Signal of Inflation Triggered Dynamics

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Work in collaboration
Gravitational wave: with Haipeng An, KunFeng Lyu, and Siyi Zhou, 2009.12381, 2201.05171
Cosmo collider: Matt Reece and Zhong-Zhi Xianyu, 2204.11869 (briefly)

CERN-CKC workshop. Jeju. June 6, 2022

Inflation: a stage for new dynamics

- High energy: H can be 10^{13} GeV.
  - Can produce heavy new physics particles. "Cosmological collider physics"
- Inflaton can travel a large distance in field space.
  - Can trigger dramatic changes in spectator sectors which couple to the inflaton.
  - Can also show up in cosmo collider.

Inflation: a stage for new dynamics

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  - Can produce heavy new physics particles. "Cosmological collider physics"

This talk

- Inflaton can travel a large distance in field space.
  - Can trigger dramatic changes in spectator sectors which couple to the inflaton.
  - Can also show up in cosmo collider.

The excursion of the inflaton

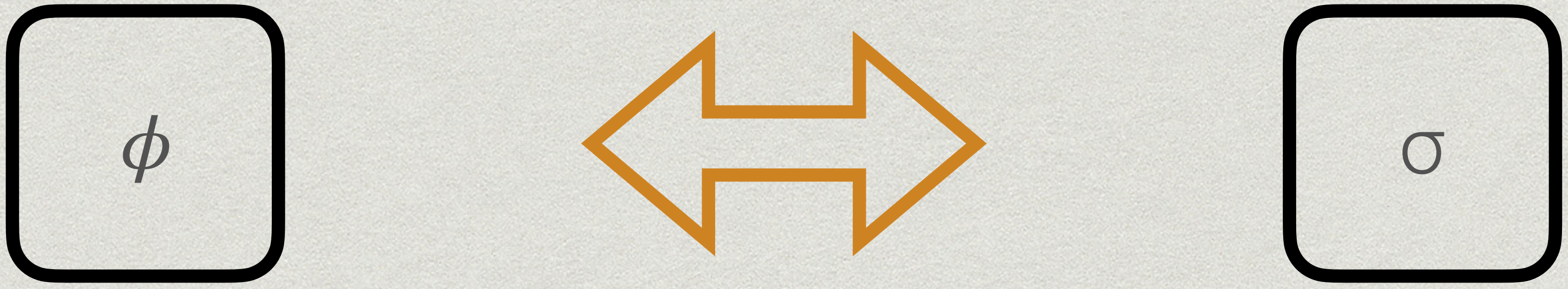
$$\Delta\phi \sim N_{\rm efold}\sqrt{\epsilon}M_{\rm Planck}$$

Large excursion of the inflaton field plausible, even if we restrict ourselves to the case where $\Delta\phi < M_{Planck}$

This is the case even for a small part of inflation with $N_{efold} \approx O(1)$

Any physics/observable effect?

Example: Inflaton + spectator



Inflaton sector
Single field slow roll
Approx. shift symmetry…

Spectator, less energy,
not driving spacetime evolution

Suppose the coupling is weak, suppressed by some high scale M, such as $M \approx M_{Planck}$

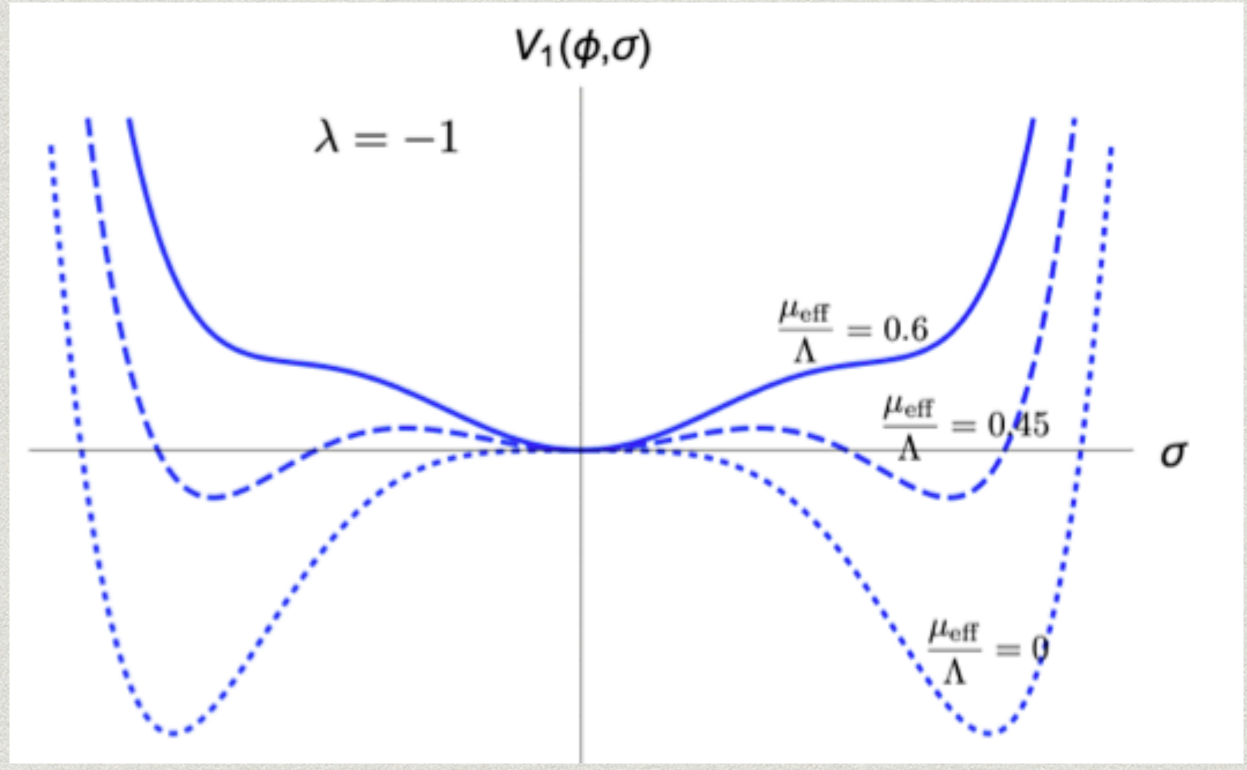
For example: $f\left(\frac{\phi}{M}\right) m_\sigma^2 \sigma^2, \quad g\left(\frac{\phi}{M}\right) \lambda \sigma^4,$ etc.

Field excursion of inflaton, $\boldsymbol{\Delta}\phi \sim M$, can change the mass and couplings in the spectator sector, leading to interesting dynamics.

For example: 1st order PT

$$V(\phi,\sigma) = -\frac{1}{2}\mu_{\rm eff}^2\sigma^2 + \frac{\lambda}{4}\sigma^4 + \frac{1}{8\Lambda^2}\sigma^6 + V_{\rm inf}(\phi), \quad \mu_{\rm eff}^2 = -\left(m_\sigma^2 - c^2\phi^2\right)$$

$$c^2 \sim \frac{m_\sigma^2}{M^2} \ll 1$$



Rolling inflaton → (1st order) phase transition in the spectator sector

1st order phase transition

Bubble nucleation rate: $$\frac{\Gamma}{V} \simeq m_\sigma^4 e^{-S_4}$$

m_σ: typical scale in the spectator sector

Efficient phase transition:

$$\int_{-\infty}^{t} dt' \frac{\Gamma}{V} \frac{1}{H^3} \simeq O(1) \ \rightarrow \ S_4 \sim \log\left(\frac{\phi H}{\dot{\phi}} \frac{m_\sigma^4}{H^4}\right) \sim \log\left(\frac{\phi}{\epsilon^{1/2} M_{\rm Pl}} \frac{m_\sigma^4}{H^4}\right)$$

Phase transition is 1st order ($S_4 \gg 1$).
Assume spectator sector does not dominate energy density:

$$H^4 \ll m_\sigma^4 \ll 3 M_{\rm Pl}^2 H^2$$

1st order phase transition

Phase transition completed with O(1) of Hubble volume in new phase:

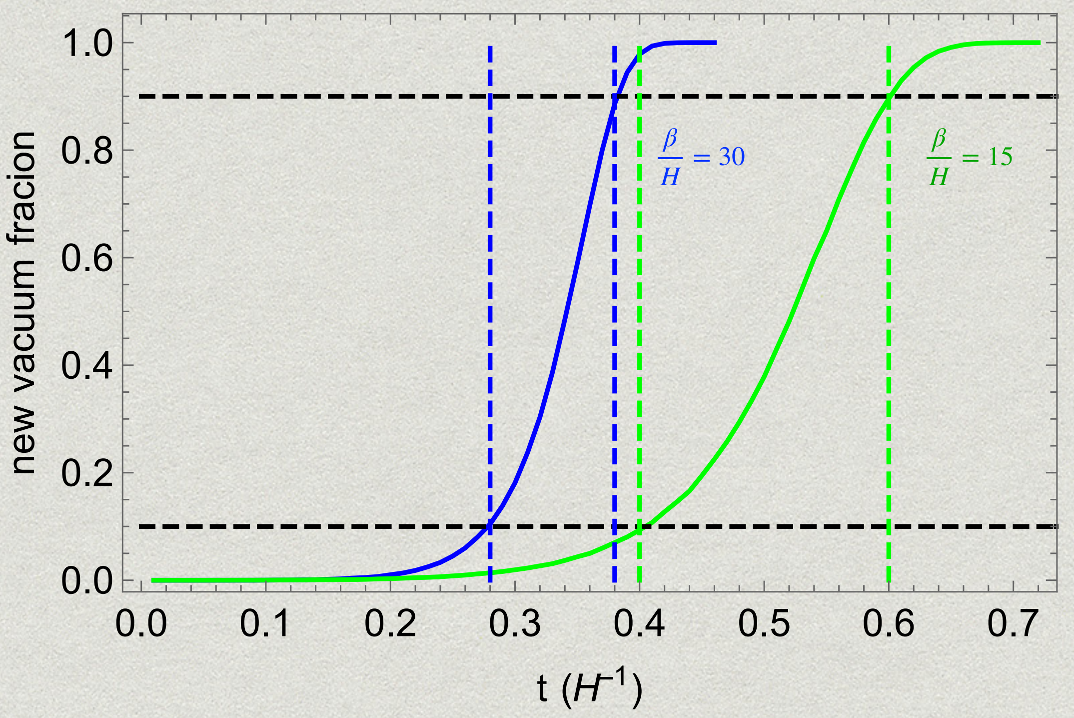
Guth and Weinberg, 83'

$$S_4(t) \simeq S(t_*) + \beta(t_* - t) + \dots$$

$$r_{\text{bubble}}^{-1} \simeq \beta = \left| \frac{dS_4}{dt} \right|$$

$$\beta^4 \ll m_\sigma^4 \ll 3M_{\text{Pl}}^2 H^2$$

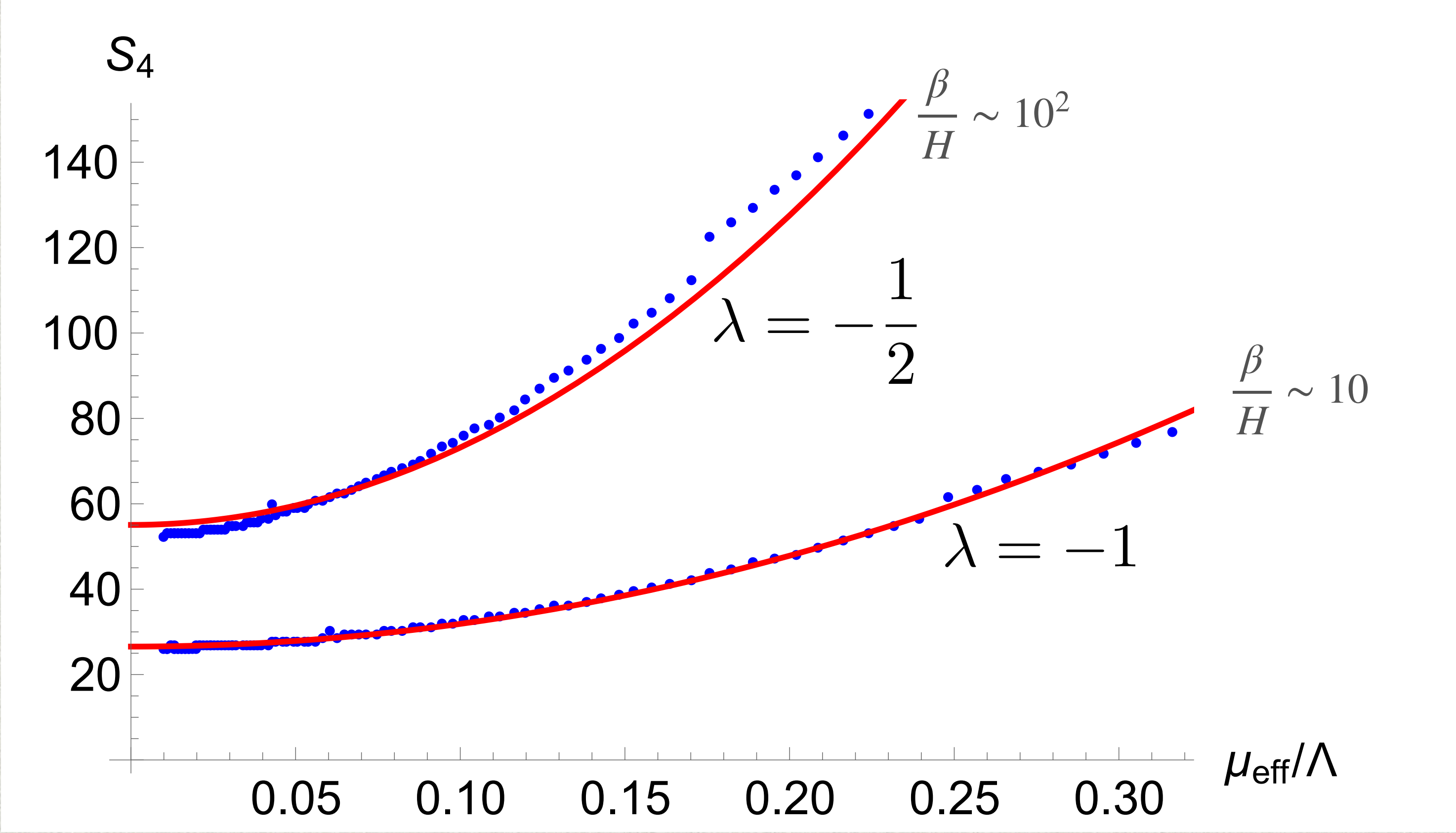
$$\beta \sim (10 - 100) \times H$$



For GW signal in the opposite limit: J. Barir, M. Geller, C. Sun, T. Volansky, 2203.00693

In our toy model:

$$\frac{\beta}{H} = \left|\frac{dS_4}{d\log\mu_{\rm eff}^2}\right|(2\epsilon)^{1/2}\times\frac{M_{\rm Pl}}{\phi\left(1-m_\sigma^2/(c^2\phi^2)\right)} \qquad \left|\frac{dS_4}{d\log\mu_{\rm eff}^2}\right| \sim O(1)$$



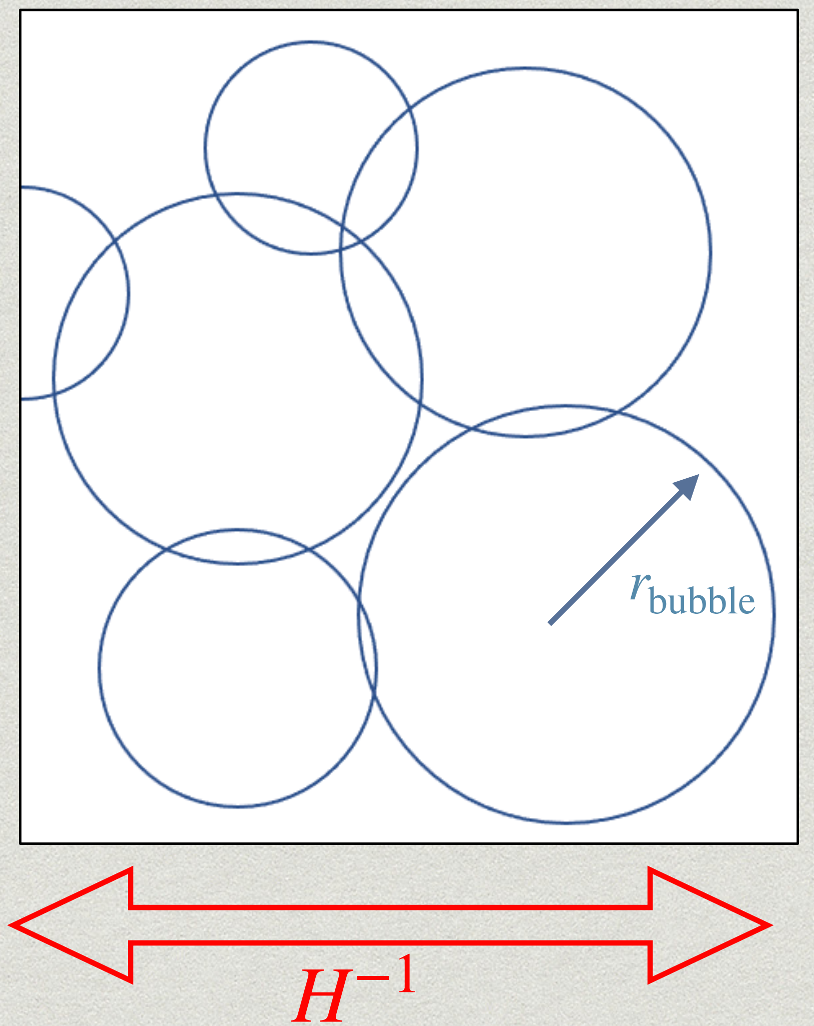
$$r_{\rm bubble}^{-1} \simeq \beta = \left|\frac{dS_4}{dt}\right|$$

CosmoTransitions

1st order phase transition

Phase transition is 1st order, and spectator sector does not dominate energy density:

$$S_4(t) \simeq S(t_*) + \beta(t_* - t) + \dots \qquad \beta^4 \ll m_\sigma^4 \ll 3M_{\rm Pl}^2 H^2$$

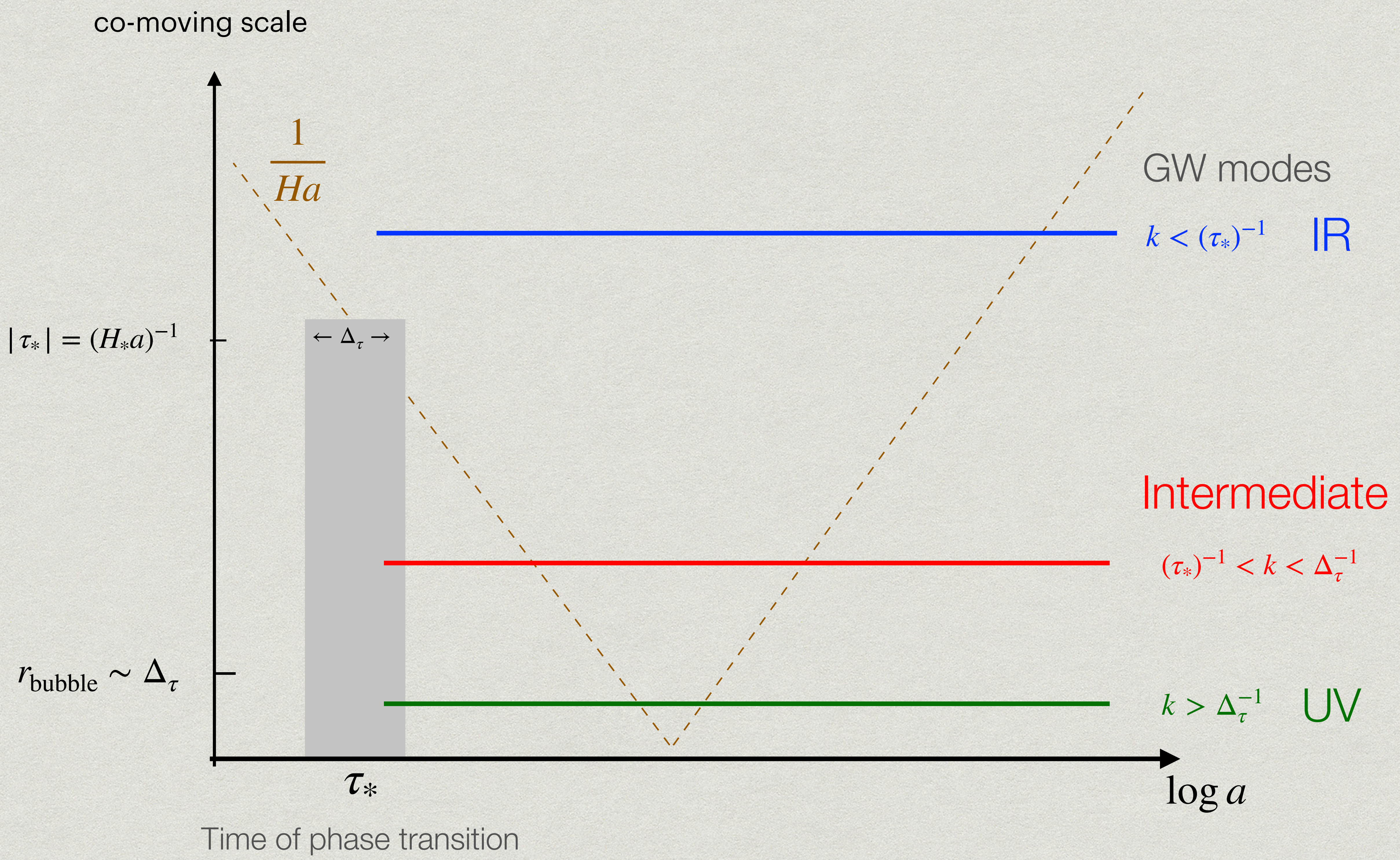


$$\beta^{-1} \sim r_{\rm bubble} \ll H^{-1}$$

$$t_{\rm bubble\ collision} \sim r_{\rm bubble} \ll H^{-1}$$

An instantaneous source of GW.

GW in three regimes



GW from instantaneous source

$$h'' + \frac{2a'}{a}h' + k^2 h = 16\pi G_{\rm N} a^3 T_{ij}$$

Instantaneous source: $T_{ij} \simeq T a^{-3}(\tau_*)\delta(\tau - \tau_*)$

Before the end of inflation:

$$h = 16\pi G_{\rm N}(-H\tau)\left[\frac{\sin k(\tau-\tau_*)}{k} + \left(\frac{1}{k^2\tau} - \frac{1}{k^2\tau_*}\right)\cos k(\tau-\tau_*) + \frac{1}{k^3\tau\tau_*}\sin k(\tau-\tau_*)\right]$$

Assume radiation domination after reheating (for now):

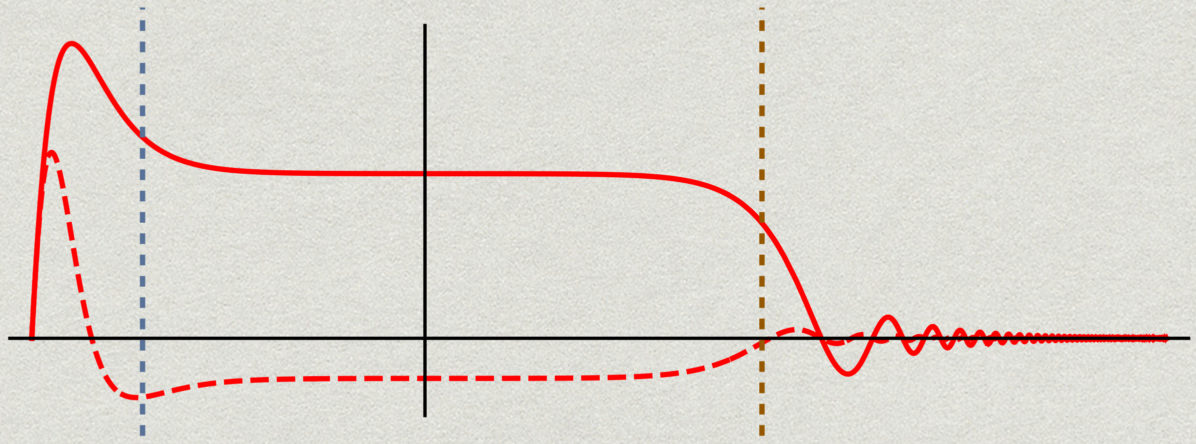
$$h \propto \frac{\sin k\tau}{k\tau}$$

GW signal

$$h''_{ij} + \frac{2a'}{a}h'_{ij} - \nabla^2 h_{ij} = 16\pi G_N a^2 \sigma_{ij}$$

Intermediate

$\tau_*^{-1} < k < \Delta_\tau^{-1}$



During inflation:

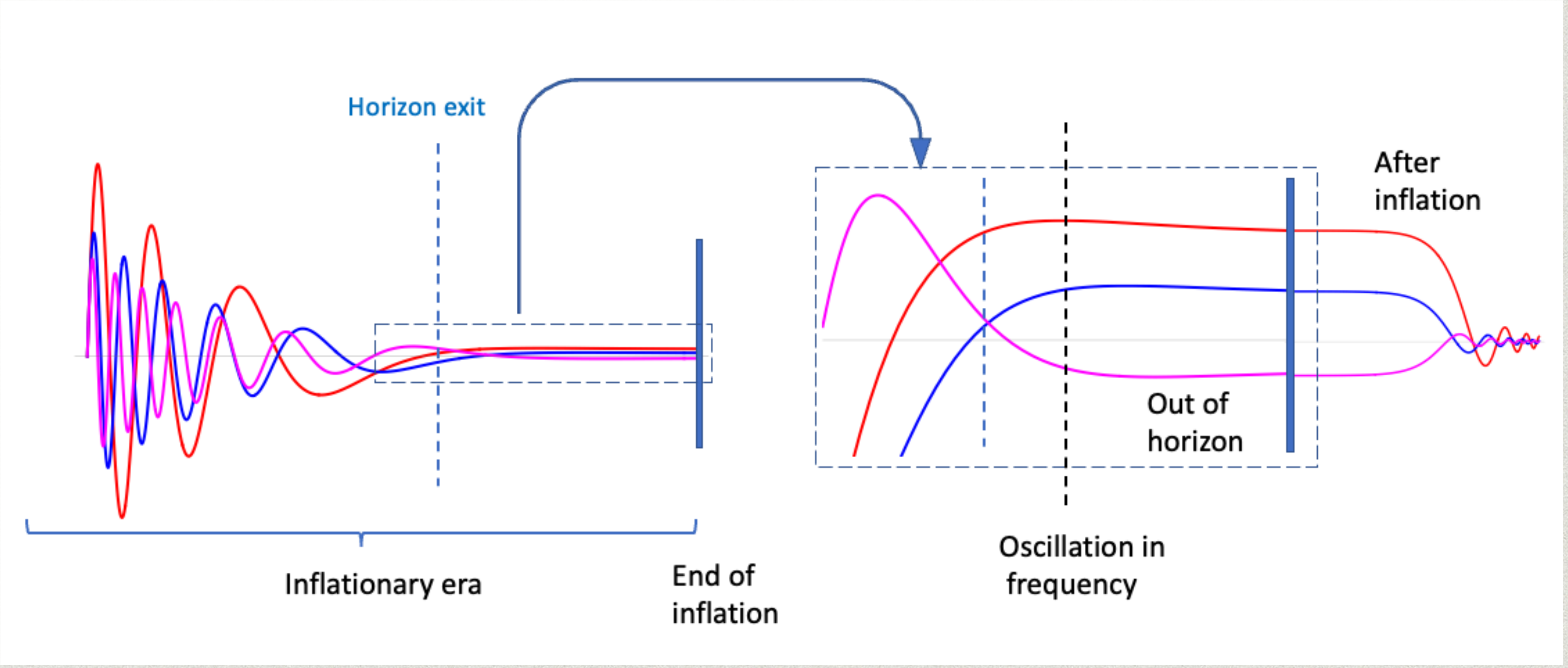
Mode starts inside horizon, oscillates till horizon exit.

➡ Amplitude depends on k.

➡ Leads to oscillatory pattern in frequency. $h \propto \frac{\cos(k\tau_*)}{k^2}$

Oscillations

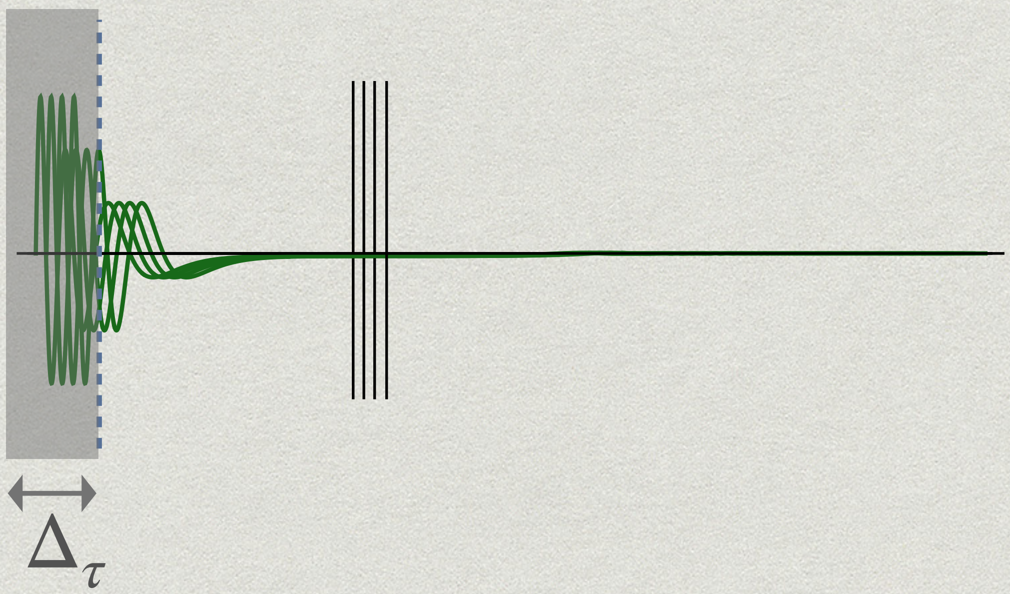
$$\tau_*^{-1} < k < \Delta_\tau^{-1}$$



GW signal

$$h''_{ij} + \frac{2a'}{a} h'_{ij} - \nabla^2 h_{ij} = 16\pi G_N a^2 \sigma_{ij}$$

UV: $k > \Delta_\tau^{-1}$



Time scale of bubble collision $\approx \Delta_\tau$.

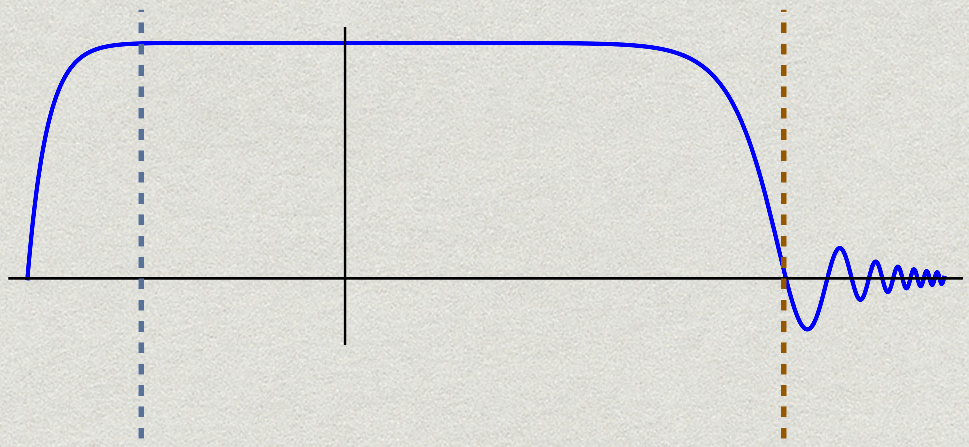
Oscillation pattern in frequency smeared out in this regime.

Spectrum depends on details of the source.

GW signal

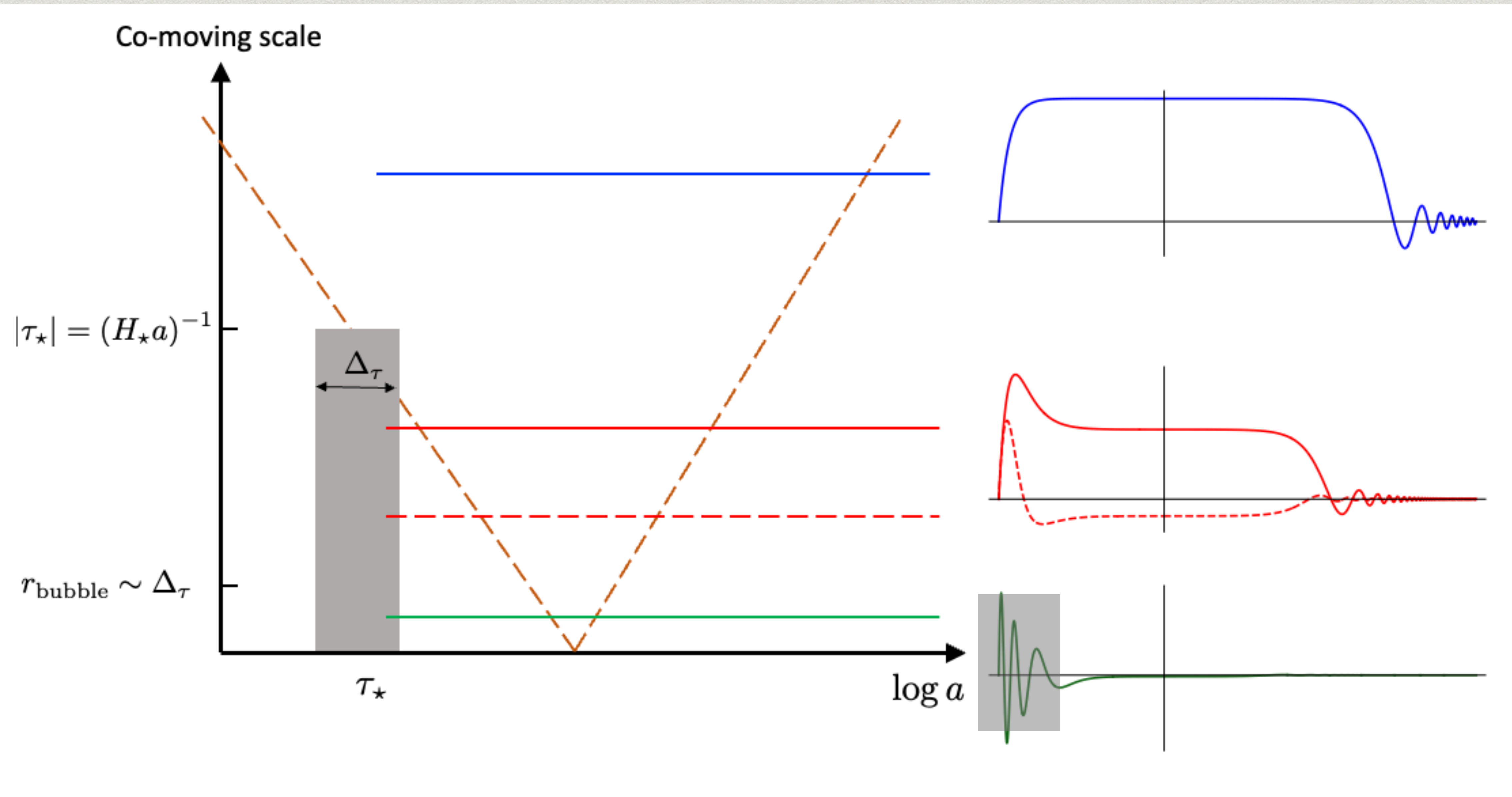
$$h''_{ij} + \frac{2a'}{a} h'_{ij} - \nabla^2 h_{ij} = 16\pi G_N a^2 \sigma_{ij}$$

IR: $k < \tau_*$

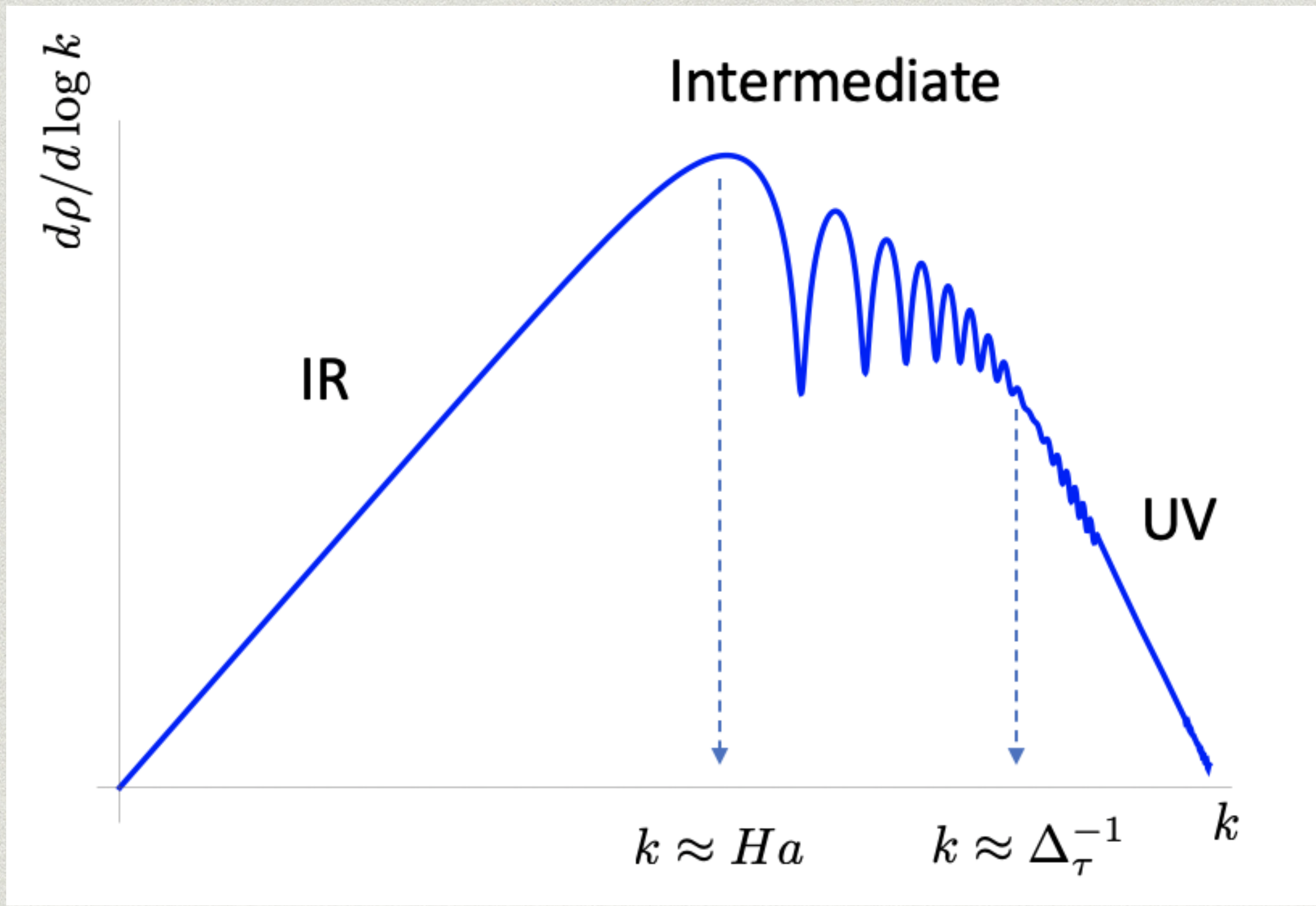


Mode outside horizon at the time of phase transition

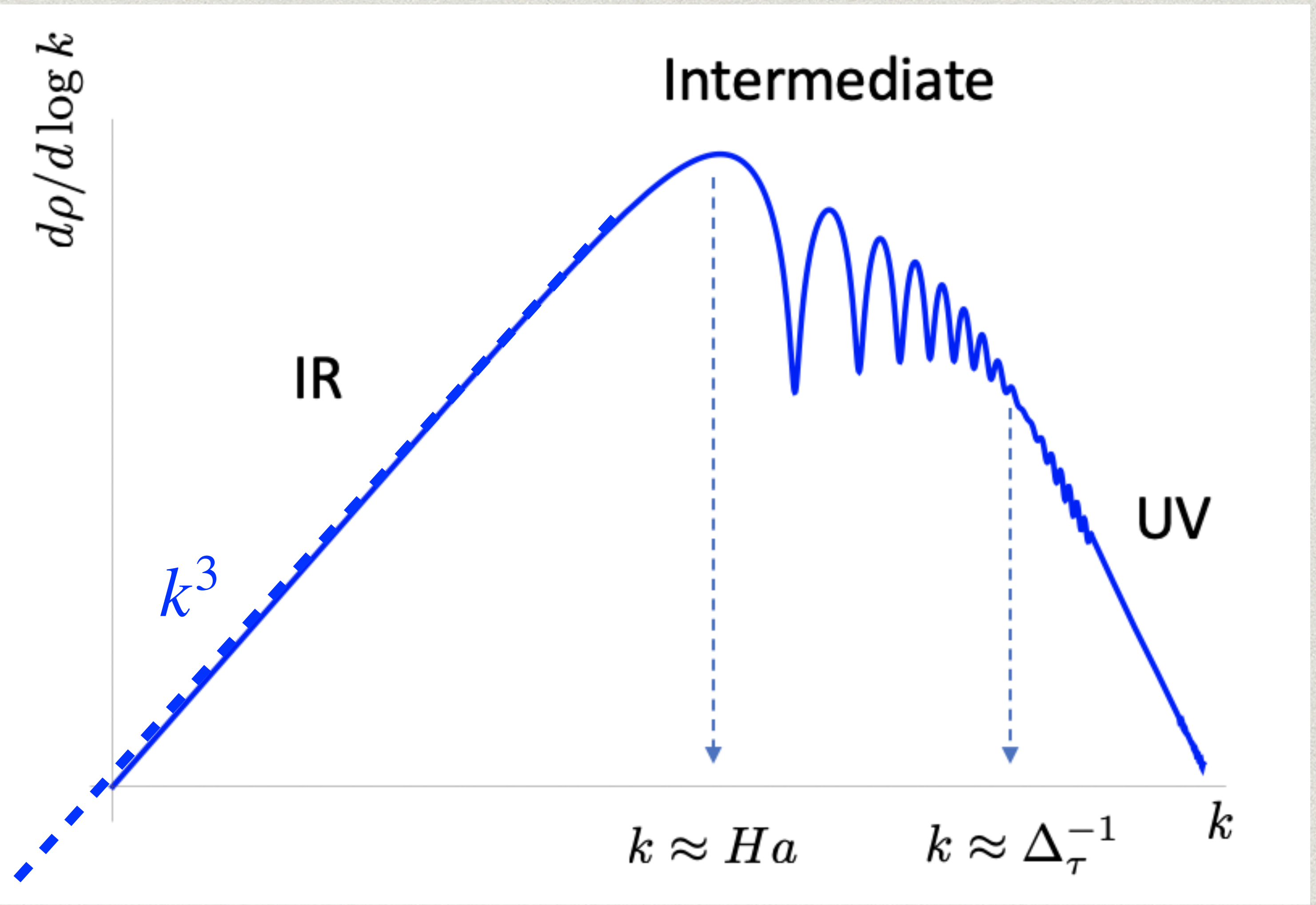
No oscillation. Can treat the GW as if it is from a point source.



Co-moving scale
$|\tau_\star| = (H_\star a)^{-1}$
Δ_τ
$r_{\text{bubble}} \sim \Delta_\tau$
$\tau_\star$
$\log a$

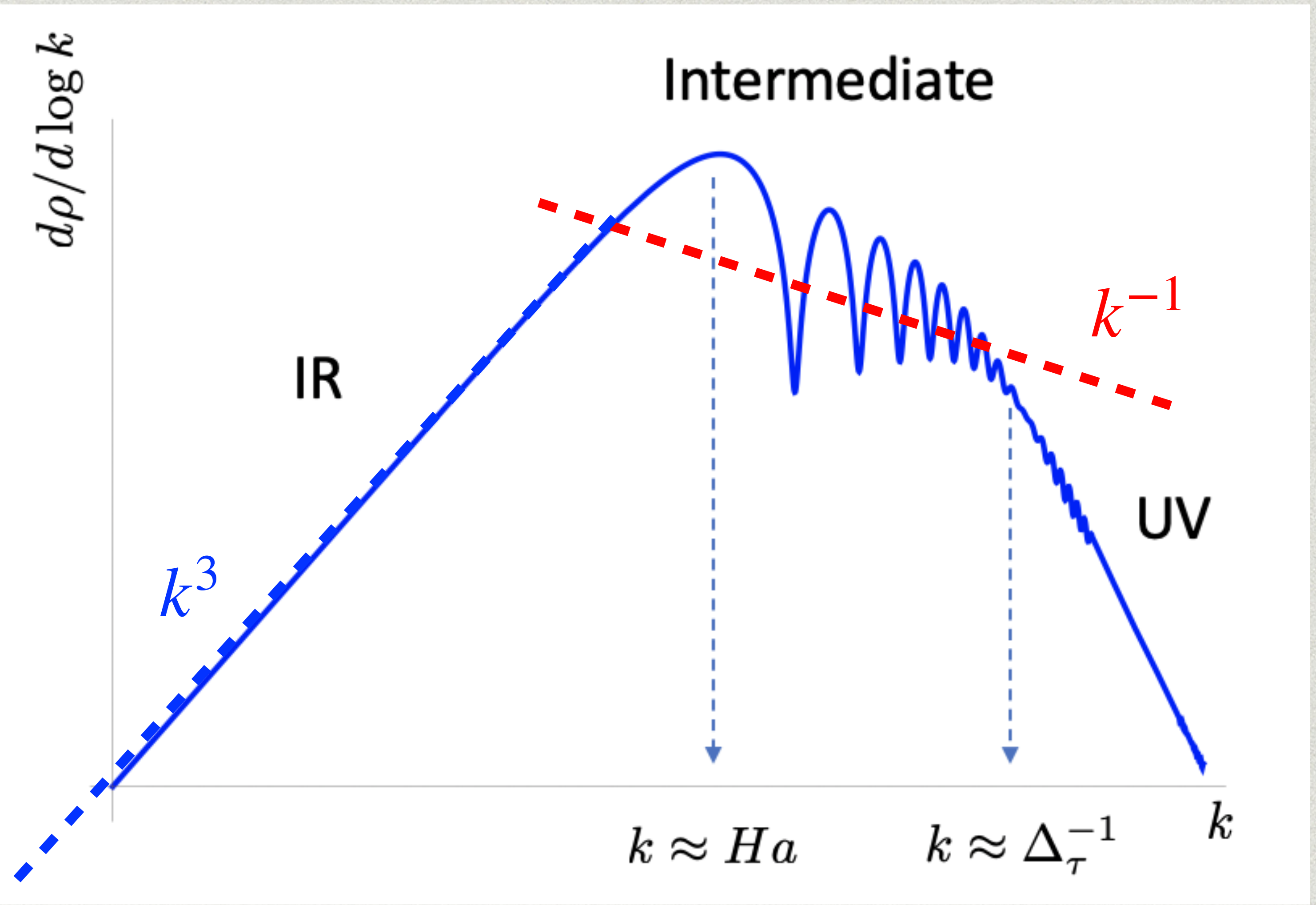


$$\frac{d\rho_{\rm GW}}{d\log k} \propto k^3 \langle (h')^2 \rangle$$



$$\frac{d\rho_{\rm GW}}{d\log k} \propto k^3 \langle (h')^2 \rangle$$

IR: $h = \text{constant} \times \dfrac{\sin k\tau}{k\tau}$



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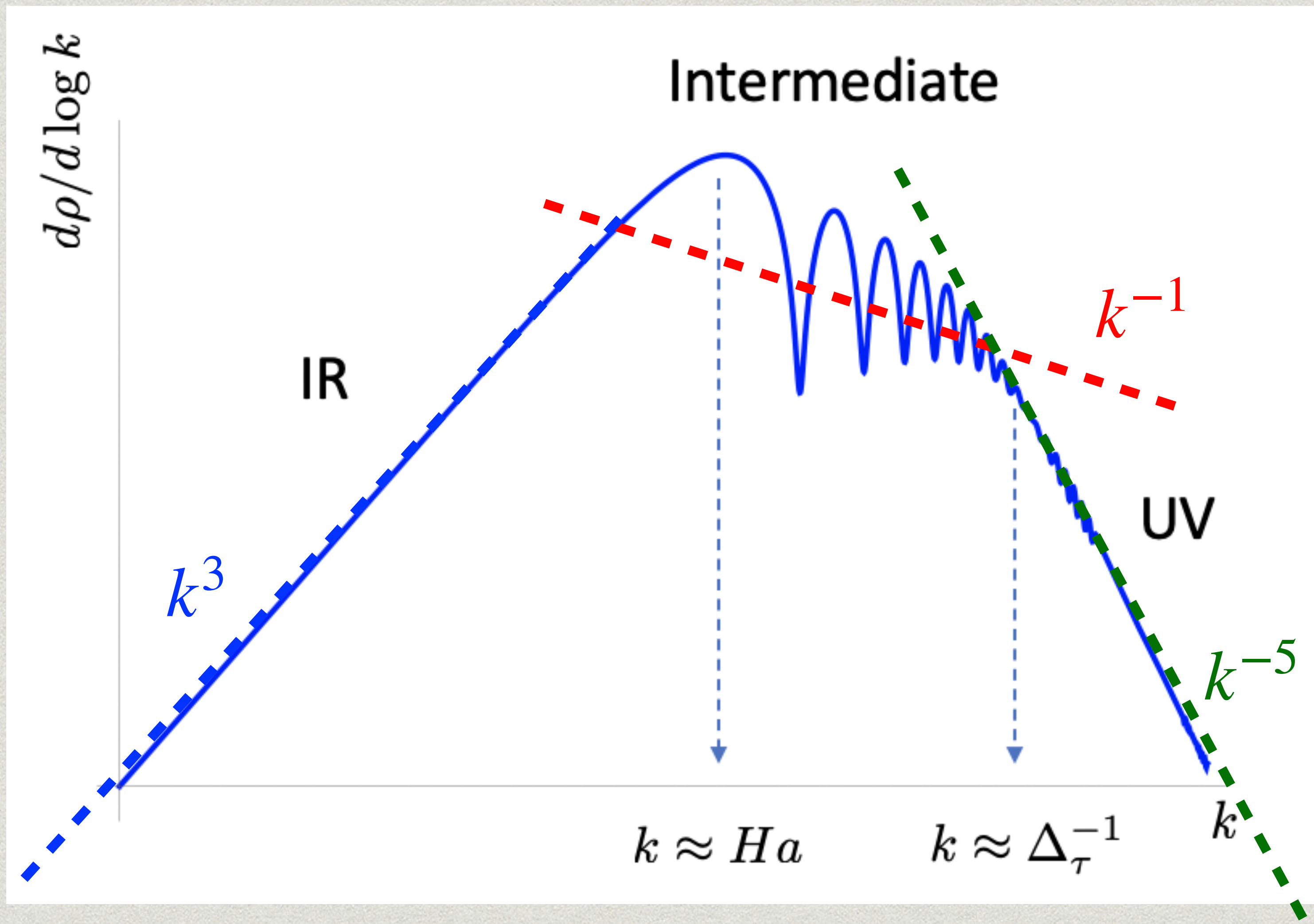
IR: $$h = \text{constant} \times \frac{\sin k\tau}{k\tau}$$

Intermediate: $$h \propto \frac{\cos(k\tau_*)}{k^2}\frac{\sin k\tau}{k\tau}$$



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IR: $h = \text{constant} \times \dfrac{\sin k\tau}{k\tau}$

Intermediate: $h \propto \dfrac{\cos(k\tau_*)}{k^2} \dfrac{\sin k\tau}{k\tau}$

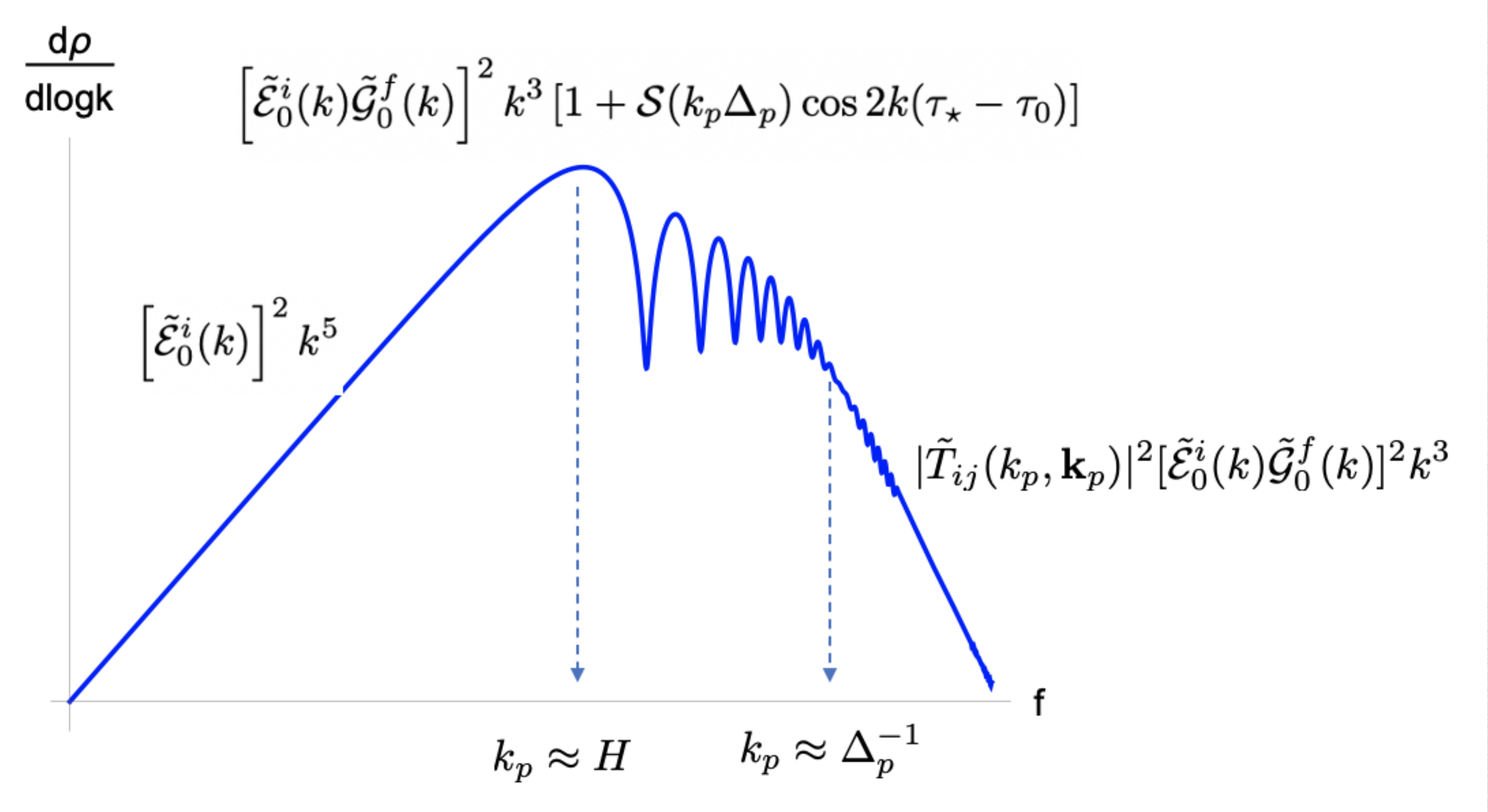
UV: $\dfrac{d\rho_{\rm GW}}{d\log k} \propto k^{-5}$

Details of source matters, determined by numerical simulation

Huber, Konstantin, 0806.1828

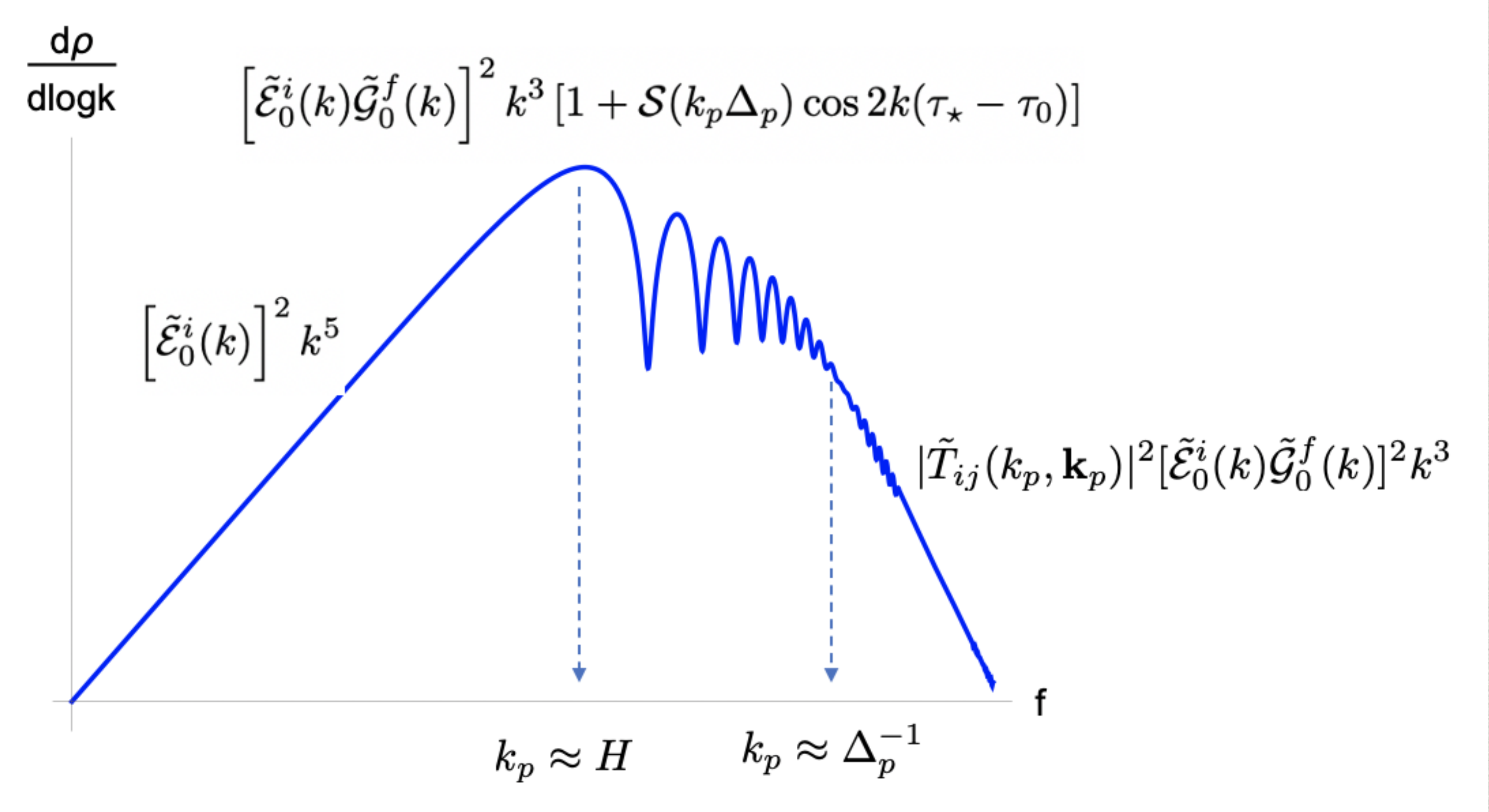
Cutting, Hindmarsh, Wier, 1802.05712

More generally



$$\frac{d\rho_{\rm GW}^{\rm osc}}{d\log k} = \frac{2G_N|\tilde{T}_{ij}(0,0)|^2}{\pi V a^4(\tau)a^2(\tau_\star)}\left\{\left[\tilde{\mathcal{E}}_0^i(k)\tilde{\mathcal{G}}_0^f(k)\right]^2 k^3\left[1+\mathcal{S}(k\Delta_\tau)\cos 2k(\tau_\star-\tau_0)\right]\right\}$$

More generally



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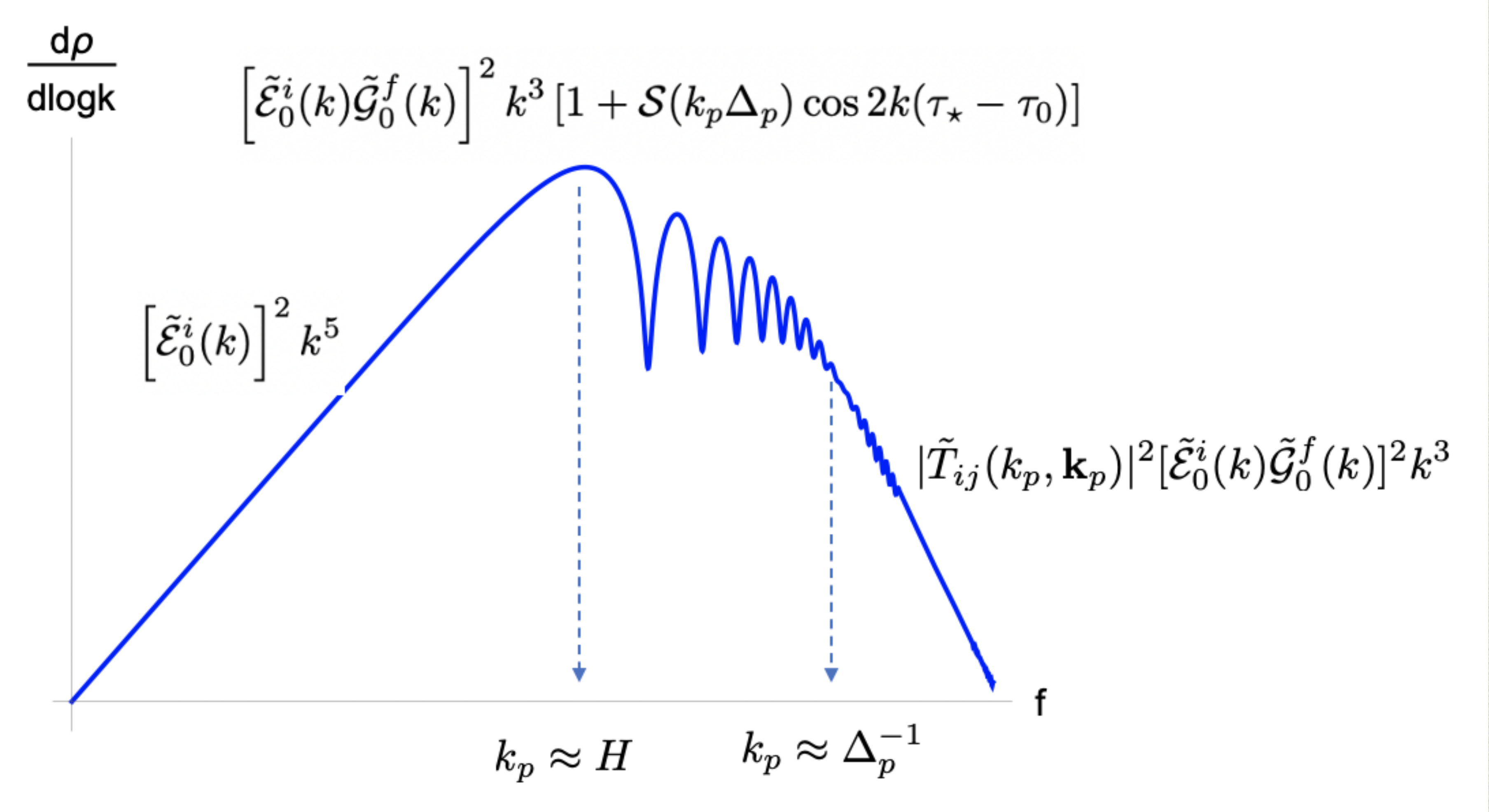
Oscillation

More generally



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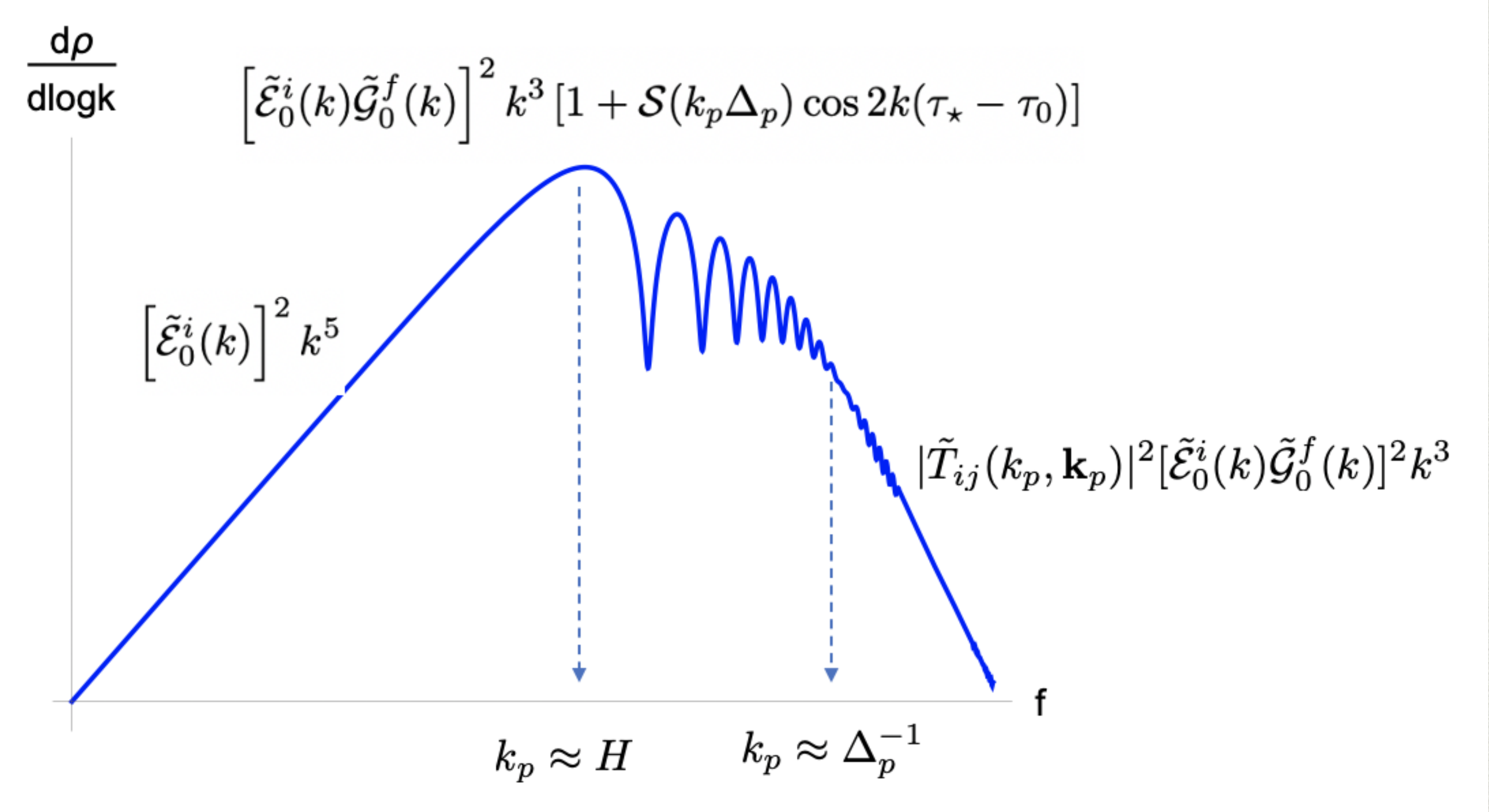
Smearing for $k\boldsymbol{\Delta_\tau} \gg 1$

Oscillation

More generally



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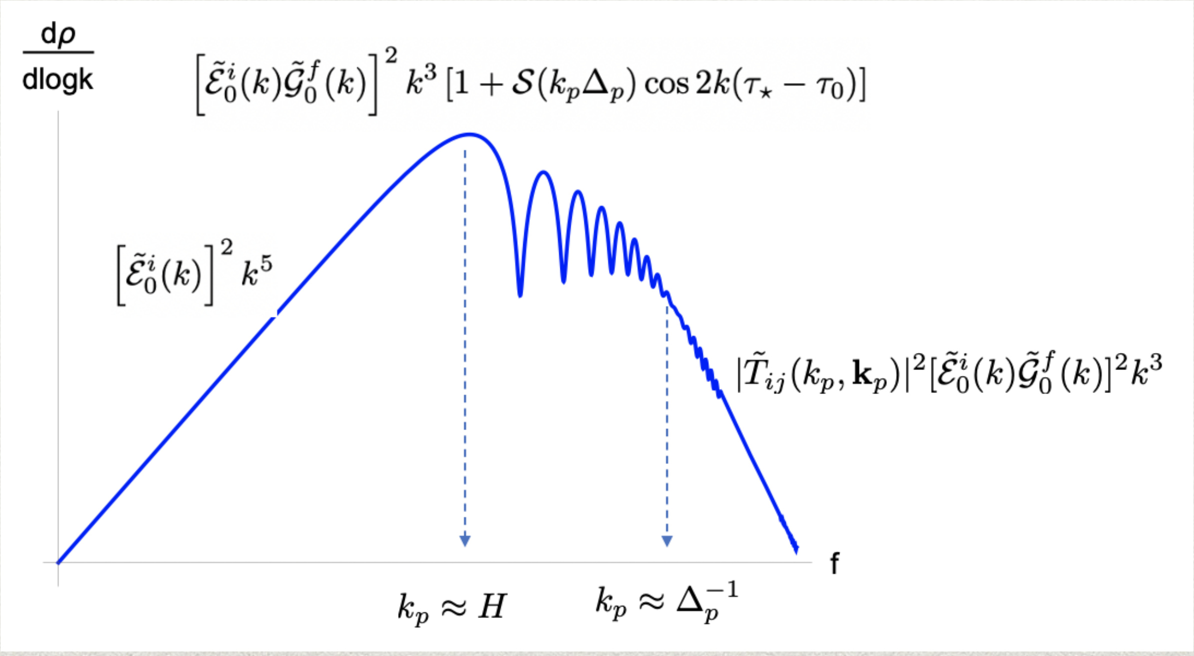
Depending on Time evolution

Smearing for $k\Delta_\tau \gg 1$

Oscillation

Dependence on later evolution



$\tilde{\mathcal{G}}_0^f(k)$ Depends on the evolution of the background spacetime during inflation

$\tilde{\mathcal{E}}_0^i(k)$ Depends on the evolution of the background spacetime after inflation

Alternative scenarios can change the shape of the GW signal!

Can be sensitive to era after the CMB mode exit the horizon and before BBN

Scenarios of inflation and its aftermath

Scenarios of inflation

Parameterized by p

Quasi de Sitter: $a(\tau) = -\frac{1}{H\tau}$

Power law: $a(t) = a_0(t/t_0)^p$

Lucchin and Matarrese, 1985

$p \to \infty$, quasi de Sitter

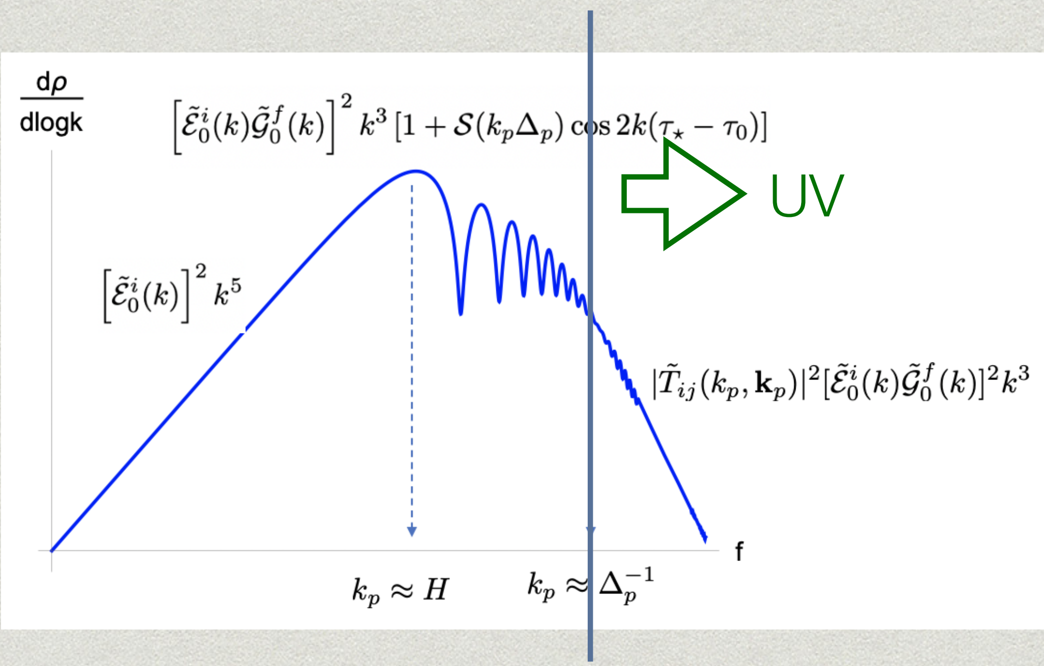
Scenarios after inflation:

Parameterized by $\tilde{p}$

$$a(t) \sim t^{\tilde{p}}$$

	w	$\rho(a)$	$\tilde{p}$	$\tilde{\alpha}$
MD	0	a^{-3}	2/3	-3/2
RD	1/3	a^{-4}	1/2	-1/2
Λ	-1	a^0	∞	3/2
Cosmic string	-1/3	a^{-2}	1	∞
Domain wall	-2/3	a^{-1}	2	5/2
kination	1	a^{-6}	1/3	0

Impact on spectrum



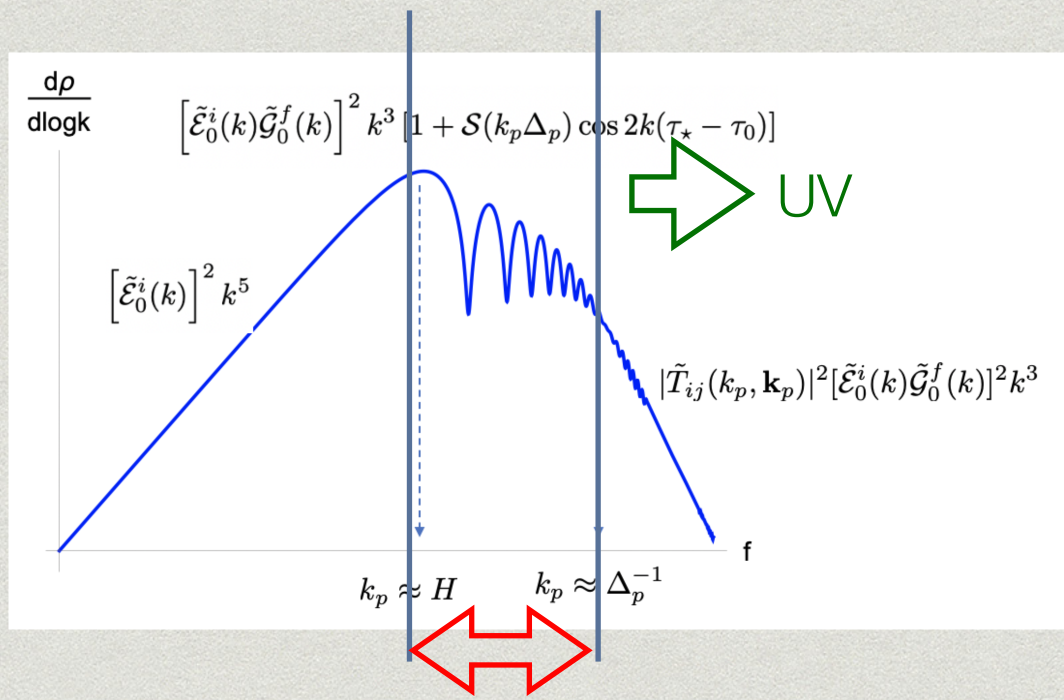
UV

Scenarios after inflation

Inflationary scenarios

	RD	MD	$t^{\tilde{p}}$
dS	k^{-5}	k^{-7}	$k^{-3+2\frac{\tilde{p}}{\tilde{p}-1}}$
t^p	$k^{-3+2\frac{p}{1-p}}$	$k^{-5+2\frac{p}{1-p}}$	$k^{-1+2\left(\frac{p}{1-p}+\frac{\tilde{p}}{\tilde{p}-1}\right)}$

Impact on spectrum



Intermediate

UV

Scenarios after inflation →

Inflationary scenarios ↓

	RD	MD	$t^{\tilde{p}}$
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Intermediate

Scenarios after inflation →

Inflationary scenarios ↓

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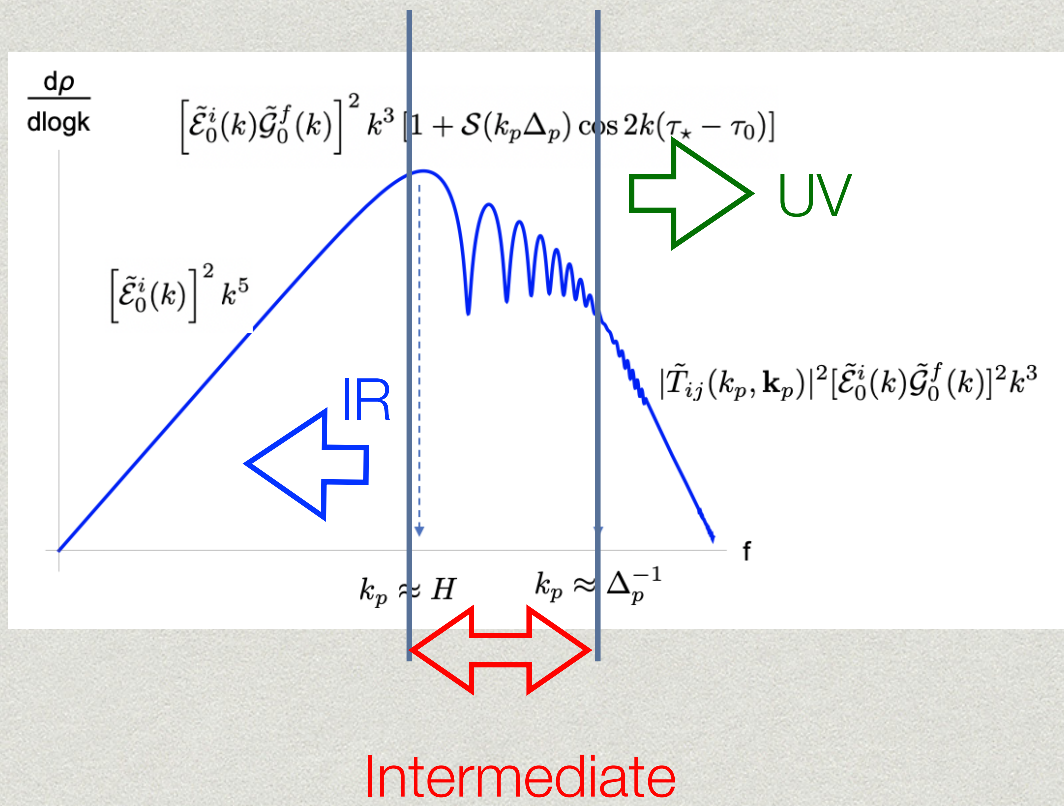
Impact on spectrum



UV

Scenarios after inflation →

Inflationary scenarios ↓

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Intermediate

Scenarios after inflation →

Inflationary scenarios ↓

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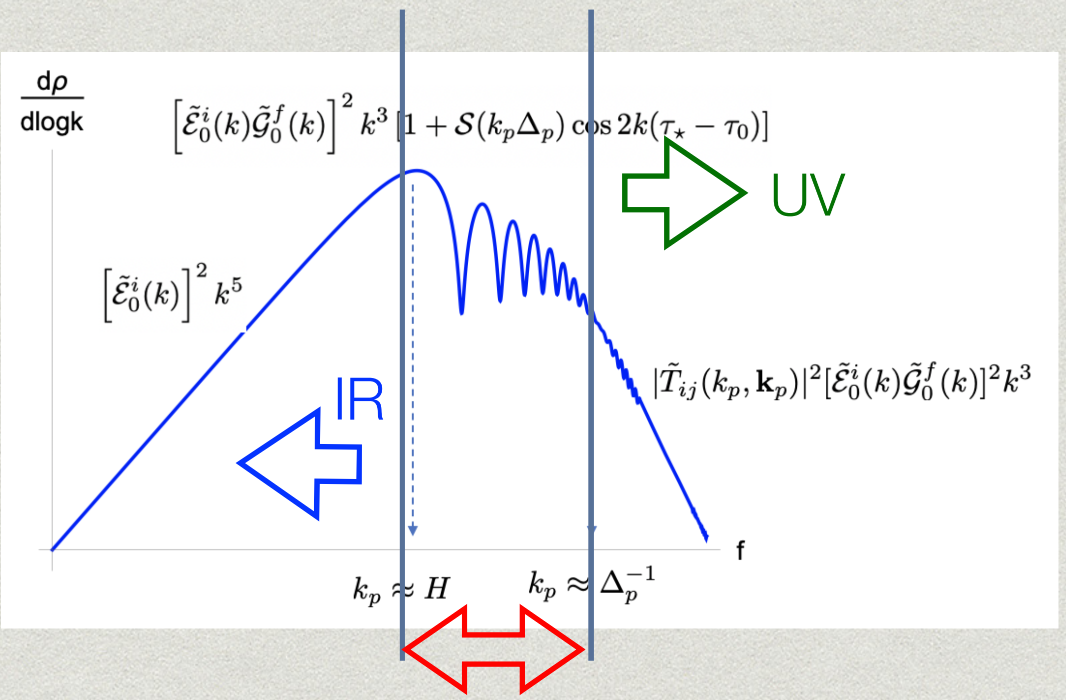
IR

Scenarios after inflation →

Inflationary scenarios ↓

	RD	MD	$t^{\tilde{p}}$
dS	k^3	k^1	$k^{5+2\frac{\tilde{p}}{\tilde{p}-1}}$
t^p	k^3	k^1	$k^{5+2\frac{\tilde{p}}{\tilde{p}-1}}$



UV

Scenarios after inflation →

Inflationary scenarios ↓

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Scenarios after inflation →

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IR

Scenarios after inflation →

Inflationary scenarios ↓

	RD	MD	$t^{\tilde{p}}$
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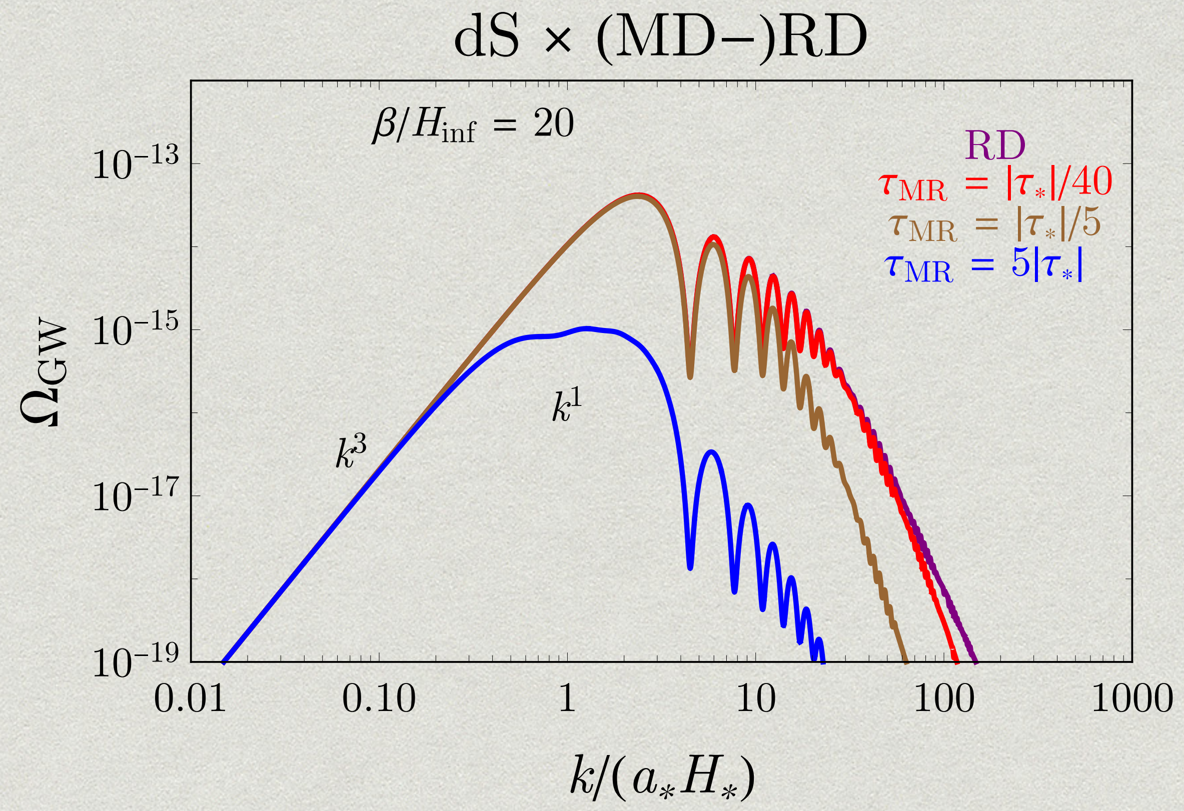
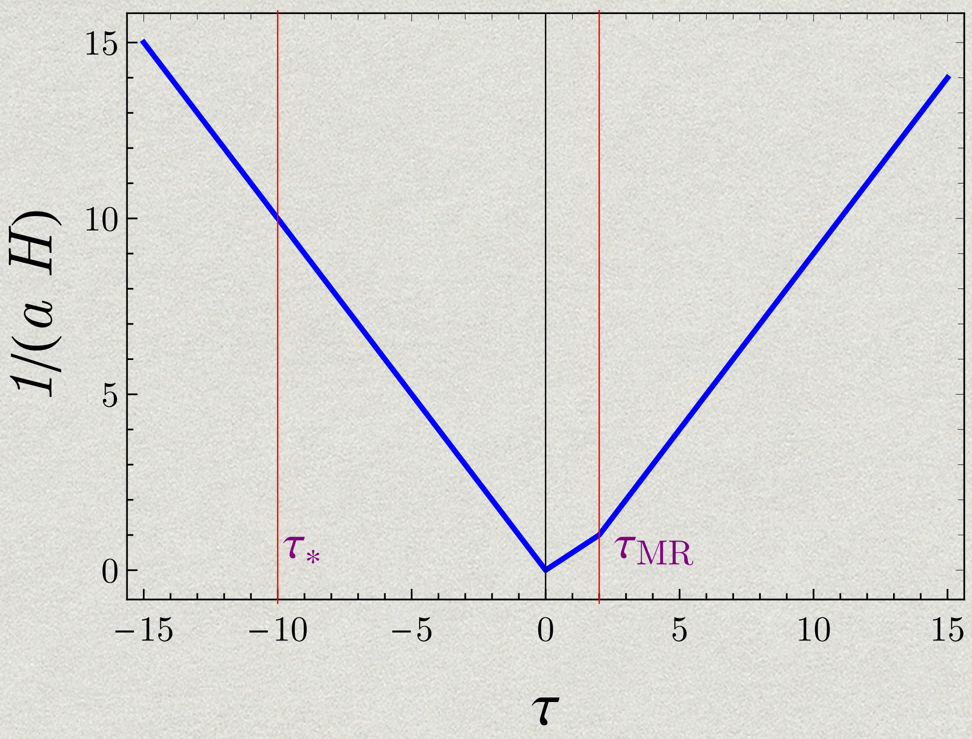
t^p	k^3	k^1	$k^{5+2\frac{\tilde{p}}{\tilde{p}-1}}$

Ideally, we will

1. Observe GW.
2. Observe Oscillation → Instantaneous source during inflation (1st order PT)
3. The spectral shape can tell us the evolution after PT.

Comparing scenarios

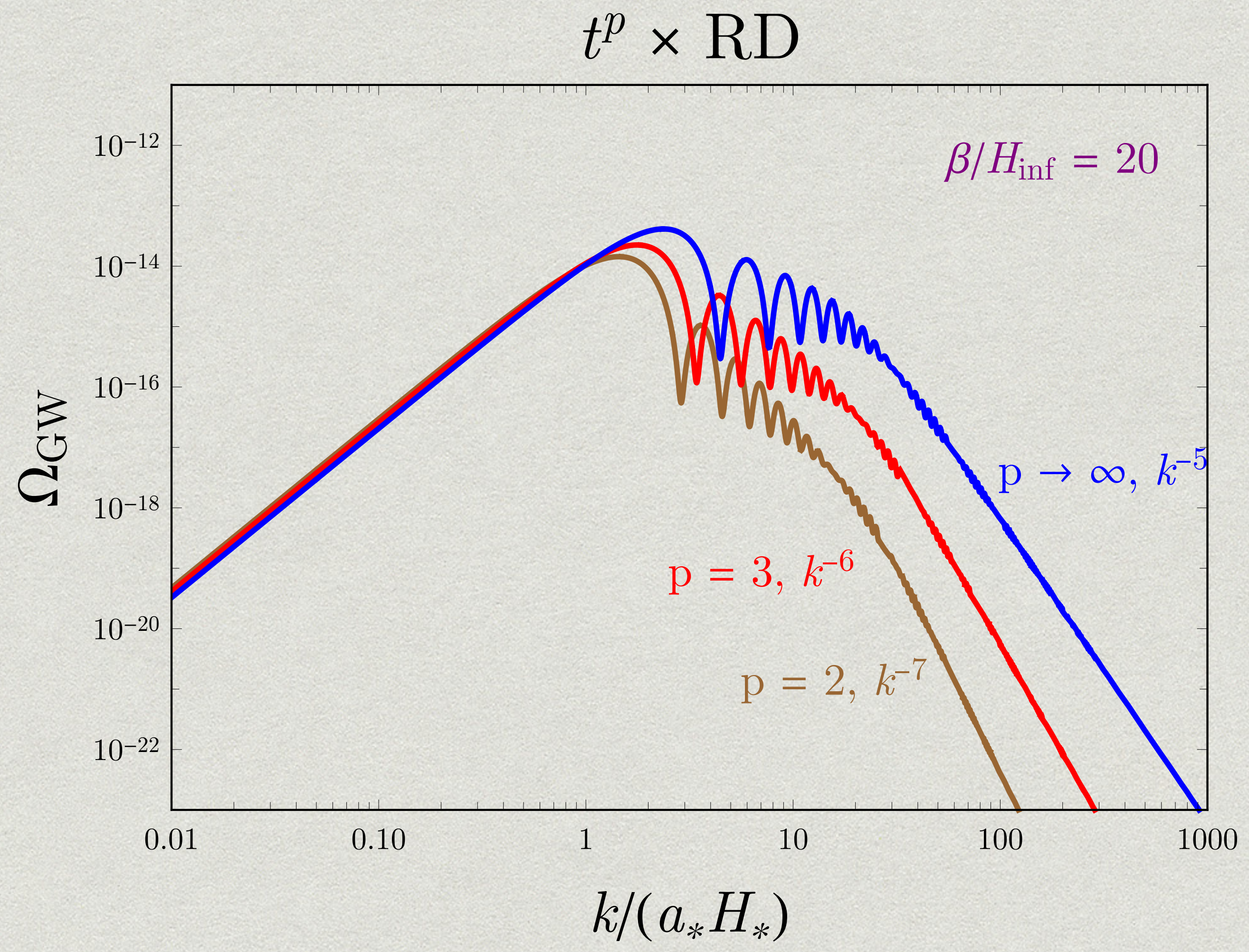




Scenarios after reheating.

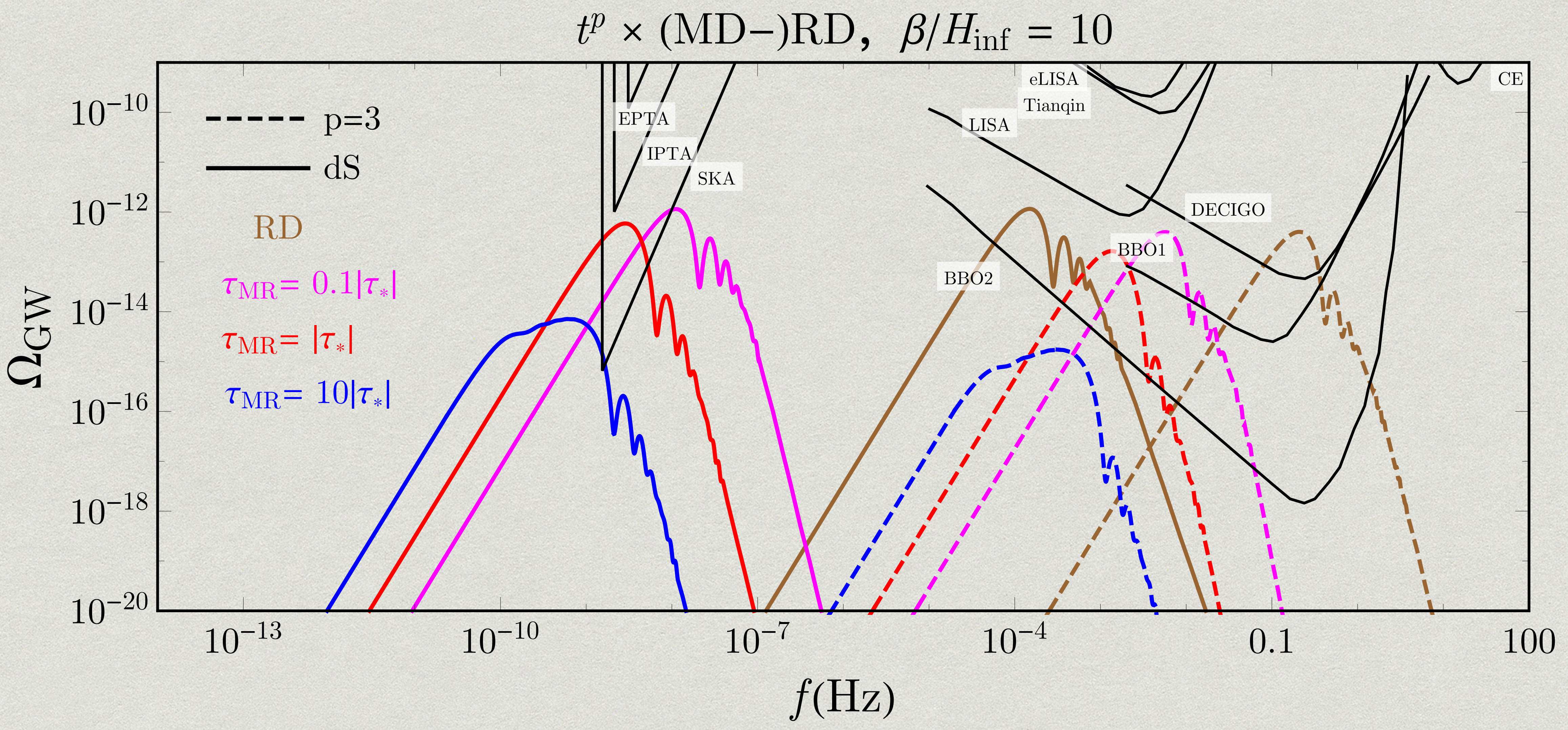
$\tau_{\rm MR}$ = MD-RD transition

Comparing scenarios



Different inflationary scenarios.
→ different slope in UV part.

Haipeng An, KunFeng Lyu, Siyi Zhou, and LTW 2009.12381, 2201.05171



$$\Omega_{\rm GW}^{\rm max} \sim \Omega_R \times \left(\frac{\Delta\rho_{\rm vac}}{\rho_{\rm inf\star}}\right)^2 \times \left(\frac{H_\star}{\beta}\right)^5 \tilde{\Delta} \times F(H_\star/H_r, a_\star/a_r, \cdots)$$

$$\approx 10^{-13} \times \left(\frac{\Delta\rho_{\rm vac}/\rho_{\rm inf\star}}{0.1}\right)^2 \times \left(\frac{H_\star/\beta}{0.1}\right)^5$$

Possible signal at cosmo collider

Matt Reece, Zhong-Zhi Xianyu, and LTW 2204.11869

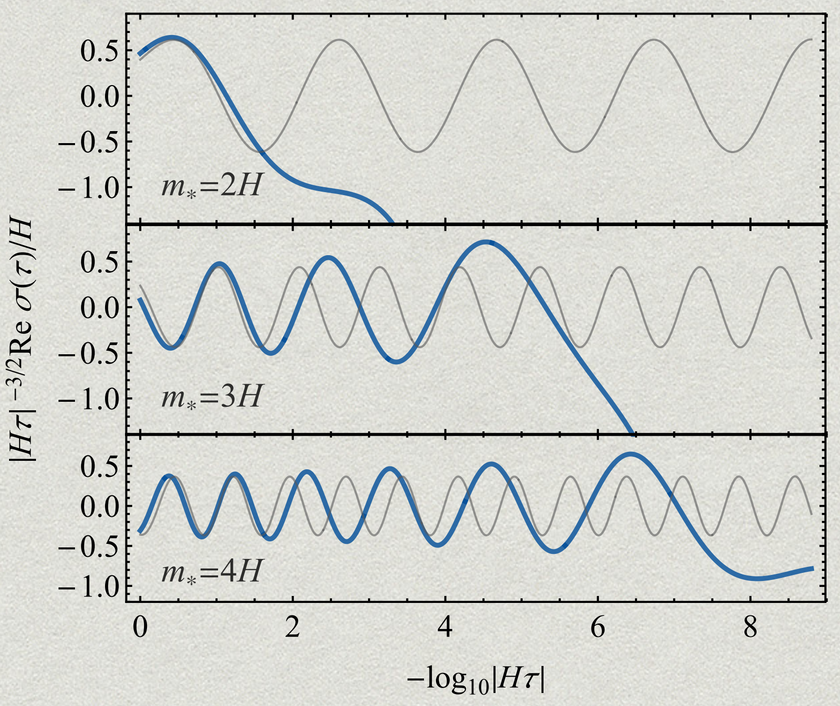
Time dependent mass at the cosmo collider

$$\mathscr{L} = -\frac{1}{2}(\partial_\mu \phi)^2 - V(\phi) - \frac{1}{2}(\partial_\mu \sigma)^2 - \frac{1}{2}e^{-2\alpha\phi/M_{\rm Pl}} m^2 \sigma^2 - \frac{1}{6}\lambda_3 \sigma^3 + \frac{1}{2}\lambda_5 (\partial_\mu \phi)^2 \sigma$$

$$m_{\rm eff}(t) = m e^{-\alpha\sqrt{2\epsilon}Ht}$$

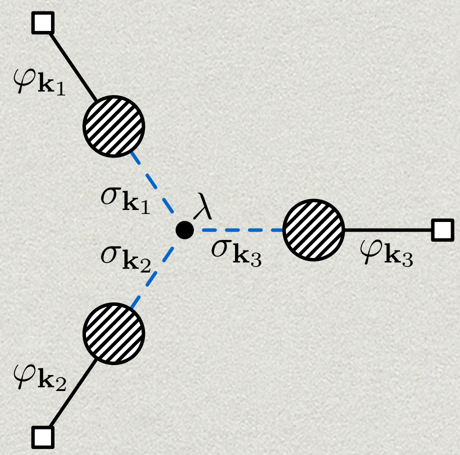
Mode function

σ



Affect oscillation pattern in cosmo collider signal

More specifically

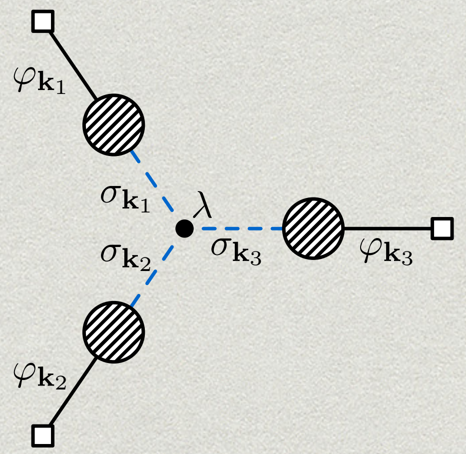


$$\mathcal{S}_{\rm signal} \simeq \frac{\lambda_3 \mu^3}{\nu^{5/2} H^4} e^{-\pi\nu} \sqrt{\frac{k_3}{k_1}} \sin\left(\nu \log \frac{k_3}{k_1} + \vartheta\right)$$

$$\nu = \sqrt{\frac{m^2}{H^2} - \frac{9}{4}}$$

For slow varying mass, still use this to give an estimate.

More specifically



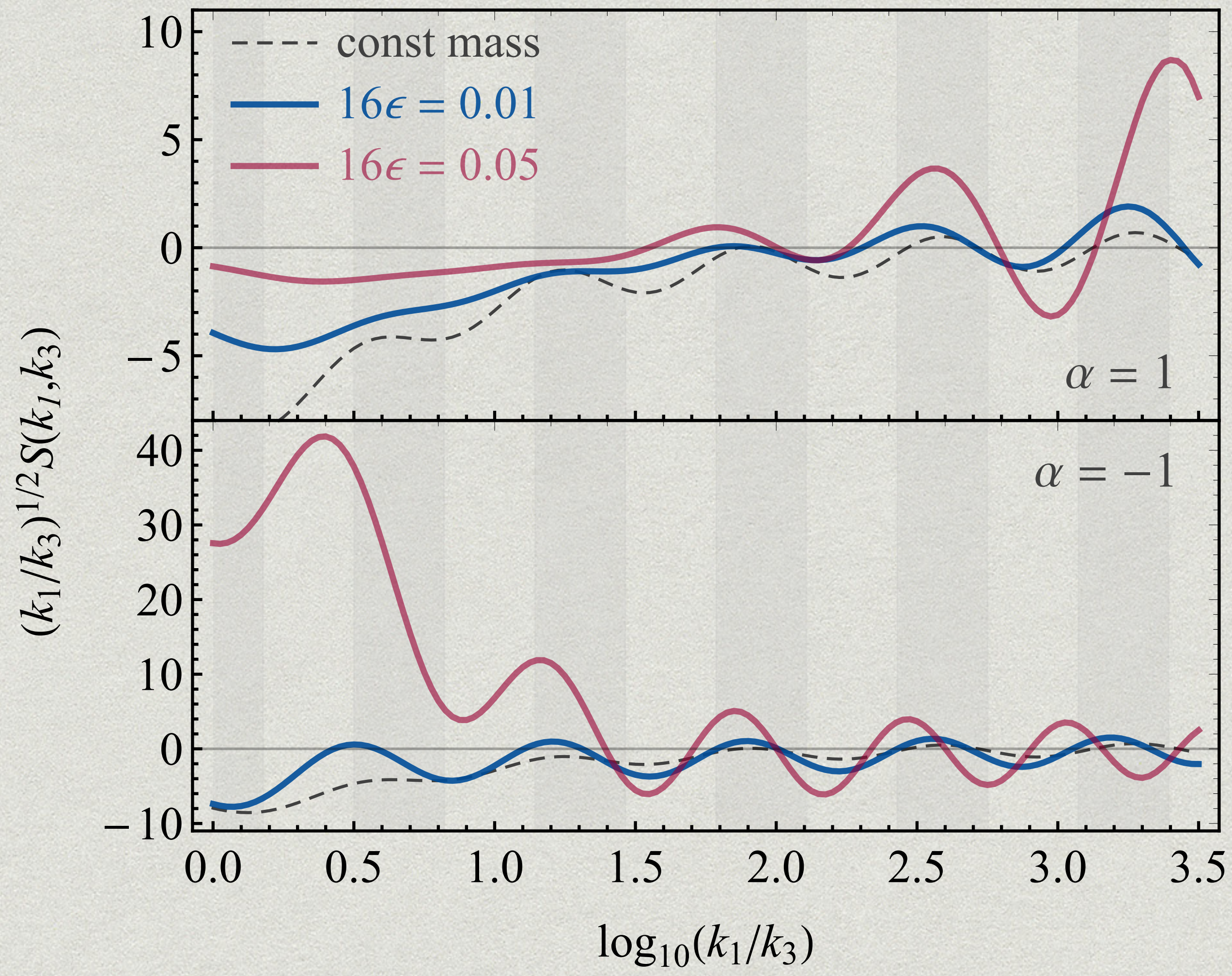
$$\mathcal{S}_{\text{signal}} \simeq \frac{\lambda_3 \mu^3}{\nu^{5/2} H^4} e^{-\pi\nu} \sqrt{\frac{k_3}{k_1}} \sin\left(\nu \log \frac{k_3}{k_1} + \vartheta\right)$$

$$\nu = \sqrt{\frac{m^2}{H^2} - \frac{9}{4}}$$

For slow varying mass, still use this to give an estimate.

Time variation affect both amplitude and frequency.

Signal shape



$$m_{\text{eff}}(t) = m e^{-\alpha\sqrt{2\epsilon}Ht}$$

Conclusions

- Cosmological observations can reveal new dynamics in the inflationary era.
- Potentially large inflaton excursion can trigger new dynamics in a spectator sector.
  - Can trigger 1st order phase transition → GW.
    - GW Can probe an era invisible from other observables, such as CMB/LSS or BBN.
  - Frequency and amplitude modulation in cosmo collider signal

Extra

Guth and Weinberg, 83'

$$\frac{\Gamma}{V} = e^{3Ht} C m_\sigma^4 e^{-S_4}$$

V: co-moving volume

$$R(t,t') = \frac{1}{H}(e^{-Ht'} - e^{-Ht})$$

Co-moving radius for bubble nucleated at t'

$$\mathcal{P}(t) = \exp\left[-\int_{-\infty}^{t} dt' \frac{4\pi}{3H^3} \times (e^{-Ht'} - e^{-Ht})^3 e^{3Ht'} C m_\sigma^4 e^{-S_4(t')}\right]$$

Fraction of space in false vacuum

For true vacuum to occupy an O(1) fraction:

$$\int_{-\infty}^{t} dt' \frac{4\pi}{3H^3}(e^{-Ht'} - e^{-Ht})^3 e^{3Ht'} C m_\sigma^4 e^{-S_4(t')} \sim \mathcal{O}(1)$$

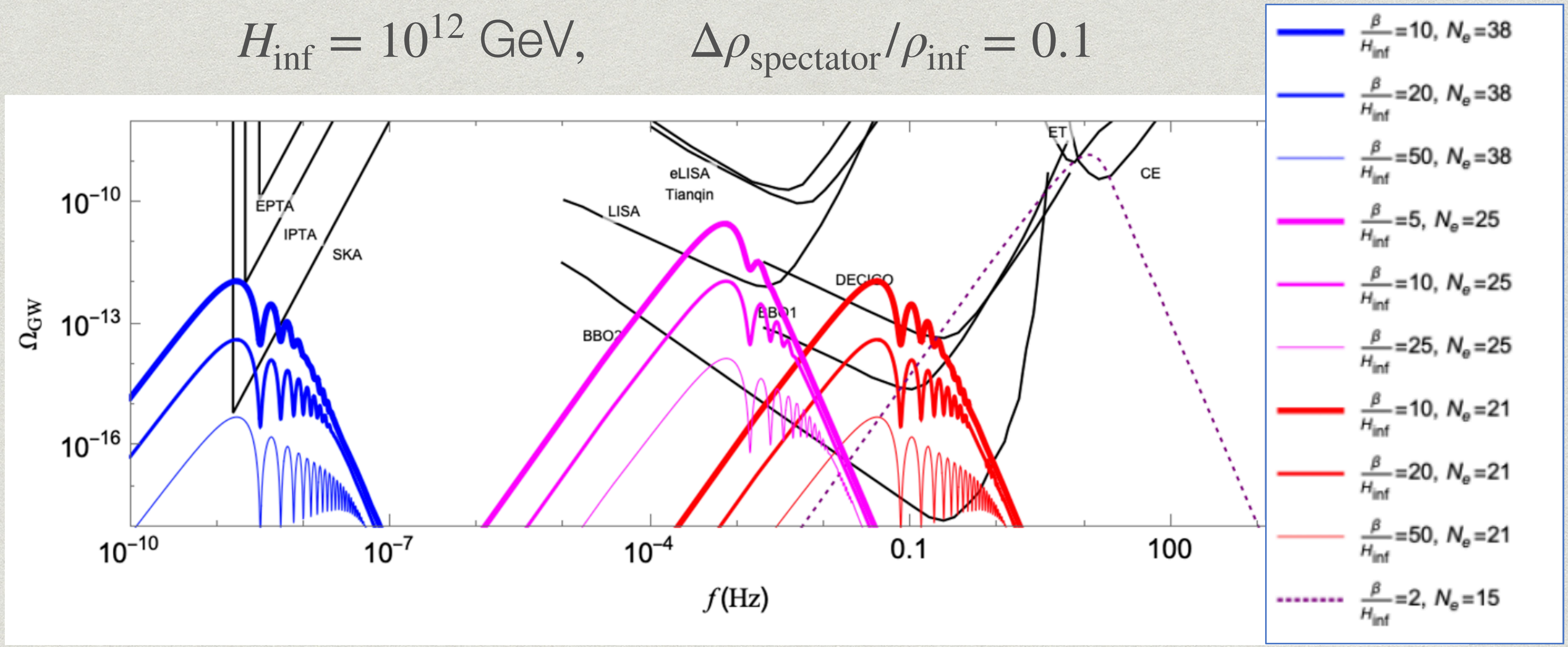
$$S_4(t') = S_4(t) + \frac{dS_4(t)}{dt}(t'-t) \equiv S_4(t) - \beta(t'-t)$$

$$\begin{aligned}\mathcal{O}(1) &\sim C m_\sigma^4 e^{-S_4(t)} \frac{4\pi}{H^3} \int_{-\infty}^{t} dt' \left(1 - e^{-H(t-t')}\right)^3 e^{-\beta(t-t')} \\ &\approx C m_\sigma^4 e^{-S_4(t)} \frac{8\pi}{\beta(\beta+H)(\beta+2H)(\beta+3H)} \\ &\approx 8\pi C e^{-S_4(t)} \frac{m_\sigma^4}{\beta^4}\,, \qquad (25)\end{aligned}$$

$$\Rightarrow \quad S_4(t_0) \approx \log\left(\frac{m_\sigma^4}{\beta^4}\right)$$

Observing the signal

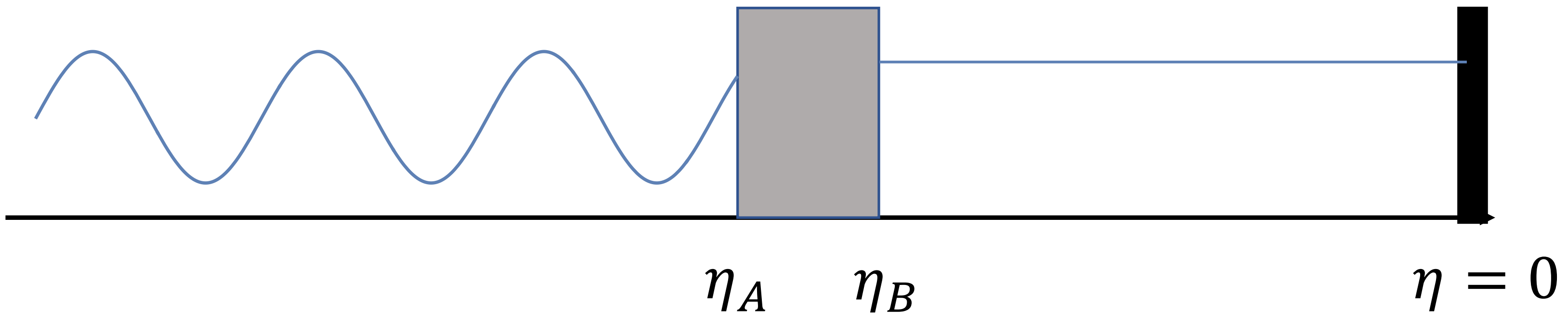
$H_{\text{inf}} = 10^{12}$ GeV, $\Delta\rho_{\text{spectator}}/\rho_{\text{inf}} = 0.1$



N_e: efold till the end of inflation = time of the phase transition

h^f in a generic inflation model

- Generic features
  - For $|k\tau'| > \eta_A$, $h^f = k^{-1}\cos k(\tau' - \tau_0')\tilde{\mathcal{G}}_0^f(k)$
  - For $|k\tau'| < \eta_B$, $h^f = \left[a(\tau')\int_{\tau'}^{0} a^{-2}(\tau_1)d\tau_1\right]$



Generic features of GW spectrum

- $k_p \ll \Delta_p^{-1}$ $\qquad \cos k_p t_p \to 1\ ,\ \ \sin k_p t_p \to 0$

$$\rho_{\mathrm{GW}}(\tau) = \int \frac{d^3k}{(2\pi)^3} \frac{8\pi G_N \left[\tilde{\mathcal{E}}_0^i(k)\tilde{\mathcal{G}}_0^f(k)\right]^2}{Va^4(\tau)a^2(\tau_\star)} \cos^2 k(\tau_\star - \tau_0)\tilde{T}_{ij}(0,\mathbf{k}_p)\tilde{T}_{ij}^*(0,\mathbf{k}_p)$$

$$\tilde{T}_{ij}(0,\mathbf{k}_p) = \int dt_p \tilde{T}_{ij}(\tau,\mathbf{k}_p)$$

$\langle \tilde{T}_{ij}\tilde{T}_{ij}^*\rangle_{k_p \ll \Delta_p^{-1}}$ independent of *k*. *Cai, Pi and Sasaki, 1909.13728*

- $k\Delta \ll 1 \ll |k\tau_\star|$, an oscillating feature in the GW spectrum

$$\frac{d\rho_{\mathrm{GW}}}{d\log k} = \frac{4G_N|\tilde{T}_{ij}(0,0)|^2}{\pi^2 Va^4(\tau)a^2(\tau_\star)} \left\{\left[\tilde{\mathcal{E}}_0^i(k)\tilde{\mathcal{G}}_0^f(k)\right]^2 k^3 \cos^2 k(\tau_\star - \tau_0)\right\}$$

Generic features of GW spectrum

- The UV part of the spectrum
  - $k_p \Delta_p \gg 1$, the oscillation pattern is completely smeared.

$$\frac{d\rho_{\rm GW}^{\rm UV}}{d\log k} = \frac{2G_N |\tilde{T}_{ij}(k_p, \mathbf{k}_p)|^2}{\pi^2 V a^4(\tau) a^2(\tau_\star)} \left\{ \left[\tilde{\mathcal{E}}_0^i(k) \tilde{\mathcal{G}}_0^f(k) \right]^2 k^3 \right\}$$

- The IR part of the spectrum $(\eta' > \eta_B, \text{ or } |\eta'| < |\eta_B|)$
  - $\tilde{\mathcal{G}}^f$ is flat, no oscillation parttern in the spectrum either,

$$\frac{d\rho_{\rm GW}^{\rm IR}}{d\log k} = \frac{4G_N |\tilde{T}_{ij}(0,0)|^2}{\pi^2 V a^4(\tau)} \left[\int_{\tau_\star}^0 a^{-2}(\tau_1) d\tau_1 \right]^2 \left\{ \left[\tilde{\mathcal{E}}_0^i(k) \right]^2 k^5 \right\}$$

First order phase transition during inflation

- Assume quasi-dS inflation, RD re-entering and fast reheating

$$\Omega_{\rm GW}(k_{\rm today}) = \Omega_R \frac{H_{\rm inf}^4}{k_p^4}\left[\frac{1}{2} + \mathcal{S}(k_p\beta^{-1})\cos\left(\frac{2k_p}{H_{\rm inf}}\right)\right]\frac{\Delta\rho_{\rm vac}}{\rho_{\rm inf}}\frac{d\rho_{\rm GW}^{\rm flat}}{\Delta\rho_{\rm vac}\, d\log k_p}$$

$\frac{H_{\rm inf}^4}{k_p^4}$ → Dilution factor

$\mathcal{S}$ → Smearing

$\frac{\Delta\rho_{\rm vac}}{\rho_{\rm inf}}$ → Suppressed by the energy faction

Redshift

$$\frac{f_{\rm today}}{f_\star} = \frac{a(\tau_\star)}{a_1}\left(\frac{g_{*S}^{(0)}}{g_{*S}^{(R)}}\right)^{1/3}\frac{T_{\rm CMB}}{\left[\left(\frac{30}{g_*^{(R)}\pi^2}\right)\left(\frac{3H_{\rm inf}^2}{8\pi G_N}\right)\right]^{1/4}}$$

$\frac{a(\tau_\star)}{a_1}$ → e^{-N_e}

N_e: e-folds before the end of inflation

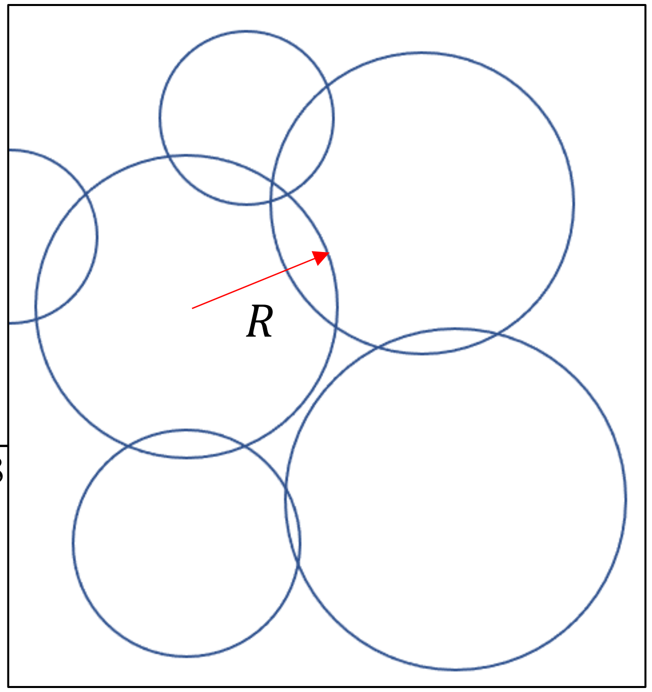
First order phase transition during inflation

- For phase transition to finish

$$R = \Delta_p \ll H_{\rm inf}^{-1}$$

$$\beta = \frac{dS_4}{dt} \sim \Delta_p^{-1} \gg H_{\rm inf}$$

$$\frac{d\rho_{\rm GW}^{flat}}{\Delta\rho_{\rm vac} d\log k_p} \approx \left(\frac{H_{\rm inf}}{\beta}\right)^2 \times \frac{\beta\ k_p^{2.8}}{\beta^{3.8} + 2.8 k_p^{3.8}}$$



Huber and Konstandin, 0806.1828

Examples

- Inflation models
  - Quasi-de Sitter inflation $\tilde{\mathcal{G}}_0^f = \left(-\frac{H}{k}\right) , \quad \eta_0' = 0$
  - t^p inflation $\tilde{\mathcal{G}}_0^f = a_0^{-1}(-k\tau_0)^{\frac{p}{1-p}} \frac{2^{\frac{p}{-1+p}}}{\sqrt{\pi}} \Gamma\left(\frac{3}{2} + \frac{1}{-1+p}\right) , \quad \eta_0' = \frac{\pi}{2-2p}$

  In t^p inflation, we have the slow-roll parameter $\epsilon = -\frac{\dot{H}}{H^2} = \frac{1}{p}$

$$\tilde{\mathcal{G}}_0^f \sim k^{-\frac{1}{1-\epsilon}}$$

- Evolution after inflation
  - In RD, $\tilde{\mathcal{E}}_0^i \sim k^{-1}$
  - In MD, $\tilde{\mathcal{E}}_0^i \sim k^{-2}$