



UNIVERSITÀ DEGLI STUDI DI NAPOLI
FEDERICO II



Understanding Heavy-ion Fusion Cross Section Data Using Novel Artificial Intelligence Approaches

Daniele Dell'Aquila

Dipartimento di Fisica "Ettore Pancini", Università degli Studi di Napoli "Federico II", Naples, Italy

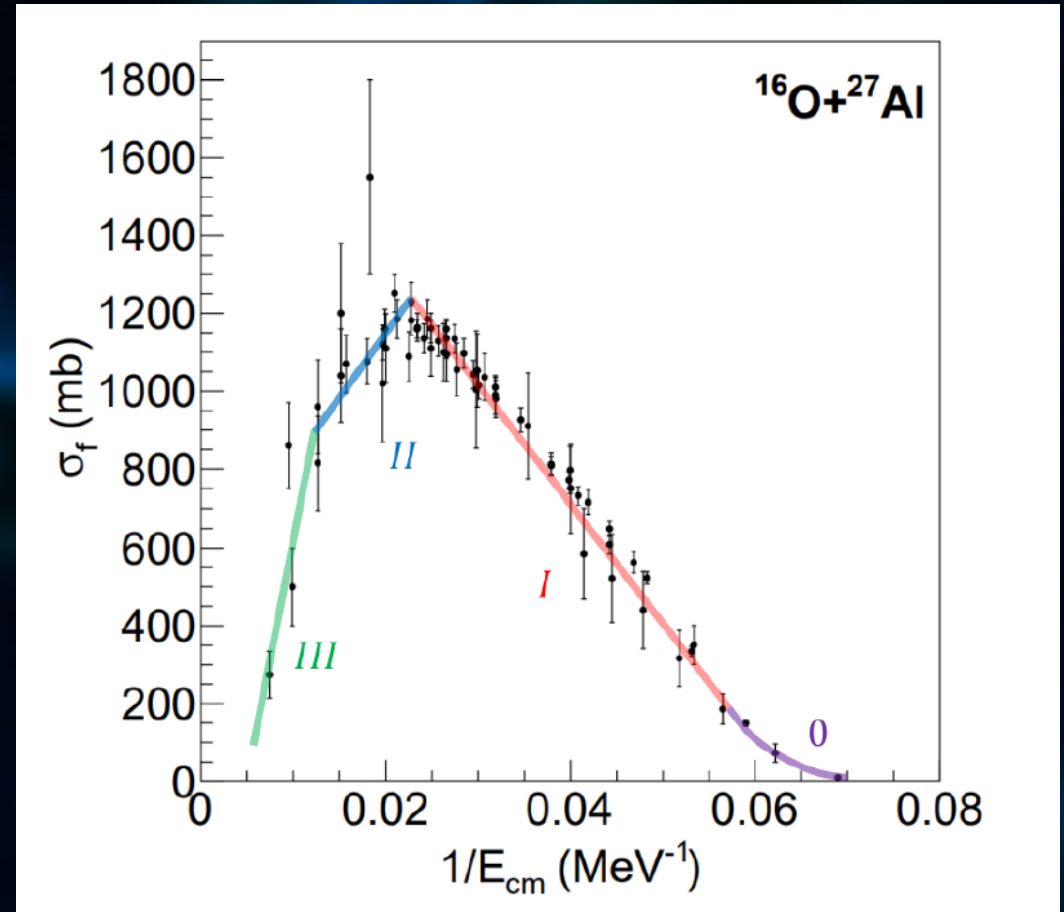
INFN – Sezione di Napoli, Naples, Italy

Complete fusion reactions: a brief overview

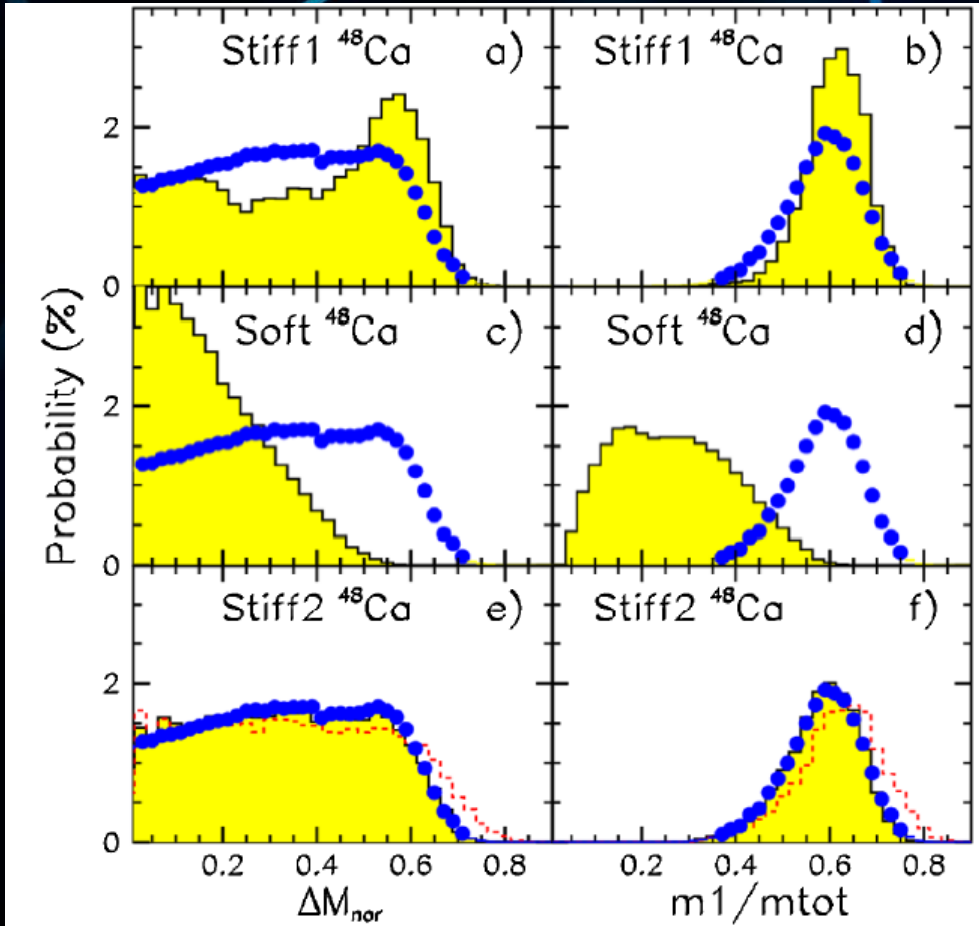
Nucleus-nucleus collisions at low and intermediate energies are the ideal playground to explore the evolution of reaction mechanisms with the collision energy and to probe the interplay of nuclear structure and dynamics (including the occurrence of collective motions in hot rotating nuclei).

Complete fusion reactions: a brief overview

- Different, complementary, experimental methods can be effectively used to estimate the yield of evaporation residues (gamma-ray analysis, time-of-flight and magnetic spectrometers, charged particle detection with telescope arrays) → heavy-ion fusion cross section from the Coulomb barrier to the onset of multi-fragmentation →
- See e.g. *P. Frobrich, Phys. Rep. 116 (1984) 337.*



Competition with incomplete fusion phenomena and isospin effects



S. Pirrone et al., Eur. J. Phys. J.A. (2019) 55: 22

→ higher probability of fusion-evaporation and fission-like processes for higher N/Z content in entrance channel.

F. Amorini et al., Phys. Rev. Lett. (2009) 102: 112701

→ Isospin effects on the competition between fusion-like processes and binary-like mechanisms.

→ Larger probability to produce heavy-residue for the systems with the largest neutron content.

→ Sensitivity to the density dependence of the symmetry energy term of the nuclear EoS using microscopic approaches.

→ Also recent results by INDRA-FAZIA support these findings!

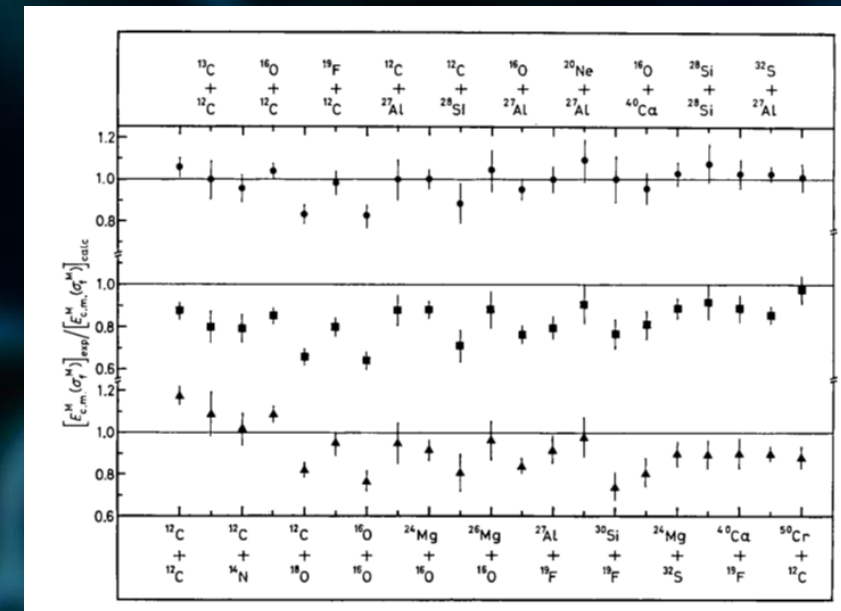
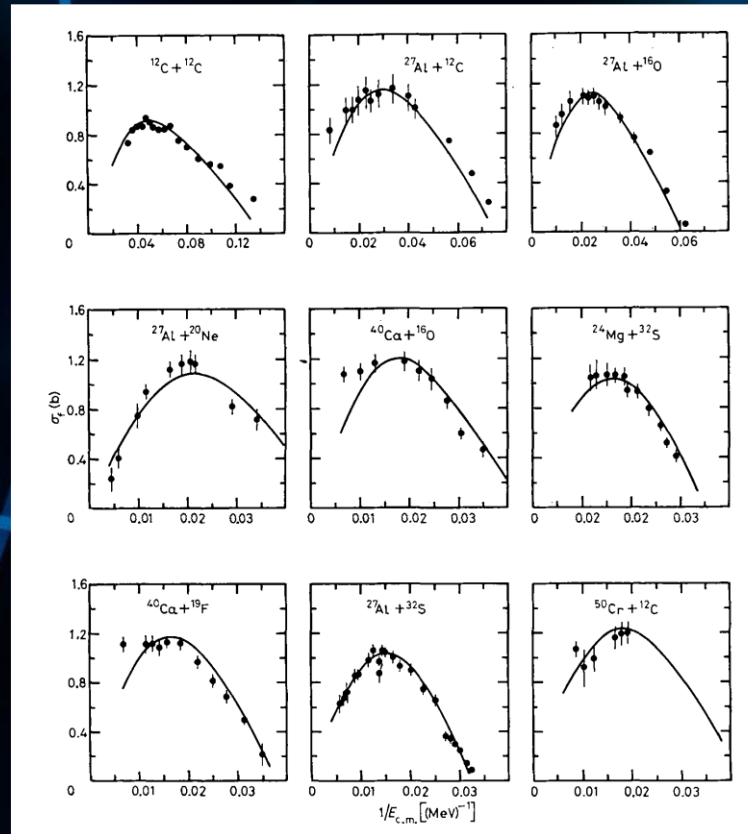
Complete fusion reactions: a brief overview

Models for the description of fusion cross section between heavy-ions:

- **Microscopical approaches:** Time-Dependent Hartree Fock (TDHF), Molecular dynamics;
- **Macroscopic models:** critical distance models, limitation to the compound nucleus model (empirical nuclear potentials from semi-classical considerations);
- **Empirical models:** starting from nuclear reaction theory and then optimizing to the experimental data.

Complete fusion reactions: a brief overview

Previous data-driven (*phenomenological*) approaches, see e.g. **Porto F and Sambataro S 1984 Nuov. Cim. 83 339** \rightarrow good description of data around the maxima of the cross section \rightarrow few datasets in Region III (high energies) and Region I (close to the Coulomb barrier).

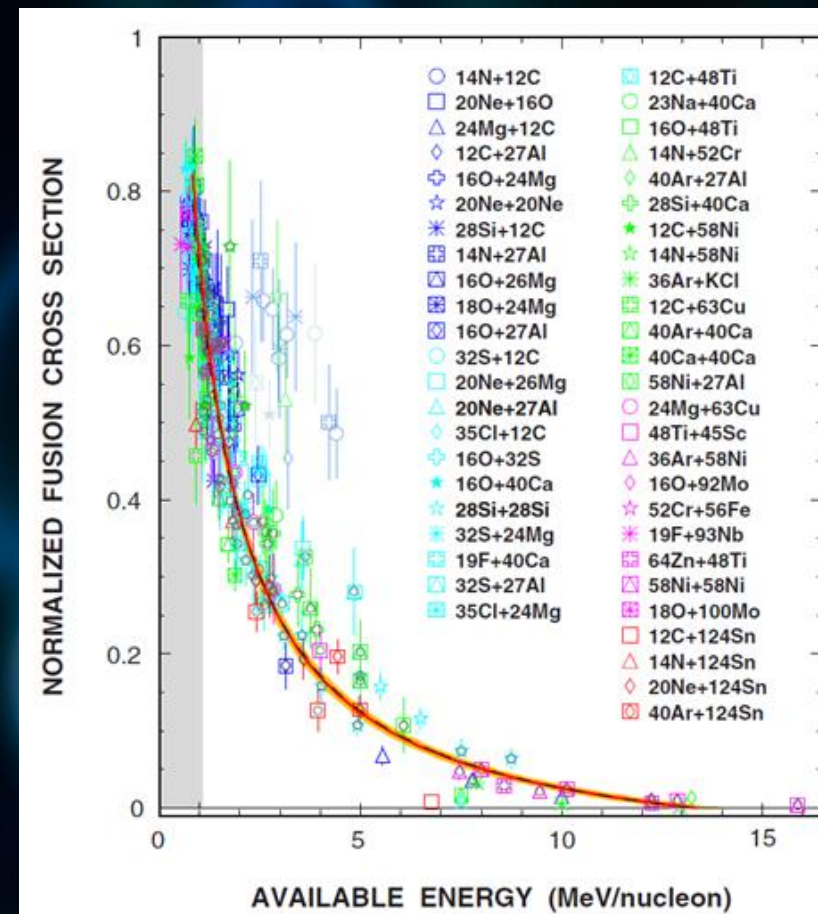


Complete fusion reactions: a brief overview

More recently → systematic study of Region III shows discrepancies for some of the systems → further investigation on both experiment and theory is required!

Fusion cross section in Region III → disagreement with the prediction of state-of-the-art for some collision systems such as:

- $^{28}\text{Si}+^{12}\text{C}$
- $^{12}\text{C}+^{27}\text{Al}$
- $^{48}\text{Ti}+^{40}\text{Ca}$
- $^{16}\text{O}+^{40}\text{Ca}$
- $^{14}\text{N}+^{12}\text{C}$
- $^{14}\text{N}+^{14}\text{N}$
- $^{14}\text{N}+^{27}\text{Al}$
- $^{14}\text{N}+^{52}\text{Cr}$
- $^{14}\text{N}+^{58}\text{Ni}$



from P. Eudes et al., Phys. Rev. C 90 (2014) 034609.

Approach, dataset, and modeling

Approach: *supervised learning* using *symbolic regression* algorithms.

Novelties:

- Deriving mathematical expressions to describe the data → support to theories and models attempting to predict the fusion cross section between heavy-ions;
- Comprehensive analysis of large amount of nuclear data → universal model for the description of the entire dataset;
- Advanced feature selection → allows to inspect the dependence on several variables (including nuclear structure variables).

Major challenges:

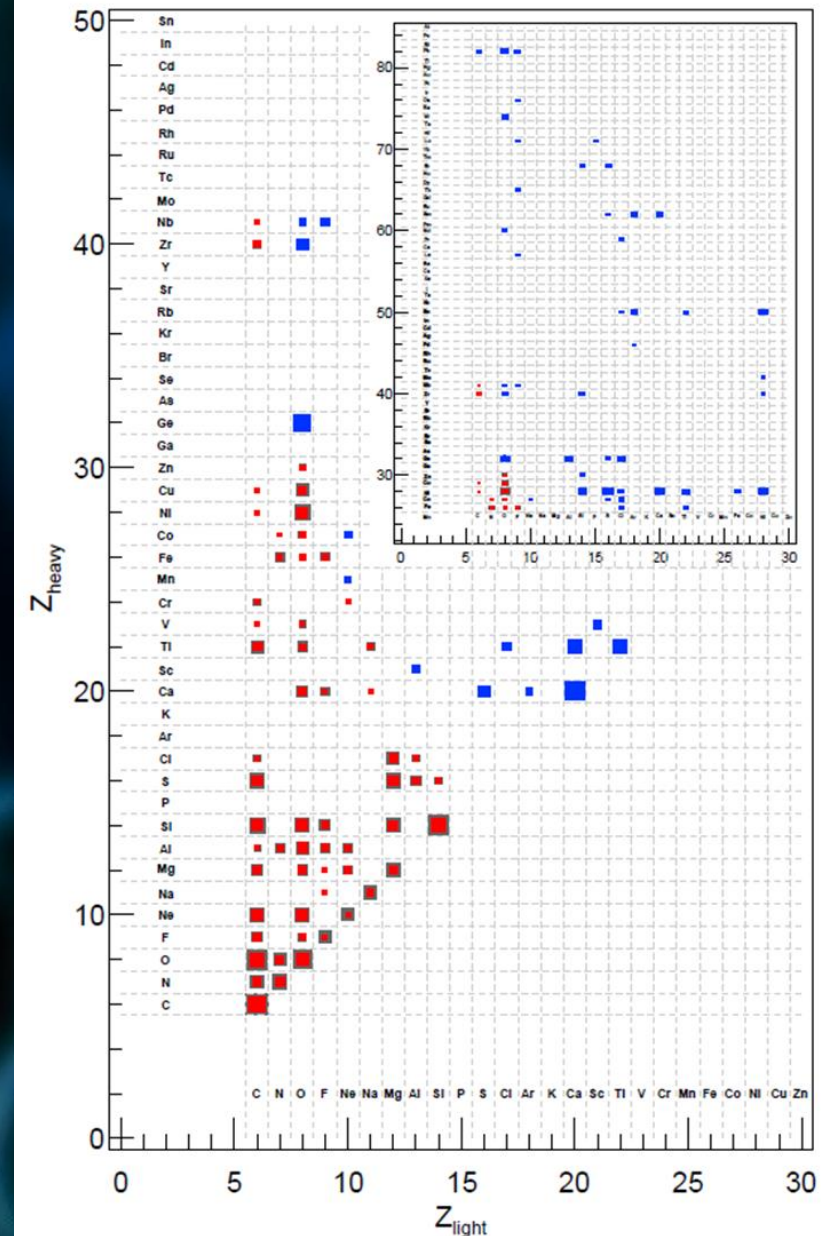
- The amplitude of the cross section varies even by several orders of magnitude with the energy;
- Experimental errors associated to each individual data point differ by several orders of magnitude for each data point;
- Resulting models must have physical boundaries and extrapolation capabilities.

We used an extensive set of *nuclear* features linked to:

- 1) The nature of the collision partners;
- 2) The energy of the collision;
- 3) The structure of the collision partners;
- 4) The structure of the compound nucleus.

Symbol	Description
E_{cm}	collision center-of-mass energy (MeV)
Z_1	charge of the first reaction partner
Z_2	charge of the second reaction partner
N_1	number of neutrons of the first reaction partner
N_2	number of neutrons of the second reaction partner
A_1	mass of the first reaction partner
A_2	mass of the second reaction partner
J_1	spin of the first reaction partner
J_2	spin of the second reaction partner
π_1	parity of the first reaction partner (1 for positive parity, -1 for negative parity)
π_2	parity of the second reaction partner (1 for positive parity, -1 for negative parity)
μ_1	magnetic dipole momentum of the first reaction partner (μ_N)
μ_2	magnetic dipole momentum of the second reaction partner (μ_N)
$\langle r^2 \rangle_1$	rms charge radius of the first reaction partner (fm)
$\langle r^2 \rangle_2$	rms charge radius of the second reaction partner (fm)
Q -value	fusion Q -value (MeV)
S_α	α separation energy of the compound nucleus (MeV)
S_n	one-neutron separation energy of the compound nucleus (MeV)
S_{2n}	two-neutron separation energy of the compound nucleus (MeV)
S_p	one-proton separation energy of the compound nucleus (MeV)
S_{2p}	two-proton separation energy of the compound nucleus (MeV)
$S_{\alpha 1}$	α separation energy of the first reaction partner
$S_{\alpha 2}$	α separation energy of the second reaction partner
S_{n1}	one-neutron separation energy of the first reaction partner
S_{n2}	one-neutron separation energy of the second reaction partner
S_{p1}	one-proton separation energy of the first reaction partner
S_{p2}	one-proton separation energy of the second reaction partner
S_{2n1}	two-neutron separation energy of the first reaction partner
S_{2n2}	two-neutron separation energy of the second reaction partner
S_{2p1}	two-proton separation energy of the first reaction partner
S_{2p2}	two-proton separation energy of the second reaction partner

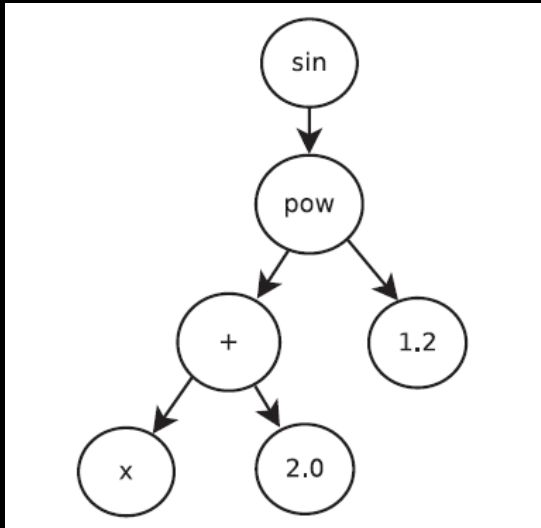
- Dataset used for model derivation \rightarrow about 4500 experimental data points.
- Learning dataset: $Z_p Z_t < 250 \rightarrow$ light-to-medium mass nuclei.
- Testing dataset 1: $Z_p Z_t \geq 250 \rightarrow$ heavy systems (test the extrapolation towards heavy systems).
- Testing dataset 2: $Z_p Z_t < 250 \rightarrow$ some of the lighter systems.



The Brain Project

Brain Project – a neural-genetic tool for the formal modeling of data

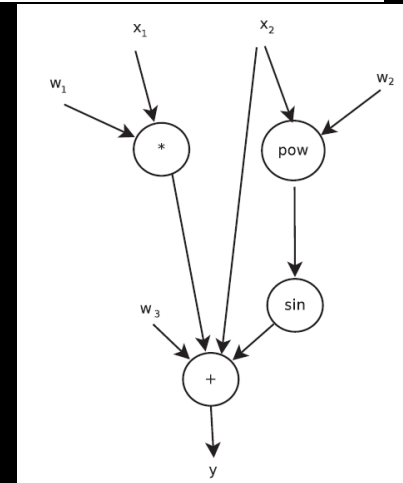
Exploits a novel hybridization of genetic programming and artificial neural network → the task is that of symbolic regression. Genetic part → foresees the evolution of tree-like structures representing mathematical expressions → deals with the global search for the maximum of a suitable fitness function; Neural part → deals with the local search for the minimum of the error when the genetic part has identified a good maximum of the fitness function.



Genetic evolution of tree-like structures representing mathematical expressions

+

$$y = 2.5 * x_1 + \sin(x_2^{3.8}) + x_2 + 1.5$$



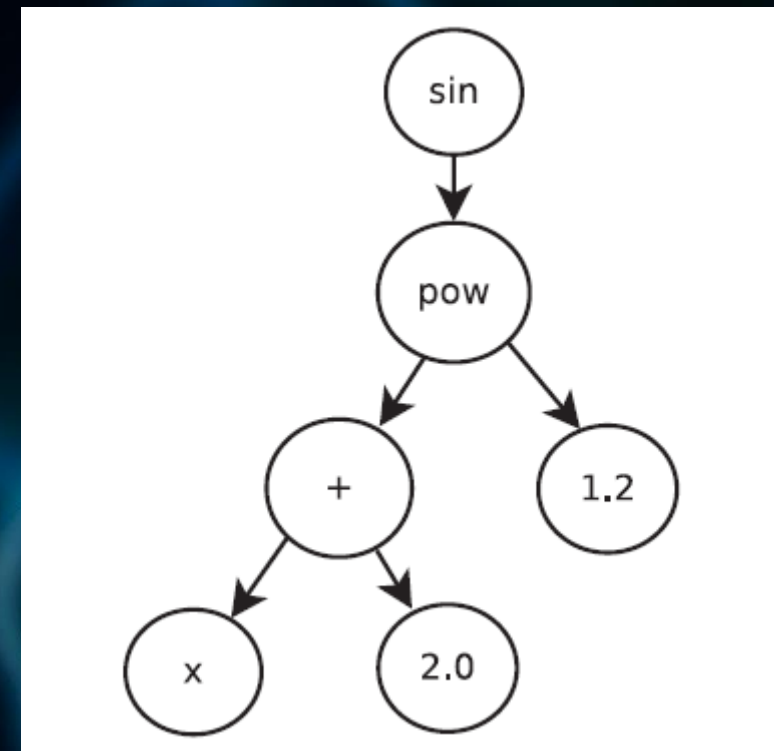
Artificial neural networks to optimize the constants (gradient descent technique)

Russo M 2016 Swarm Evo. Comput. 27 145
Russo M 2020 Soft Comput. 24 16885–16894

The Brain Project: genetic mechanism

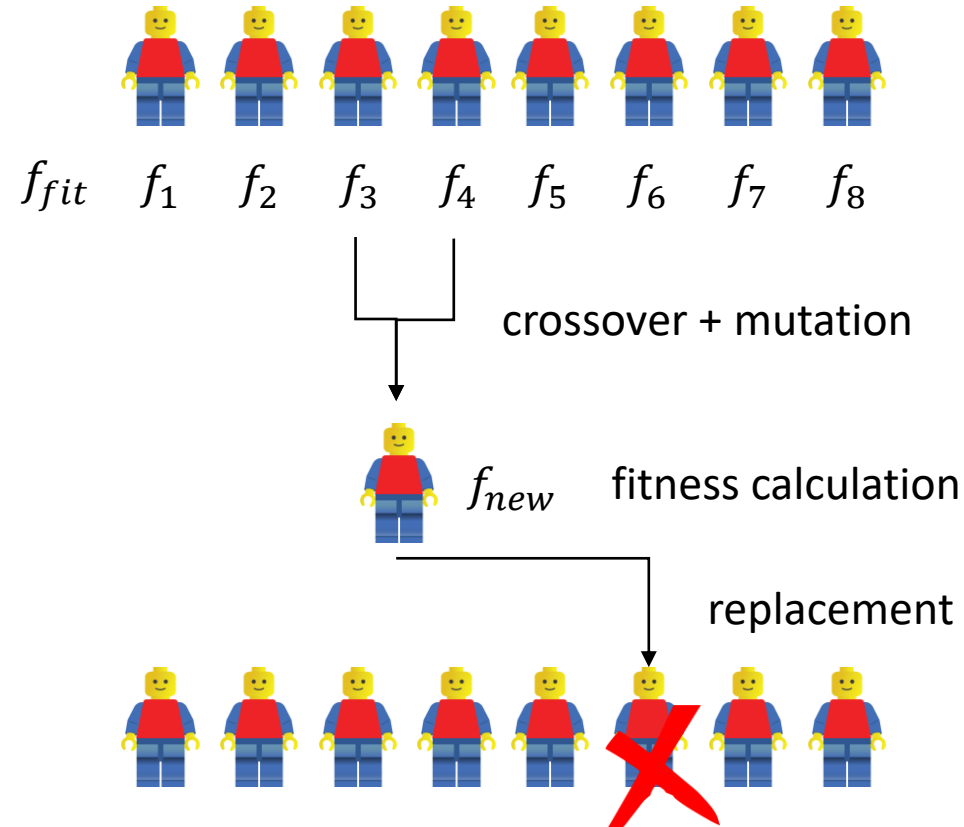
Brain Project: the genetic part forseees the evolution of tree-like structures representing mathematical expressions to model the data. In these structures, a node is a mathematical function or operation, a constant or a variable. The number of nodes is used to evaluate the complexity of the model.

→ See e.g. John R. Koza, *Genetic Programming – On the Programming of Computers by Means of Natural Selection*, A Bradford Book – The MIT Press (Cambridge, Massachussets; London, England)



The Brain Project: genetic mechanism

1. A set of possible solutions to the optimization problem, encoded according to a predefined scheme, is generated (often randomly). Each of such solutions is called *individual*. A set of individuals forms a *population*.
2. A numerical value, called *fitness*, is associated to each individual. The fitness quantifies how much a given solution is *suitable* to the problem to solve. Generally speaking, the higher is the fitness associated to an individual the more promising is the individual itself. This is a crucial quantity for the success of the optimization procedure.
3. Until a predefined convergence criterion is reached, the following steps are iterated:
 - (a) Some individuals are selected (*parents*) to be used as a starting point for the generation of new individuals (*offsprings*).
 - (b) Offsprings are obtained through a suitable mechanism of parents encoding recombination (*crossover*). In this phase, the chromosomes of the parents, i.e. their encoding, are suitably combined to generate new individuals. A valid crossover should produce individuals whose genetic code is, to some extent, similar to that of the parents. Crossover is usually followed by a random variation, with low probability, of some portions of the derived encoding. Such a process is called *mutation* and has a crucial importance as it allows to introduce missing genetic code and to keep genetic diversity in the population. The fitness is finally calculated for all newly obtained individuals.
 - (c) Some offsprings live sufficiently long to replace other pre-existing individuals.



The Brain Project: fitness function

Fitness function → is the function to maximize → it suitably contains the prediction error and a term related to the complexity of the model and/or feature costs.

to tune the desired trade-off between accuracy and complexity

$$f_{\text{fit}} = 100.0 * \frac{f_e u(f_e) + \alpha f_n u(f_n)}{1 + \alpha} e^{f_e(1-u(f_e))} e^{f_n(1-u(f_n))}$$

$$f_e = \frac{e_{\text{max}} - e}{e_{\text{max}}}$$

related to the accuracy of the model

$$e = 100 \sqrt{\frac{\sum_{o=1}^{N_o} \sum_{p=1}^{N_p} (w_{\text{pat}_p} w_{\text{out}_o} (y_{op}^d - y_{op}^c))^2}{N_o N_{p_{\text{eq}}}}}$$

$$f_n = \frac{n_{\text{max}} - n}{n_{\text{max}} - 1}$$

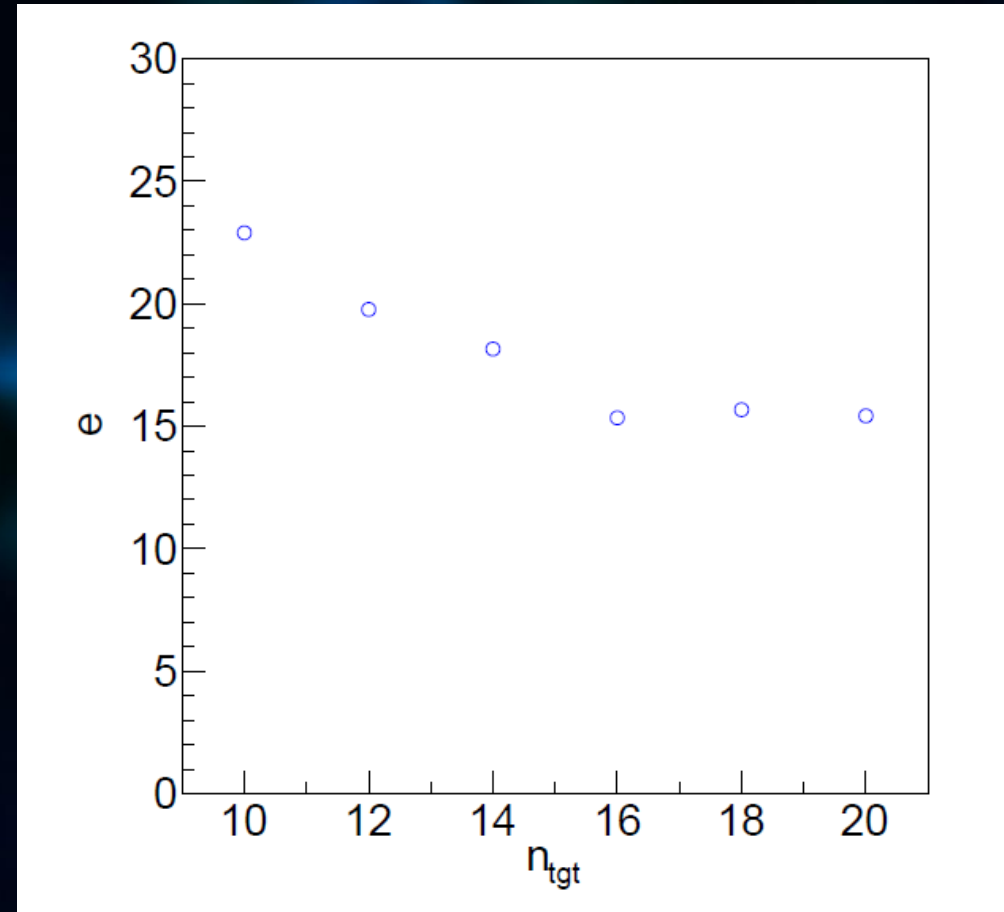
related to the complexity of the model

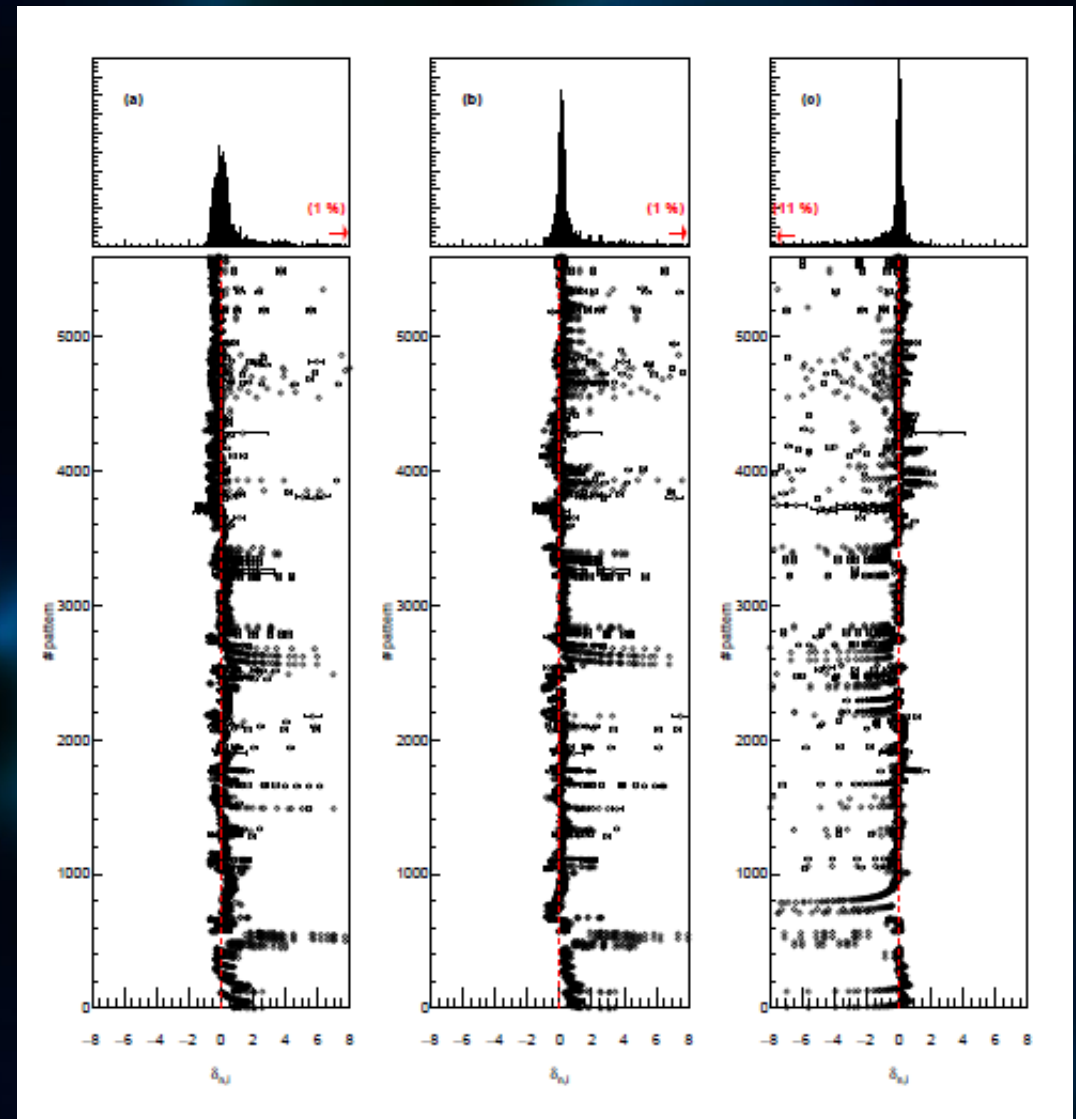
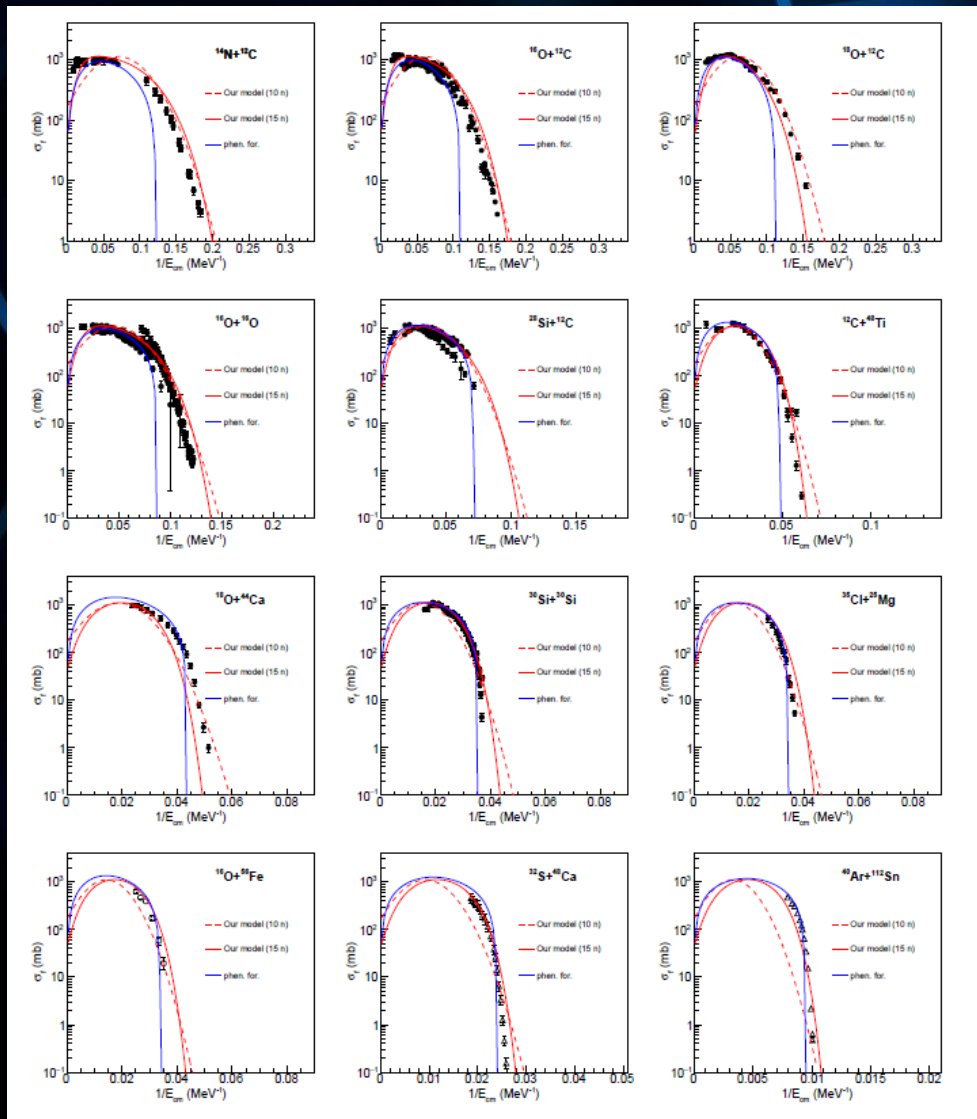
Results

$f_{fit} = f_{fit} \cdot e^{\frac{n_{tgt}-n}{n_{tgt}}}$ → required to reach a predefined, target, number of nodes. Brain Project usually tries to optimize the error with a given number of nodes → interesting to more easily tune the complexity of the desired model.

$$\sigma_{fus}^{(n_{tgt}=10)}(E_{cm}) = 1103 \cdot \exp \left[- \left(1.387 - 0.468 \cdot \frac{Z_2 \cdot Z_1}{E_{cm}} \right)^2 \right]$$

$$\sigma_{fus}^{(n_{tgt}=15)}(E_{cm}) = 1116 \cdot \exp \left[- \sinh^2 \left(-1.359 + \operatorname{erf} \left(\frac{S_{2n}}{E_{cm}} \right) + 0.061 \cdot \frac{A_1 \cdot A_2}{E_{cm}} \right) \right]$$





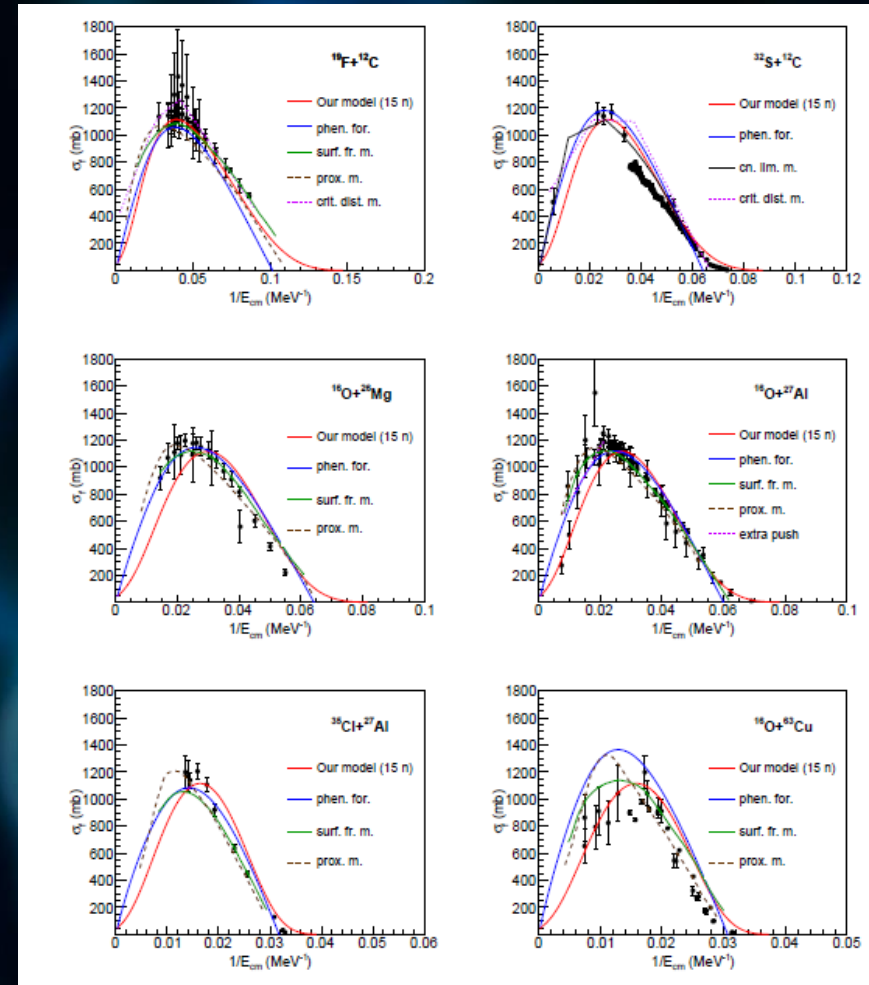
Comparison with other models

Our model slightly overestimates data in region 0;

Regions I-III → good overall description of data;

The positions of maxima are well reproduced, for lighter systems the maximum is predicted at slightly lower energies;

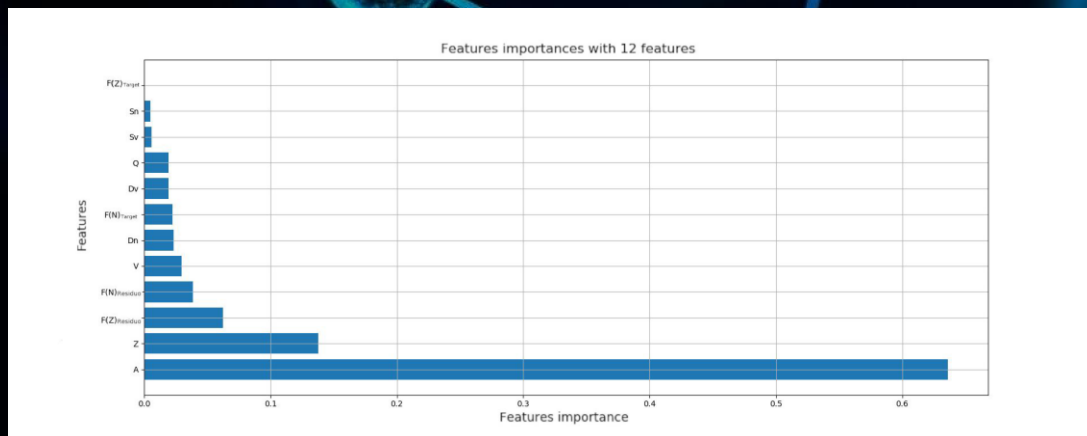
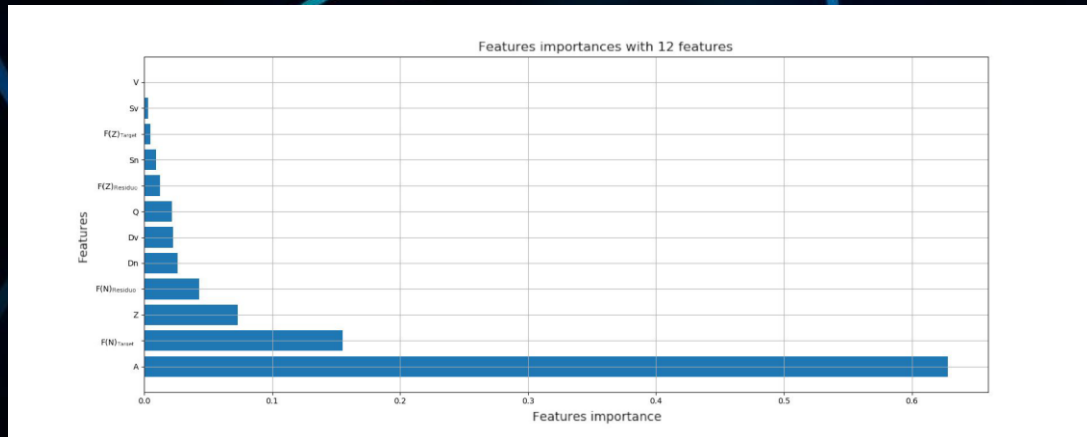
Nearly-symmetric systems → variances in agreement with surface friction model and slightly larger than those obtained with the proximity model.



Other applications...

Other decay channels \rightarrow (n,p) , and (n,α) reactions

\rightarrow The *feature selection* can help to inspect shell effects in the reaction cross sections... Difficult with *traditional* analysis methods!



Some Conclusions...

- Artificial Intelligence approaches based on hybridization of genetic programming and artificial neural networks are promising to help the description and understanding of large datasets of nuclear variables;
- One of the key aspects of these methods is the advanced feature selection, implemented via a programming simulation of the natural selection → help to probe the existence of correlations between variables;
- The present application focused on modeling an unprecedentedly large dataset of heavy-ion fusion cross section data and a broad body of nuclear variables;
- The newly developed models are capable to describe the entire systematics with a few variables, which are found to contain the whole informative content of a larger set of variables;
- These methods are readily applicable to numerous datasets in the nuclear domain!



Thank you for your attention!

**Daniele Dell'Aquila^{2,3}, Brunilde Gnoffo¹, Ivano Lombardo^{1,5},
Francesco Porto^{4,5}, Luigi Redigolo^{1,5} and Marco Russo^{1,5}**

1 INFN - Sezione di Catania, Catania, Italy

2 Dipartimento di Fisica "Ettore Pancini", Università degli Studi di Napoli "Federico II", Naples, Italy

3 INFN – Sezione di Napoli, Napoli, Italy

4 INFN - Laboratori Nazionali del Sud, Catania, Italy

5 Dipartimento di Fisica e Astronomia, Università degli Studi di Catania, Catania, Italy

Modeling heavy-ion fusion cross section data via a novel artificial intelligence approach

Daniele Dell'Aquila^{1,2,5,*}, Brunilde Gnoffo³,
Ivano Lombardo^{3,4,*}, Francesco Porto^{2,4} and
Marco Russo^{3,4}

¹ Dipartimento di Scienze Chimiche, Fisiche, Matematiche e Naturali, Università degli Studi di Sassari, Sassari, Italy

² INFN—Laboratori Nazionali del Sud, Catania, Italy

³ INFN—Sezione di Catania, Catania, Italy

⁴ Dipartimento di Fisica e Astronomia, Università degli Studi di Catania, Catania, Italy

E-mail: ddellaquila@uniss.it and ivano.lombardo@ct.infn.it

Received 27 January 2022, revised 24 September 2022

Accepted for publication 17 October 2022

Published 24 November 2022



Recently published results.

→ An interesting by-product of this study → refined dataset to investigate fusion cross section even with traditional methods → a new Sum-of-Difference (SoD) approach to unveil structure and shell effects on over-barrier fusion.



ELSEVIER

Contents lists available at ScienceDirect

Physics Letters B

journal homepage: www.elsevier.com/locate/physletb



Nuclear structure effects on over-barrier fusion reactions investigated with a new phenomenological model

Daniele Dell'Aquila^{b,e,*}, Brunilde Gnoffo^{a,d}, Ivano Lombardo^{a,d,*}, Luigi Redigolo^{a,d},
Francesco Porto^{c,d}

^a INFN - Sezione di Catania, via S. Sofia 64, 95123, Catania, Italy

^b Dipartimento di Fisica "Ettore Pancini", Università degli Studi di Napoli "Federico II", via Cinthia 21, 80126, Napoli, Italy

^c INFN - Laboratori Nazionali del Sud, via S. Sofia 62, 95123, Catania, Italy

^d Dipartimento di Fisica e Astronomia, Università degli Studi di Catania, via S. Sofia 64, 95123, Catania, Italy

^e INFN - Sezione di Napoli, via Cinthia 21, 80126, Napoli, Italy

ARTICLE INFO

Article history:

Received 25 May 2022

Received in revised form 17 November 2022

Accepted 20 December 2022

Available online 29 December 2022

Editor: D.F. Geesaman

Keywords:

Heavy-ion fusion

Excitation functions

Shell effects

Clustering

ABSTRACT

We investigate the occurrence of nuclear structure effects in the cross section of heavy-ion fusion reactions at energies above the Coulomb barrier. To this end, we initially develop a universal phenomenological model capable to reproduce, with an unprecedented accuracy, all previously published experimental fusion excitation functions with a few parameters. The new model, which foresees exclusively charge, mass, and energy of the colliding systems, shows a clear saturation of the critical angular momentum and avoids analytical non-regularities. The predictions of the newly developed model are then inspected to pin down residual discrepancies with the data, which could be ascribed to the structure of the colliding systems. In this framework, we obtain the following findings: (1) for the first time, we suggest an anomaly in the optimum value of the fusion cross section for systems having nearly-zero fusion Q -values; (2) we point out the occurrence of shell closure effects in the fusion of light systems; (3) we suggest that shell effects are washed-out at relative velocities $v_{rel} \gtrsim 0.07c$; (4) in the higher energy part of the fusion excitation function, the cross section for colliding systems involving fluorine or neon isotopes impinging on $2p_{3/2}$ - $1f_{5/2}$ nuclei is suppressed, possibly due to the occurrence of α -clustering effects enhancing α -transfer reactions.

© 2022 The Author(s). Published by Elsevier B.V. This is an open access article under the CC BY license (<http://creativecommons.org/licenses/by/4.0/>). Funded by SCOAP³.