Understanding Heavy-ion Fusion Cross Section Data Using Novel Artificial Intelligence Approaches

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Abstract. We modeled an unprecedentedly large dataset of complete fusion cross section data using a novel artificial intelligence approach. Our analysis aims especially to unveil, in a data-driven way, nuclear structure effects on the fusion between heavy ions and to suggest a universal formula capable to describe all previously available data. The study focused on light-to-medium-mass nuclei, where incomplete fusion phenomena are more difficult to occur and less likely to contaminate the data. The method used to derive the models exploits a state-of-the-art hybridization of genetic programming and artificial neural networks and is capable to derive an analytical expression that serves to predict integrated cross section values. For the first time, we analyzed a comprehensive set of nuclear variables, including quantities related to the nuclear structure of projectile and target. In this manuscript, we describe the derivation of two computationally simple models that can satisfactorily describe, with a reduced number of variables and only a few parameters, a large variety of light-to-intermediate-mass collision systems in an energy domain ranging approximately from the Coulomb barrier to the oncet of multi-fragmentation phenomena. The underlying methods are particularly innovative and are of potential use for a broad domain of applications in the nuclear field.

1 Introduction

Nuclear fusion is a key reaction mechanisms when one wants to investigate heavy-ion collisions at energies around the Coulomb barrier [1–5]. In the literature, there are numerous attempts to fully understand the corresponding cross section, its variation with the collision energy, and the possible link with nuclear structure variables.

In the literature, there are numerous models, mainly semi-classical, attempting to describe the fusion cross section between heavy ions. They benefited from large systematics of data [6]. If one focuses on light-to-medium-mass systems, i.e. systems in which the mass of the compound nucleus is not too large, $A_{tot} \simeq 20 - 140$, the fusion cross section above barrier has a peculiar trend with the center-of-mass energy $1/E_{cm}$. Such a trend can be schematically

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described subdividing data into three main regions [2, 7]. The so-called Region I, which starts approximately at the Coulomb barrier, i.e. the lowest energy region, exhibits a quasi linear increase of the cross section for decreasing $1/E_{cm}$ values [1, 2, 7–9]. In this region, multi-nucleon-transfer processes compete with the fusion reaction mechanisms [10–12], and also quasi-fission processes can occur (especially for heavy systems) [13–15]. In the successive energy, i.e. in Region II, at higher energies, the fusion cross section is usually characterized by a smooth fall, due to the competition with deep inelastic phenomena [16, 17] and quasi-fission. It is often quite complex to define sharp boundaries between Region I and Region II. In addition, due to the complicated reaction mechanisms, experimental data are often affected by relatively large uncertainties. In the higher energy region, called Region III, there is a sharp fall of the cross section, as fusion-evaporation is supplanted by incomplete fusion mechanisms [18–20], multi-fragmentation [21–23], and, in general, to much more complicated reaction scenarios [5]. For very high E_{cm} values, the fusion cross section is expected to reach negligibly small values, as a result of mechanical and thermodynamical instabilities of the transient system formed in the collisions [22–28].

The present theoretical and experimental knowledge of the fusion cross section between heavy ions leaves a number of yet open questions. For example, those related to the existence of a limiting angular momentum at high energies, which was explicitly investigated only for a few systems [29]. There are two main families of macroscopic models that attempt to describe this energy region: critical distance models [7, 30] and models based on limitations to compound nucleus [31]. Besides macroscopic models, also microscopical (Time-Dependent Hartree-Fock, TDHF, see e.g. [32, 33]), molecular dynamics (see e.g. [34, 35]), and phenomenological models (see, e.g., [1, 27, 28, 36–38]) have been proposed. The latter, in particular, reached a good level of accuracy in describing experimental function cross section excitation functions, see e.g. Refs. [28, 36–39]. However, present phenomenological models have been focused especially to the portion of the excitation function close to the optimal value of E_{cm}, i.e. close to the maximum of the fusion cross section. Only a few phenomenological models, such as Ref. [27], have directly explored the highest energy region (III), and there is very limited literature for energies close to the Coulomb barrier. Furthermore, previous phenomenological models often lack of physical boundaries, i.e. they predict divergent or even negative value of the fusion cross section outside of the range constrained by experimental data. There are also a number of unanswered questions regarding possible effects of nuclear structure parameters (i.e. characteristics of projectiles, targets and compound nuclei) on the fusion cross section.

In this work, we analyze an unprecedentedly large dataset of complete fusion cross section data [48, 49], aiming to derive a data-driven model capable to describe the entire dataset. In our approach, we inspect simultaneously the correlation of the fusion cross section with a large number of variables, also linked to the structure of projectile, target, and the compound nucleus. To this end, we use a novel artificial intelligence tool named Brain Project [40, 41], which is based on a state-of-the-art hybridization of genetic programming and artificial neural networks.

2 Methods

2.1 The Brain Project

The present data-driven models are derived using the Brain Project (BP) [40, 41], a machine learning tool for the formal modeling of data. BP is based on a novel hybridization of genetic programming and neural networks, and was previously successfully exploited in a number of investigations involving many research fields (see, for example, Refs. [42–46]). In the

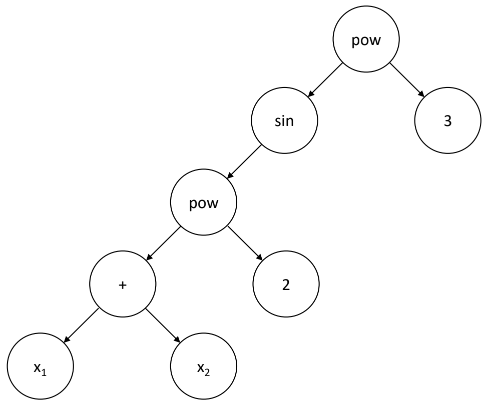

Figure 1. The tree-like representation of the expression $\sin^3[(x_1 + x_2)^2]$.

scheme implemented by BP, the genetic part and the neural part cooperate to derive formal mathematical expressions to describe some input data.

The genetic part of BP follows a well-known approach in *evolutionary computation* that foresees the evolution of tree-like structures representing mathematical expressions [47]. A tree-like structure is composed by *nodes*, which are suitably *interconnected*. A node can contain a mathematical function, a variable, or a constant. For example, Fig. 1 shows the representation of the expression $\sin^3[(x_1+x_2)^2]$, where x_1 and x_2 are two given variables, this expression comprises 8 nodes. In the framework of BP, the number of nodes in an expression is used to quantify its *mathematical complexity*. A number of mathematical expressions, often generated randomly, are initially created and subdivided into *populations*. They are treated as *individuals*. Within each population, a computer implementation of the *natural selection* in biological systems is then used to maximize the *fitness* of the individuals. The fitness is a measure of the *quality* of an individual. In the present approach, it contains a combination of the prediction error (i.e. the amplitude of the deviations between the model predictions and the experimental data), and the mathematical complexity (i.e. the number of nodes exploited by the model). More details about the choice of the fitness function can be found in Ref. [46].

2.2 Dataset and Data Splitting

The models derived in the present work are obtained exploiting data from 124 light-to-medium-mass collision systems ($Z_1 \cdot Z_2 \leq 250$) extracted from the dataset described in Refs. [48, 49]. In turn, we used about 4500 experimental points, which is a much larger body of data compared to previous phenomenological studies on the fusion cross section between heavy ions. Data associated with 63 heavier systems (with $Z_1 \cdot Z_2 > 250$) were excluded from the fit data, but were used to check the extrapolation of the models towards heavier collision systems. In addition, experimental points randomly extracted from the $Z_1 \cdot Z_2 \leq 250$ where excluded from the fit data and used to test the generalization capabilities of the models as described in Ref. [46].

A interesting point of the method adopted in this study is the advanced *feature selection*, i.e. the capability of a modeling tool to automatically select the input variables with the largest importance to predict the output (i.e. the cross section of the fusion between heavy ions, in the present case), neglecting other poorly important variables. This is natively done by BP [43]. In the present study, we exploited the advanced feature selection operated by BP to probe the dependence of the fusion cross section on several variables, including nuclear structure variables that identify projectile, target, and compound systems, of the fusion cross section between heavy ions (see Ref. [46] for a detailed list of such variables).

3 Results

We derived models with different complexity choosing different numbers of target nodes n_{tgt}, ranging between 10 and 20, as discussed in details in Ref. [46]. In other words, given a certain n_{tgt}, BP will try to identify mathematical expressions with a number of nodes as close as possible to n_{tgt}, while minimizing their error. We thus obtain gradually more complex models, up to a maximum of $n_{tgt} = 20$. The prediction accuracy is found to increase up to about $n_{tgt} \approx 15$, and then we observe a saturation, even if additionally increasing n_{tgt}. Driven by these preliminary checks, we derived two different models, using $n_{tgt} = 10$ and $n_{tgt} = 15$, which have the following mathematical expressions:

$$\sigma_{fus}^{(n_{tgt}=10)}(E_{cm}) = 1103 \cdot \exp\left[-\left(1.387 - 0.468 \cdot \frac{Z_2 \cdot Z_1}{E_{cm}}\right)^2\right] \quad (1)$$

$$\sigma_{fus}^{(n_{tgt}=15)}(E_{cm}) = 1116 \cdot \exp\left[-\sinh^2\left(-1.359 + \mathrm{erf}\left(\frac{S_{2n}}{E_{cm}}\right) + 0.061 \cdot \frac{A_1 \cdot A_2}{E_{cm}}\right)\right] \quad (2)$$

where both formulas are expressed in units of mb. The first formula is outstandingly simple, and can be easily used for a fast evaluation of the fusion cross section. This model exploits only 3 variables: E_{cm}, Z_1, and Z_2. We identify the product $Z_1 \cdot Z_2$, which is clearly related to the Coulomb term. Several other phenomenological models show a clear dependence on $Z_1 \cdot Z_2$ (e.g., [28, 37–39]). The complex model, derived using $n_{tgt} = 15$, exploits 4 variables: E_{cm}, A_1, A_2, and S_{2n}. In this case, one has the production $A_1 \cdot A_2$, which has a similar dependence than $Z_1 \cdot Z_2$ found in the simplest equation. In the complex model, due to the dependence on the two-neutron separation energy of the compound S_{2n}, one finds a slight increase of the complete fusion cross section at energies larger than the optimal fusion energy, for neutron-rich systems. This finding is in qualitative agreement with both the phenomenological model of Ref. [39] and with experiments at the onset of multi-fragmentation [24, 50]. With this approach, we do not find any significant correlation on other variables related to nuclear structure properties. This fact suggests that the informative content of present nuclear fusion datasets might be essentially contained in the set of variables E_{cm} and Z_1Z_2 (or A_1A_2 and S_{2n}).

The results of our models are shown in Fig. 2 ($n_{tgt} = 10$, red dashed line; $n_{tgt} = 20$, red solid line) for some typical collision systems in our dataset. For comparison, we report also the results of Ref. [39] (blue dashed line). In the figure, open circles represent experimental points of light-to-medium-mass systems not used for the fit, while open triangles are used to indicate data from heavy systems used to test the extrapolation capabilities of our models. By carefully inspecting the figure, it is seen that the accuracy of the new models is improved compared to previous state-of-the-art phenomenological models, especially in the regions close to the Coulomb barrier, where, for example, the model of Ref. [39] falls to rapidly to zero and even to negative values of the cross section. The cross section at low energy predicted by the newly derived models falls nearly exponentially in the proximity of the Coulomb barrier, closely following the typical fall caused by the penetrability through the Coulomb barrier. At higher energies, especially in Region III, there are discrepancies between

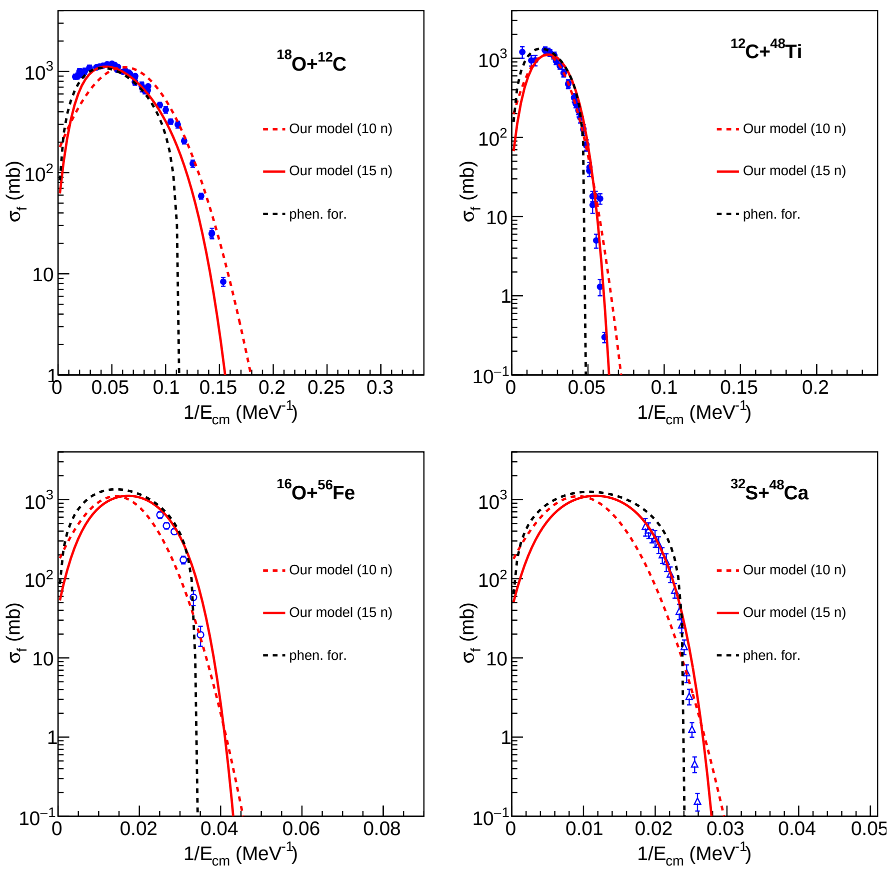

Figure 2. Results of the newly derived models (n_{tgt} = 15, red solid line; n_{tgt} = 10, red dashed line) for some collision systems in the dataset, compared to the results of the phenomenological model of Ref. [39]. Open circles indicate data from light systems excluded from the fit data, while open triangles are from heavier systems not included in the fit data.

experimental data and model prediction. Unfortunately, understanding this region is made complex by the poor statistics of data and their large uncertainties. This clearly calls for new experimental investigations in such a complicated energy region.

4 Conclusions and Perspectives

A broad body of fusion cross section data for light-to-medium mass systems is analyzed with a novel artificial intelligence approach based on a hybridization of genetic programming and neural networks. This approach made it possible, for the first time, to inspect the dependence of the fusion cross section on numerous variables that describe both the collision system and the structure of the nuclei involved in the reaction. We derive two computationally simple formulas that can be used as universal models to describe the fusion between heavy ions.

The new findings seem to call for new experimental data well above the Coulomb barrier (especially in regions II and III), which was largely neglected after the 90s, leaving a number of unanswered questions that could be addressed with the help of modern detection systems for charged particles [52–60].

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