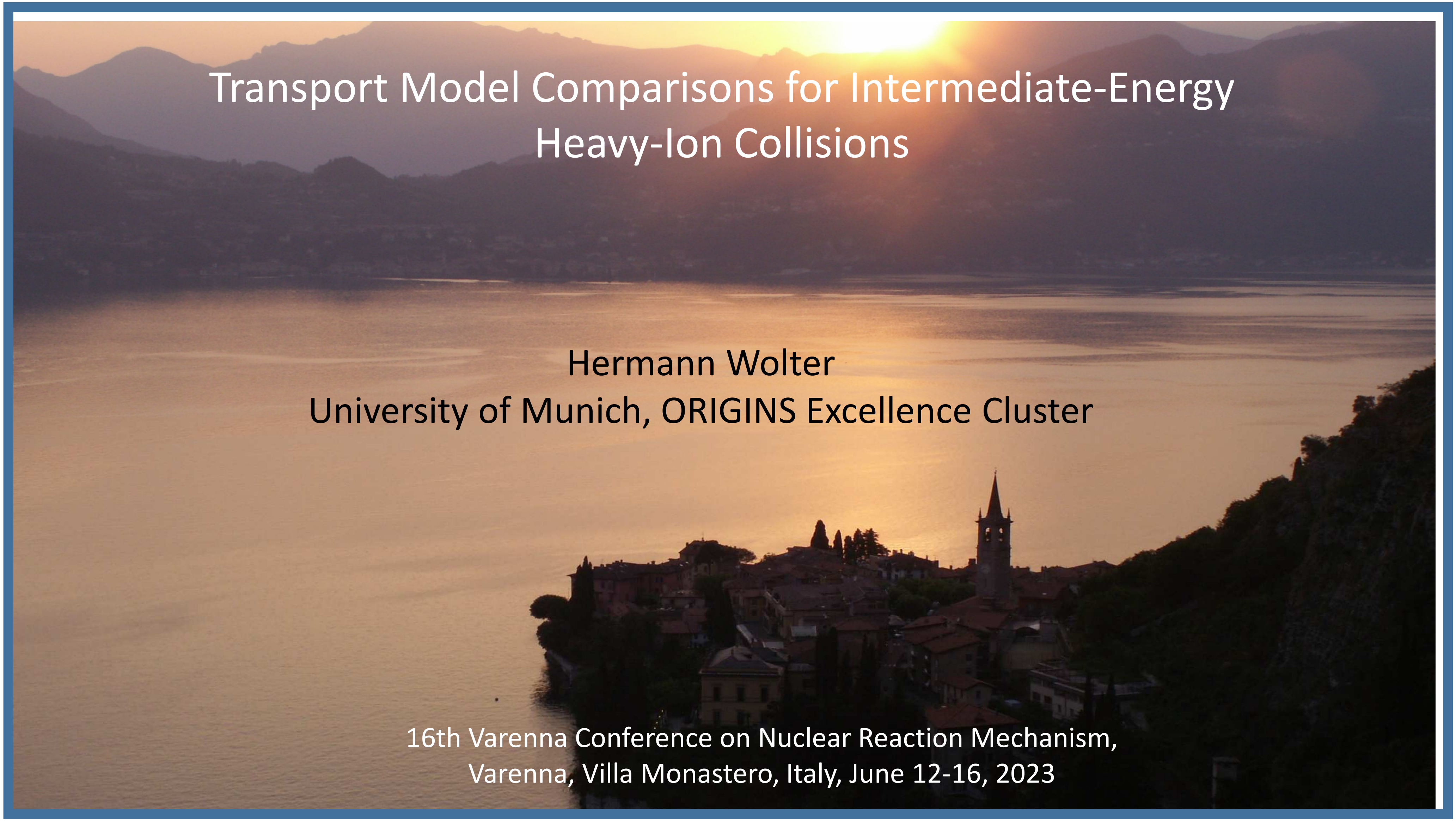
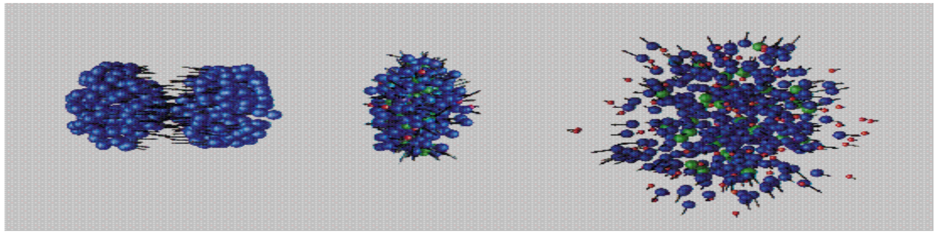


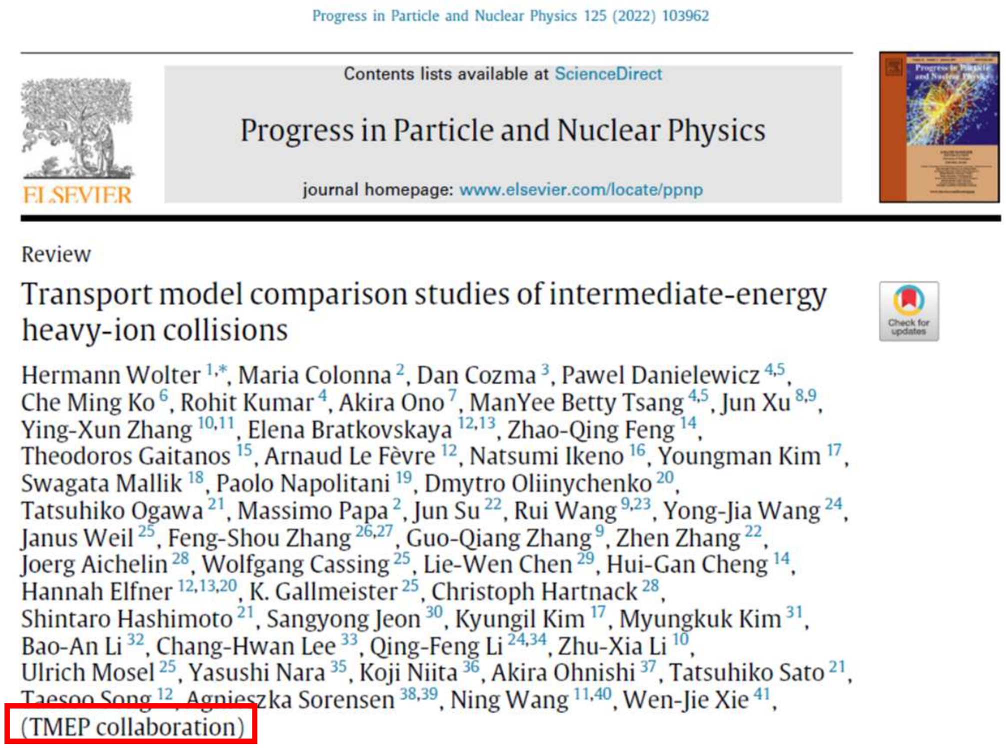
Transport Model Comparisons for Intermediate-Energy Heavy-Ion Collisions

Hermann Wolter
University of Munich, ORIGINS Excellence Cluster

16th Varenna Conference on Nuclear Reaction Mechanism,
Varenna, Villa Monastero, Italy, June 12-16, 2023

On behalf of the Transport Model Evaluation Project (TMEP) Collaboration





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Review

Transport model comparison studies of intermediate-energy heavy-ion collisions

Hermann Wolter [1,*], Maria Colonna [2], Dan Cozma [3], Pawel Danielewicz [4,5], Che Ming Ko [6], Rohit Kumar [4], Akira Ono [7], ManYee Betty Tsang [4,5], Jun Xu [8,9], Ying-Xun Zhang [10,11], Elena Bratkovskaya [12,13], Zhao-Qing Feng [14], Theodoros Gaitanos [15], Arnaud Le Fèvre [12], Natsumi Ikeno [16], Youngman Kim [17], Swagata Mallik [18], Paolo Napolitani [19], Dmytro Oliinychenko [20], Tatsuhiko Ogawa [21], Massimo Papa [2], Jun Su [22], Rui Wang [9,23], Yong-Jia Wang [24], Janus Weil [25], Feng-Shou Zhang [26,27], Guo-Qiang Zhang [9], Zhen Zhang [22], Joerg Aichelin [28], Wolfgang Cassing [25], Lie-Wen Chen [29], Hui-Gan Cheng [14], Hannah Elfner [12,13,20], K. Gallmeister [25], Christoph Hartnack [28], Shintaro Hashimoto [21], Sangyong Jeon [30], Kyungil Kim [17], Myungkuk Kim [31], Bao-An Li [32], Chang-Hwan Lee [33], Qing-Feng Li [24,34], Zhu-Xia Li [10], Ulrich Mosel [25], Yasushi Nara [35], Koji Niita [36], Akira Ohnishi [37], Tatsuhiko Sato [21], Taesoo Song [12], Agnieszka Sorensen [38,39], Ning Wang [11,40], Wen-Jie Xie [41], (TMEP collaboration)

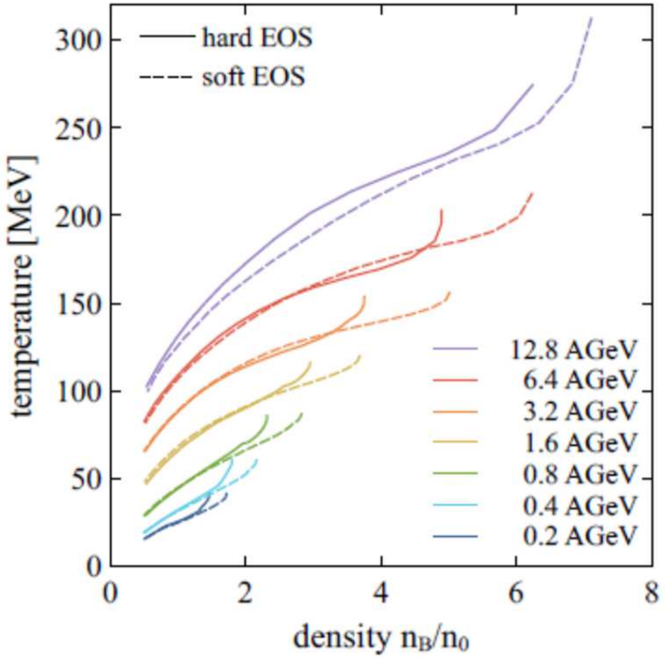
Outline:

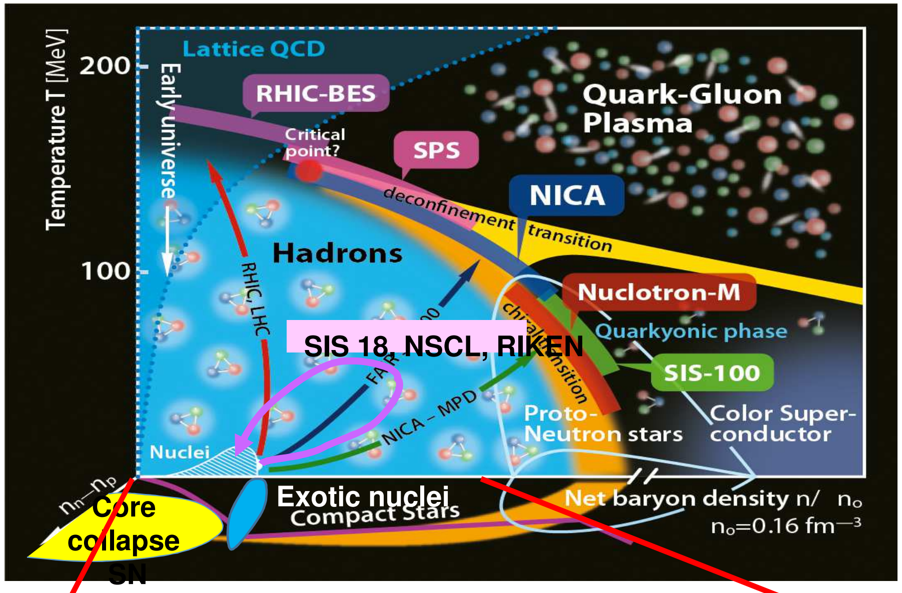
- Motivation: Importance of Heavy-Ion Collisions (HICs) for the exploration of the EOS
- Model dependence of results of HIC transport simulations
- Transport model comparisons under controlled conditions
  - box calculations,
  - HICs
- Lessons, future projects, conclusions

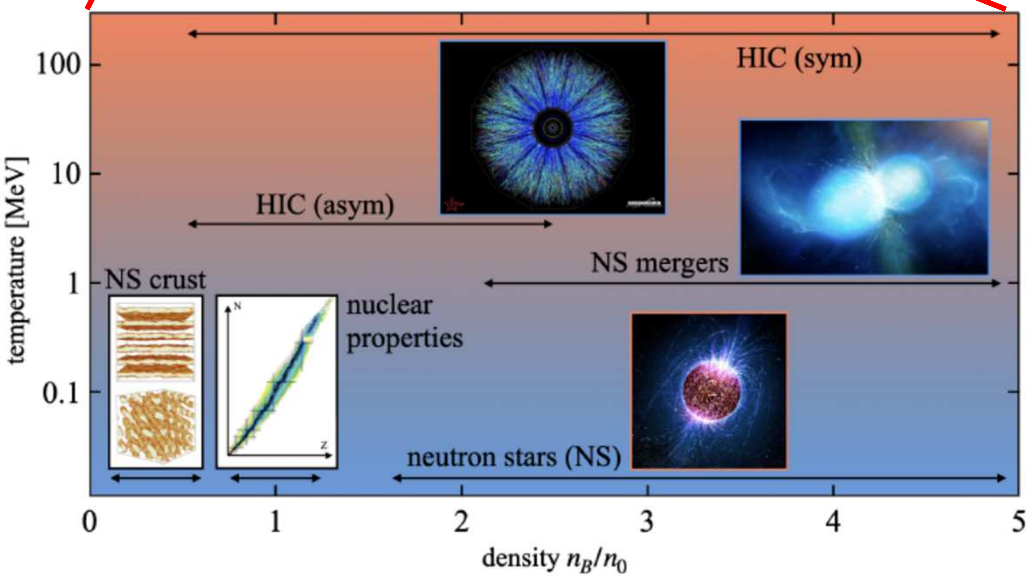
Importance of intermediate-energy heavy-ion collisions for the exploration of equation-of-state (EOS), i.e. E(density ρ, temp T, asymmetry β)

→ filling the gap between information from nuclear structure ($\rho \leq \rho_0$) and neutron star observations ($\rho \geq 2.5\ \rho_0$)

densities attainable (transport model simulation)





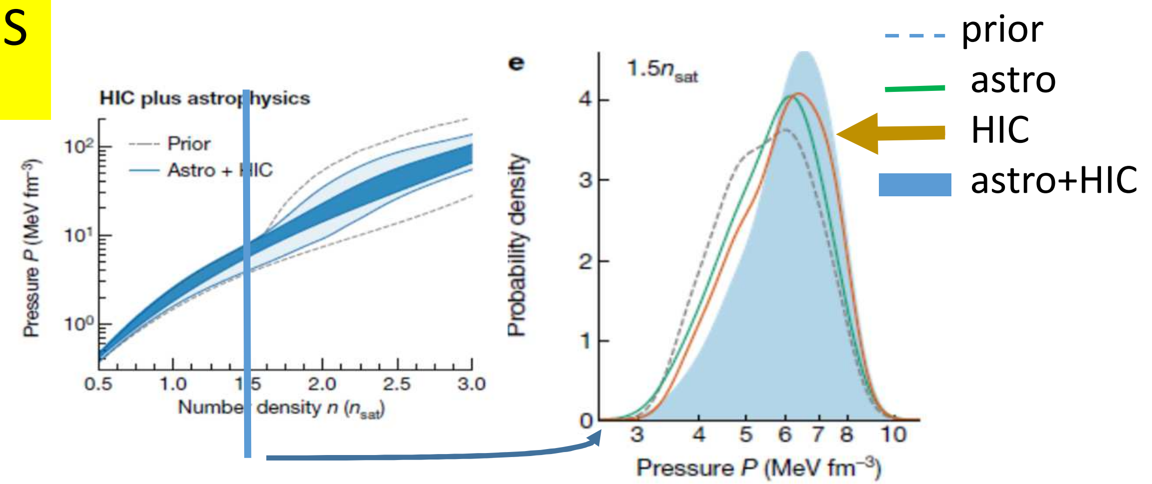


Contribution of HICs to constraints on the EOS

(Bayesian analysis from several sources)

Huth, et al., Nature 606 (2022): neutron star matter EOS
Using: nuclear many-body theory (χEFT, prior)
astrophysical multi-mess (LIGO, NICER)
HIC (FOPI, ASY-EOS)

controversial

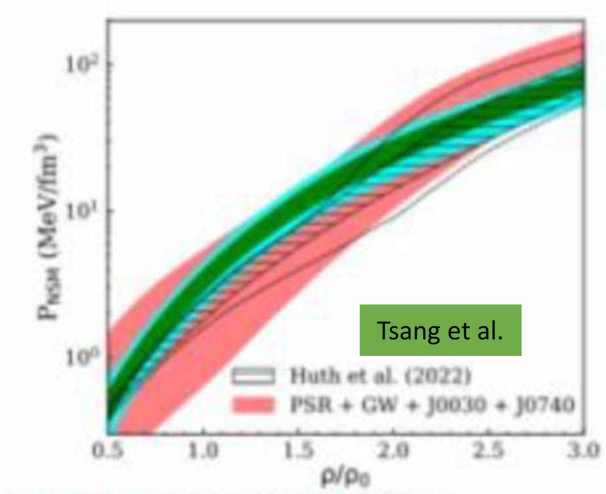


C.Y. Tsang, R. Kumar, M.B.Tsang, W. Lynch, et al., in preparation
Using: symmetric nm: (masses, GMR, HIC flow)

asymmetric nm:
structure (masses, neutron skins, α_d, IAS),
HIC (isodiff,n/p ratios and flow, pion ratios)

astrophysics (GW (LIGO), pulsar pulse shape(NICER))

not: many-body theory



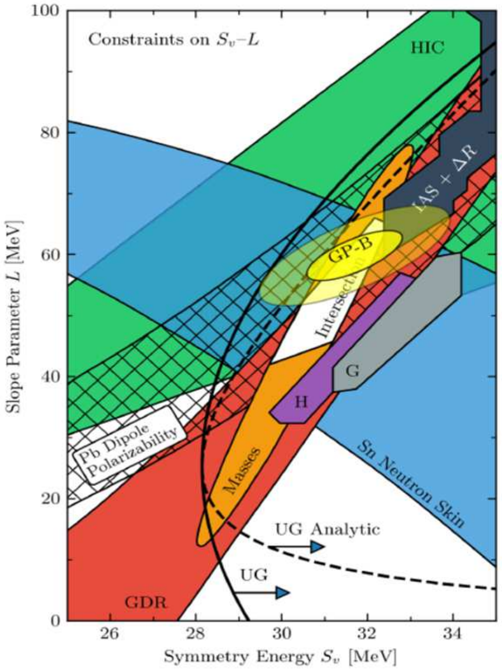
→ HIC data can make important contributions to the symmetry energy, but still with uncertainties, usually using one model, uncertainty of model-dependence not included

→ Need quality control of transport codes → **code comparisons**

Constraints on the density dependence of the Symmetry Energy: Lynch, Tsang, PLB 830 (2022)

$$E(\rho_B,\delta) = E_{nm}(\rho_B) + E_{sym}(\rho_B)\,\delta^2 + O(\delta^4) + \ldots \qquad \delta = (\rho_n - \rho_p)/\rho_B$$

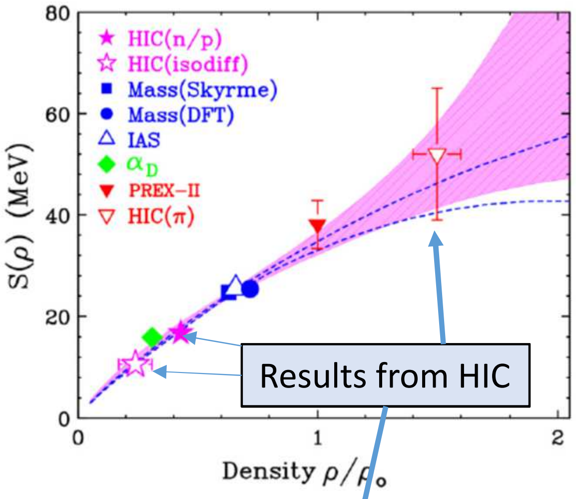
$$E_{sym}(\rho_B) = S_0 + \frac{L}{3}\left(\frac{\rho_B - \rho_0}{\rho_0}\right) + \frac{K_{sym}}{18}\left(\frac{\rho_B - \rho_0}{\rho_0}\right)^2 + \ldots$$



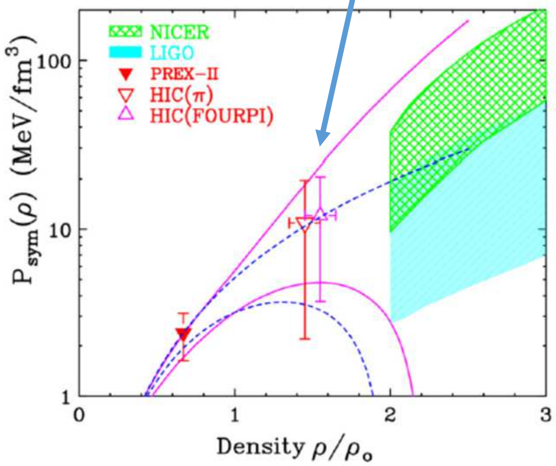
Usual representation of S_0 vs. L from various observables.
Not so informative, since involve model-dep. extrapolation to ρ_0, and are thus correlated.

Determine sensitive density for each observable by diff. methods

HIC contribute at supra-saturation density, but with large uncertainites.
In each case the analysis was performed using one transport code.



Results from HIC

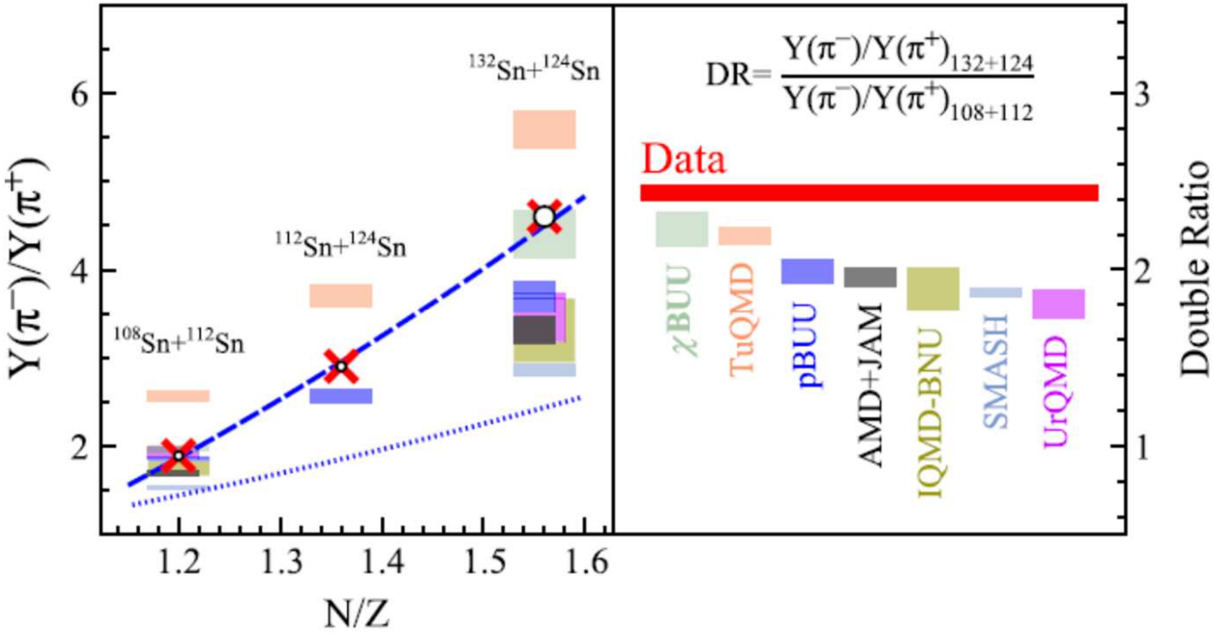


Uncertainties in predictions of HICs

Example:

SπRIT data, Sn+Sn, 270 MeV/A, Jhiang, et al., PLB 813 (21)

predictions: calculations with best model of each code



Large variation between codes,
Sensitivity to SE (width of boxes) smaller than differences between codes and difference to experiment
difficult to give an error to transport conclusions

Competing but complementary methods to constrain EoS, and, in particular, the symmetry energy:

Nuclear structure: masses, collective motion, dipole polarizability, isobaric resonances, neutron skins

Heavy-ion collisions: isospin diffusion, n/p ratios, momentum distribution (“flow”), pion ratios

Astrophysics: Neutron stars: mass-radius relation ↔ EOS,
NS mergers: GW signal → Λ deformability ↔ radius, kilonova → nucleosynthesis
Supernovae: explodability, neutrino opacity and nucleosynthesis

	density	asymm. β=N/Z	temp	equilibr	composition	accuracy
Nuclear structure	$\rho<\rho_0$	$\beta\leq 1.2$	≈ 0	yes	(yes)	high
HIC	$0\leq\rho\leq 3\rho_0$	$\beta\leq 1.6$	$(2\text{–}50)$	no	yes	discussed here
astrophysics	$\rho>\rho_0$	$\beta\approx 10$	0	yes	(yes)	improving

bridge gap between information from structure and astrophysics

inherent complexity of heavy-ion collisions

Remarks on derivation of transport theory for HIC

(e.g. P. Danielewicz, Ann. Phys. 152, 239 (1984), and Transport 2019 workshop, ECT*)

Real-time Green function method: non-equilibrium, many-body

$$G_1(r_1,t_1;r_1',t_1') \leftrightarrow G_2(r_1,t_1,r_2,t_2;r_1',t_1',r_2',t_2') \leftrightarrow G_3(1,2,3;1',2',3') \leftrightarrow \cdots$$ BBGKY-Hierarchy

non-equilibrium-> 2 indep. Greenfcts

Truncation on 1-body level and definition of self energy

$$\Sigma i G_1(1,1') = \int d2d2'd3'\, G_1(1,2)\langle 23'|V|2'3'\rangle G_2(2'3';1'3')$$

$$\approx \int d2d2' G_1(1,2)\boxed{\Sigma(2,2')}G_1(2',1')$$

This neglects higher order correlation effects,

they have to re-introduced: - in the form of fluctuations (for fragment production)

- explicitely (for light clusters)

Quasi-particle approx.: under slow spatial and temporal changes of the system the Wigner transform of $G^<$ becomes a 1-body phase space density

$$f(r,p;t) = \int dr' e^{ipr'} G_1^<(r+\tfrac{r'}{2}, r-\tfrac{r'}{2};t), \quad r = \tfrac{1}{2}(r_1+r_2),\ r' = (r_1-r_2)$$

This obeys an evolution equation of the Boltzmann-Vlasov type:
Mean field evolution plus collision term

$$\boxed{\frac{df}{dt} = \frac{\partial f}{\partial t} + \frac{\vec{p}}{m}\vec{\nabla}^{(r)} f - (\vec{\nabla}^{(r)}U(r,p)\,\vec{\nabla}^{(p)} + \vec{\nabla}^{(p)}U(r,p)\vec{\nabla}^{(r)})f(\vec{r},\vec{p};t) = I_{coll}}$$

Transport theory: kinetic equation Boltzmann-Uehling-Uhlenbeck (BUU)

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \frac{\vec{p}}{m}\vec{\nabla}^{(r)} f - \left(\vec{\nabla}^{(r)} U(r,p)\, \vec{\nabla}^{(p)} + \vec{\nabla}^{(p)} U(r,p)\, \vec{\nabla}^{(r)}\right) f(\vec{r},\vec{p};t) = I_{coll}$$

$$I_{coll} = \int d\vec{p}_2 d\vec{p}_{1'} d\vec{p}_{2'}\, v_{21}\, \sigma_{12}^{in-med}(\Omega)\, (2\pi)^3 \delta(p_1 + p_2 - p_{1'} - p_{2'}) \left[f_{1'} f_{2'} \bar{f}_1 \bar{f}_2 - f_1 f_2 \bar{f}_{1'} \bar{f}_{2'} \right]$$

$\bar{f}_i := (1 - f_i)$ Pauli blocking factors,

Physical model:
mean field->EOS,
in-medium xsec

Two main reasons of model dependence: **1) fluctuations, 2) simulation strategies**

1) Two families, depending on representation of phase space density $f(\vec{r},\vec{p};t)$, different philosophies about fluctuation

BUU phase space density represented by test particles (TP) of shape *g*

$$f(\vec{r},\vec{p};t) = \sum_{TP\ i} g(\vec{r} - \vec{r}_i(t))\, \tilde{g}\big(\vec{p} - \vec{p}_i(t)\big),$$

deterministic and exact for #TP→∞

introduce fluctuations explicitely, Boltzmann-Langevin, add term $+\delta I_{fluc}$

QMD product of wave packets in coordinate space $f(\vec{r},\vec{p};t) = \left(\frac{\hbar}{\sqrt{L}}\right)^3 \sum_i \exp\left[-\frac{(\vec{r} - \vec{R}_i(t))^2}{2L}\right] \delta(\vec{p} - \vec{P}_i(t))$

fluctuation on classical level by ansatz, fluctuations parametrized by width parameter L,
„events“

-> difference in fluctuations, influences many aspects of simulation

Transport theory: kinetic equation Boltzmann-Uehling-Uhlenbeck (BUU)

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \frac{\vec{p}}{m}\vec{\nabla}^{(r)} f - \left(\vec{\nabla}^{(r)} U(r,p)\,\vec{\nabla}^{(p)} + \vec{\nabla}^{(p)} U(r,p)\,\vec{\nabla}^{(r)}\right) f(\vec{r},\vec{p};t) = I_{coll}$$

$$I_{coll} = \int d\vec{p}_2 d\vec{p}_{1'} d\vec{p}_{2'}\, v_{21}\, \sigma_{12}^{in-med}(\Omega)\, (2\pi)^3 \delta(p_1 + p_2 - p_{1'} - p_{2'}) \left[f_{1'} f_{2'} \bar{f}_1 \bar{f}_2 - f_1 f_2 \bar{f}_{1'} \bar{f}_{2'} \right]$$

$\bar{f}_i := (1 - f_i)$ Pauli blocking factors,

Physical model:
mean field->EOS,
in-medium xsec

Two main reasons of model dependence: **1) fluctuations, 2) simulation strategies**

2) Transport equation solved by simulations, involves strategies

mf dynamics: - Hamiltonan equations-of-motion (for TP or nucleons)
- finite-size TP, use of lattice Hamiltonian
- non-linear density functionals $\propto \rho^{\sigma}$, often approximated in QMD

collision term: - stochastic two-body collisions
- geometric coll criterion $d < \sqrt{\sigma(\sqrt{s})/\pi}$ $\leftrightarrow$ local thermal equilibrium
- blocking: need for averaging, coarse graining, surface effects
- Markovian collisions, not ensured in geometric prescription

Different strategies:
Reason for code differences,
even with controlled physical model

Quality Control, to increase predictive power of HIC

→ Transport Model Evaluation Project (TMEP): Compare transport codes with controlled conditions (alternative: develop universal code?)

Brief summary of efforts so far: review, H. Wolter, et al., Progr. Part. Nucl. Phys. 125 (2022)

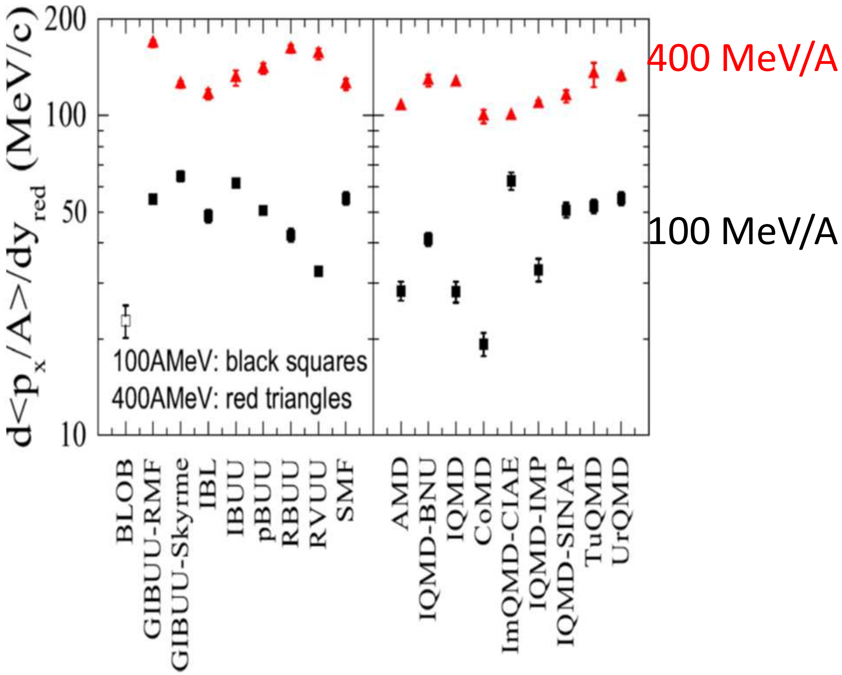
History:

2009/2014: Au+Au@100, 400 MeV/A: (J. Xu, et al., PRC 93 (2016))

density evolution and nucleonic observables (stopping, flow)

considerable differences (dep. on energy)

→ difficult to identify exact reasons (e.g. blocking, initialization)



2018-2021 Box calculations: controlled calculations in a periodic box, simple initialization, near equilibrium, exact limits

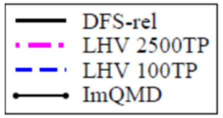
check separately ingredients of transport:

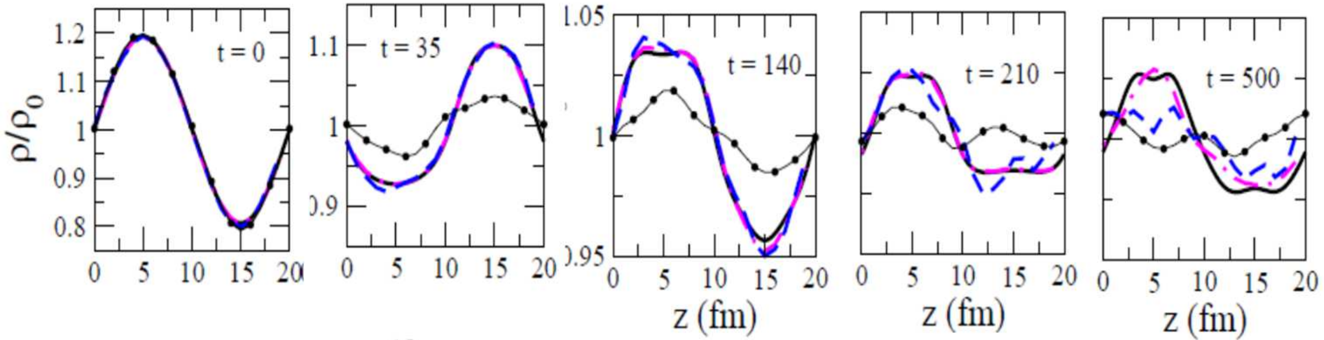
- Mean field propagation (Vlasov)
- Collision term (cascade)
- Pion production in cascade

2023 Back to HICs; Sn+Sn@270 MeV/A, SPIRIT Collaboration

Mean field evolution (M. Colonna, et al., PRC104 (2021))

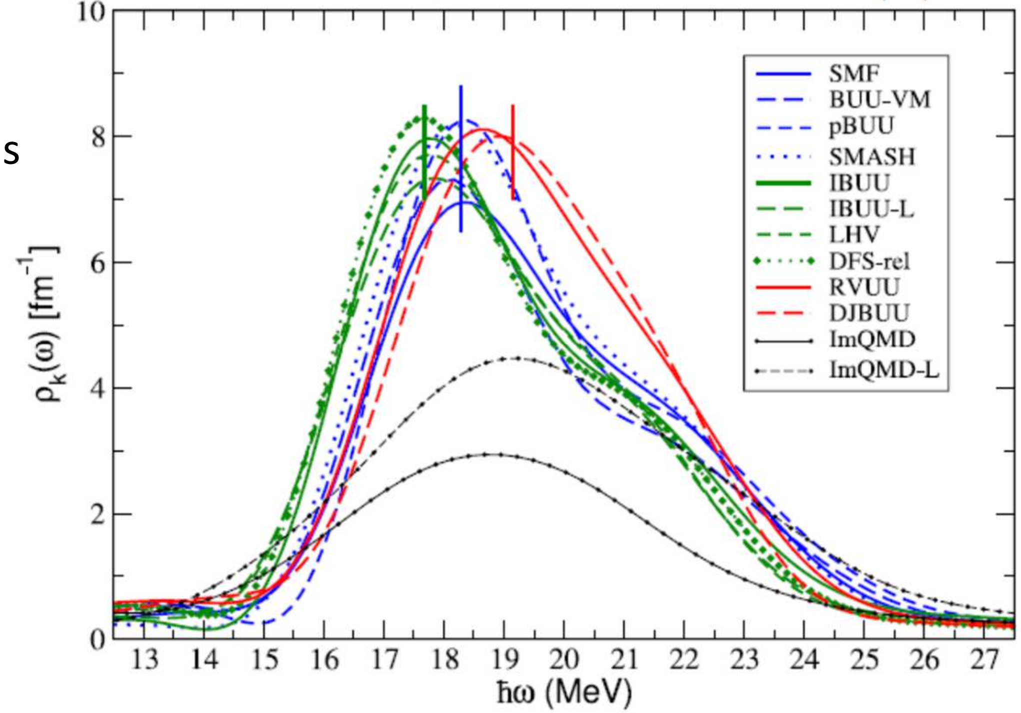
evolution of a standing wave





strength function, power spectrum

exact results from Landau theory: horizontal lines



Understanding diff's:

treatment of relativity (diff colors)

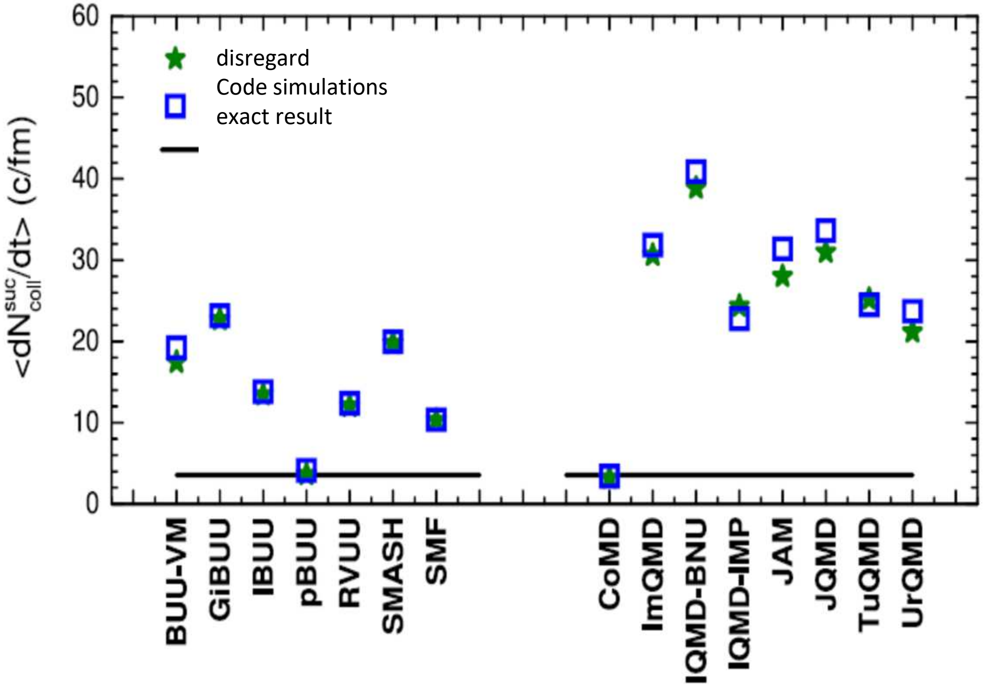
fluctuations affects forces, damping

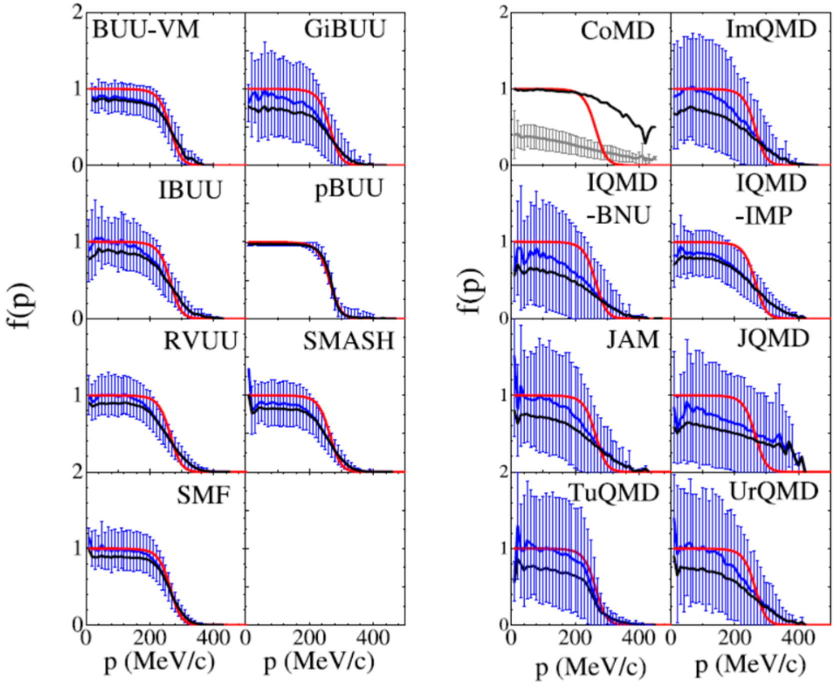
treatment of non-linear term (QMD)

Collision intergral (only nucleons, with Pauli blocking, initialize at T=5 MeV)

(YX. Zhang, et al., PRC 97 (2018))

Collision rates, compared to exact result:
Systematic difference between BUU and QMD results





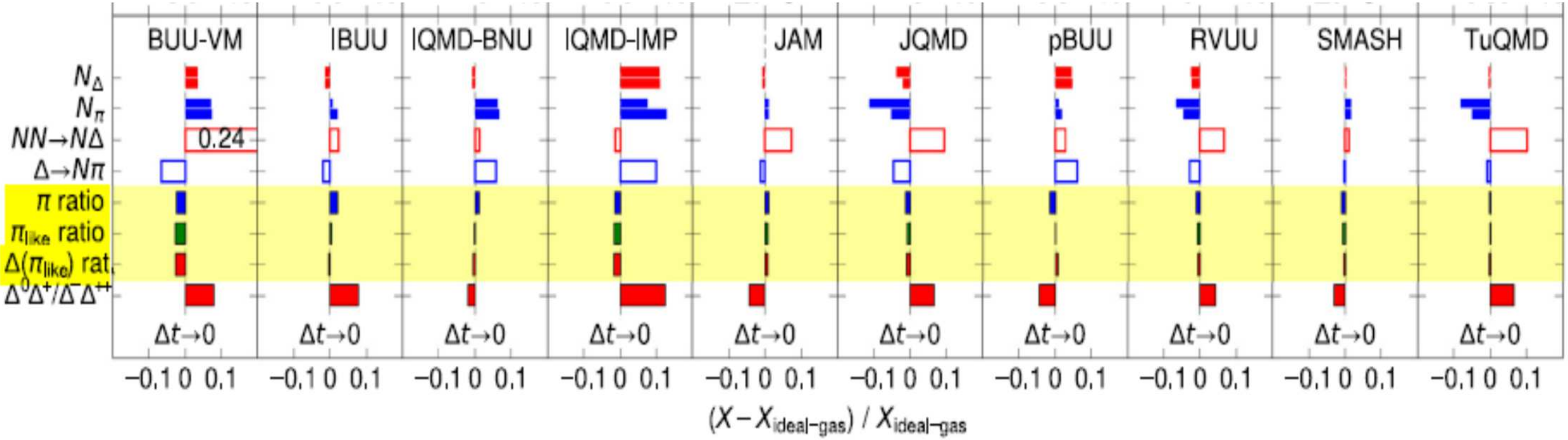
Reason: Fluctuations in Pauli blocking factor (1-f)
exact: red
average: blue
effective (enforce f≤1): black

generally underblocking (black ↔ red)

Pion production in a box (w/o Pauli blocking), (A. Ono, et al., PRC 100 (2019))

extrapolation to time step zero

multiplicities and multiplicity ratios (relative difference to exact result)



determines
π–/π+ ratio
in a HIC

origins of differences:

- independence of collisions (Markov)
- strategies in handling elastic and inelastic collisions

Cancel rather well in ratios (underlaid in yellow)

Back to HIC: Sn+Sn@270 MeV/A (SπRIT setup) (J. Xu, et al., in preparation)

similar to earlier Au+Au@100,400 MeV/A, plus calculations of pion observables

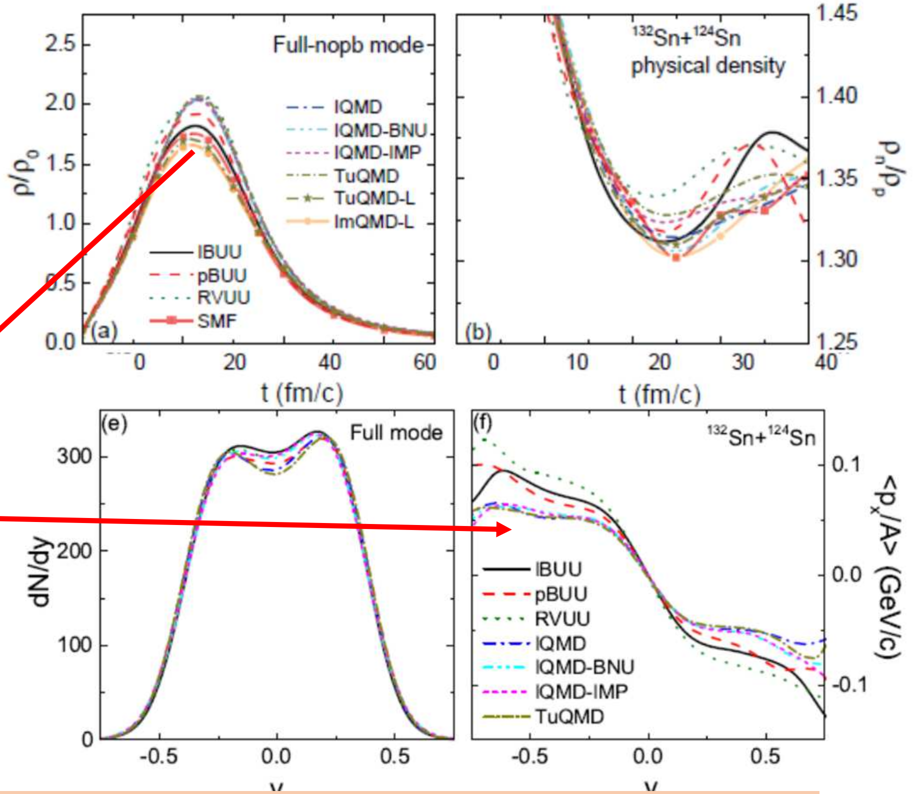
controlled input: common initializ., simple mom.-indep. EOS, σ_{el}=const, $\sigma_{NN\leftrightarrow N\Delta}$, $\sigma_{\Delta\leftrightarrow N\pi}$

Nucleon evolution

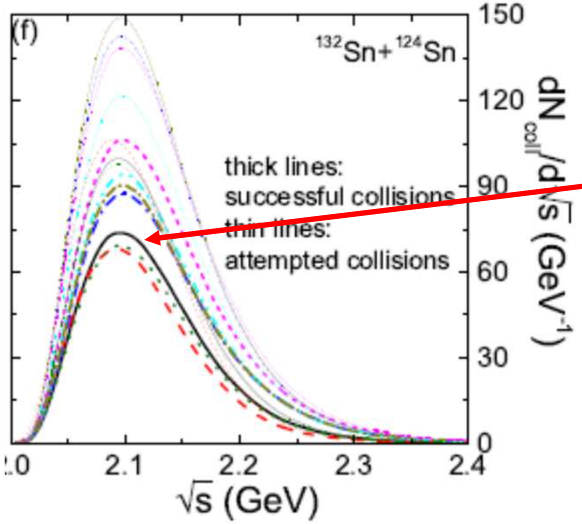
Not really identical,
e.g. systematic BUU-QMD difference (as in box-Vlasov)
non-linear repuls. forces, fluctuations, Pauli blocking.

Influences stopping and transverse flow, and also collision rates (here NN→NΔ)

density asymmetry (in central 5^3 fm^3 cell



BUU codes
smaller density,
more flow,
smaller collision rates



An identical evolution of the collision in the different codes is not achieved (partly intrinsic, partly possible to improve), affects pion observables

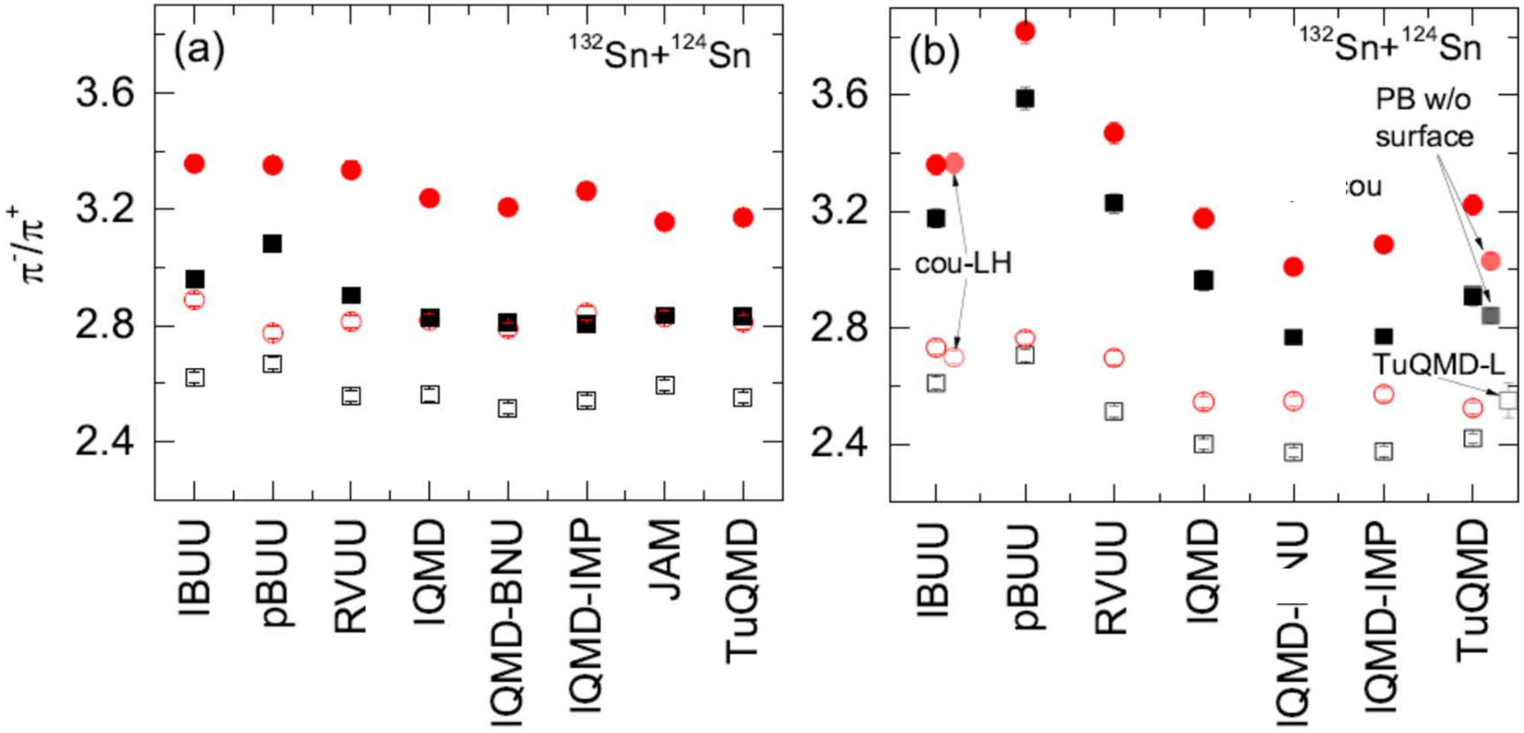
Sn+Sn@270 MeV/A (SπRIT setup) charged pion ratios

π-/π+ ratio

○ Open symbol w/o Pauli
● Full symbol with Pauli
Black squares
w/o Coulomb (ok)

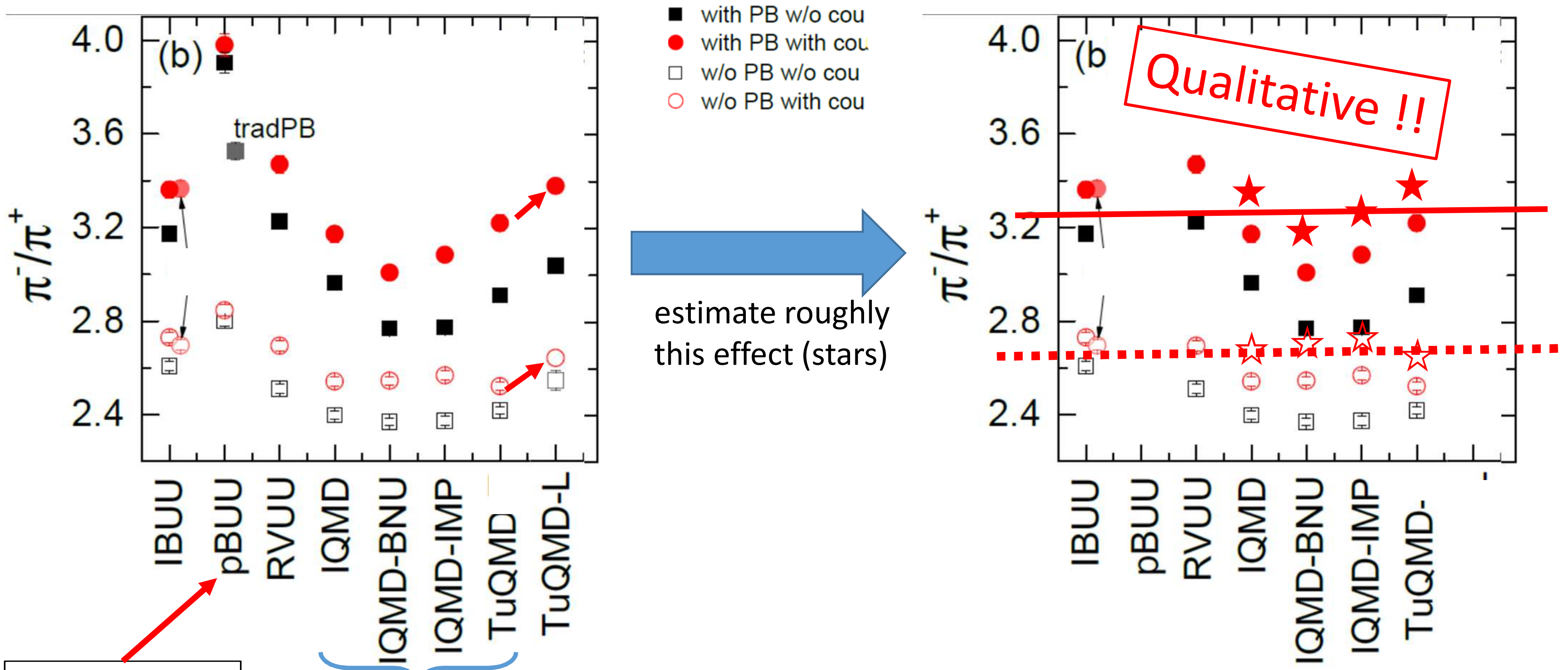
Only collisons (Cascade)

Full calc (mean field + collisions)



Rather good convergence w/o mean-field.
Not so good with mean-field,
but can be explained and related to differences in the nucleon evolution (in most cases)→next slide,

Discussion of differences



estimate roughly this effect (stars)

π, Δ feel symmetry energy not comparble

QMD codes, approx. for ρ^γ weaker repulsion, reduced pion ratio, correct by TuQMD diff.

agreement w/o Pauli-blocking very good
with Pauli-blocking differences <≈ 10%
→ due to differences in simulation of PB

Discussion:

- It is difficult to reach complete convergence of all codes.
  Different strategies →different evolution (density, asymmetry)→different observables (open system!)
- Possible in most cases to understand the reasons for differences, at least qualitatively.
- **Differences can be argued to reduce to below 10%**

- How to control and assess uncertainty of simulations?

a) check important ingredients in box calculations,
done: mf evolution, NN collisions, pion production,
future: momentum dependence, threshold effects, pion and Delta potentials, clusterization,
fragmentation (instabilities)

b) minimize effect of evolution of system, esp. late time evolution

- consider high-energy spectra
- study particles that do not interact strongly with medium: anti-strange K mesons, photons
- study correlations, e.g. between nucleon asymmetry and pion ratio

c) Check correct globa description of the evolution of the collision
→ measure and describe many nucleon observables:

d) Bayesian analysis using several codes. The width of posterior will then include the systematic
theoretical error

Conclusions:

Goal of TMEP, to increase the predictive power of transport simulations for HICs
Advantage of HICs: Control thermodynamic conditions, i.e. thermalization, clusterization, etc
Access to an important density region of the EOS for astrophysics in the lab.

Code differences have their origin on different simulation strategies:

Essential:

- Fluctuations. Different philosophies in QMD and BUU. Correct amount of fluct. at issue
- Compare QMD not with BUU but with Boltzmann-Langevin (BL).

Solvable:

- treatment of non-linear density functionals
- Pauli blocking with local statistical method (fit final state phase space locally)
- avoid non-Markovian effects in the collision integral, geometric prescription not optimal
- treatment of Coulomb potential

Estimating some of these issues the agreement could be within about 10%

The goal may not be the convergence of all codes,
but a solid **uncertainty quantification of transport analyses.**

Thank you
for your attention