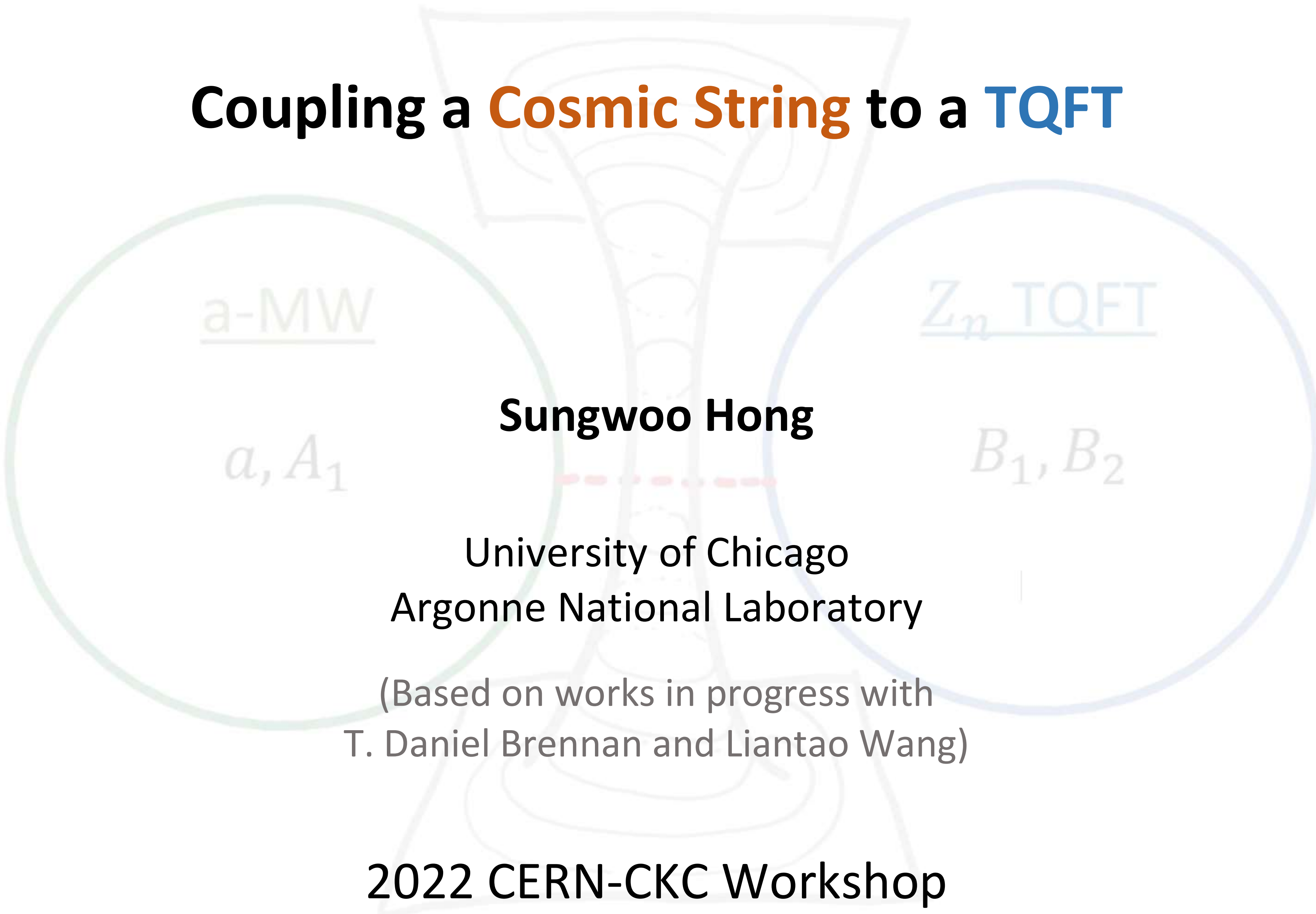
Coupling a Cosmic String to a TQFT



Sungwoo Hong

University of Chicago
Argonne National Laboratory

(Based on works in progress with
T. Daniel Brennan and Liantao Wang)

2022 CERN-CKC Workshop

Symmetry

- **Symmetry** plays the central role in theoretical physics!
  - SM: $SU(3)_C \times SU(2)_L \times U(1)_Y/\Gamma$
  - ChPT: $SU(3)_L \times S(3)_R \times U(1)_B/SU(3)_V$

- **Symmetry** plays the central role in theoretical physics!
  - SM: $SU(3)_C \times SU(2)_L \times U(1)_Y/\Gamma$
  - ChPT: $SU(3)_L \times S(3)_R \times U(1)_B/SU(3)_V$
- **Global Symmetry** can be broken, anomalous, gauged
  - `t Hooft anomaly matching
    - $\mathcal{L}_{ChPT} \to \mathcal{L}_{ChPT} + \boldsymbol{\mathcal{L}_{WZW}}$

- **Symmetry** plays the central role in theoretical physics!
  - SM: $SU(3)_C \times SU(2)_L \times U(1)_Y/\Gamma$
  - ChPT: $SU(3)_L \times S(3)_R \times U(1)_B/SU(3)_V$
- **Global Symmetry** can be broken, anomalous, gauged
  - `t Hooft anomaly matching
    - $\mathcal{L}_{ChPT} \to \mathcal{L}_{ChPT} + \boldsymbol{\mathcal{L}_{WZW}}$
      - CP-violation: $\pi\pi\pi \to KK$
      - $\pi^0 \to \gamma\gamma$
      - Skyrmion statistics
      - Many others

✹ Recently, notion/concept of **Symmetry** has gone through explosive **generalizations**!

○ **Ordinary Symmetry** → **Higher-form Symmetry**

Ordinary Symmetry		Higher-form Symmetry
0-form	→	1-form, 2-form, …
particle or local operator		extended object: line, surface, …

✹ Recently, notion/concept of **Symmetry** has gone through explosive **generalizations**!

- **Ordinary Symmetry** → **Higher-form Symmetry**

  0-form → 1-form, 2-form, …

  particle or local operator → extended object: line, surface, …

- **Group** → **Higher-Group**, **Non-Invertible Symmetries**

✹ Recently, notion/concept of **Symmetry** has gone through explosive **generalizations**!

- **Ordinary Symmetry** → **Higher-form Symmetry**
  0-form → 1-form, 2-form, …
  particle or local operator → extended object: line, surface, …
- **Group** → **Higher-Group**, **Non-Invertible Symmetries**
- **Conserved in entire Spacetime** → **Subsystem Symmetries** (Fracton)
- More…

- Generalized Global Symmetries (GGS) have shown to be extremely powerful in deepening our understanding of QFT

  - Aharony, Seiberg, Tachikawa '13
  - Kapustin, Seiberg '14
  - Gaiotto, Kapustin, Seiberg, Willett '14
  - Gaiotto, Kapustin, Komargodski, Seiberg '17
  - Anber, Poppitz '18
  - Cordova, Dumitrescu '18
  - Cordova, Dumitrescu, Intriligator '18
  - Benini, Cordova, Po-Shen-Hsin '18
  - Cordova, Ohmori '19
  - …. Anber, **Hong**, Son '21 ….
  - Kaidi, Ohmori, Zheng '21
  - Choi, Cordova, Po-Shen Hsin, Ho Tat Lam, Shu-Heng Shao '21
  - Many many more

(Q1) Are there **generalized symmetries** in **(3+1)d QFTs** that relevant for **particle physics**?

(Q2) Can there be **observable signals** (even in principle) associated with (due to) the presence of those **generalized symmetries**?

(Q3) Can **generalized symmetry** provide **novel or meaningful solutions** to problems in **particle physics**?

Today's Talk

(Q1) Are there **generalized symmetries** in **(3+1)d QFTs** that relevant for **particle physics**?

(Q2) Can there be **observable signals** (even in principle) associated with (due to) the presence of those **generalized symmetries**?

(Q3) Can **generalized symmetry** provide **novel or meaningful solutions** to problems in **particle physics**?

Today's Talk

(A1) A very familiar theory:
Axion-Maxwell Theory and its **Topological Modifications**

(Q2) Can there be **observable signals** (even in principle) associated with (due to) the presence of those **generalized symmetries**?

(Q3) Can **generalized symmetry** provide **novel or meaningful solutions** to problems in **particle physics**?

Today's Talk

(A1) A very familiar theory:

Axion-Maxwell Theory and its **Topological Modifications**

(A2) Coupling **Axion-MW** to a Z_n **TQFT**

case 1) bulk (4d) Chern-Simons Coupling to Z_n **TQFT**

↓

Modifications of **local** but **2d string-worldsheet QFT**

case 2) Gauging a discrete subgroup of axion shift (0-form)

↓

Modification of **cosmic string spectrum**

Today's Talk

(A1) A very familiar theory:

Axion-Maxwell Theory and its **Topological Modifications**

(A2) Coupling **Axion-MW** to a Z_n **TQFT**

case 1) bulk (4d) Chern-Simons Coupling to Z_n **TQFT**

↓

Modifications of **local** but **2d string-worldsheet QFT**

case 2) Gauging a discrete subgroup of axion shift (0-form)

↓

Modification of **cosmic string spectrum**

Outline

I. Axion-Maxwell Theory

Generalized Symmetries (0-, 1-, 2-form → 3-group)

II. Coupling **Axion-MW** to a Z_n **TQFT**

III. IR-Universal Observables from **TQFT-Coupling**

Anomaly-Inflow, Fermion Zero Modes

IV. UV-Completion from **Standard QFT**

Extended KSVZ Construction

Outline

I. Axion-Maxwell Theory

Generalized Symmetries (0-, 1-, 2-form → 3-group)

II. Coupling **Axion-MW** to a Z_n **TQFT**

III. IR-Universal Observables from **TQFT-Coupling**

Anomaly-Inflow, Fermion Zero Modes

IV. UV-Completion from **Standard QFT**

Extended KSVZ Construction

I. Axion-Maxwell Theory

$$S = \int \frac{1}{2} \left(\partial_\mu a\right)^2 + \int \frac{1}{2g^2} F_{\mu\nu}F^{\mu\nu} - \int \frac{iK}{16\pi^2}\frac{a}{f} F_{\mu\nu}\tilde{F}^{\mu\nu}$$

cf) $F_{\mu\nu}\tilde{F}^{\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu}F_{\rho\sigma}$

I. Axion-Maxwell Theory

$$S = \int \frac{1}{2} \left(\partial_\mu a\right)^2 + \int \frac{1}{2g^2} F_{\mu\nu}F^{\mu\nu} - \int \frac{iK}{16\pi^2} \frac{a}{f} F_{\mu\nu}\tilde{F}^{\mu\nu}$$

$$\downarrow$$

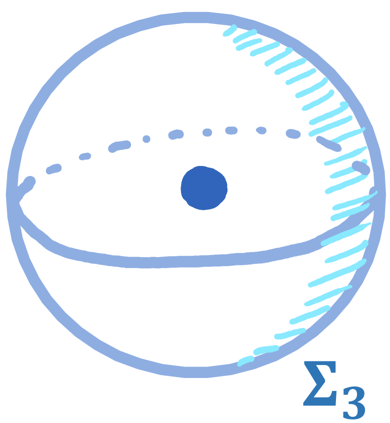
$$S = \int \frac{1}{2} da \wedge * \, da + \int \frac{1}{2g^2} F \wedge * \, F - \int \frac{iK}{8\pi^2} \frac{a}{f} F \wedge F$$

cf) $F_{\mu\nu}\tilde{F}^{\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu}F_{\rho\sigma}$

I. Axion-Maxwell Theory

$$S = \int \frac{1}{2}\, da \wedge * \, da \;+ \int \frac{1}{2g^2}\, F \wedge * \, F \;- \int \frac{iK}{8\pi^2}\frac{a}{f} F \wedge F$$

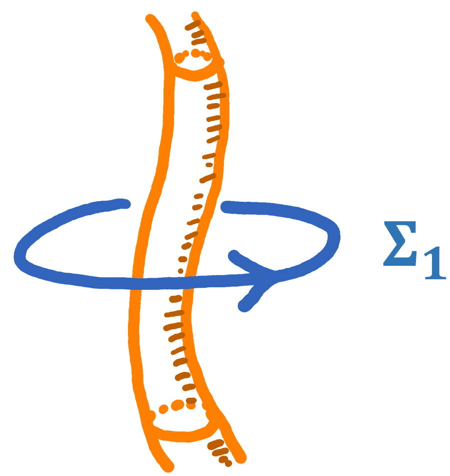
- This very familiar theory enjoys a large set of GGS:
  - 0-form axion shift
    - $\theta = \frac{a}{f} \rightarrow \theta + c$
    - $* J_1 = if * da - \frac{K}{8\pi^2} A_1 \wedge F \rightarrow [U(1)^{(0)} \rightarrow Z_K{}^{(0)}]$



I. Axion-Maxwell Theory

$$S = \int \frac{1}{2}\, da \wedge * \, da \; + \int \frac{1}{2g^2}\, F \wedge * \, F \; - \int \frac{iK}{8\pi^2}\frac{a}{f} F \wedge F$$

- This very familiar theory enjoys a large set of GGS:
  - 0-form axion shift
  - 2-form axion winding
    - $\oint_{\Sigma_1} d\theta = 2\pi m$
    - $* J_3 = \dfrac{1}{2\pi f} da \quad \rightarrow \quad [U(1)^{(2)}]$
    - Charged object: cosmic string

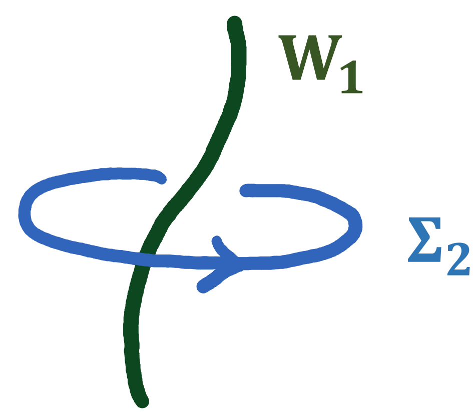


I. Axion-Maxwell Theory

$$S = \int \frac{1}{2}\, da \wedge * \, da \; + \int \frac{1}{2g^2}\, F \wedge * \, F \; - \int \frac{iK}{8\pi^2} \frac{a}{f} F \wedge F$$

- This very familiar theory enjoys a large set of GGS:
  - 0-form axion shift
  - 2-form axion winding
  - 1-form electric
    - $* J_2^E = \frac{i}{g^2} * F + \frac{K}{4\pi^2} da \wedge F \quad \rightarrow \quad [U(1)_E^{(1)} \rightarrow Z_K^{(1)}]$
    - Charged object: Wilson lines

$$W_1(\Sigma_1, m) = e^{im \oint A_1}$$

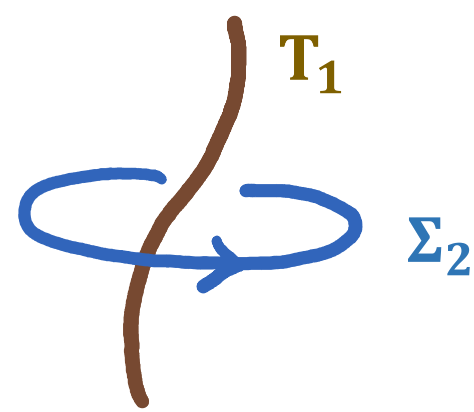


I. Axion-Maxwell Theory

$$S = \int \frac{1}{2}\, da \wedge * \, da \; + \int \frac{1}{2g^2}\, F \wedge * \, F \; - \int \frac{iK}{8\pi^2} \frac{a}{f} F \wedge F$$

- This very familiar theory enjoys a large set of GGS:
  - 0-form axion shift
  - 2-form axion winding
  - 1-form electric
  - 1-form magnetic
    - $* J_2^M = \frac{1}{2\pi} F \quad \rightarrow \quad [U(1)_M^{(1)}]$
    - Charged object: `t Hooft lines

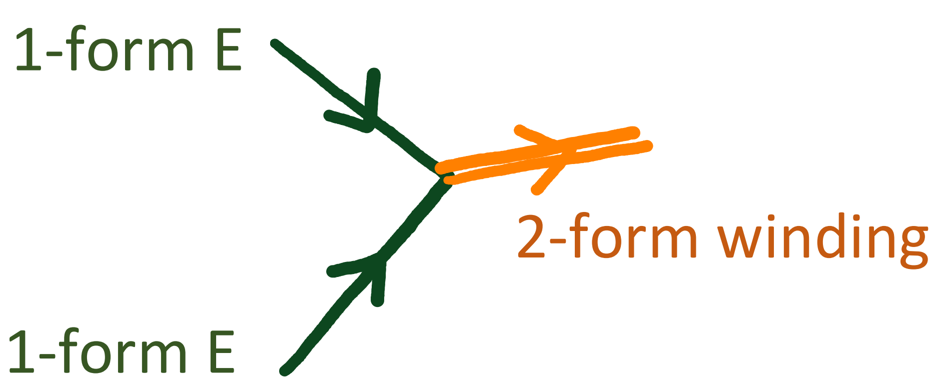
$$T_1(\Sigma_1, m) = e^{im \int_{\Sigma_2} * F}$$

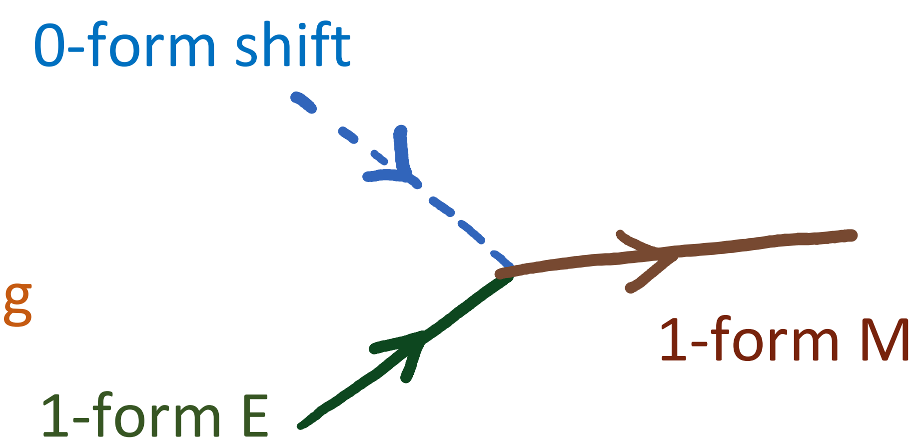


I. Axion-Maxwell Theory

$$S = \int \frac{1}{2}\, da \wedge * \, da \; + \int \frac{1}{2g^2}\, F \wedge * \, F \; - \int \frac{iK}{8\pi^2}\frac{a}{f} F \wedge F$$

- This very familiar theory enjoys a large set of GGS:
  - 0-form axion shift
  - 2-form axion winding
  - 1-form electric
  - 1-form magnetic
  - ✸ 3-group





I. Axion-Maxwell Theory

$$S = \int \frac{1}{2}\, da \wedge * \, da \; + \int \frac{1}{2g^2}\, F \wedge * \, F \; - \int \frac{iK}{8\pi^2} \frac{a}{f} F \wedge F$$

- This very familiar theory enjoys a large set of GGS:
  - 0-form axion shift
  - 2-form axion winding
  - 1-form electric
  - 1-form magnetic
  - ✸ 3-group
  - ✸ Non-invertible symmetries

$$\mathcal{D}_k = \mathcal{C}_k \times \mathcal{A}^{N,p}\left(\frac{f}{2\pi}\right)$$

Outline

I. Axion-Maxwell Theory

Generalized Symmetries (0-, 1-, 2-form → 3-group)

II. Coupling **Axion-MW** to a Z_n **TQFT**

III. IR-Universal Observables from **TQFT-Coupling**

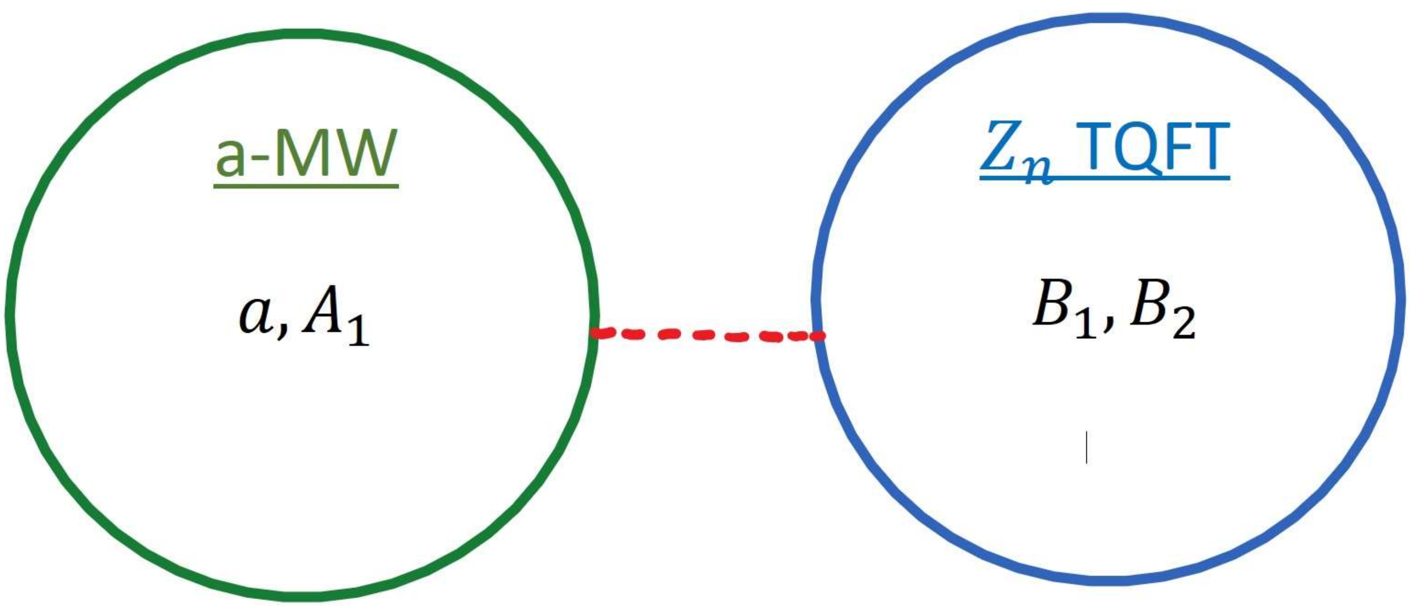
Anomaly-Inflow, Fermion Zero Modes

IV. UV-Completion from **Standard QFT**

Extended KSVZ Construction

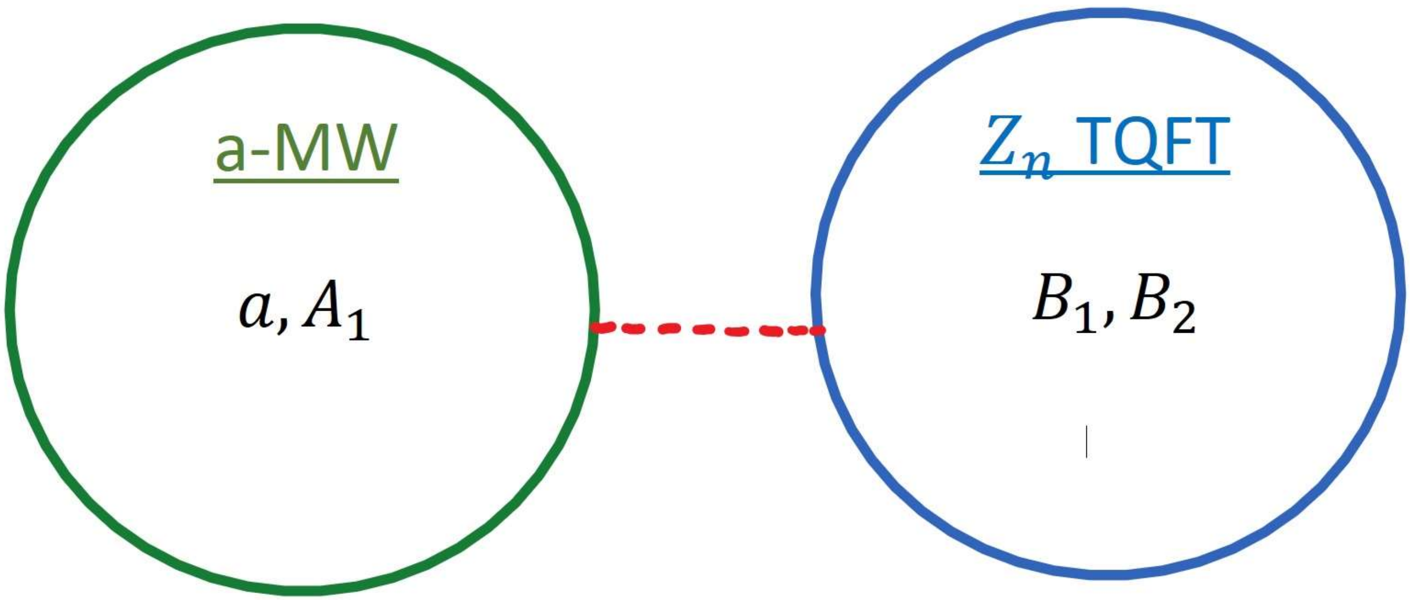
II. Coupling **Axion-MW** to a $\mathbb{Z}_n$ **TQFT**

$$S = \int \frac{1}{2}\, da \wedge * \, da \; + \int \frac{1}{2g^2}\, F \wedge * \, F \; - \int \frac{iK}{8\pi^2}\frac{a}{f} F \wedge F$$



II. Coupling **Axion-MW** to a $\mathbb{Z}_n$ **TQFT**

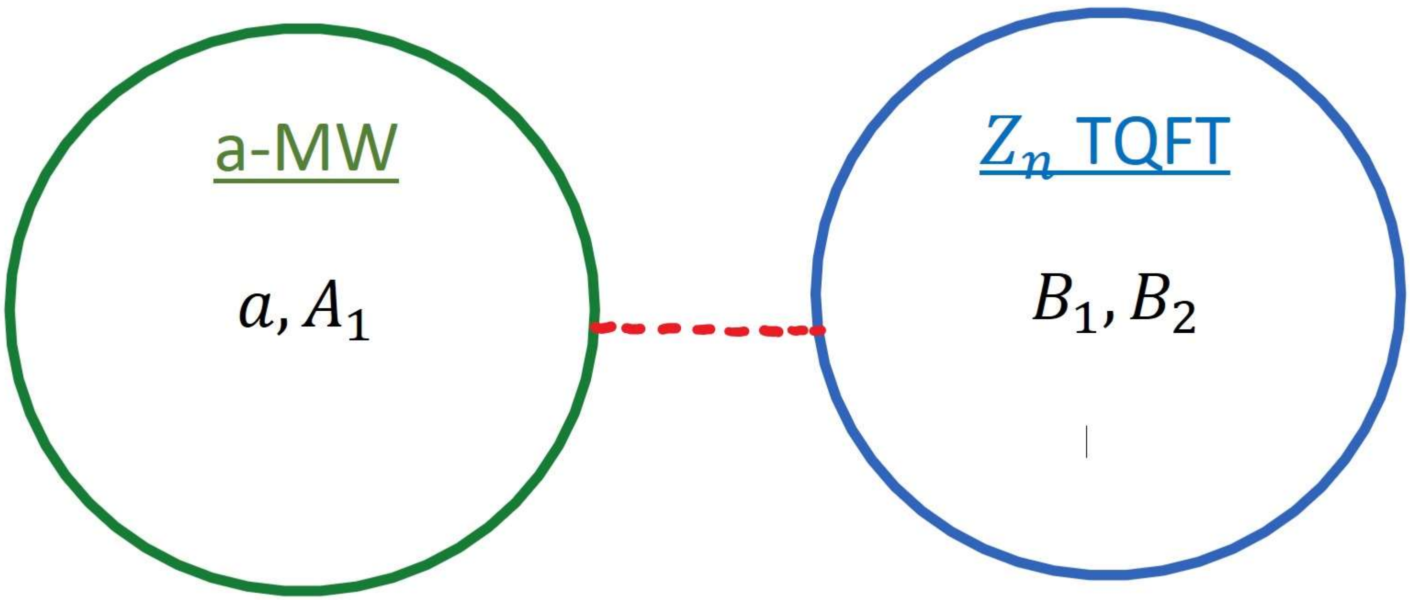
$$S = \int \frac{1}{2}\, da \wedge {*}\, da \; + \int \frac{1}{2 g_A^2}\, F_A \wedge {*}\, F_A \; - \int \frac{i K_A}{8\pi^2} \frac{a}{f} F_A \wedge F_A$$



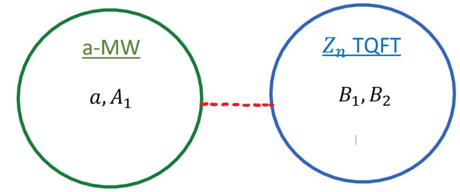
II. Coupling **Axion-MW** to a Z_n **TQFT**

$$S = \int \frac{1}{2}\, da \wedge^* da \;+ \int \frac{1}{2g_A^2}\, F_A \wedge^* F_A \;- \int \frac{iK_A}{8\pi^2}\frac{a}{f} F_A \wedge F_A$$

$$+ \int \frac{in}{2\pi}\, B_2 \wedge dB_1 \;- \int \frac{iK_{AB}}{8\pi^2}\frac{a}{f} F_A \wedge F_B - \int \frac{iK_B}{8\pi^2}\frac{a}{f} F_B \wedge F_B$$



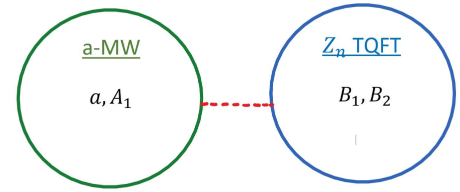
II. Z_n Gauge Theory (BF theory)



$$S = \int \frac{in}{2\pi} B_2 \wedge dB_1$$

(i) Describes discrete (Z_n) gauge theory

II. $\boldsymbol{Z_n}$ Gauge Theory (BF theory)



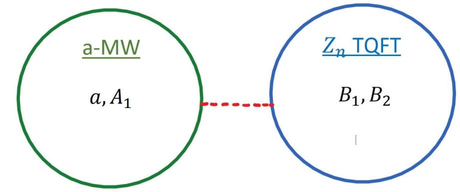
$$S = \int \frac{in}{2\pi}\, B_2 \wedge dB_1$$

(i) Describes discrete ($\boldsymbol{Z_n}$) gauge theory

(ii) Can be obtained from Abelian Higgs Model (AHM)

$$\mathcal{L} \sim \Lambda^2 (d\varphi - nB_1) \wedge * \, (d\varphi - nB_1) + \frac{1}{2g_B^2}\, F_B \wedge * \, F_B$$

II. $\mathbb{Z}_n$ Gauge Theory (BF theory)



$$S = \int \frac{in}{2\pi} B_2 \wedge dB_1$$

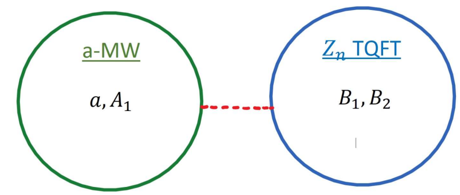
(i) Describes discrete ($\mathbb{Z}_n$) gauge theory

(ii) Can be obtained from Abelian Higgs Model (AHM)

$$\mathcal{L} \sim \Lambda^2 (d\varphi - nB_1) \wedge * (d\varphi - nB_1) + \frac{1}{2g_B^2} F_B \wedge * F_B$$

$$\mathcal{L} \sim \Lambda^2 (nB_1) \wedge * (d\varphi) \sim n\, B_1 \wedge d\, B_2 \qquad (dB_2 \sim * \, d\varphi)$$

II. Z_n Gauge Theory (BF theory)



$$S = \int \frac{in}{2\pi} B_2 \wedge dB_1$$

(i) Describes discrete (Z_n) gauge theory

(ii) Can be obtained from Abelian Higgs Model (AHM)

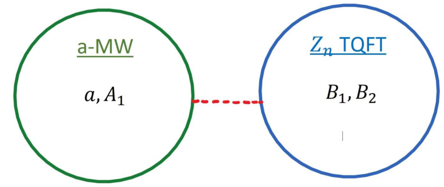
(iii) Describes non-trivial vacuum structure and selection rules in terms of topological degrees of freedom B_1 , B_2

$$dB_1 = 0, \quad dB_2 = 0$$

$$\langle W_1(\Sigma_1, m)\, W_2(\Sigma_2, \ell)\rangle \sim e^{i\frac{2\pi}{n} m\ell\, Link(\Sigma_1, \Sigma_2)}$$

$$W_1\,(\Sigma_1, m) = e^{im \oint_{\Sigma_1} B_1}\ , \quad W_2(\Sigma_2, \ell) = e^{i\ell \oint_{\Sigma_2} B_2}$$

II. Coupling **Axion-MW** to a $\boldsymbol{Z_n}$ **TQFT**



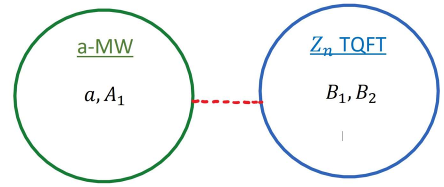
$$S = \int \frac{1}{2}\, da \wedge * \, da \; + \int \frac{1}{2g_A^2}\, F_A \wedge * \, F_A \; - \int \frac{iK_A}{8\pi^2} \frac{a}{f} F_A \wedge F_A$$

$$+ \int \frac{in}{2\pi}\, B_2 \wedge dB_1 \; - \int \frac{iK_{AB}}{8\pi^2} \frac{a}{f} F_A \wedge F_B - \int \frac{iK_B}{8\pi^2} \frac{a}{f} F_B \wedge F_B$$

(Q1) Can there be any **IR-Universal** (local) observable effect?

(Q2) Is this very exotic / pure academic setup?
Or can this arise as IR-EFT of some **standard UV QFT** relevant for particle physics?

II. Coupling **Axion-MW** to a $\boldsymbol{Z_n}$ **TQFT**



$$S = \int \frac{1}{2}\, da \wedge * \, da \; + \int \frac{1}{2g_A^2}\, F_A \wedge * \, F_A \; - \int \frac{iK_A}{8\pi^2} \frac{a}{f} F_A \wedge F_A$$

$$+ \int \frac{in}{2\pi}\, B_2 \wedge dB_1 \; - \int \frac{iK_{AB}}{8\pi^2} \frac{a}{f} F_A \wedge F_B - \int \frac{iK_B}{8\pi^2} \frac{a}{f} F_B \wedge F_B$$

(Q1) Can there be any **IR-Universal** (local) observable effect?

(Q2) Is this very exotic / pure academic setup?
Or can this arise as IR-EFT of some **standard UV QFT** relevant for particle physics?

✓ Illustrate importance of studying carefully the effects of remnant TQFT-couplings (GGS = essential tools)

Outline

I. Axion-Maxwell Theory

Generalized Symmetries (0-, 1-, 2-form → 3-group)

II. Coupling **Axion-MW** to a Z_n **TQFT**

III. IR-Universal Observables from **TQFT-Coupling**

Anomaly-Inflow, Fermion Zero Modes

IV. UV-Completion from **Standard QFT**

Extended KSVZ Construction

Outline

I. Axion-Maxwell Theory
Generalized Symmetries (0-, 1-, 2-form → 3-group)

II. Coupling **Axion-MW** to a Z_n **TQFT**

III. IR-Universal Observables from **TQFT-Coupling**
Anomaly-Inflow, Fermion Zero Modes

IV. UV-Completion from **Standard QFT**
Extended KSVZ Construction

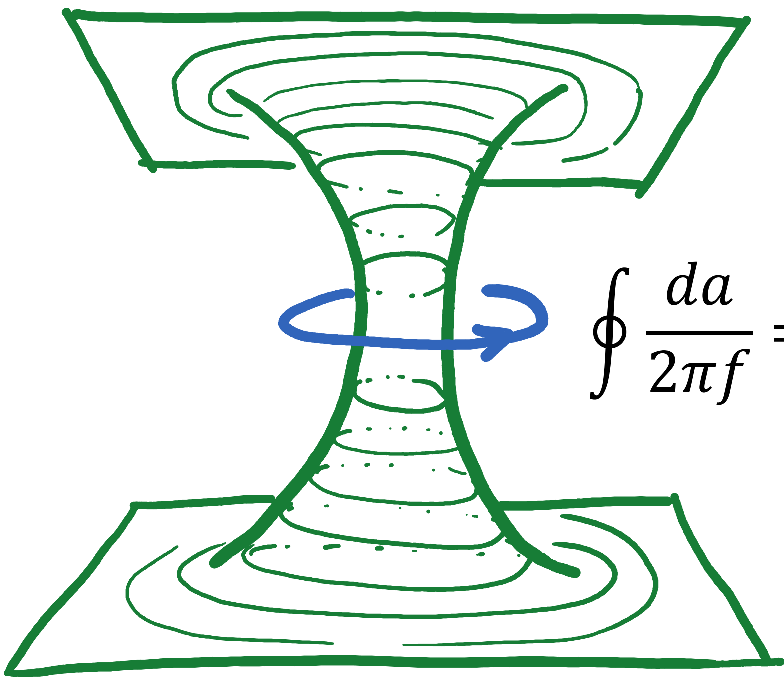
III. IR-Universal Observables from TQFT-Coupling

✸ **Anomaly Inflow**

III. IR-Universal Observables from TQFT-Coupling

✸ **Anomaly Inflow** : W/O TQFT-Coupling

$$S = \int \frac{1}{2}\, da \wedge {*}\, da + \int \frac{1}{2g_A^2}\, F_A \wedge {*}\, F_A - \int \frac{iK_A}{8\pi^2} \frac{a}{f} F_A \wedge F_A$$

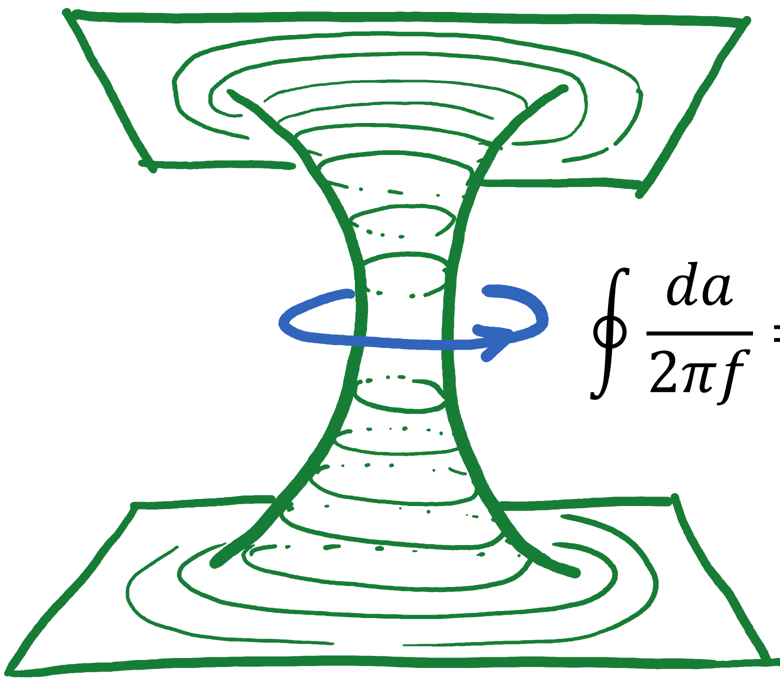


$$\oint \frac{da}{2\pi f} = 1$$

III. IR-Universal Observables from TQFT-Coupling

✸ **Anomaly Inflow** : W/O TQFT-Coupling

$$S = \int \frac{1}{2}\, da \wedge {}^{*}\, da \; + \int \frac{1}{2g_A^2}\, F_A \wedge {}^{*} F_A \; - \int \frac{iK_A}{8\pi^2} \frac{a}{f} F_A \wedge F_A$$



$$\oint \frac{da}{2\pi f} = 1$$

○ Consistency with $U(1)_A$ invariance :

$$A_1 \to A_1 + d\lambda$$

○ $S \supset \frac{iK_A}{8\pi^2} \int da \wedge A_1 \wedge F_A$

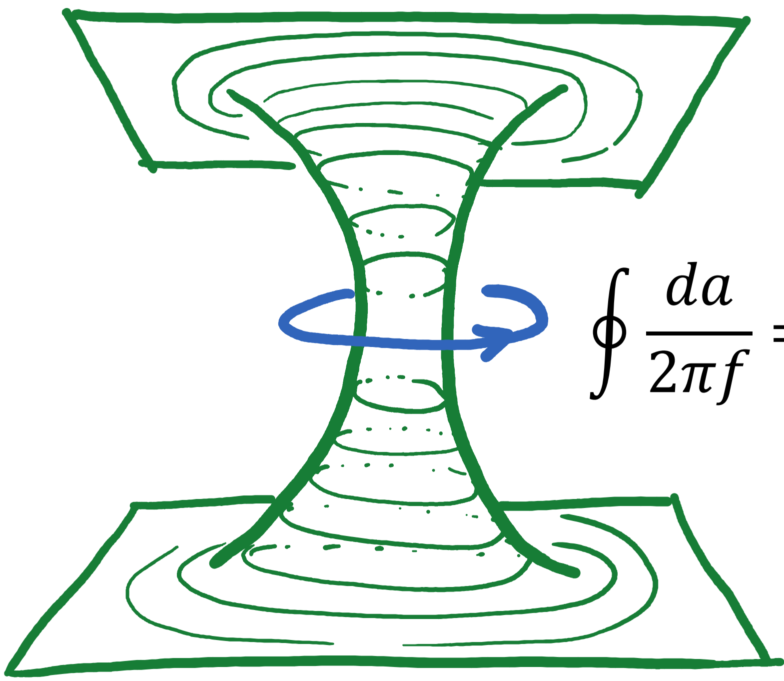
$$\downarrow$$

○ $\delta S = i \int \delta^{(2)}\left(M_2^{st}\right) \wedge \left(\lambda \frac{K_A}{4\pi} F_A\right)$

III. IR-Universal Observables from TQFT-Coupling

✸ Anomaly Inflow : W/O TQFT-Coupling

$$S = \int \frac{1}{2}\, da \wedge^* da \;+ \int \frac{1}{2g_A^2}\, F_A \wedge^* F_A \;- \int \frac{iK_A}{8\pi^2} \frac{a}{f} F_A \wedge F_A$$



$$\oint \frac{da}{2\pi f} = 1$$

$$\circ\ \delta S = i \int \delta^{(2)}\left(M_2^{st}\right) \wedge \left(\lambda \frac{K_A}{4\pi} F_A\right)$$

- Violation of $U(1)_A$ - inv localized on the string core

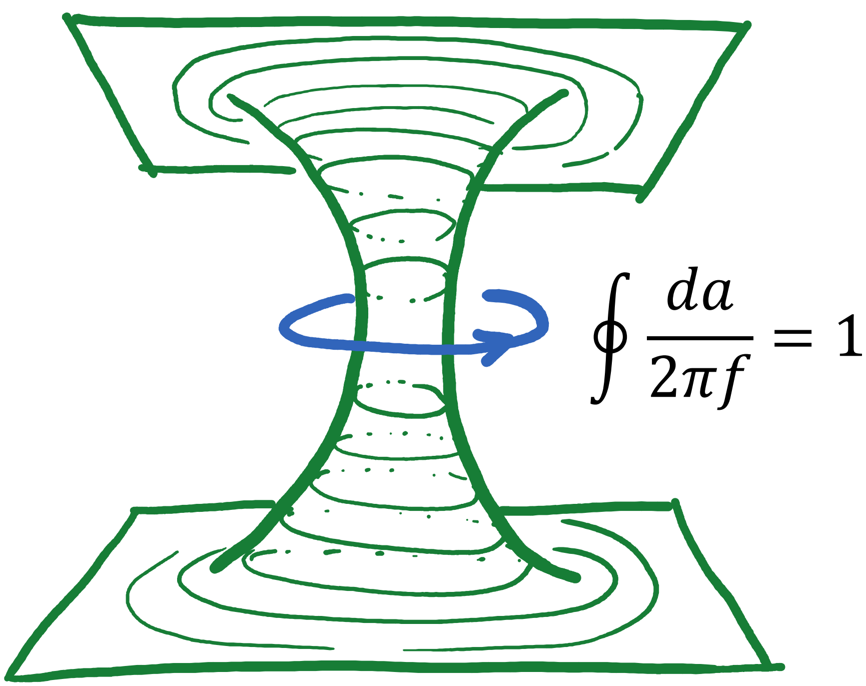
$$d * J_1 = \frac{K_A}{4\pi}\, F_A \wedge \delta^{(2)}\left(M_2^{st}\right)$$

- This 'gauge anomaly' must be cancelled by an anomalous 2d QFT on M_2^{st}
- Anomalous $TQFT_4$ in "bulk" + Anomalous QFT_2 on the "boundary" = 0

III. IR-Universal Observables from TQFT-Coupling

✸ **Anomaly Inflow** : W/O TQFT-Coupling

$$S \supset \frac{iK_A}{8\pi^2} \int da \wedge A_1 \wedge F_A = i \int A_1 \wedge * J_1$$



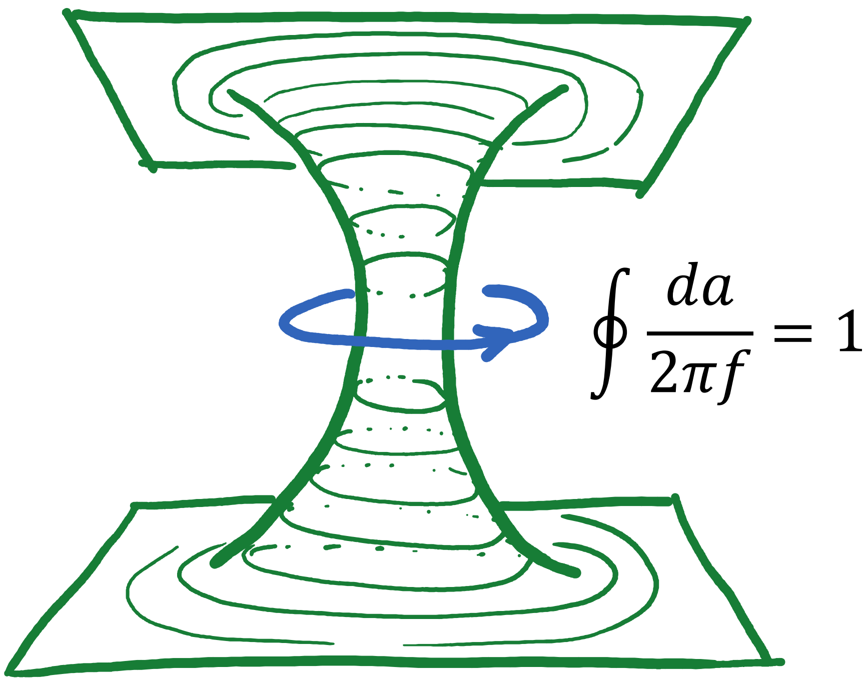
○ $* J_1 = \frac{K_A}{4\pi} da \wedge F_A$

○ $\mathrm{d} * J_1(bulk) = \frac{K_A}{4\pi} F_A \wedge \delta^{(2)}(M_2^{st})$

III. IR-Universal Observables from TQFT-Coupling

✸ **Anomaly Inflow** : W/O TQFT-Coupling

$$S \supset \frac{iK_A}{8\pi^2} \int da \wedge A_1 \wedge F_A \; = i \int A_1 \wedge * J_1$$

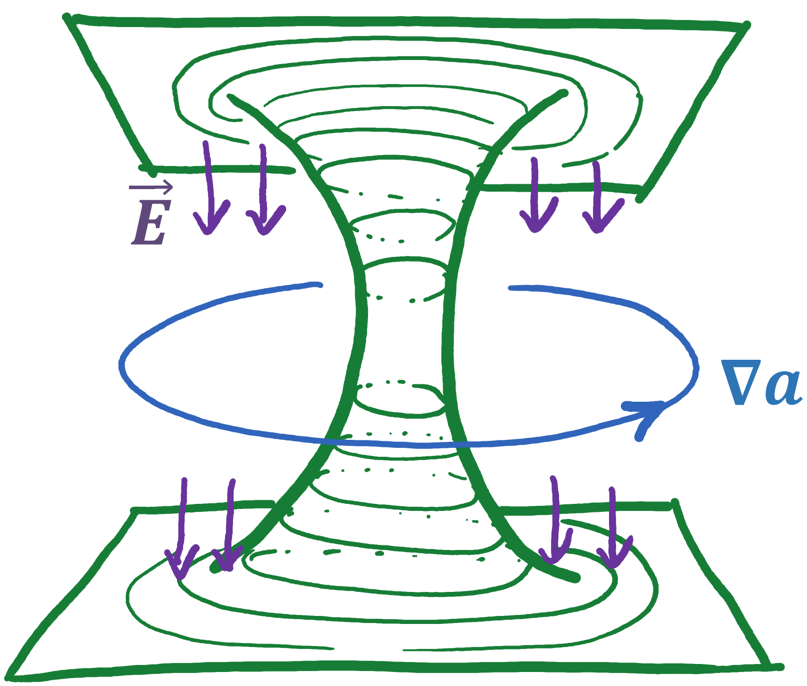


- $* J_1 = \frac{K_A}{4\pi} \, da \wedge F_A$
- $\mathrm{d} * J_1(bulk) = \frac{K_A}{4\pi} \, F_A \wedge \delta^{(2)}(M_2^{st})$
- $\vec{J}_1 \sim \nabla a \times \vec{E}$ (Hall-like current)

III. IR-Universal Observables from TQFT-Coupling

✸ **Anomaly Inflow** : W/O TQFT-Coupling

$$S \supset \frac{iK_A}{8\pi^2} \int da \wedge A_1 \wedge F_A = i \int A_1 \wedge * J_1$$

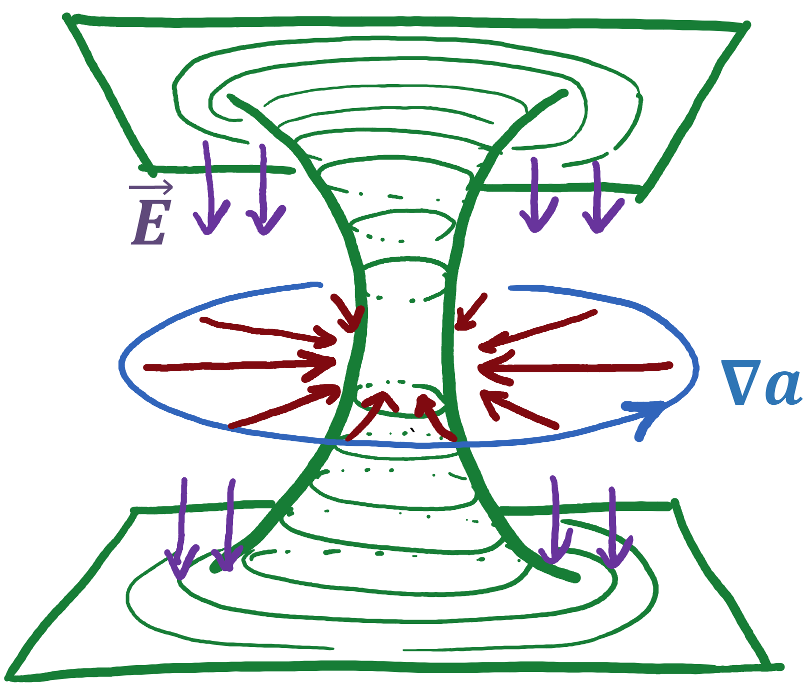


- $* J_1 = \frac{K_A}{4\pi}\, da \wedge F_A$
- $\mathrm{d} * J_1(bulk) = \frac{K_A}{4\pi}\, F_A \wedge \delta^{(2)}(M_2^{st})$
- $\vec{J}_1 \sim \nabla a \times \vec{E}$ (Hall-like current)

III. IR-Universal Observables from TQFT-Coupling

✸ **Anomaly Inflow** : W/O TQFT-Coupling

$$S \supset \frac{iK_A}{8\pi^2} \int da \wedge A_1 \wedge F_A = i \int A_1 \wedge * J_1$$

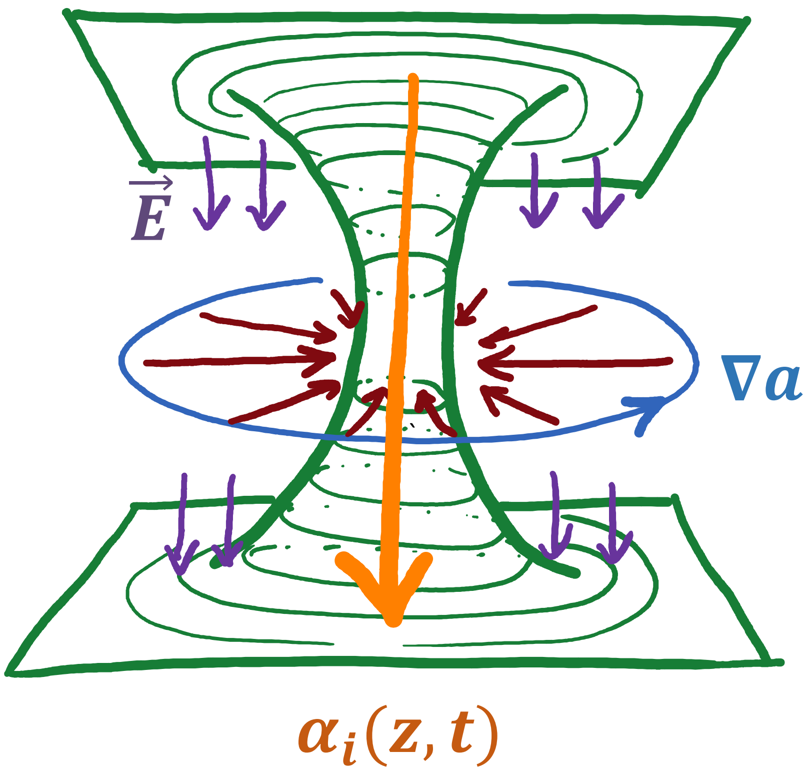


- $* J_1 = \frac{K_A}{4\pi}\, da \wedge F_A$
- $\mathrm{d} * J_1(bulk) = \frac{K_A}{4\pi}\, F_A \wedge \delta^{(2)}(M_2^{st})$
- $\vec{j}_1 \sim \nabla a \times \vec{E}$ (Hall-like current)

III. IR-Universal Observables from TQFT-Coupling

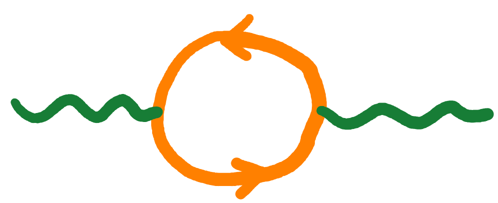
✸ **Anomaly Inflow** : W/O TQFT-Coupling

$$S \supset \frac{iK_A}{8\pi^2}\int da \wedge A_1 \wedge F_A \;= i\int A_1 \wedge * J_1$$



- $* J_1 = \frac{K_A}{4\pi}\, da \wedge F_A$
- $\mathrm{d} * J_1(bulk) = \frac{K_A}{4\pi}\, F_A \wedge \delta^{(2)}(M_2^{st})$
- $\vec{j}_1 \sim \nabla a \times \vec{E}$ (Hall-like current)
- 2d chiral fermions $\{\alpha_i\,(z,t)\}$

$$\mathrm{d} * J_1(2d) = -\frac{K_A}{4\pi}\, F_A$$



$$\sum_i Q_i^2 = K_A$$

III. IR-Universal Observables from TQFT-Coupling

✸ **Anomaly Inflow** : With **TQFT-Coupling**

$$S = \int \frac{1}{2}\, da \wedge * \, da \ + \int \frac{1}{2g_A^2}\, F_A \wedge * \, F_A \ - \int \frac{iK_A}{8\pi^2}\frac{a}{f} F_A \wedge F_A$$

$$+ \int \frac{in}{2\pi}\, B_2 \wedge dB_1 \ - \int \frac{iK_{AB}}{8\pi^2}\frac{a}{f} F_A \wedge F_B - \int \frac{iK_B}{8\pi^2}\frac{a}{f} F_B \wedge F_B$$

III. IR-Universal Observables from TQFT-Coupling

✸ **Anomaly Inflow** : With **TQFT-Coupling**

$$S = \int \frac{1}{2}\, da \wedge {}^{*} da \;+ \int \frac{1}{2g_A^2}\, F_A \wedge {}^{*} F_A \;- \int \frac{iK_A}{8\pi^2}\frac{a}{f} F_A \wedge F_A$$

$$+ \int \frac{in}{2\pi}\, B_2 \wedge dB_1 \;- \int \frac{iK_{AB}}{8\pi^2}\frac{a}{f} F_A \wedge F_B - \int \frac{iK_B}{8\pi^2}\frac{a}{f} F_B \wedge F_B$$

1. $A_1 \to A_1 + d\lambda_A$

$$\delta_A S = i \int \delta^{(2)}\left(M_2^{st}\right) \wedge \lambda_A \left(\frac{K_A}{4\pi} F_A + \frac{K_{AB}}{2\pi} F_B\right)$$

III. IR-Universal Observables from TQFT-Coupling

✸ **Anomaly Inflow** : With **TQFT-Coupling**

$$S = \int \frac{1}{2}\, da \wedge^* da \;+ \int \frac{1}{2g_A^2}\, F_A \wedge^* F_A \;- \int \frac{iK_A}{8\pi^2}\frac{a}{f} F_A \wedge F_A$$

$$+ \int \frac{in}{2\pi}\, B_2 \wedge dB_1 \;- \int \frac{iK_{AB}}{8\pi^2}\frac{a}{f} F_A \wedge F_B - \int \frac{iK_B}{8\pi^2}\frac{a}{f} F_B \wedge F_B$$

1. $A_1 \to A_1 + d\lambda_A$

$$\delta_A S = i \int \delta^{(2)}\left(M_2^{st}\right) \wedge \lambda_A \left(\frac{K_A}{4\pi} F_A + \frac{K_{AB}}{2\pi} F_B\right)$$

2. $B_1 \to B_1 + d\lambda_B\,, \qquad \lambda_B = \frac{2\pi}{n}\kappa\,, \qquad \kappa = 0,1,\cdots,n-1$

$$\delta_B S = i \int \delta^{(2)}\left(M_2^{st}\right) \wedge \lambda_B \left(\frac{K_{AB}}{2\pi} F_A + \frac{K_B}{4\pi} F_B\right)$$

III. IR-Universal Observables from TQFT-Coupling

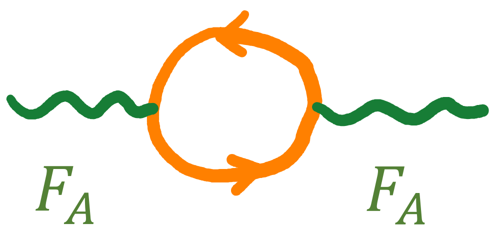
✸ **Anomaly Inflow** : With **TQFT-Coupling**

1. $\delta_A S = i \int \delta^{(2)}\left(M_2^{st}\right) \wedge \lambda_A \left(\frac{K_A}{4\pi} F_A + \frac{K_{AB}}{2\pi} F_B \right)$

III. IR-Universal Observables from TQFT-Coupling

✸ **Anomaly Inflow** : With **TQFT-Coupling**

1. $\delta_A S = i \int \delta^{(2)}(M_2^{st}) \wedge \lambda_A \left(\frac{K_A}{4\pi} F_A + \frac{K_{AB}}{2\pi} F_B \right)$



III. IR-Universal Observables from TQFT-Coupling

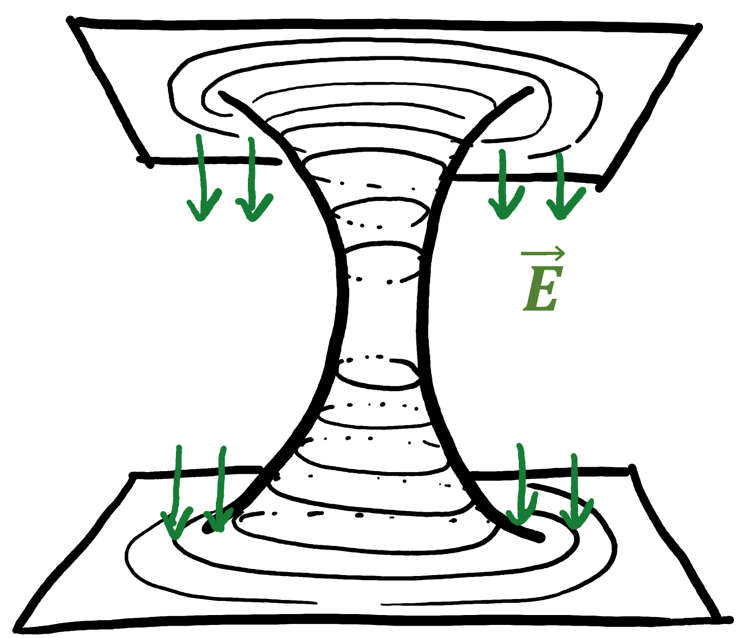
✸ **Anomaly Inflow** : With **TQFT-Coupling**

1. $$\delta_A S = i \int \delta^{(2)}(M_2^{st}) \wedge \lambda_A \left(\frac{K_A}{4\pi} F_A + \frac{K_{AB}}{2\pi} F_B \right)$$

III. IR-Universal Observables from TQFT-Coupling

✸ Anomaly Inflow : With TQFT-Coupling

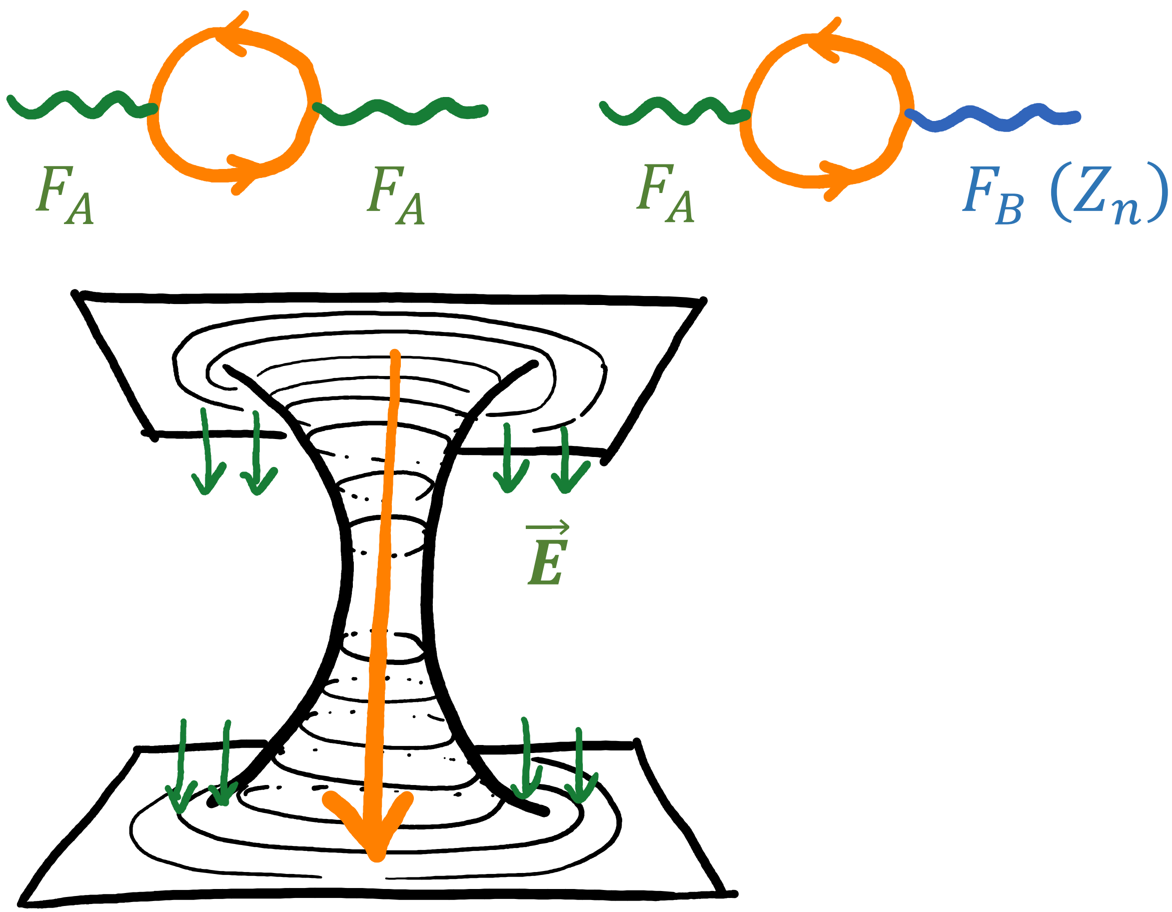
1. $\delta_A S = i \int \delta^{(2)}(M_2^{st}) \wedge \lambda_A \left(\frac{K_A}{4\pi} F_A + \frac{K_{AB}}{2\pi} F_B \right)$



III. IR-Universal Observables from TQFT-Coupling

✸ Anomaly Inflow : With TQFT-Coupling

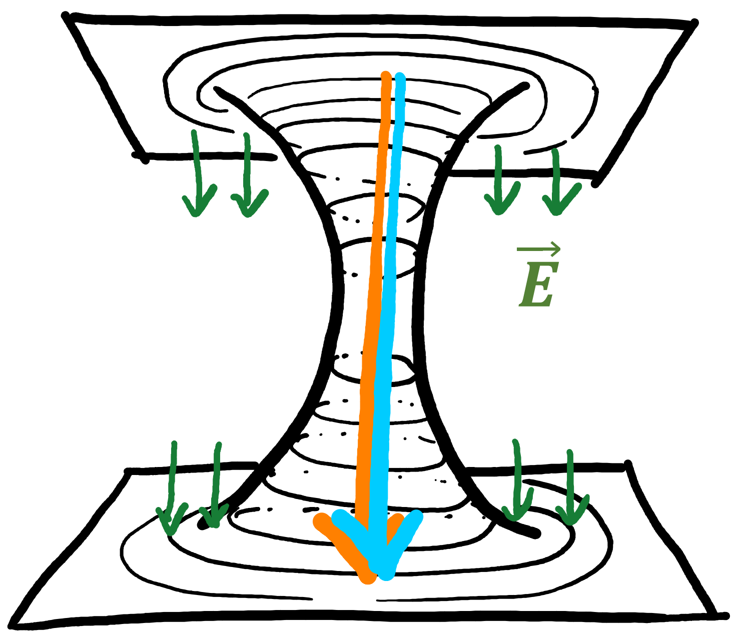
1. $\delta_A S = i \int \delta^{(2)}(M_2^{st}) \wedge \lambda_A \left(\frac{K_A}{4\pi} F_A + \frac{K_{AB}}{2\pi} F_B \right)$



III. IR-Universal Observables from TQFT-Coupling

✸ Anomaly Inflow : With TQFT-Coupling

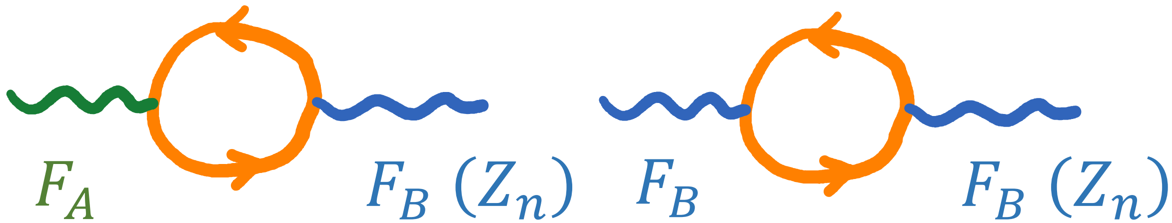
1. $\delta_A S = i \int \delta^{(2)}(M_2^{st}) \wedge \lambda_A \left(\frac{K_A}{4\pi} F_A + \frac{K_{AB}}{2\pi} F_B \right)$



III. IR-Universal Observables from TQFT-Coupling

✸ **Anomaly Inflow** : With **TQFT-Coupling**

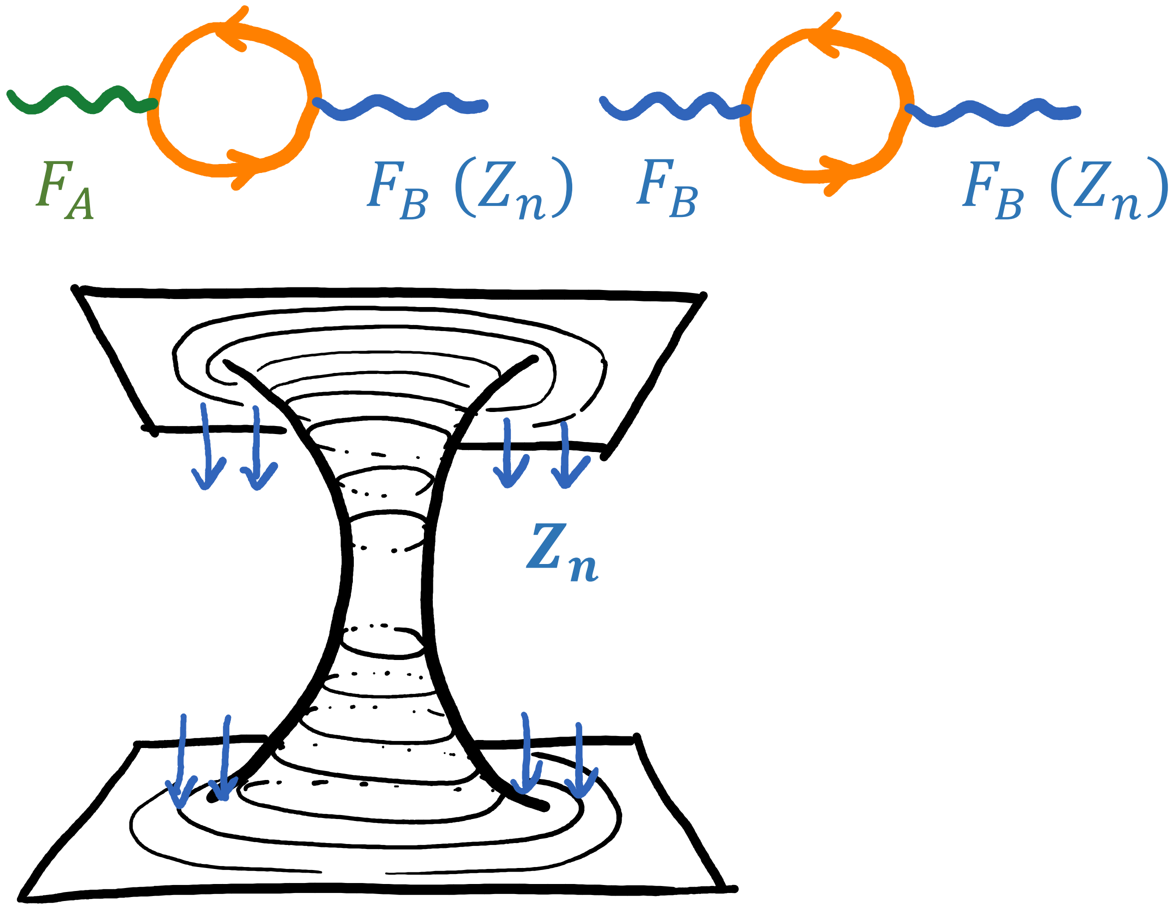
2. $\delta_B S = i \int \delta^{(2)}(M_2^{st}) \wedge \lambda_B \left(\frac{K_{AB}}{2\pi} F_A + \frac{K_B}{4\pi} F_B \right)$



III. IR-Universal Observables from TQFT-Coupling

✸ **Anomaly Inflow** : With **TQFT-Coupling**

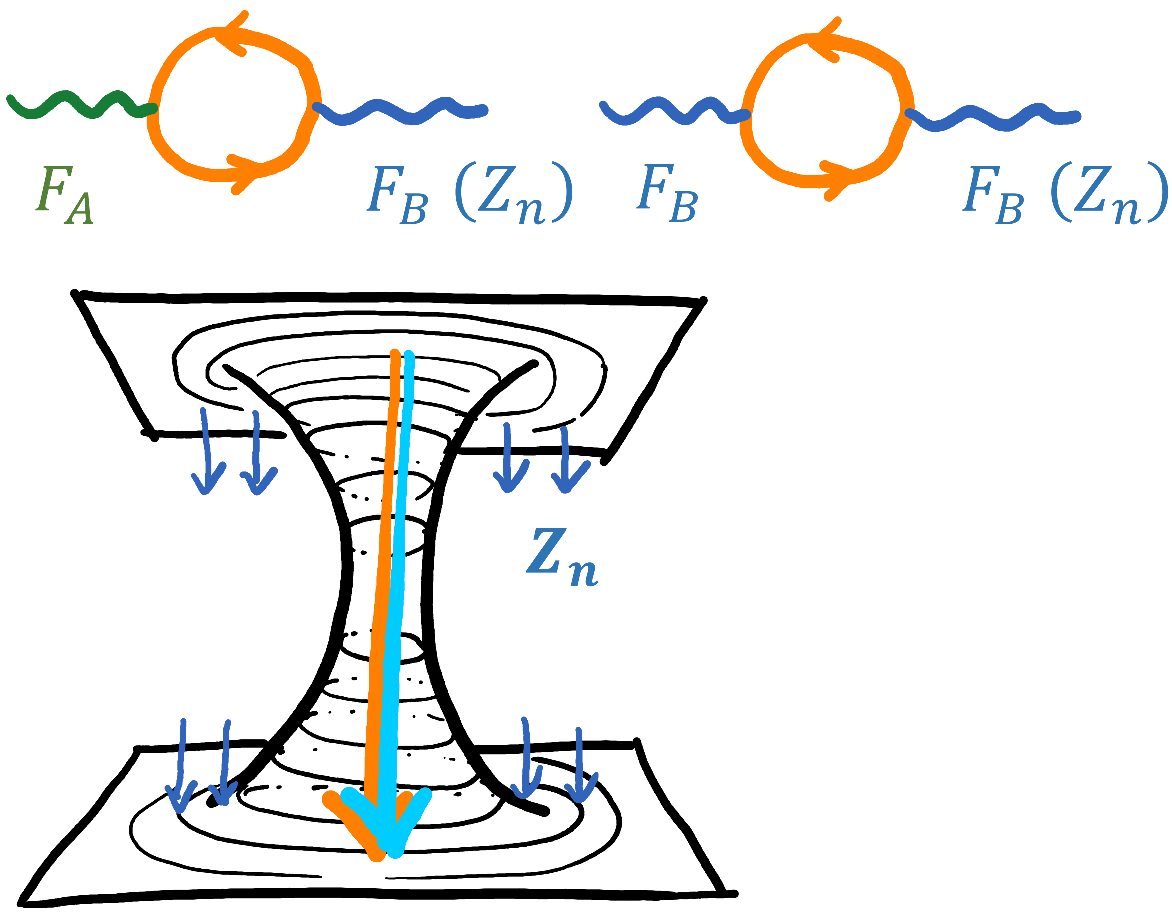
2. $\delta_B S = i \int \delta^{(2)}(M_2^{st}) \wedge \lambda_B \left(\frac{K_{AB}}{2\pi} F_A + \frac{K_B}{4\pi} F_B \right)$



III. IR-Universal Observables from TQFT-Coupling

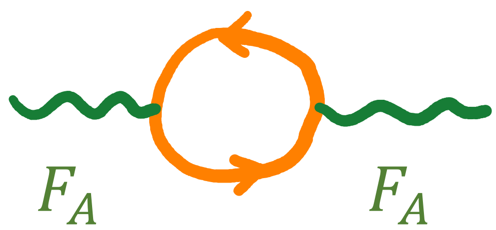
✸ **Anomaly Inflow** : With **TQFT-Coupling**

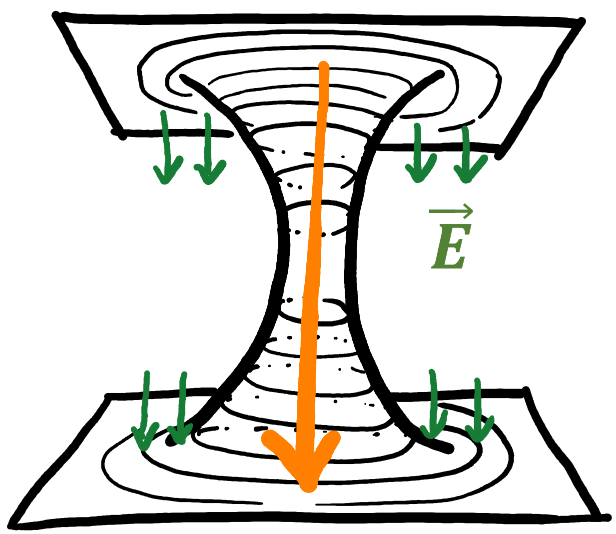
2. $\delta_B S = i \int \delta^{(2)}(M_2^{st}) \wedge \lambda_B \left(\frac{K_{AB}}{2\pi} F_A + \frac{K_B}{4\pi} F_B \right)$



III. IR-Universal Observables from TQFT-Coupling

W/O TQFT-Coupling

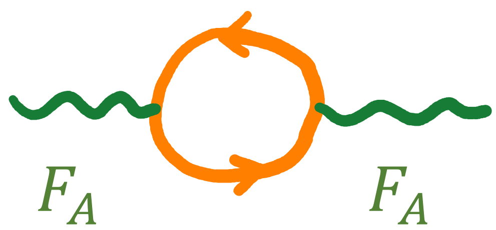


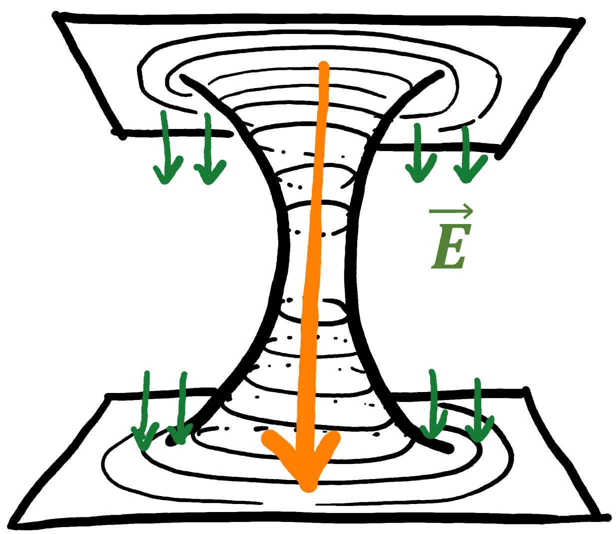


With TQFT-Coupling

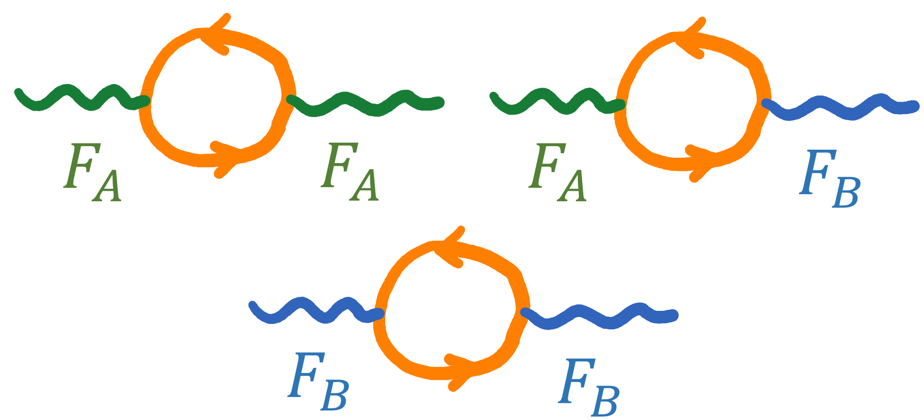
III. IR-Universal Observables from TQFT-Coupling

W/O TQFT-Coupling



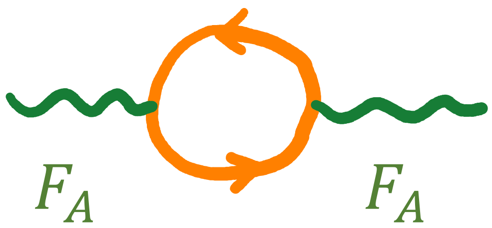


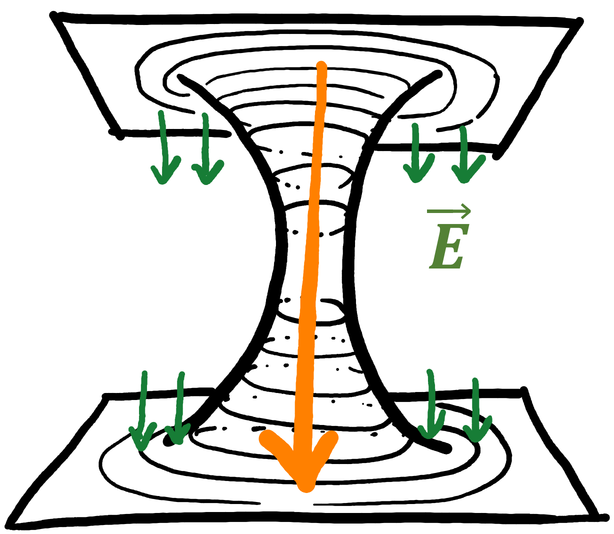
With TQFT-Coupling



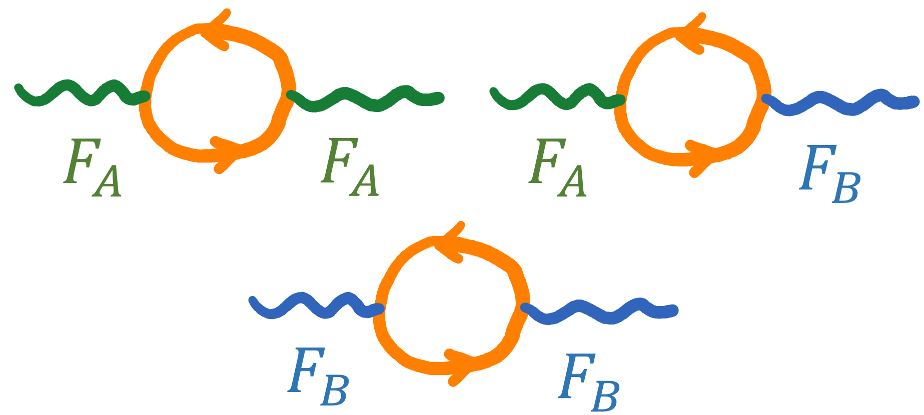
III. IR-Universal Observables from TQFT-Coupling

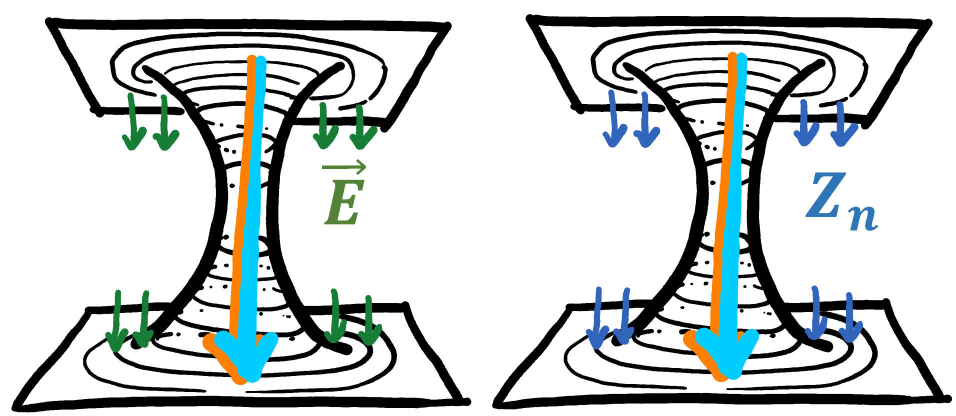
W/O TQFT-Coupling





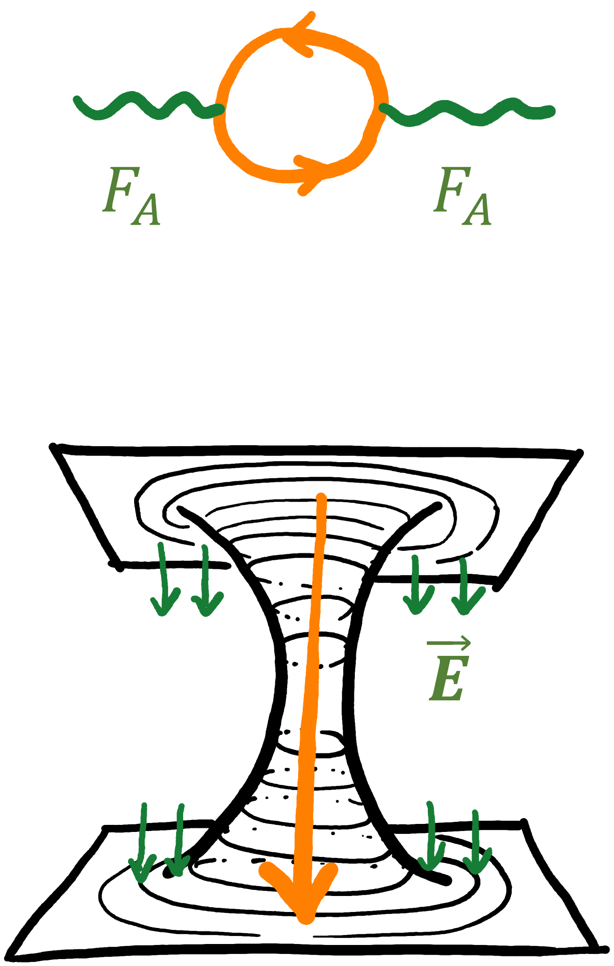
With TQFT-Coupling



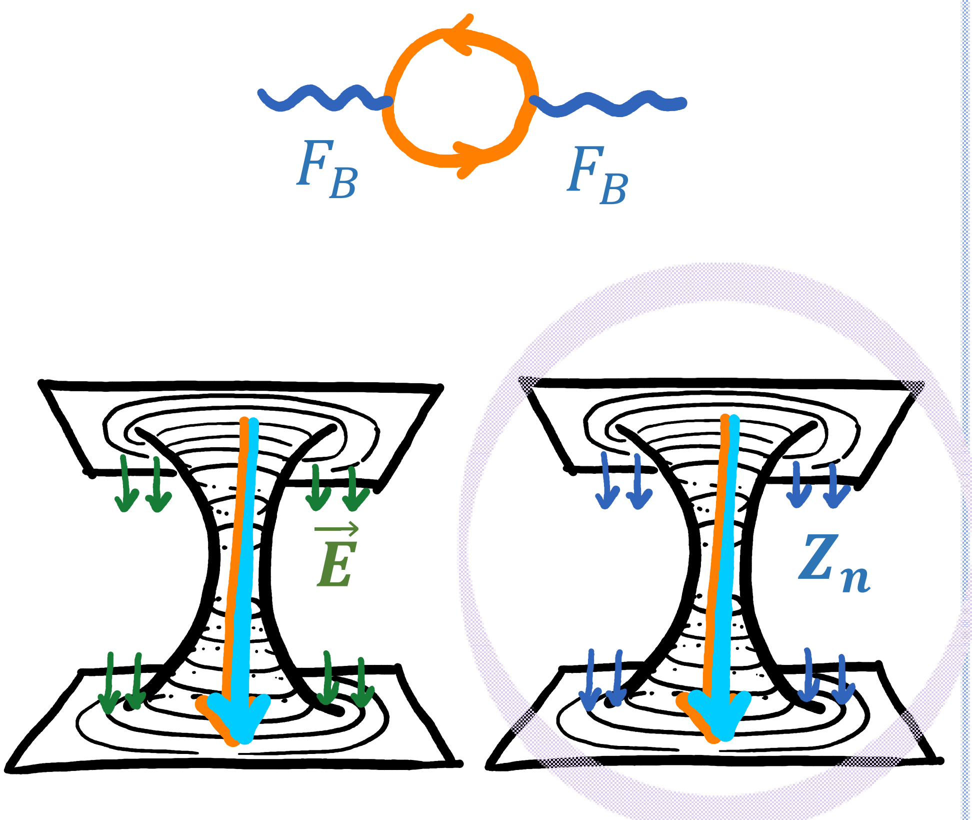


III. IR-Universal Observables from TQFT-Coupling

W/O TQFT-Coupling



With TQFT-Coupling



Outline

I. Axion-Maxwell Theory
Generalized Symmetries (0-, 1-, 2-form → 3-group)

II. Coupling **Axion-MW** to a Z_n **TQFT**

III. IR-Universal Observables from **TQFT-Coupling**
Anomaly-Inflow, Fermion Zero Modes

IV. UV-Completion from **Standard QFT**
Extended KSVZ Construction

IV. UV-Completion from Standard QFT

(i) We've seen that **TQFT-coupling** leads to

   a. Direct modification of 2d string worldsheet QFT
   b. Different nature of superconducting current along the string
   c. New mechanism to trigger superconducting current

IV. UV-Completion from **Standard QFT**

(i) We've seen that **TQFT-coupling** leads to

a. Direct modification of 2d string worldsheet QFT
b. Different nature of superconducting current along the string
c. New mechanism to trigger superconducting current

(ii) These non-trivial TQFT-coupling can arise quite easily from **standard UV QFT**

a. Important to figure out remnant TQFT effects
b. Incorporate in the EXP search strategies its impact on the phenomenology

IV-1. Standard KSVZ [Kim '79 / Shifman,Vainshtein, Zakharov '80]

$$\mathcal{L} = -\frac{1}{2g_A^2} F_A \wedge * F_A + \overline{\psi_1} i\gamma^\mu D_\mu \psi_1 + \overline{\chi_1} i\gamma^\mu D_\mu \chi_1 - \lambda_1 \Phi_1^\dagger \psi_1 \chi_1 + V(\Phi_1)$$

	$U(1)_{PQ}$	$U(1)_A$
Φ_1	1	0
ψ_1	1	1
χ_1	0	-1

IV-1. Standard KSVZ [Kim '79 / Shifman,Vainshtein, Zakharov '80]

$$\mathcal{L} = -\frac{1}{2g_A^2} F_A \wedge * F_A + \overline{\psi_1} i\gamma^\mu D_\mu \psi_1 + \overline{\chi_1} i\gamma^\mu D_\mu \chi_1 - \lambda_1 \Phi_1^{+} \psi_1 \chi_1 + V(\Phi_1)$$

	$U(1)_{PQ}$	$U(1)_A$
Φ_1	1	0
ψ_1	1	1
χ_1	0	-1

- To integrate out heavy fermions, $\psi_1 \rightarrow e^{\frac{ia}{f}} \psi_1 \qquad \Phi = e^{\frac{ia}{f}} \varphi$

IV-1. **Standard KSVZ** [Kim '79 / Shifman,Vainshtein, Zakharov '80]

$$\mathcal{L} = -\frac{1}{2g_A^2}\, F_A \wedge * F_A + \overline{\psi_1} i\gamma^\mu D_\mu \psi_1 + \overline{\chi_1} i\gamma^\mu D_\mu \chi_1 - \lambda_1 \Phi_1^\dagger \psi_1 \chi_1 + V(\Phi_1)$$

	$U(1)_{PQ}$	$U(1)_A$
Φ_1	1	0
ψ_1	1	1
χ_1	0	-1

- To integrate out heavy fermions, $\psi_1 \to e^{\frac{ia}{f}} \psi_1 \qquad \Phi = e^{\frac{ia}{f}} \varphi$
- $U(1)_{PQ} U(1)_A^2$ $[ABJ]$: $N_f = K_A$
- $S \to S - \frac{iK_A}{8\pi^2} \int \frac{a}{f} F_A \wedge F_A$

IV-1. Standard KSVZ [Kim '79 / Shifman,Vainshtein, Zakharov '80]

$$\mathcal{L} = -\frac{1}{2g_A^2}\, F_A \wedge * F_A + \overline{\psi_1} i\gamma^\mu D_\mu \psi_1 + \overline{\chi_1} i\gamma^\mu D_\mu \chi_1 - \lambda_1 \Phi_1^\dagger \psi_1 \chi_1 + V(\Phi_1)$$

	$U(1)_{PQ}$	$U(1)_A$
Φ_1	1	0
ψ_1	1	1
χ_1	0	-1

- To integrate out heavy fermions, $\psi_1 \to e^{\frac{ia}{f}} \psi_1$ $\quad \Phi = e^{\frac{ia}{f}} \varphi$
- $U(1)_{PQ} U(1)_A^2\ [ABJ] : N_f = K_A$
- $S \to S - \frac{iK_A}{8\pi^2} \int \frac{a}{f}\, F_A \wedge F_A$
- $S_{IR} = \int \frac{1}{2}\, da \wedge * da + \int \frac{1}{2g_A^2}\, F_A \wedge * F_A - \int \frac{iK_A}{8\pi^2} \frac{a}{f} F_A \wedge F_A$

IV-2. Extended KSVZ with TQFT-Coupling [Brennan, **Hong**, Wang '22]

$$\mathcal{L} = -\frac{1}{2g_A^2} F_A \wedge * F_A + \overline{\psi_1} i\gamma^\mu D_\mu \psi_1 + \overline{\chi_1} i\gamma^\mu D_\mu \chi_1 - \lambda_1 \Phi_1^\dagger \psi_1 \chi_1 + V(\Phi_1)$$

$$-\frac{1}{2g_B^2} F_B \wedge * F_B$$

	$U(1)_{PQ}$	$U(1)_A$	$U(1)_B$
Φ_1	1	0	n
ψ_1	1	1	q
χ_1	0	-1	n-q

IV-2. Extended KSVZ with TQFT-Coupling [Brennan,**Hong**,Wang '22]

$$\mathcal{L} = -\frac{1}{2g_A^2} F_A \wedge * F_A + \overline{\psi_1} i\gamma^\mu D_\mu \psi_1 + \overline{\chi_1} i\gamma^\mu D_\mu \chi_1 - \lambda_1 \Phi_1^\dagger \psi_1 \chi_1 + V(\Phi_1)$$

$$-\frac{1}{2g_B^2} F_B \wedge * F_B$$

	$U(1)_{PQ}$	$U(1)_A$	$U(1)_B$
Φ_1	1	0	n
ψ_1	1	1	q
χ_1	0	-1	n-q

$U(1)_B U(1)_A^2$

$U(1)_A U(1)_B^2$

$U(1)_B^3$

IV-2. Extended KSVZ with TQFT-Coupling [Brennan, **Hong**, Wang '22]

$$\mathcal{L} = -\frac{1}{2g_A^2} F_A \wedge * F_A + \overline{\psi_1} i\gamma^\mu D_\mu \psi_1 + \overline{\chi_1} i\gamma^\mu D_\mu \chi_1 - \lambda_1 \Phi_1^\dagger \psi_1 \chi_1 + V(\Phi_1)$$

$$-\frac{1}{2g_B^2} F_B \wedge * F_B + \overline{\psi_2} i\gamma^\mu D_\mu \psi_2 + \overline{\chi_2} i\gamma^\mu D_\mu \chi_2$$

	$U(1)_{PQ}$	$U(1)_A$	$U(1)_B$
Φ_1	1	0	n
ψ_1	1	1	q
χ_1	0	-1	n-q
ψ_2	0	1	q-n
χ_2	0	-1	-q

$U(1)_B U(1)_A^2$

$U(1)_A U(1)_B^2$

$U(1)_B^3$

IV-2. Extended KSVZ with TQFT-Coupling [Brennan,**Hong**,Wang '22]

$$\mathcal{L} = -\frac{1}{2g_A^2} F_A \wedge * F_A + \overline{\psi_1} i\gamma^\mu D_\mu \psi_1 + \overline{\chi_1} i\gamma^\mu D_\mu \chi_1 - \lambda_1 \Phi_1^\dagger \psi_1 \chi_1$$

$$-\frac{1}{2g_B^2} F_B \wedge * F_B + \overline{\psi_2} i\gamma^\mu D_\mu \psi_2 + \overline{\chi_2} i\gamma^\mu D_\mu \chi_2 - \lambda_2 \Phi_2 \psi_2 \chi_2 + V(\Phi_1, \Phi_2)$$

	$U(1)_{PQ}$	$U(1)_A$	$U(1)_B$
Φ_1	1	0	n
Φ_2	0	0	n
ψ_1	1	1	q
χ_1	0	-1	n-q
ψ_2	0	1	q-n
χ_2	0	-1	-q

$U(1)_B U(1)_A^2$

$U(1)_A U(1)_B^2$

$U(1)_B^3$

IV-2. Extended KSVZ with TQFT-Coupling [Brennan, **Hong**, Wang '22]

(i) Integrating out heavy fermions yields

$$S = \int \frac{1}{2}\, da \wedge * \, da + \int \frac{1}{2g_A^2}\, F_A \wedge * \, F_A - \int \frac{iK_A}{8\pi^2} \frac{a}{f} F_A \wedge F_A$$

$$+ \int \frac{in}{2\pi}\, B_2 \wedge dB_1 - \int \frac{iK_{AB}}{8\pi^2} \frac{a}{f} F_A \wedge F_B - \int \frac{iK_B}{8\pi^2} \frac{a}{f} F_B \wedge F_B$$

(ii) This is a pretty modest extension of the minimal KSVZ

(iii) Non-minimal charges of $\Phi_{1,2}$ $\rightarrow$ remnant Z_n TQFT

(iv) We've seen that this TQFT-coupling leads to several, distinct physics

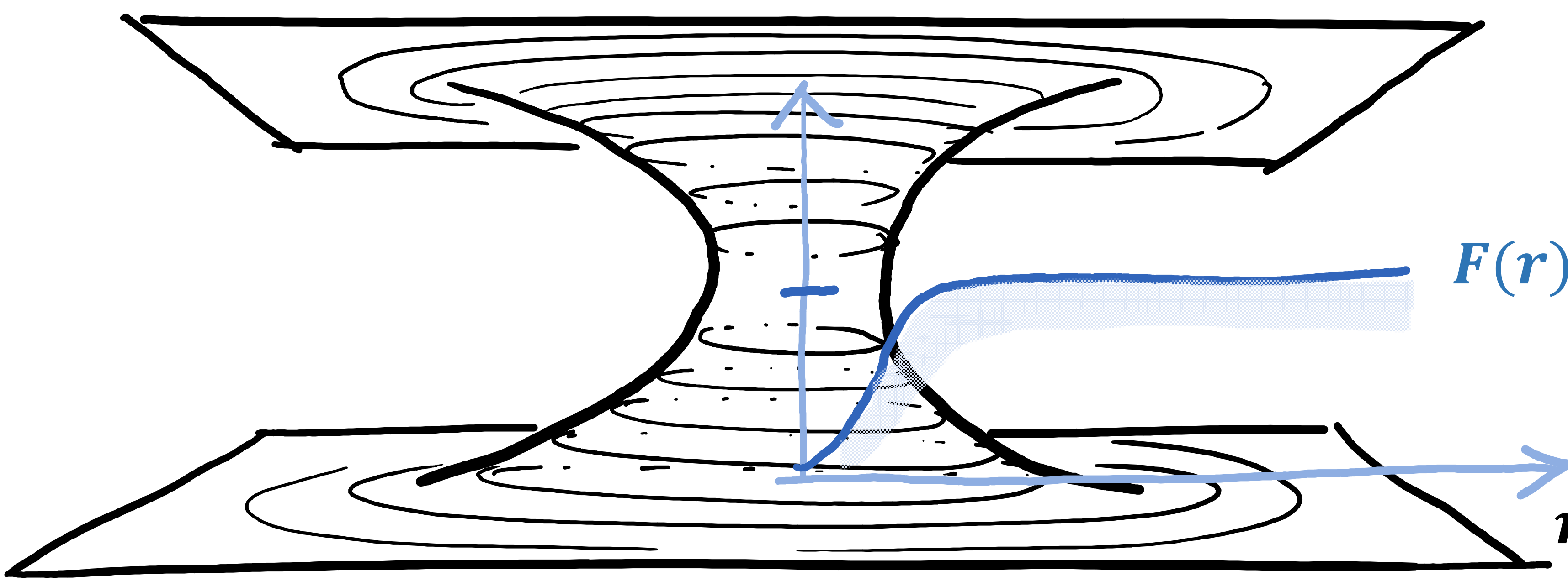
IV-3. Fermion Zero Modes [Brennan, **Hong**, Wang '22]

(i) Within the UV theory, the fermion (=boson in 2d) charge carriers along the string can be explicitly determined

(ii) They are 2d fermion string zero modes of 4d Weyl fermions

(iii) Solving Dirac equation in the string/vortex background

$$\Phi_i = \varphi_i e^{i\theta_i}, \qquad \varphi_i = F_i(r) e^{i m_i \phi}$$



IV-3. Fermion Zero Modes [Brennan, **Hong**, Wang '22]

(i) Within the UV theory, the fermion (=boson in 2d) charge carriers along the string can be explicitly determined

(ii) They are 2d fermion string zero modes of 4d Weyl fermions

(iii) Solving Dirac equation in the string/vortex background

$$\Phi_1^\dagger \psi_1 \chi_1$$

$\{\psi_1, \chi_1\}$ pair coupled to anti-Φ_1-string

$$\Phi_2 \psi_2 \chi_2$$

$\{\psi_2, \chi_2\}$ pair coupled to Φ_2-string

IV-3. Fermion Zero Modes [Brennan, **Hong**, Wang '22]

(i) Within the UV theory, the fermion (=boson in 2d) charge carriers along the string can be explicitly determined

(ii) They are 2d fermion string zero modes of 4d Weyl fermions

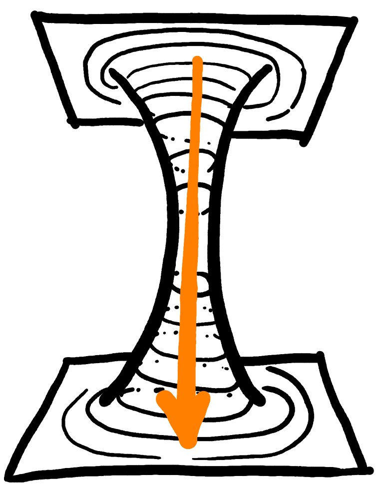
(iii) Solving Dirac equation in the string/vortex background

$\Phi_1^{+}\psi_1\chi_1$

$\{\psi_1, \chi_1\}$ pair coupled to anti-Φ_1-string

$\Phi_2\psi_2\chi_2$

$\{\psi_2, \chi_2\}$ pair coupled to Φ_2-string



Φ_1-String

$\{\psi_1, \chi_1\}$

$\downarrow$

$\alpha_1(z,t) = g(z+t)$

single LH weyl

$(0_{PQ}, 1_A, (q-n)_B)$

IV-3. Fermion Zero Modes [Brennan,**Hong**,Wang '22]

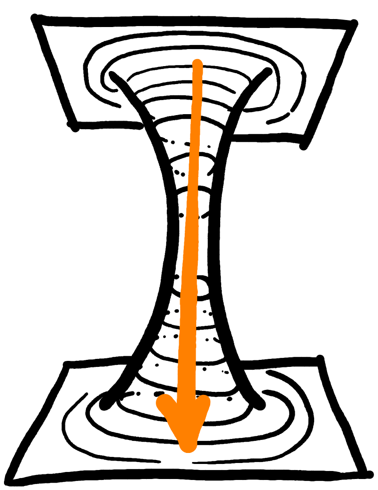
(i) Within the UV theory, the fermion (=boson in 2d) charge carriers along the string can be explicitly determined

(ii) They are 2d fermion string zero modes of 4d Weyl fermions

(iii) Solving Dirac equation in the string/vortex background

$\Phi_1^{+}\psi_1\chi_1$

$\{\psi_1, \chi_1\}$ pair coupled to anti-Φ_1-string



$\{\psi_1, \chi_1\}$

$\downarrow$

$\alpha_1(z,t) = g(z+t)$

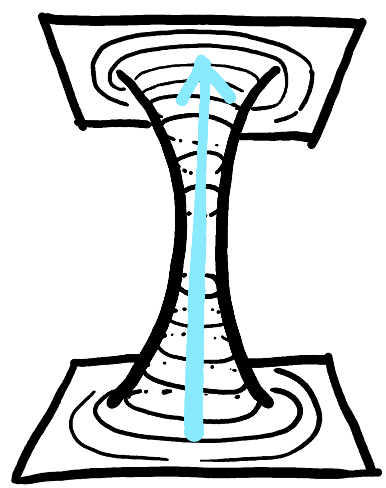
single LH weyl

$(0_{PQ}, 1_A, (q-n)_B)$

Φ_1-String

$\Phi_2\psi_2\chi_2$

$\{\psi_2, \chi_2\}$ pair coupled to Φ_2-string



$\{\psi_2, \chi_2\}$

$\downarrow$

$\alpha_2(z,t) = f(z-t)$

single RH weyl

$(0_{PQ}, 1_A, (q-n)_B)$

Φ_2-String

Conclusion

Conclusion and Outlook

- ✓ Non-trivial TQFT-Coupling or Topological Modification of a QFT can lead to interesting observable consequences.
- ✓ Generalized Global Symmetry and their anomalies play the central role!
- ✓ This is just the beginning!
  - ➢ Applications: **DM** charged under topological Z_n force
  - ➢ Wealth of **cosmic string physics**
    - > Axion-string + BF-string + Composite-string
    - > Non-Kibble mechanism for production?
  - ➢ Axion-YM (QCD) coupled to a TQFT : very rich!
  - ➢ Non-invertible symmetries

Thank you!