First measurement of $K^{\pm} \to \pi^0\pi^0\mu^{\pm}\nu$ ($K_{\mu4}^{00}$) decay

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on behalf of the NA48/2 Collaboration

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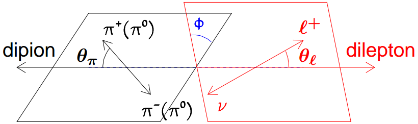
Outline

$K^\pm \to \pi^0\pi^0\mu^\pm\nu$ ($K^{00}_{\mu 4}$) state of the art

$K \to \pi\pi\mu\nu(K_{l4})$ depends on F, G, R, H form-factors.
Cabibbo-Maksymowicz variables: S_π (dipion mass squared), S_l (dilepton mass squared) and angles θ_π (in the dipion frame), θ_l (in the dilepton frame), ϕ.



- For $K^{00}_{\mu 4}$, s-wave for $\pi^0\pi^0$, there are no dependence on $\cos\theta_\pi, \phi$, and only F and R contribute.
- Unlike K^{00}_{e4} case, R plays some role due to μ mass.

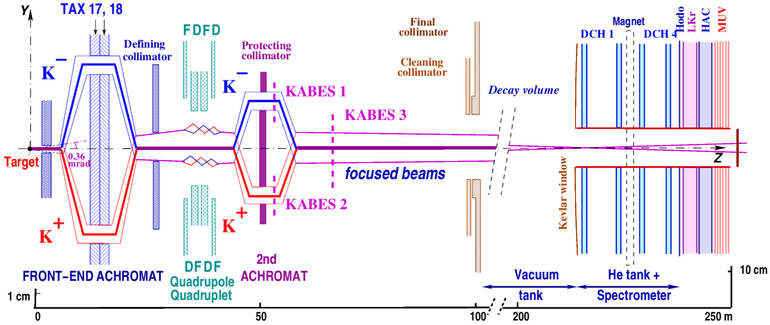
K_{l4} mode	BR $[10^{-5}]$	N_{cand}	
${K_{e4}}^\pm$	4.26 ± 0.04	1108941	NA48/2 (2012)
${K_{e4}}^{00}$	2.55 ± 0.04	65210	NA48/2 (2014)
${K_{\mu 4}}^\pm$	1.4 ± 0.9	7	Bisi et al. (1967)
${K_{\mu 4}}^{00}$	?	0	

$K^{00}_{\mu 4}$: first observation, ChPT test, check of R presence, potential study of $\pi\pi$ rescattering effects in the $F(S_\pi)$.

$K_{\mu 4}$: huge bkg $K^\pm \to \pi\pi(\pi^\pm \to \mu^\pm\nu)$.

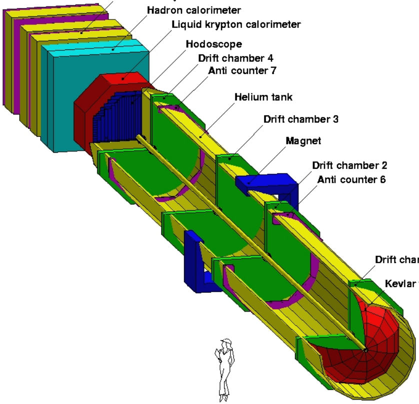
- According to lepton universality, experimental $F(S_\pi, S_l)$ parameterization from K^{00}_{e4} [NA48/2 JHEP 08 (2014) 159] may be used for $K^{00}_{\mu 4}$.
- The only available source of $R(S_\pi, S_l)$ is ChPT calculation [J.Bijnens, G.Colangelo, J.Gasser, Nucl.Phys.B 427 (1994) 427].

NA48/2 main goal were $K_{3\pi}$ charge asymmetry studies; additional rare decys program.



- Two charged beams:
  - 6% of $K^{\pm}$
  - $\langle P_K \rangle \approx 60$ GeV/c
  - $\Delta P_K / \langle P_K \rangle \approx \pm 3.8\%$
- KABES (Kaon Beam Spectrometer) resolutions:
  - $\sigma(X, Y) \sim 800\ \mu$m
  - $\sigma(P_K)/P_K \sim 1\%$
  - $\sigma(T) \sim 600$ ps

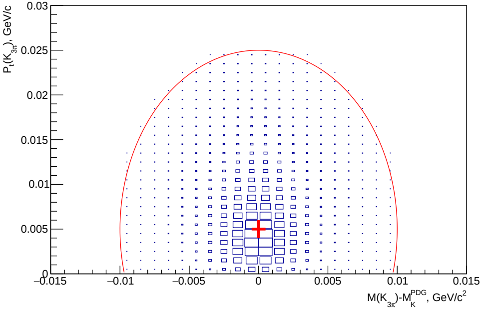
NA48/2 setup (CERN SPS, 2003-2004)



- Magnetic spectrometer (drift chambers DCH1–DCH4):
  - $\sigma(X, Y) \sim 90\ \mu$m per chamber
  - $\sigma(P_{DCH})/P_{DCH} = (1.02 \oplus 0.044 \cdot P_{DCH})\%$ (P_{DCH} in GeV/c)
- Scintillator hodoscope (HOD):
  - $\sigma(T) \sim 150$ ps
- Liquid Krypton EM calorimeter (LKr):
  - $\sigma_x = \sigma_y = (0.42/\sqrt{E_\gamma} \oplus 0.06)$ cm
  - $\sigma(E_\gamma)/E_\gamma = (3.2/\sqrt{E_\gamma} \oplus 9.0/E_\gamma \oplus 0.42)\%$ (E_γ in GeV)
- Hadronic calorimeter, muon system MUV.

Events selection

- Signal $K_{\mu 4}$ is $K^{\pm} \to \pi^0 \pi^0 \mu^{\pm} \nu$
- Normalization $K_{3\pi}$ is $K^{\pm} \to \pi^{\pm} \pi^0 \pi^0$
- Trigger chain: L1 trigger using HOD and LKr, followed by L2 trigger using DCH for online momentum calculation.
- Event selection: 4 isolated photons consistent with $2\pi^0$ in time-spatial matching with a KABES beam track and a DCH track.



Normalization $K_{3\pi}$ kinematic selection ellipse:

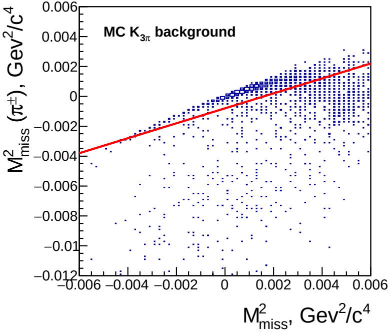
- center:
  - $M(K_{3\pi}) = M_K^{PDG}$
  - $P_t = 5$ MeV$/c$
- semi-axes:
  - $\Delta M(K_{3\pi}) = 10$ MeV$/c^2$
  - $\Delta P_t = 20$ MeV$/c$
- 72.99×10^6 $K_{3\pi}$ selected data events.

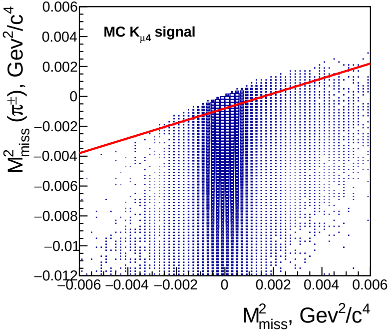
$K^{\pm} \to \pi^0\pi^0\mu^{\pm}\nu$ signal events selection

- Off the $K_{3\pi}$ kinematic ellipse
- DCH track has associated MUV response

$M^2_{miss} = (\mathbf{P}_K - \mathbf{P}(\pi^0_1) - \mathbf{P}(\pi^0_2) - \mathbf{P}(\mu^{\pm}))^2$

$M^2_{miss}(\pi^{\pm}) = (\mathbf{P}_K - \mathbf{P}(\pi^0_1) - \mathbf{P}(\pi^0_2) - \mathbf{P}(\pi^{\pm}))^2$

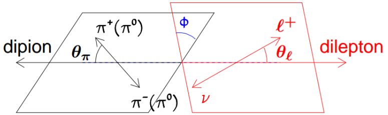


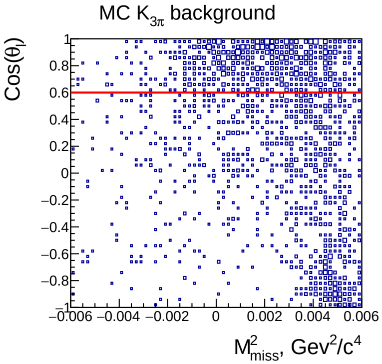


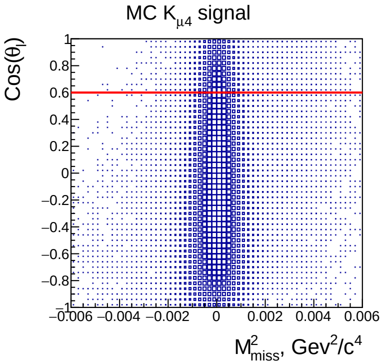
- $M^2_{miss}(\pi^{\pm}) < 0.5 M^2_{miss} - 0.0008\ \mathrm{GeV}^2/c^4$

$K^\pm \to \pi^0\pi^0\mu^\pm\nu$ signal events selection

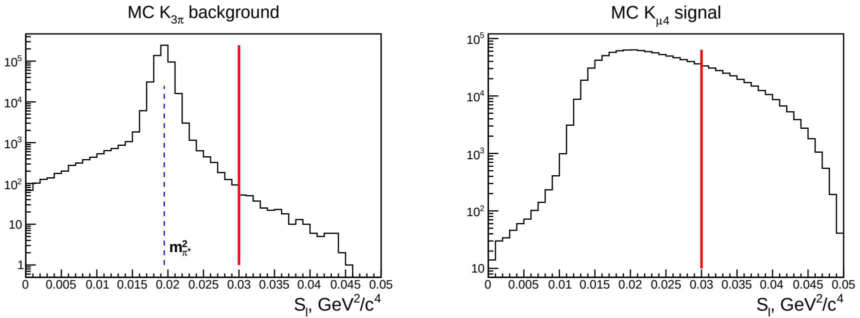
- $cos(\Theta_l) < 0.6$





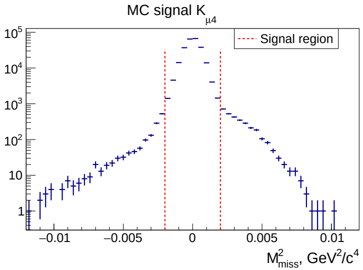


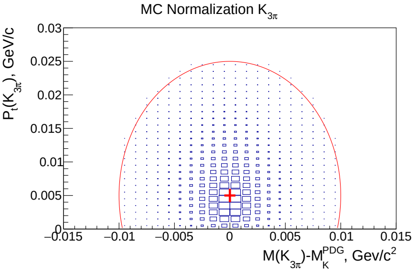
$K^\pm \to \pi^0\pi^0\mu^\pm\nu$ signal events selection



- $S_l = m(\mu\nu)^2 > 0.03$ GeV$^2/c^4$ (to reject $\pi^\pm \to \mu^\pm\nu$).
- 3718 $K_{\mu 4}$ data candidates selected
- 2437 data candidates in M^2_{miss} signal region [-0.002,0.002] GeV$^2/c^4$
- The MC M^2_{miss} signal region contains 98.2% of all selected MC events

Acceptances





- $K^{00}_{\mu4}$ signal accepance is

$$A_S = \frac{N^{MC}_{Selected\ \ in\ \ signal\ \ region}}{N^{MC}_{Generated}(all\ \ S_l^{true})} = (0.651 \pm 0.001)\%,$$

- However, for the restricted phase space region $S_l^{true} > 0.03\ \mathrm{GeV}^2/c^4$, the signal acceptance is

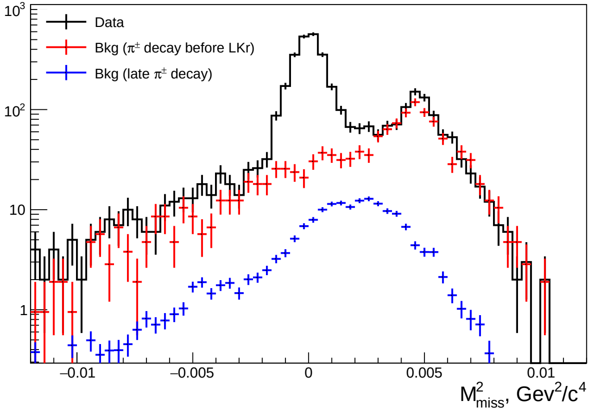
$$A^r_S = \frac{N^{MC}_{Selected\ \ in\ \ signal\ \ region}}{N^{MC}_{Generated}(S_l^{true}>0.03)} = (3.453 \pm 0.007)\%.$$

- $K_{3\pi}$ normalization channel acceptance is

$$A_N = \frac{N^{MC}_{Selected}}{N^{MC}_{Generated}} = (4.477 \pm 0.002)\%$$

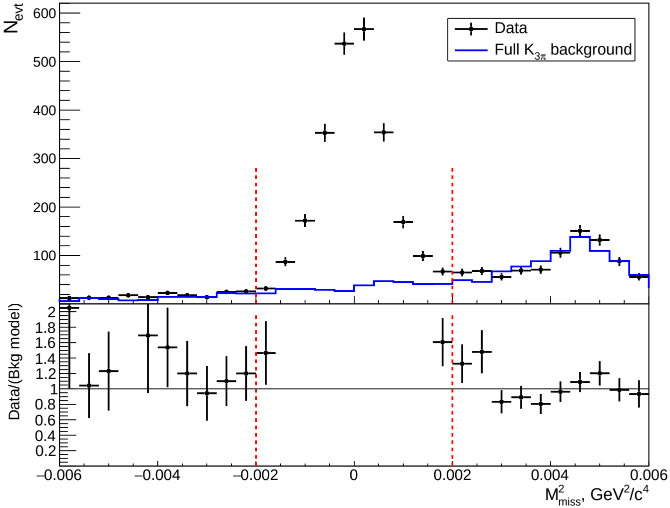
Residual background

- $K^{\pm} \to \pi^{\pm}\pi^0\pi^0$,
- followed by $\pi^{\pm} \to \mu\nu$ before MUV with a probability $\approx 10\%$ for $P(\pi^{\pm}) \approx 10$ GeV$/c$.



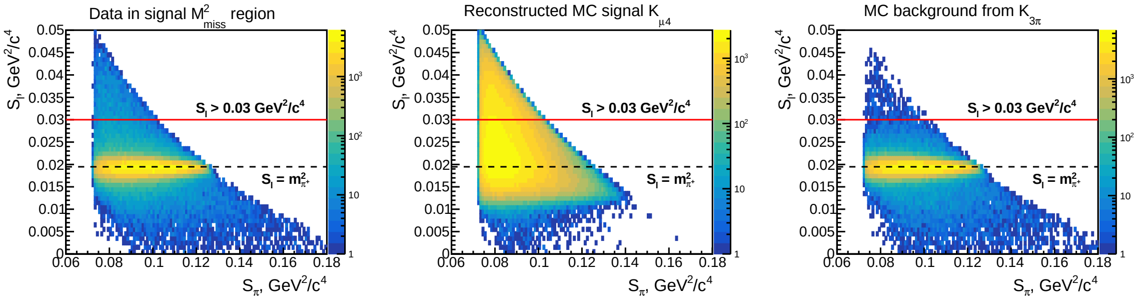
- $K_{3\pi}$ background with $\pi^{\pm}$ decay before LKr: from MC.
- $K_{3\pi}$ background with late $\pi^{\pm}$ decay or muon emission in a late hadron shower:
  - Can not be easily simulated
  - Data-driven method of estimation
  - Background-enhansed control sample, selected using E_{LKr} and P_{DCH}

$K^{00}_{\mu 4}$ signal extraction fit



- 2437 candidates in the signal region.
- Fit in the M^2_{miss} interval [-0.003,0.006] GeV$^2/c^4$, ignoring the signal region to decrease sensitivity to the imperfect MC resolution.
- Data fit by a linear combination of background and MC signal tails.
- $354 \pm 33_{stat} \pm 62_{syst}$ background events.
- The background-related systematics are determined by varying the way the background is estimated.

Signal versus S_π, S_l



- The branching ratio is measured for the restricted phase space $S_l^{true} > 0.03$ GeV^2/c^4.
- Extrapolation to the full phase space depends on the theory.

Signal versus S_π, S_l

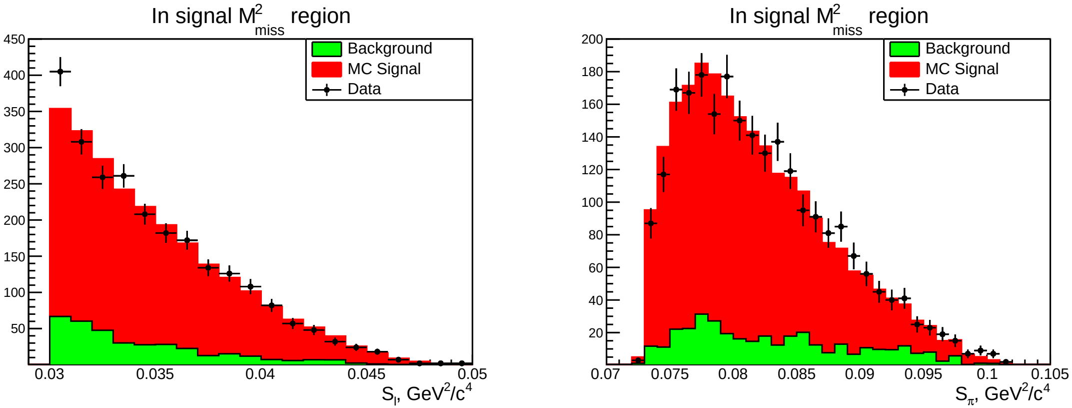


Figure: 1D projections comparison for $S_l > 0.03$ GeV^2/c^4

Acceptance variations due to $K^{00}_{\mu 4}$ generator modifications

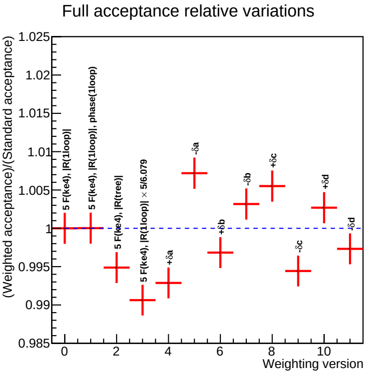
NA48/2 JHEP 08 (2014) 159:

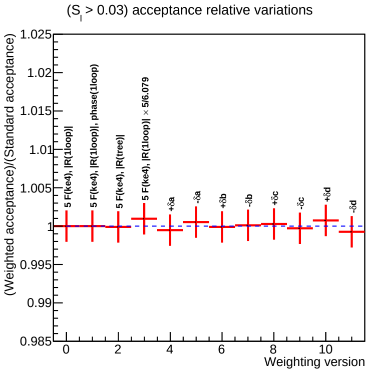
The absolute normalization is also measured $F = f \times F(K_{e4})$, $f = 6.079 \pm 0.055$.

$$F(K_{e4}) = \begin{cases} (1 + aq^2 + bq^4 + c \cdot S_l/4m^2_{\pi^+}) \text{ for } q^2 \geq 0 \\ (1 + d\sqrt{|q^2/(1+q^2)|} + c \cdot S_l/4m^2_{\pi^+}) \text{ for } q^2 < 0 \end{cases}$$

where $q^2 = S_\pi/4m^2_{\pi^+} - 1$.

- Decay generator was modified by MC events weighting.
- The acceptance spread is taken as systematics.





Preliminary result: Ingredients

$$BR(K^{00}_{\mu 4}) = \frac{N_S}{N_N} \cdot \frac{A_N}{A_S} \cdot K_{trig} \cdot BR(K^{00}_{3\pi}).$$

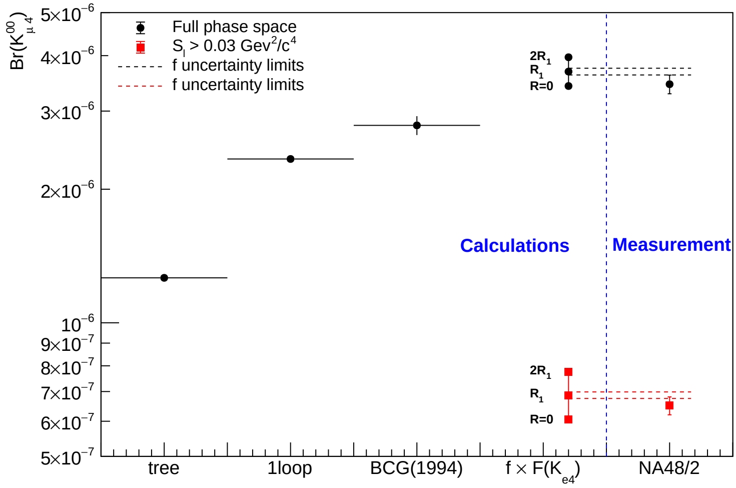
- Extracted signal $N_S = N_{Sign.\ cand.} - N_{Bkg} = 2437 - (354 \pm 33_{stat}) = 2083 \pm 59_{stat}$ events;
  - Signal/Background is $5.89 \pm 0.66_{stat}$;
- Number of normalization events $N_N = 72.99 \times 10^6$;
- Normalization acceptance $A_N = (4.477 \pm 0.002)\%$;
- Signal acceptance for the restricted phase space $A^r_S = (3.453 \pm 0.007)\%$;
- Signal acceptance for the full phase space $A_S = (0.651 \pm 0.001)\%$;
- Trigger correction (extracted with control triggers) $K_{trig} = K_{CHT} \cdot K_{NUT} = (0.998 \pm 0.002) \cdot (1.0007 \pm 0.0007) = 0.999 \pm 0.002$;
- PDG $BR(K^{00}_{3\pi}) = (1.760 \pm 0.023)\%$;

Preliminary result: Central values and errors budget

	Full phase space		$S_l > 0.03$ GeV^2/c^4	
$BR(K_{\mu 4})$ central value $[10^{-6}]$	3.45		0.651	
	$\delta BR[10^{-6}]$	$\delta BR/BR$	$\delta BR[10^{-6}]$	$\delta BR/BR$
Data stat. error	0.10	2.85%	0.019	2.85%
MC stat. error	0.01	0.21%	0.001	0.21%
Trigger	0.01	0.18%	0.001	0.18%
Background	0.10	2.96%	0.019	2.96%
Accidentals	0.01	0.32%	0.002	0.32%
MUV inefficiency	0.06	1.65%	0.011	1.65%
Form Factor modelling	0.05	1.37%	0.001	0.14%
$BR(K_{3\pi})$ error (external)	0.05	1.31%	0.009	1.31%
Total error	0.17	4.83%	0.030	4.64%

- Accidentals obtained from side bands of time distributions;
- MUV inefficiency uncertainty taken as full inefficiency effect.

Preliminary result: Comparison to theory



Theory:

- J. Bijnens, G. Colangelo, J. Gasser, Nucl. Phys. B, 427 (1994) 427:
  - Tree approximation;
  - 1-loop;
  - BCG(1994): 'beyond 1-loop' with measured F from [Rosselet etc. Phys. Rev. D 15 (1977) 574].
- Re-calculated now:
  - $F(K_{e4})$ from NA48/2 (2015);
  - $R_1 = R(1loop)$;
  - 1-loop (F,R) phase;
  - 2020 PDG constants.

Conclusion

A first observation and branching fraction measurement of $K^{\pm} \rightarrow \pi^0\pi^0\mu^{\pm}\nu$ decay is performed by NA48/2 experiment at SPS in CERN

- We observe 2437 signal candidates with an estimated background of $354 \pm 33_{stat} \pm 62_{syst}$ events, Signal/Background ratio is $5.9 \pm 1.4_{tot}$
- Preliminary result for restricted phase space ($S_l > 0.03$) is

$$BR(K^{00}_{\mu 4}, S_l > 0.03) = (0.65 \pm 0.019_{stat} \pm 0.024_{syst}) \times 10^{-6} = (0.65 \pm 0.03) \times 10^{-6};$$

- Preliminary full phase space result is

$$BR(K^{00}_{\mu 4}) = (3.4 \pm 0.10_{stat} \pm 0.13_{syst}) \times 10^{-6} = (3.4 \pm 0.2) \times 10^{-6}.$$

- The results are consistent with a contribution of the R form factor, as computed at 1-loop ChPT.

Spare slides

SPARE SLIDES

Theoretical framework: decay width

$K^{00}_{\mu 4}$ matrix element is [J.Bijnens, G.Colangelo, J.Gasser, Nucl.Phys.B 427, 427 (1994)]:

$$T = \frac{G_f}{\sqrt{2}} \cdot V^*_{us} \cdot \underbrace{\overline{u}(p_\nu) \cdot \gamma_\mu \cdot (1-\gamma_5) \cdot v(p_l)}_{Lepton\ part} \cdot \underbrace{(V_\mu - A_\mu)}_{Hadron\ part} \quad (1)$$

where

$$V_\mu = \frac{-H}{M_K^3} \varepsilon_{\mu,\nu,\rho,\sigma} L^\nu P^\rho Q^\sigma, \ A_\mu = -i \frac{1}{M_K} [P_\mu F + Q_\mu G + L_\mu R], \quad (2)$$

$\varepsilon_{0,1,2,3}$=1 and the four-momenta are defined as $P = p_1 + p_2$, $Q = p_1 - p_2$ (p_1 and p_2 are the two pion momenta) and $L = p_l + p_\nu$. The form factors F,G,R,H are analytic functions of the decay kinematic variables.
In general, decay width is a function of five Cabibbo-Maksymowicz variables. But for $K^{00}_{\mu 4}$, in s-wave approximation for $\pi^0\pi^0$, there are no dependence on $\cos\theta_\pi, \phi$, and so $G = 0, H = 0$:

$$d\Gamma_3 = \frac{G_f^2 |V_{us}|^2 (1-z_l)^2 \sigma_\pi X}{2^{11}\pi^5 M_K^5} (I_1 + I_2(2(\cos\theta_l)^2 - 1) + I_6 \cos\theta_l) dS_\pi dS_l d\cos\theta_l. \quad (3)$$

where $I_1 = \frac{1}{4}\{(1+z_l)|F_1|^2 + 2z_l|F_4|^2\}$, $I_2 = -\frac{1}{4}(1-z_l)|F_1|^2$, $I_6 = z_l Re(F_1^* F_4)$, $F_1 = X \cdot F$, $F_4 = -(PL)F - S_l R$, $z_l = \frac{m_l^2}{S_l}$, $\sigma_\pi = \sqrt{1 - \frac{4M_\pi^2}{s_\pi}}$, $X = \frac{1}{2}\sqrt{\lambda(M_K^2, S_\pi, S_l)}$, $\lambda(x,y,z) = x^2 + y^2 + z^2 - 2(xy + xz + yz)$.
Only three variables $(S_\pi, S_l, \cos\theta_l)$ and two form factors (F, R) matter.

Theoretical framework: form factors

According to lepton universality, $F_s(S_\pi, S_l)$ s-wave form factor measured in K_{e4}^{00} may be used for $K_{\mu 4}^{00}$ decay MC simulation, while the only available source of $R_s(S_\pi, S_l)$ is ChPT calculation.

- We have ChPT 1-loop generator code [BCG(1994)] with $F(1loop), R(1loop)$.
- For the predictions beyong 1-loop (central value), BCG(1994) used $F = 5.59(1 + 0.08q^2)$, where $q^2 = S_\pi/4m_{\pi^+}^2 - 1$ [L.Rosselet et al., Phys. Rev. D15 (1977) 574].
- Additionally, from the NA48/2 K_{e4} analysis [J.R. Batley et al. JHEP 08 (2014) 159] we have the best measurement of F shape:

$$F(K_{e4}) = (1 + aq^2 + bq^4 + c \cdot S_l/4m_{\pi^+}^2), q^2 \geq 0$$
$$F(K_{e4}) = (1 + d\sqrt{|q^2/(1+q^2)|} + c \cdot S_l/4m_{\pi^+}^2), q^2 \leq 0, \quad (4)$$

  where $q^2 = S_\pi/4m_{\pi^+}^2 - 1$, $a = 0.149 \pm 0.033 \pm 0.014$, $b = -0.070 \pm 0.039 \pm 0.013$, $c = 0.113 \pm 0.022 \pm 0.007$, $d = -0.256 \pm 0.049 \pm 0.016$.
  For systematic studies we take into account the combined (both statistical and systematic) uncertainties of these parameters. Moreover, the absolute value was also measured $F = 6.079F(K_{e4})$.

Nevertheless, for MC production we have used $5F(K_{e4})$ (to be close to the 1-loop F), $R(1loop)$, $H = G = 0$ and zero phase between F and R.

F form factor

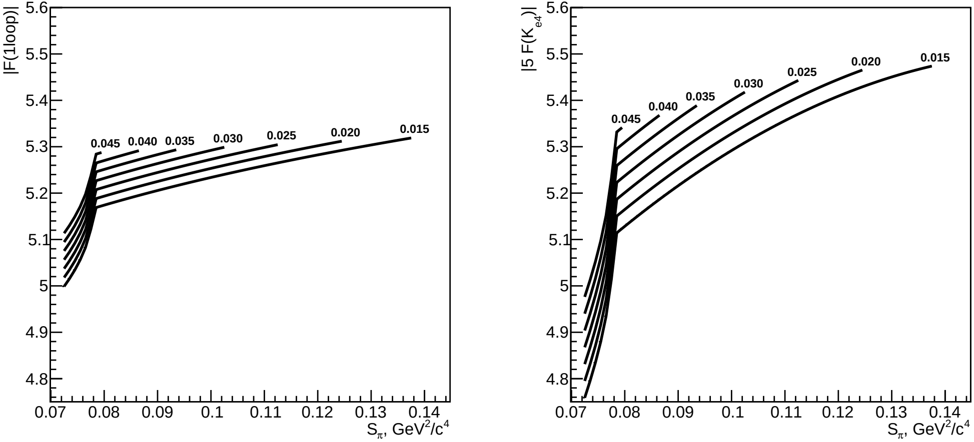


Figure: Values near the lines: S_l $[GeV^2/c^4]$

R form factor

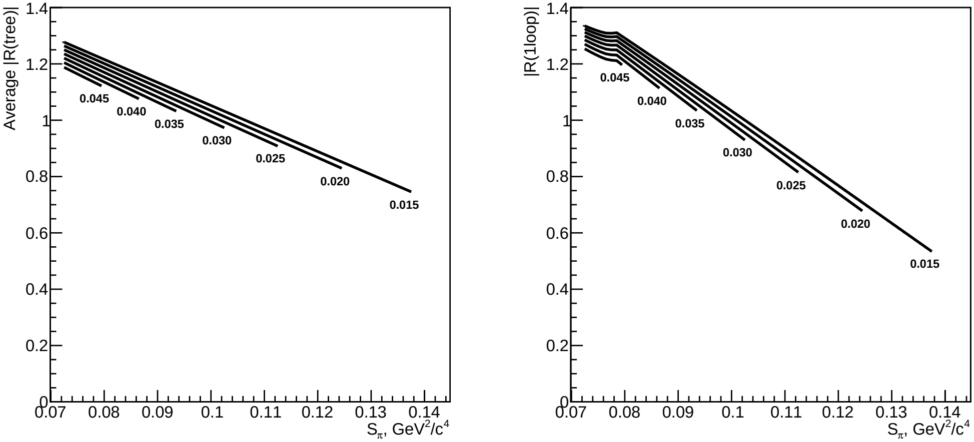
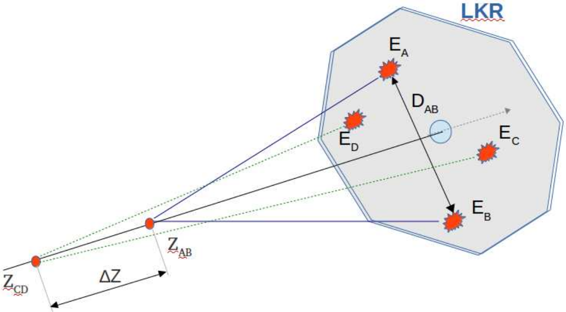


Figure: Values near the lines: S_l $[\mathrm{GeV}^2/c^4]$

Common selection criteria for signal and normalization channels

NA48/2 data collected in 2003-2004.
Signal (S) $K_{\mu 4}$ is $K^{\pm} \to \mu^{\pm}\nu\pi^0\pi^0$. Normalization (N) $K_{3\pi}$ is $K^{\pm} \to \pi^{\pm}\pi^0\pi^0$.
A common trigger chain: L1 trigger using HOD and LKr, followed by L2 trigger using DCH (MBX).
Combination: 4 isolated photons in time with a KABES track and with a DCH track.
Vertex 'charged position' Z_c is taken from the space matching of the KABES track and the DCH track.



- P_{KABES} in [54,67] GeV/c; P_{DCH} in [5,35] GeV/c; $E_\gamma > 3$ GeV;
- For π^0_i $(i = 1, 2)$:
  - $Z_1 = Z_{AB} = D_{AB}\sqrt{E_A E_B}/m_{\pi^0}$;
  - $Z_2 = Z_{CD} = D_{CD}\sqrt{E_C E_D}/m_{\pi^0}$.
- $-1600\ cm < Z_n = Z_{LKR} - (Z_1 + Z_2)/2 < 9000\ cm$ (Z_n is the vertex 'neutral position');
- Flunge cut: $R_\gamma @DCH1 > 11$ cm, assuming $Z_n + 400$ cm decay position;
- $|Z_1 - Z_2| < 500$ cm; $|Z_n - Z_c| < 600$ cm.

The best combination of each mode in the event has the most compatible Z_1,Z_2,Z_c.

Normalization $K^{\pm} \to \pi^{\pm}\pi^{0}\pi^{0}$ channel selection

No missing momentum is expected.

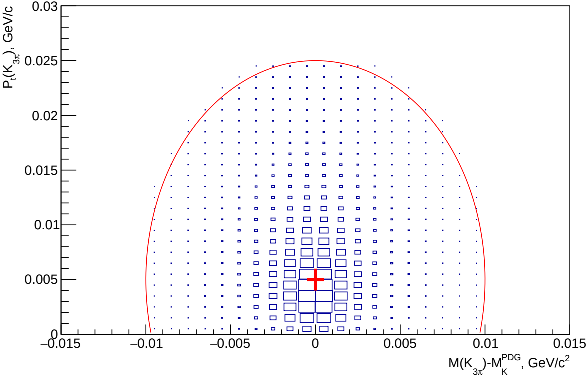


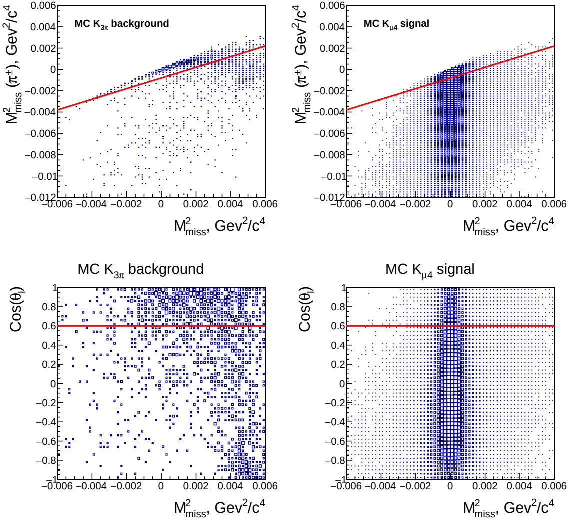
Figure: $K_{3\pi}$ kinematic selection area

$K_{3\pi}$ kinematic selection ellipse:

- center:
  - $M(K_{3\pi}) = M_K^{PDG}$;
  - $P_t = 5$ MeV$/c$;
- semi-axes:
  - $\Delta M(K_{3\pi}) = 10$ MeV$/c^2$;
  - $\Delta P_t = 20$ MeV$/c$.

Normalization channel selection output is $N_N = 72.99 \times 10^6$ $K_{3\pi}$ reconstructed events.

Signal $K^{\pm} \to \mu^{\pm}\nu\pi^0\pi^0$ channel selection



- Off the $K_{3\pi}$ kinematic ellipse;
- Good $K-\mu$ tracks matching;
- $P_{DCH} > 10$ GeV$/c$ for MUV efficiency;
- DCH track is in restricted MUV acceptance with high MUV efficiency;
- DCH track has associated MUV hits in the first two planes;
- $M^2_{miss}(assuming\ \pi^{\pm}) < 0.5M^2_{miss} - 0.0008\ \mathrm{GeV}^2/c^4$;
- $cos(\Theta_l) < 0.6$;
- $S_l > 0.03$ GeV$^2/c^4$ to reject $\pi^{\pm} \to \mu^{\pm}\nu$.

3718 $K_{\mu4}$ candidates selected (any M^2_{miss}).

$K^{00}_{\mu 4}$ signal extraction procedure

We fit $K_{\mu 4}$ data M^2_{miss} spectrum with with a linear combination of the simulated signal, simulated background and unsimulated background spectra by minimizing the χ^2:

$$\chi^2 = \Sigma_{i=min}^{max} \frac{(Data_i - p_0 \cdot S_i - p_1((1-p_2) \cdot Bg_i + p_2 \cdot UBg_i))^2}{\delta Data_i^2 + p_0^2 \cdot \delta S_i^2 + p_1^2(1-p_2)^2 \cdot \delta Bg_i^2 + p_1^2 p_2^2 \cdot \delta UBg_i^2}, \quad (5)$$

where i is the M^2_{miss} bin number; $Data_i$ is the bin content of the data histogram; S_i, Bg_i, UBg_i are the bin contents of the simulated signal, simulated background and unsimulated background model, correspondingly. $\delta Data_i$, δS_i, δBg_i and δUBg_i are their statistical uncertainties, and p_0, p_1, p_2 are free parameters of the fit.
Prior to fit, the S_i, Bg_i, UBg_i are scaled to make their integrals in the signal region equal to one. An interval of [0,1] is allowed for p_2 values during the fit. In such a way, p_0 represents the best fit MC signal in terms of events amount, p_1 is the total background in the signal region and p_2 is the share of unsimulated background.
But we don't consider p_0 parameter (fit of the peak) as a measured signal size, as it may depend on the peak resolution simulation quality. The signal is extracted as a difference between the data histogram content in the signal region and p_1 representing the total background:

$$N_S = \Sigma_j Data_j - p_1, \quad (6)$$

where j bins interval corresponds to the signal region.
For the present result, we avoid the resolution simulation problem by ignoring the signal region in χ^2 calculation. The tails of MC simulated signal are taken into account outside the signal region, that requires an estimation of MC signal using the relation (6) during the fit.

$K_{\mu4}^{00}$ acceptance vs R form factor contribution

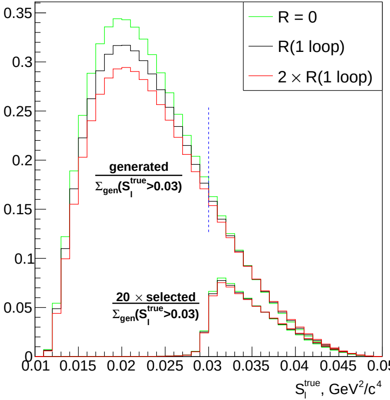


Figure: MC acceptance ingredients for different R contributions

- We change R(1 loop) form factor normalization between 0% and 200% to illustrate different R-sensitivity of
  - the full phase space acceptance $A_S = \frac{N^{MC}_{Selected}}{N^{MC}_{Generated}(all\ \ S_l^{true})}$ and
  - the restricted phase space acceptance $A^r_S = \frac{N^{MC}_{Selected}}{N^{MC}_{Generated}(S_l^{true}>0.03)}$.
- For our systematic uncertainty, we consider only 20% R variation.

Trigger efficiency

- A common trigger chain for the signal (S) and normalization (N) modes: a first level (L1) trigger using signals from HOD (Q1+Q2) and LKr (NUT), followed by a second level (L2) trigger using DCH (MBX).
- It is possible to measure separately NUT efficiency using the $Q1 \cdot [E_{LKr} > 10\ GeV] \cdot MBX$ control trigger and the 'charged trigger chain' $CHT = (Q1 + Q2) \cdot MBX$ efficiency using a special control trigger $nx > 2 || ny > 2$ for 2003 and $nx > 3 || ny > 3$ for 2004 data.
- The ratio $K_{trig} = \frac{\mathcal{E}_N}{\mathcal{E}_S}$ is a multiplicative trigger correction to the measured branching ratio. We decompose it to the neutral trigger and charged trigger corrections, that may be measured separately: $K_{trig} = (K_{NUT} = \frac{\mathcal{E}_N^{NUT}}{\mathcal{E}_S^{NUT}}) \times (K_{CHT} = \frac{\mathcal{E}_N^{CHT}}{\mathcal{E}_S^{CHT}})$.
- Direct $\mathcal{E}_S$ measurement has only 2% precision due to small statistics of the control samples for the rare decay.
- $\mathcal{E}_S^{CHT}$ is estimated using MC trigger simulation and taking into account comparison between MC and measured $\mathcal{E}_N^{CHT}$;
- $\mathcal{E}_S^{NUT}$ is recalculated from $\mathcal{E}_N^{NUT}$ using MC simulated $(S_\pi, Min(E_\gamma), Z_c)$ distributions.

As a result, we estimate the charged trigger correction factor as $K_{CHT} = 0.998 \pm 0.002$ and the neutral trigger correction factor as $K_{NUT} = 1.0007 \pm 0.0007$.