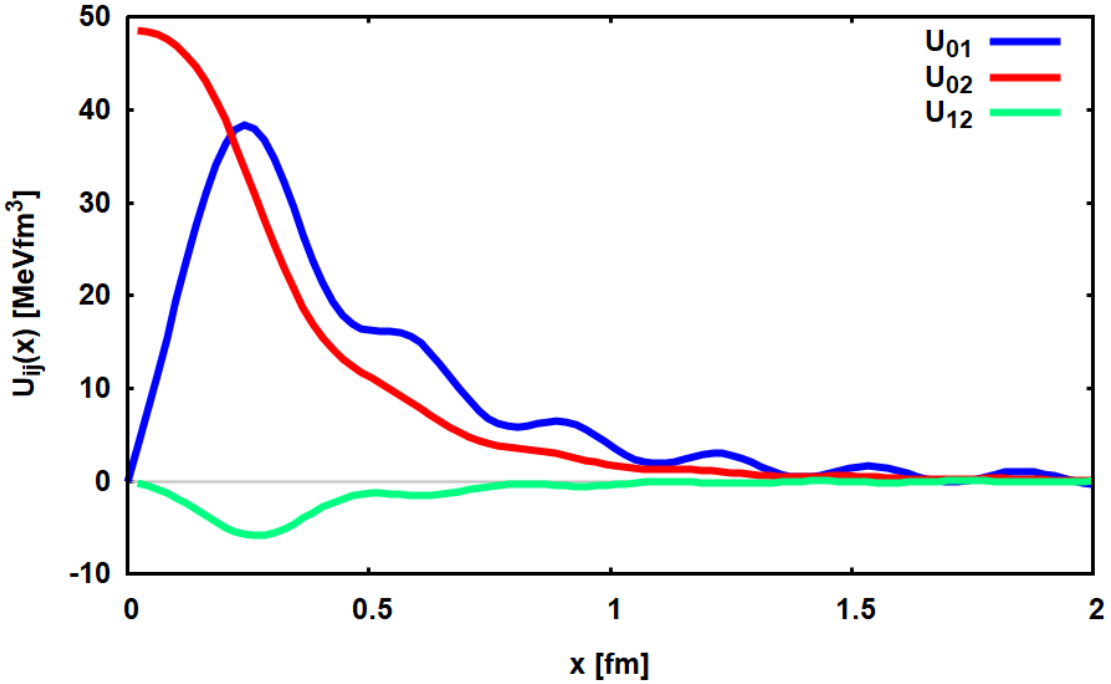
$$T_{\pi N}(p_j,k) = T_0(w) + \frac{1}{m_\pi^2}\left(T_1(w)\,p_j \bullet k + iT_2(w)\sigma_j \bullet (p_j \times k)\right)$$

→ 6 potentials: $U_{00}(x)$, $U_{11}(x)$, $U_{22}(x)$, $U_{01}(x)$, $U_{02}(x)$ and $U_{12}(x)$

Pion Potentials at low Momentum Transfer
$p_1=p_2=30$ MeV/c



Diagonal $U_{ii}(x)$

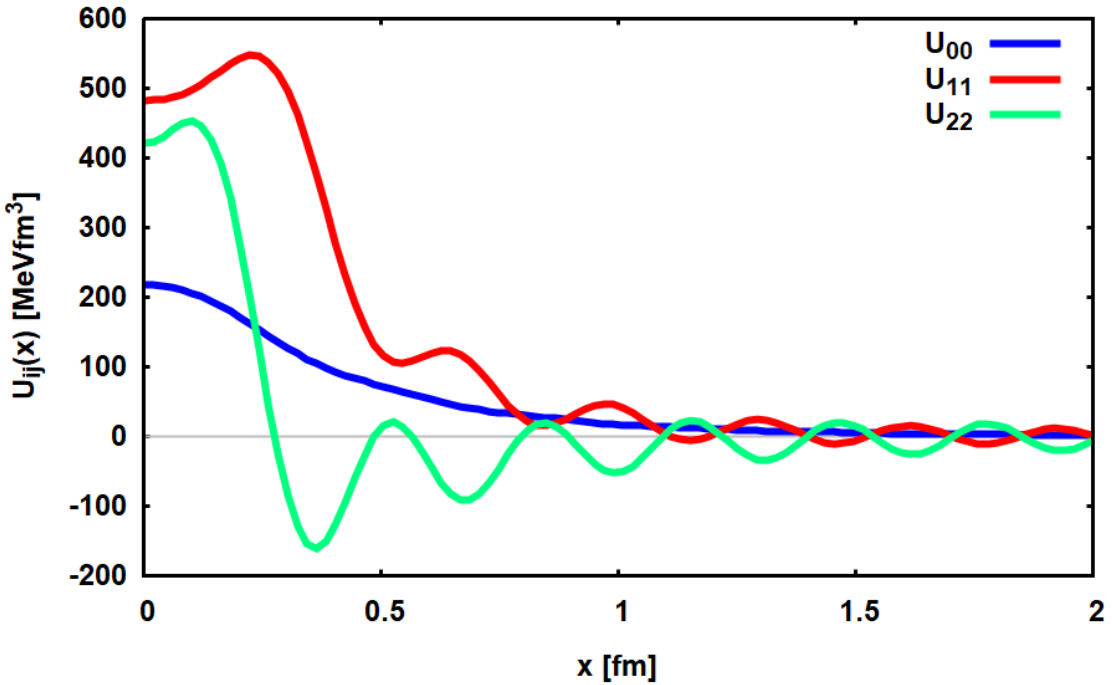


Mixed $U_{ij}(x)$

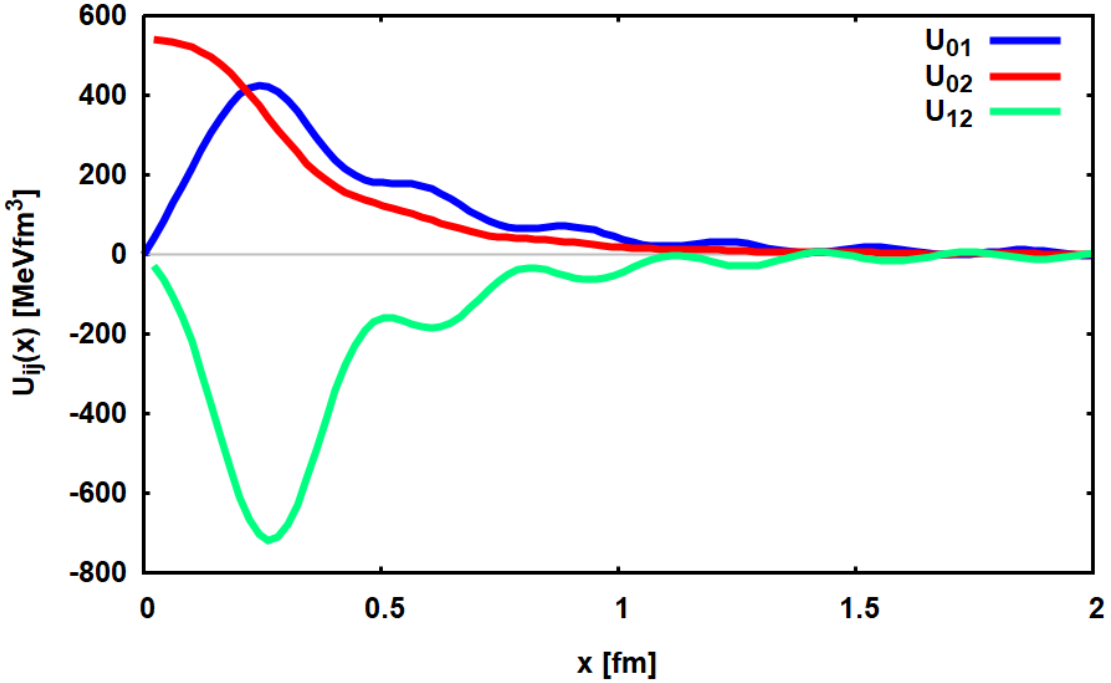
Dominance of S-wave Components

Pion Potentials at high Momentum Transfer
$p_1=p_2=300$ MeV/c

General Feature:
Short-Range NN-Correlation – $x_{NN} \sim 0.5$ fm



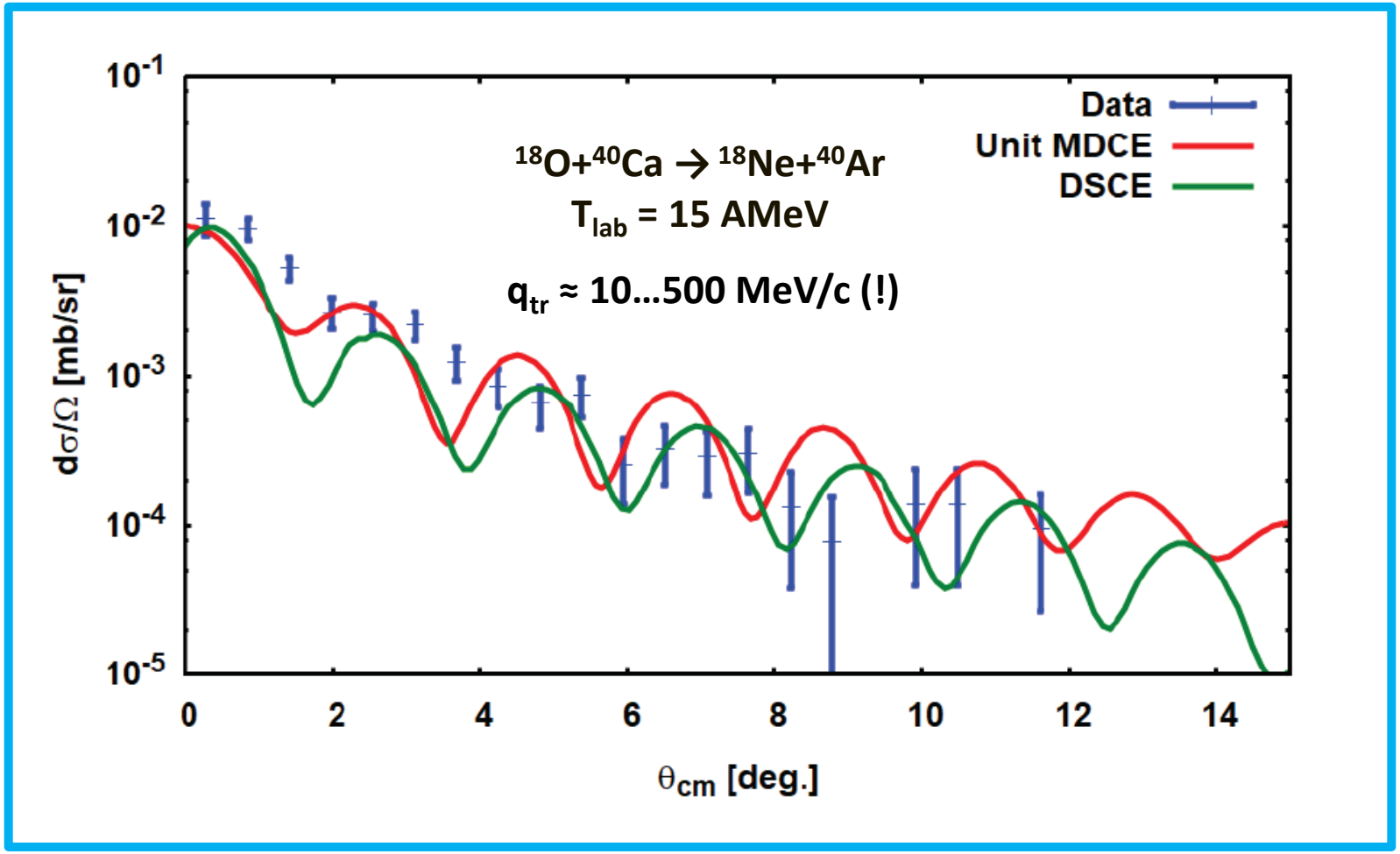
Diagonal $U_{ii}(x)$



Mixed $U_{ij}(x)$

Dominance of P-wave Components

Heavy Ion DCE Reactions



2-step DSCE: intermediate states with $J^{\pi} \leq 5^{\pm}$

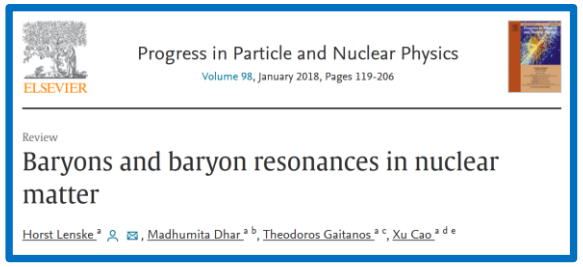
1-step MDCE: $^{40}Ca(0^+) \rightarrow {}^{40}Ar([n^{-2}p^2]0^+)$: J=0+ with L=S=0 & $[L=2 \times S=2]_{0+}$

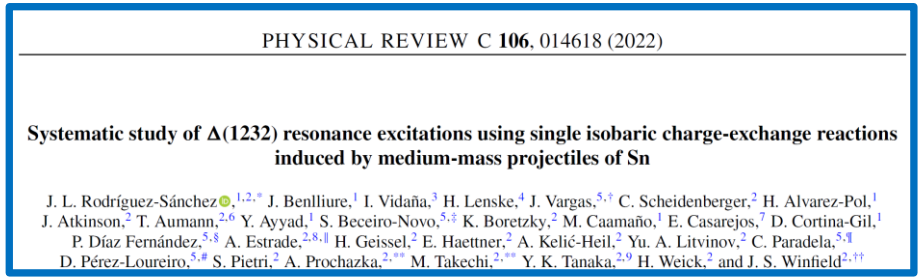
Data: F. Cappuzzello et al., EPJ A51 (2015)

MDCE Preliminary!

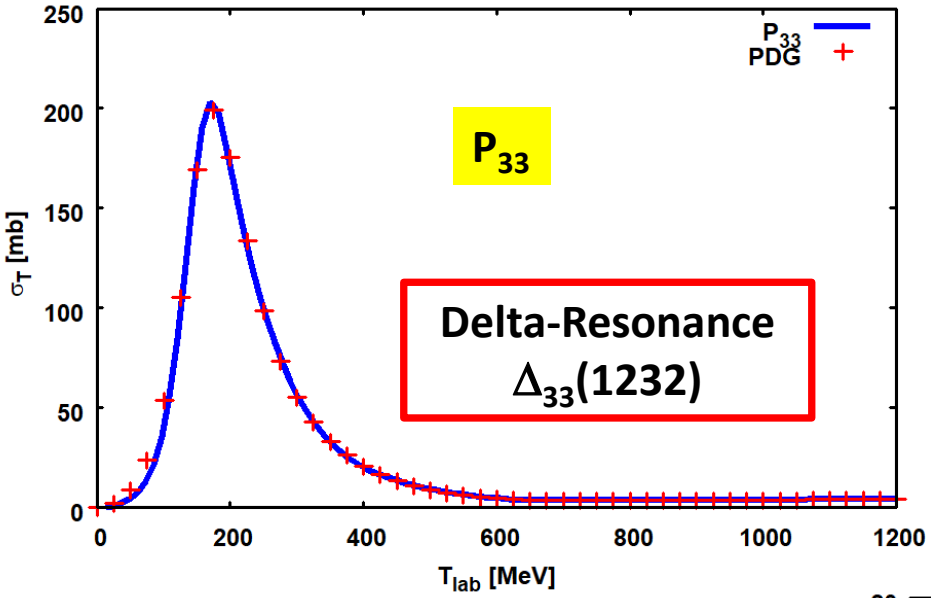
Pion-Nucleon Interactions

$$T_{\pi N}(p_j,k) = T_0(w) + \frac{1}{m_\pi^2}\left(T_1(w)\,p_j \bullet k + iT_2(w)\sigma_j \bullet (p_j \times k)\right)$$





π^-N total Cross Sections



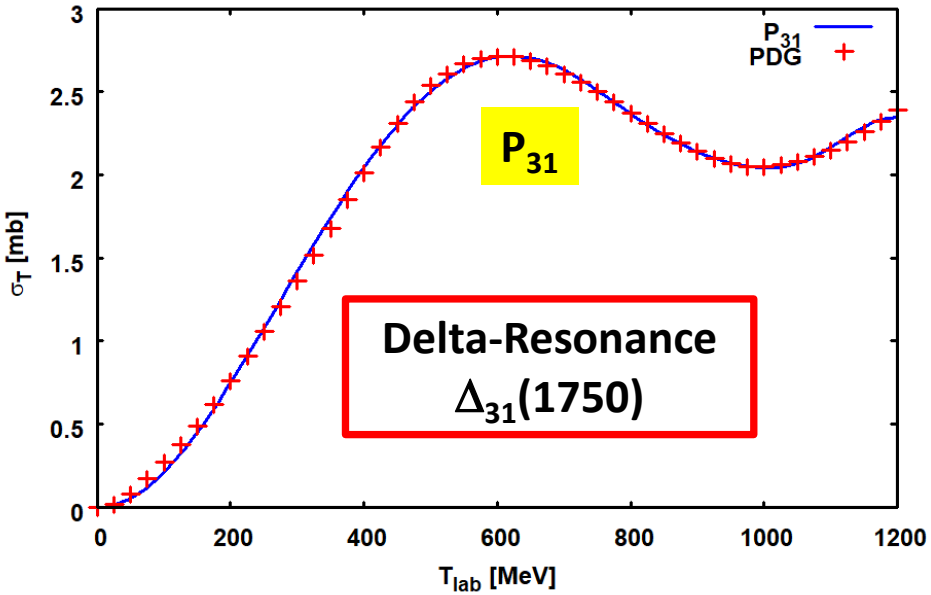
Relevant partial waves:

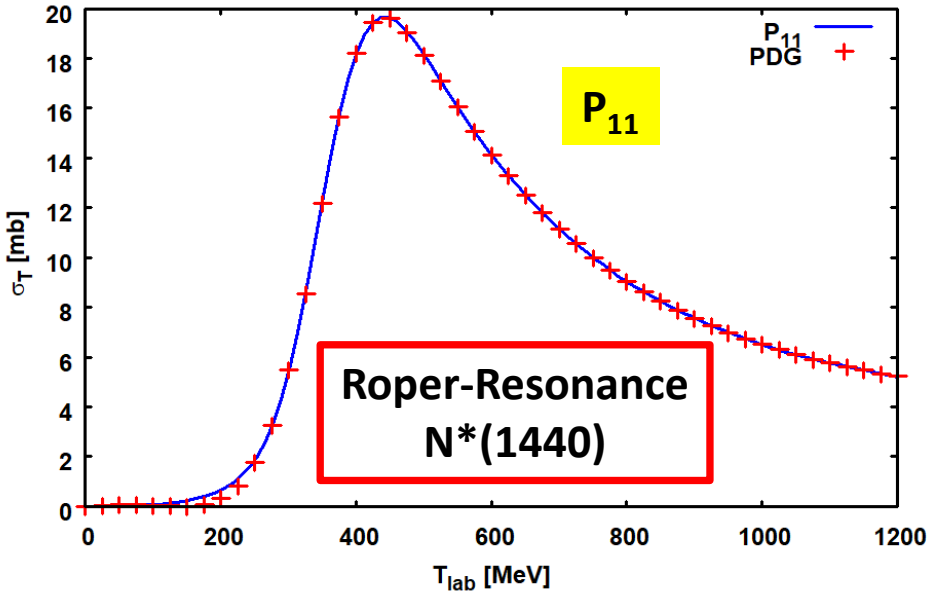
$P_{33}, P_{11}, P_{31}, S_{11}, S_{31}$

(Notation: L_{2I2J})

$T_0 \sim \{S_{11}, S_{31}\}$

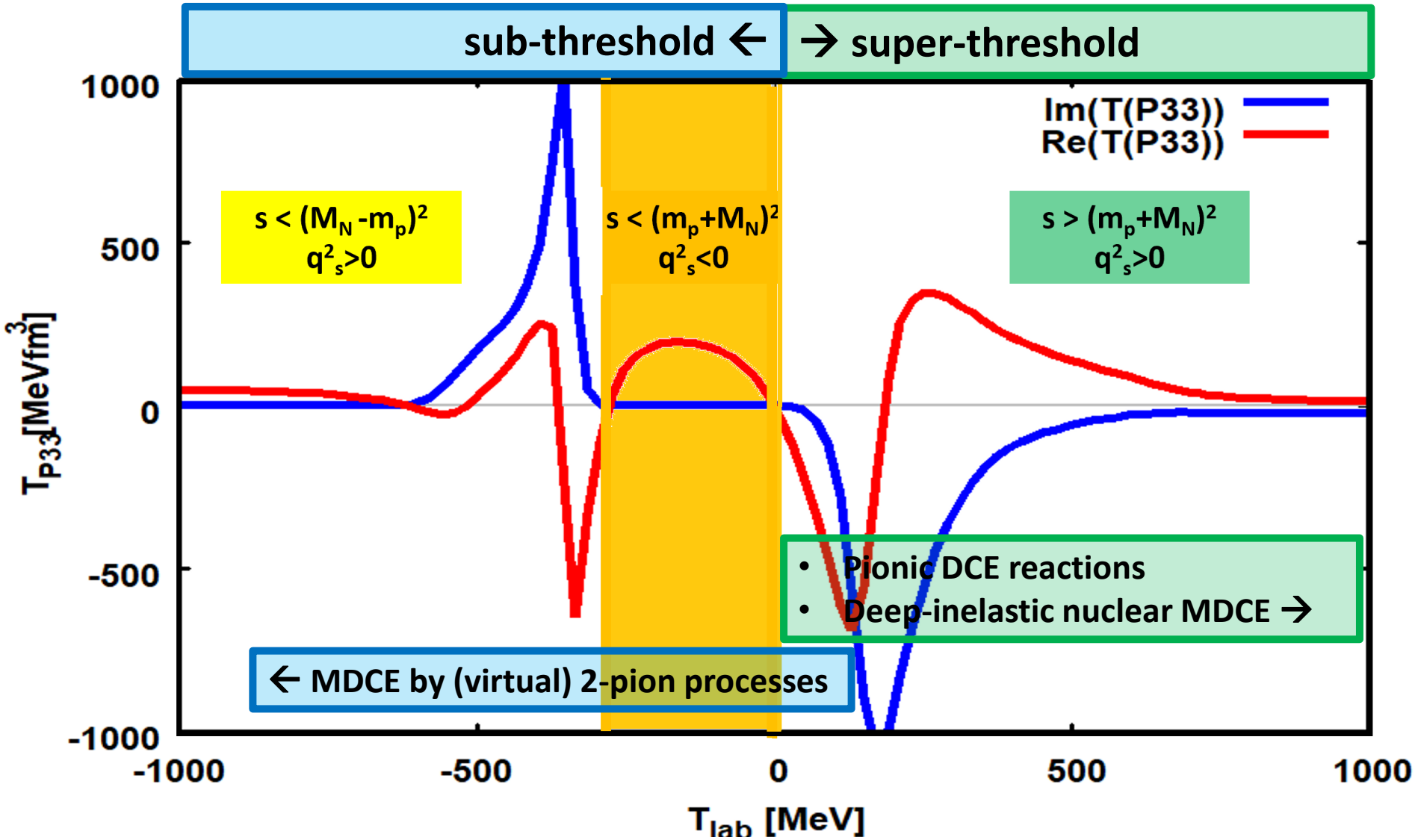
$T_{1,2} \sim \{P_{11}, P_{31}, P_{31}\}$





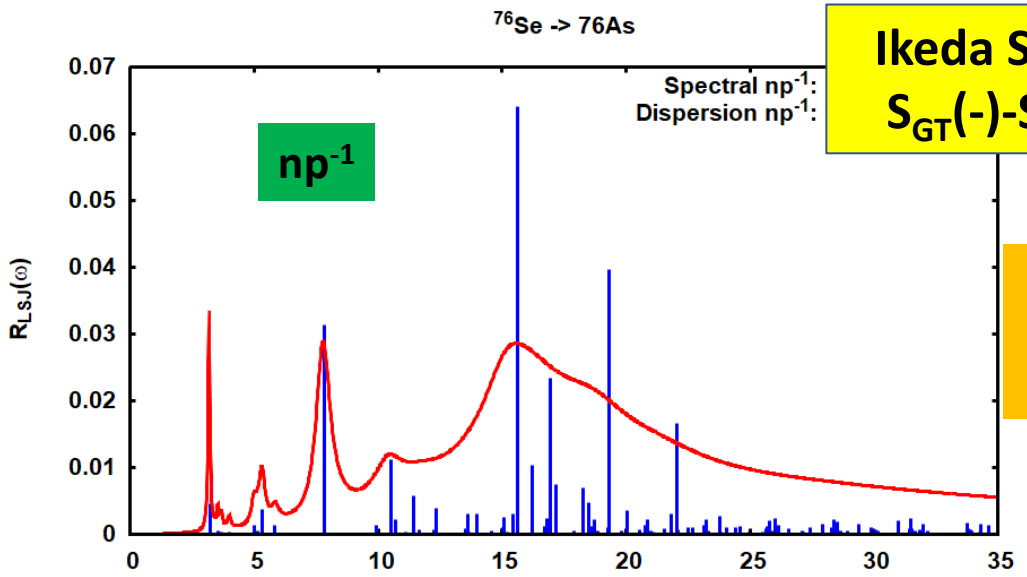
Review on the Giessen π+N CC approach:
H.L. et al., PPNP 98 (2018) 119

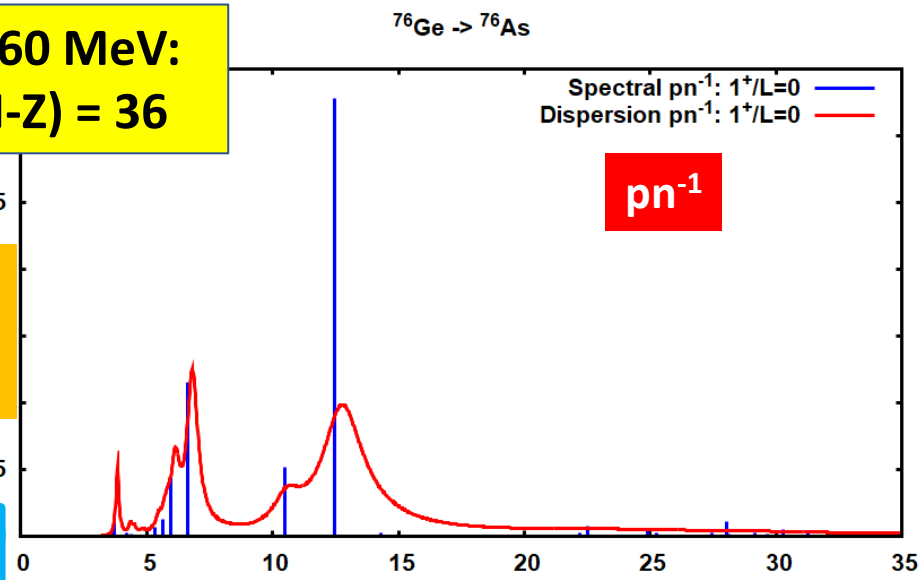
Kinematical Regions of Importance for Hadronic DCE Reactions



Nuclear Structure Aspects of DCE Physics

Ikeda Spin Sumrule – ω ≤ 60 MeV:
$S_{GT}(-)-S_{GT}(+) = 33.96 \leq 3(N-Z) = 36$



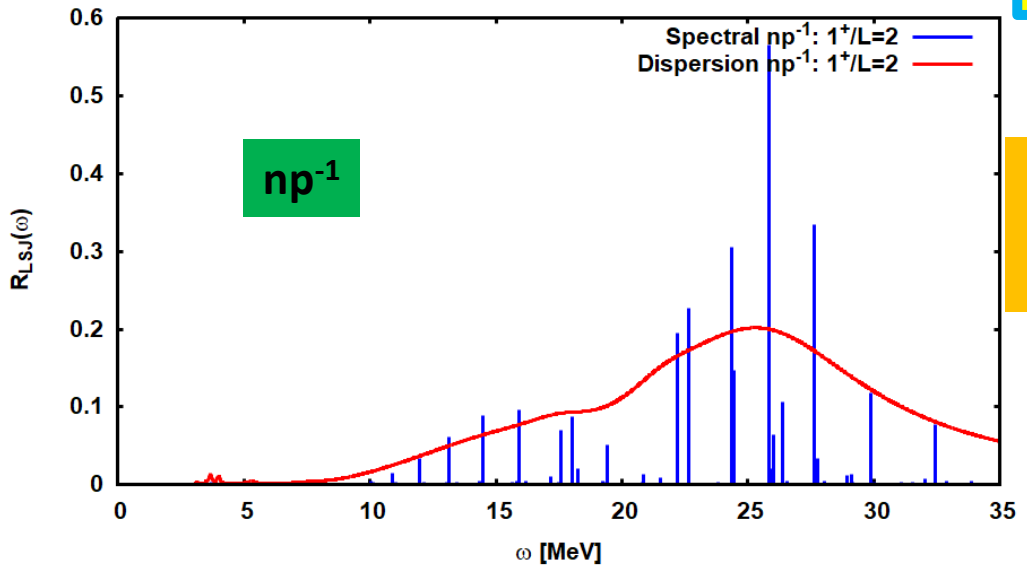


$J^\pi = 1^+$
L=J-1

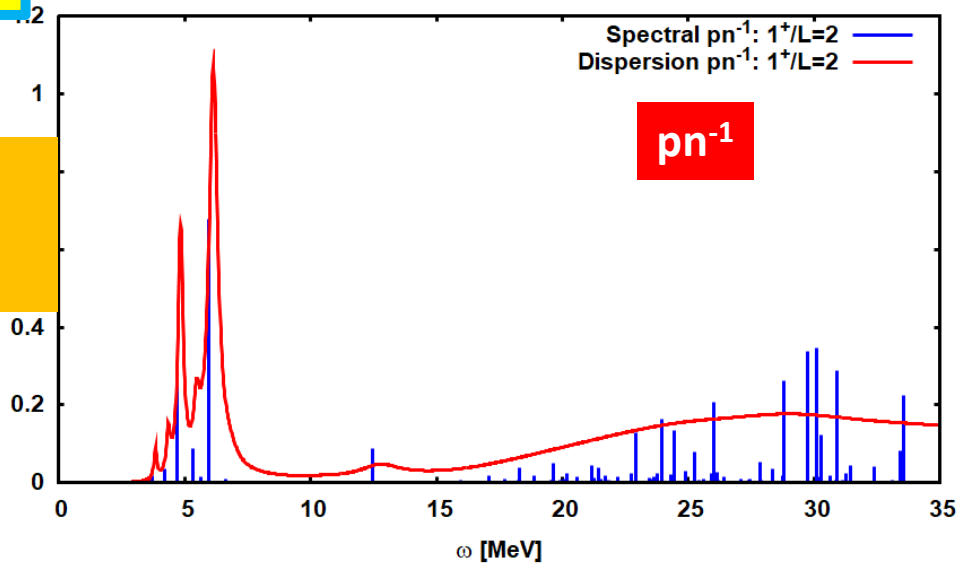
GiEDF+HFB+QRPA
e.g. *Eur.Phys.J.A* 57 (2021) 3, 89

^{76}Se → ^{76}As

^{76}Ge → ^{76}As

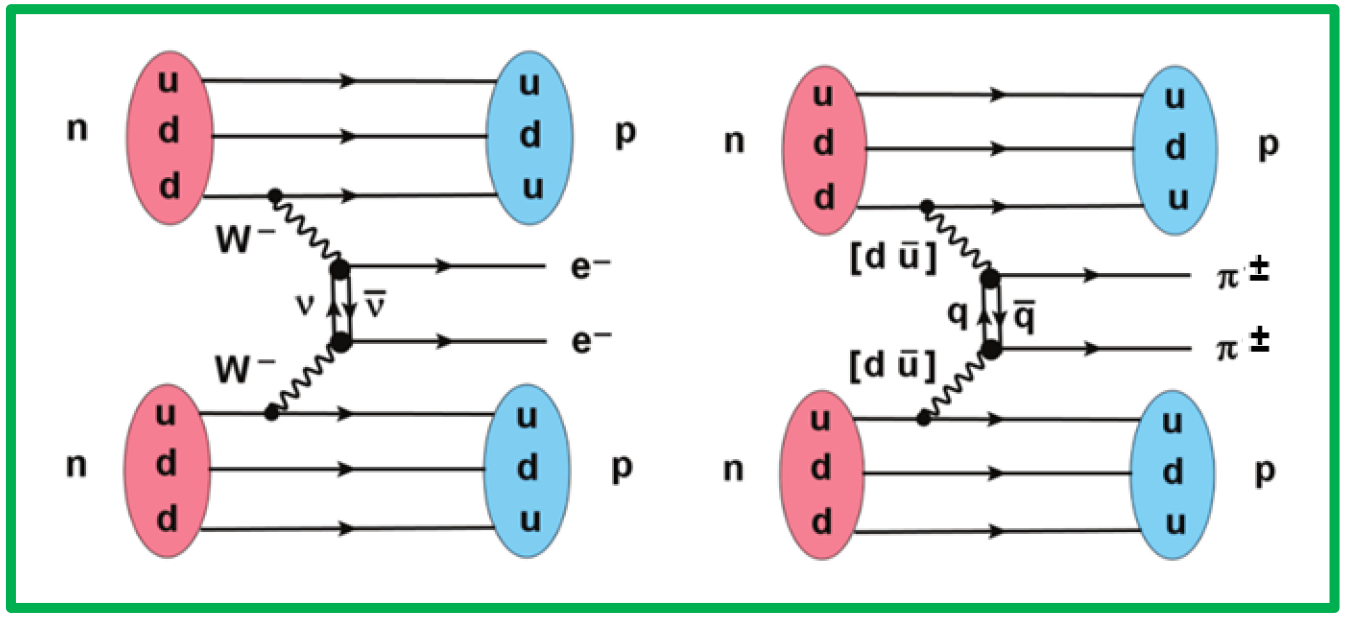


$J^\pi = 1^+$
L=J+1



MDCE and 0ν2β Double Beta-Decay

The Majorana Aspect

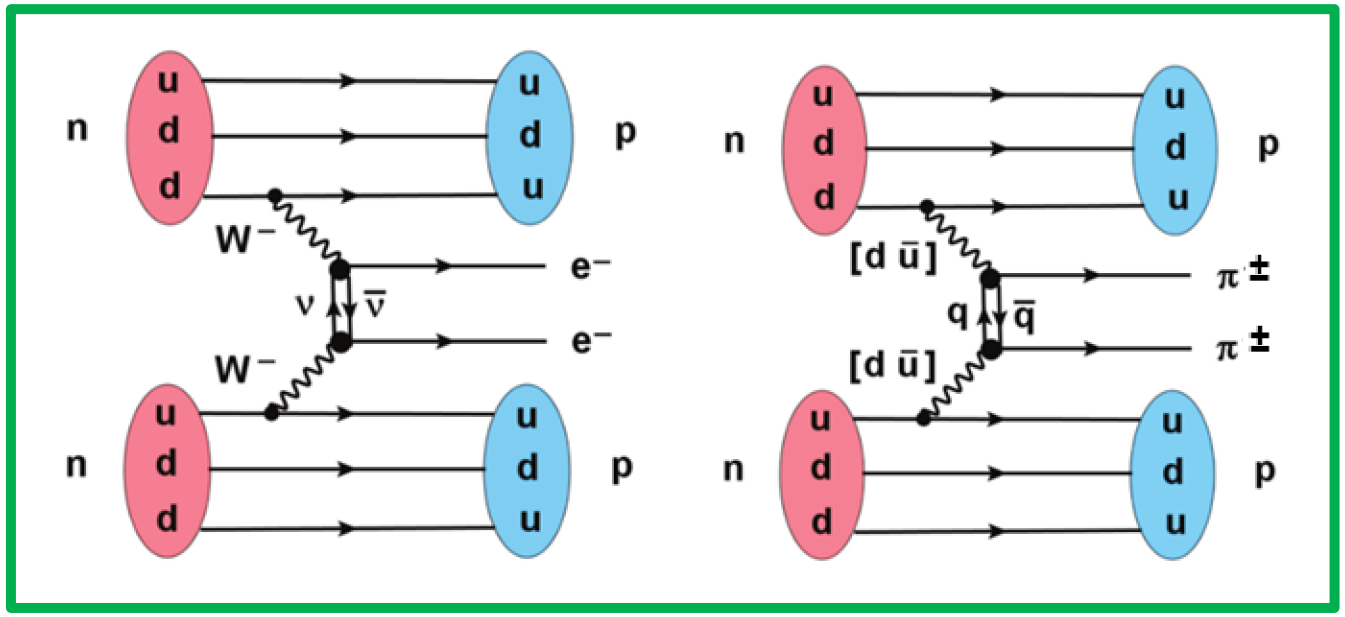


Ettore Majoran
*Aug., 5, 1906, at Catania
disappeared, Mar, 1938
+ 1959 in Venezuela?
+ in a Sicilian monastery?

- **Topological Correspondence on the Diagrammatic Level**
- **MDCE as Probe for Nuclear for Wave Functions in 0ν2β-NME**

MDCE and 0ν2β Double Beta-Decay

The Majorana Aspect



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Summary and Outlook

- **DSCE reactions ↔ analogue of $2\nu2\beta$ – decay**
- **MDCE reactions ↔ analogue of $0\nu2\beta$ – decay**
- **Induced rank-2 IsoTensor interaction ↔ generic for heavy ion DCE reactions**
- **MDCE box diagrams and pion potentials**
- **Pion-nucleon scattering and MDCE interactions**
- **Nuclear structure aspects of DCE physics – „Multi-Methods“ Approach**

Precision Spectroscopy with heavy ion beams

…in collaboration with the NUMEN theory group:
J. Bellone, M. Colonna, D. Gambacurta
and A. Gargano (Naples), J.-A. Lay (Valencia), and E. Santopinto (Genova)

Supported by DFG, grant Le439/16, INFN, and AvH

Backup

Question;
How to fix the Invariant Pion-Nucleon Energy?

Consider Fermi-motion ($k \le k_F$) and motion in the P+T c.m.-frame ($q = q_{cm}/A$)
plus
pion off-shell 4-momentum $p_\pi=(\omega,p)$

$$s_{\pi N}(k,p,q\,|\,k_F) = \left\langle \left(E_N(\vec{k}+\vec{q})+\omega \right)^2 - (\vec{k}+\vec{q}+\vec{p})^2 \right\rangle_{K_F}$$

$$= \left(M_N + m_\pi \right)^2 + \omega^2 - \varepsilon_\pi^2(p) + 2\left(\omega \left\langle E_N(\vec{k}+\vec{q}) \right\rangle_{K_F} - m_\pi M_N \right)$$

→ **On-shell only for $\omega=\varepsilon_\pi$**

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