

The SM- Like Higgs

Alignment

& the W Mass

Nausheen R. Shah

WAYNE STATE
UNIVERSITY



CERN – CKC

Physics Beyond the SM

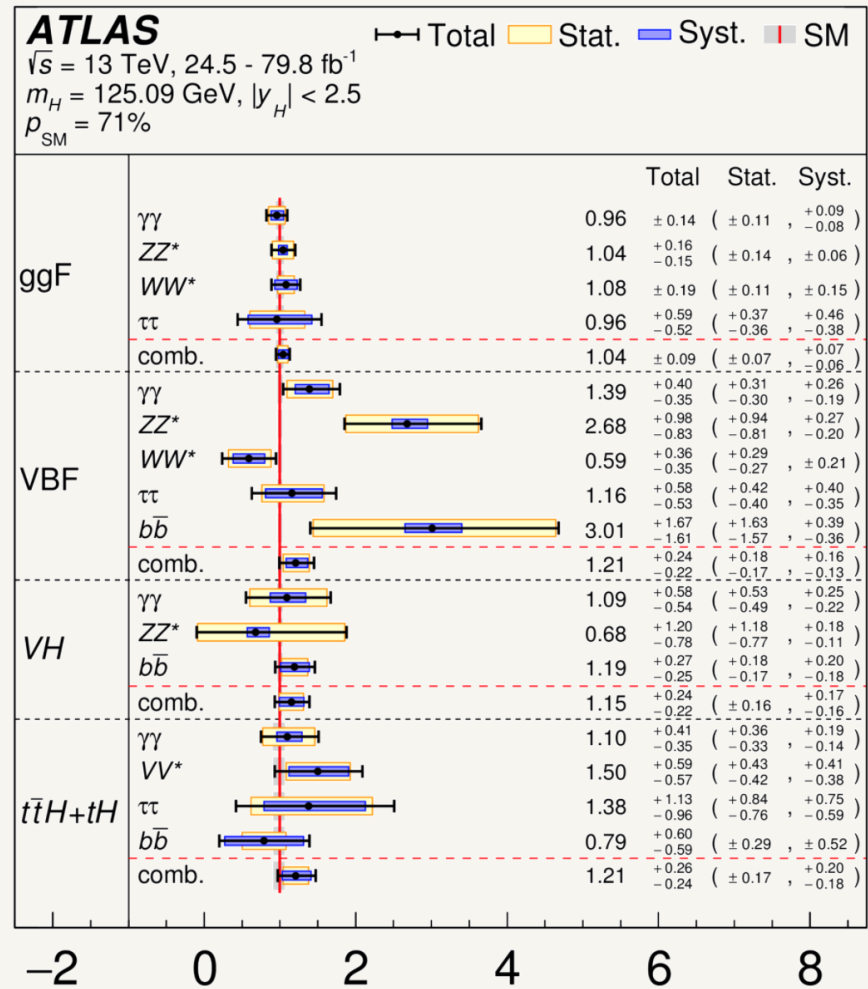
Jeju Island

Tuesday June 07, 2022

Still large uncertainties in couplings... but compatible with SM expectations.

125 GeV SM-Like Higgs

*Observed Higgs
Production x Branching Ratios
as a ratio to SM expectation*



$\sigma \times \text{BR}$ normalized to SM



Beyond the Standard Model with the Higgs.

SM Higgs is a Doublet

- The Higgs *FIELD* is a two component weakly charged doublet.
- h is the neutral particle we think we have observed at the LHC: h_{125}
- v is the SM vev: 174 GeV.
- $G^{+/-}$ and G^0 are “eaten” by the W and Z gauge bosons to give them mass.

$$H_{SM} = \begin{bmatrix} G^{\pm} \\ \frac{1}{\sqrt{2}}(h + iG^0) + v \end{bmatrix}$$

What if there were more???

But we
SEE
a SM-like Higgs!

2 Higgs Doublet Model (2HDM).

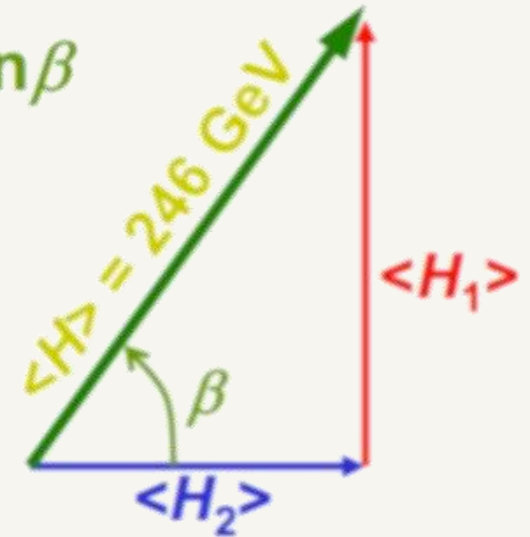
$$\langle H_1 \rangle, \langle H_2 \rangle \rightarrow \langle H \rangle, \tan \beta$$

In SUSY Need 2 Higgs doublets:

H_u – Couples only to up-type quarks

H_d – Couples only to down-type quarks and leptons.

$$m_A \sim m_H$$
$$\tan \beta = v_u / v_d$$




5 Physical Higgs bosons:

CP-Even: h, H

CP-Odd: A

Charged Higgs: $H^{+,-}$

$\langle H \rangle$  246 GeV
 $\langle h \rangle \sim \langle H_{SM} \rangle$

$$\begin{aligned}
 H_{SM} &= \sin \beta H_u + \cos \beta H_d \leftarrow v \cos^2 \beta \\
 H_{NSM} &= -\cos \beta H_u + \sin \beta H_d
 \end{aligned}$$

$v \sin^2 \beta$ ↓

SM: Only 1 Higgs which then acquires a vev and leads to EWSB.

This is what we want!

Lighter (h) is 125 GeV SM-like Higgs.

Additional states can exist!

Additional States can be light!

Haber and Gunion, '03, M. Carena, I. Low, N.R.S. & C. Wagner, '13, A. Delgado, G. Nardini & M. Quiros, '13, N. Craig, J. Galloway & S. Thomas, '13, P. Dev, A. Pilaftsis '14, M. Carena, H. Haber, I. Low, N.R.S. & C. Wagner '14 & '15 etc....

$$\begin{aligned}
 \langle H_d \rangle &= v \cos \beta \\
 \langle H_u \rangle &= v \sin \beta \\
 \Rightarrow \langle H_{SM} \rangle &= v \\
 \langle H_{NSM} \rangle &= 0
 \end{aligned}$$

SM-like HIGGS

ALIGNMENT

Higgs Basis

SM-Like Higgs: h_{125}

$$c_{\beta-\alpha} \ll 1, s_{\beta-\alpha} \approx 1$$

$$H_{SM} = \sin \beta H_u + \cos \beta H_d$$

$$H_{NSM} = -\cos \beta H_u + \sin \beta H_d$$

$$\mathcal{V} \supset \dots + \frac{1}{2} Z_1 (H_1^\dagger H_1)^2 + \dots + [Z_5 (H_1^\dagger H_2)^2 + Z_6 (H_1^\dagger H_1) H_1^\dagger H_2 + \text{h.c.}] + \dots$$

$$Z_1 \equiv \lambda_1 c_\beta^4 + \lambda_2 s_\beta^4 + \frac{1}{2} (\lambda_3 + \lambda_4 + \lambda_5) s_{2\beta}^2 + 2s_{2\beta} [c_\beta^2 \lambda_6 + s_\beta^2 \lambda_7],$$

$$Z_5 \equiv \frac{1}{4} s_{2\beta}^2 [\lambda_1 + \lambda_2 - 2(\lambda_3 + \lambda_4 + \lambda_5)] + \lambda_5 - s_{2\beta} c_{2\beta} (\lambda_6 - \lambda_7),$$

$$Z_6 \equiv -\frac{1}{2} s_{2\beta} [\lambda_1 c_\beta^2 - \lambda_2 s_\beta^2 - (\lambda_3 + \lambda_4 + \lambda_5) c_{2\beta}] + c_\beta c_{3\beta} \lambda_6 + s_\beta s_{3\beta} \lambda_7.$$

λ_i : Z_2 Quartics

(Fermion couplings defined)

$$Z_1 v^2 \sim (125 \text{ GeV})^2$$

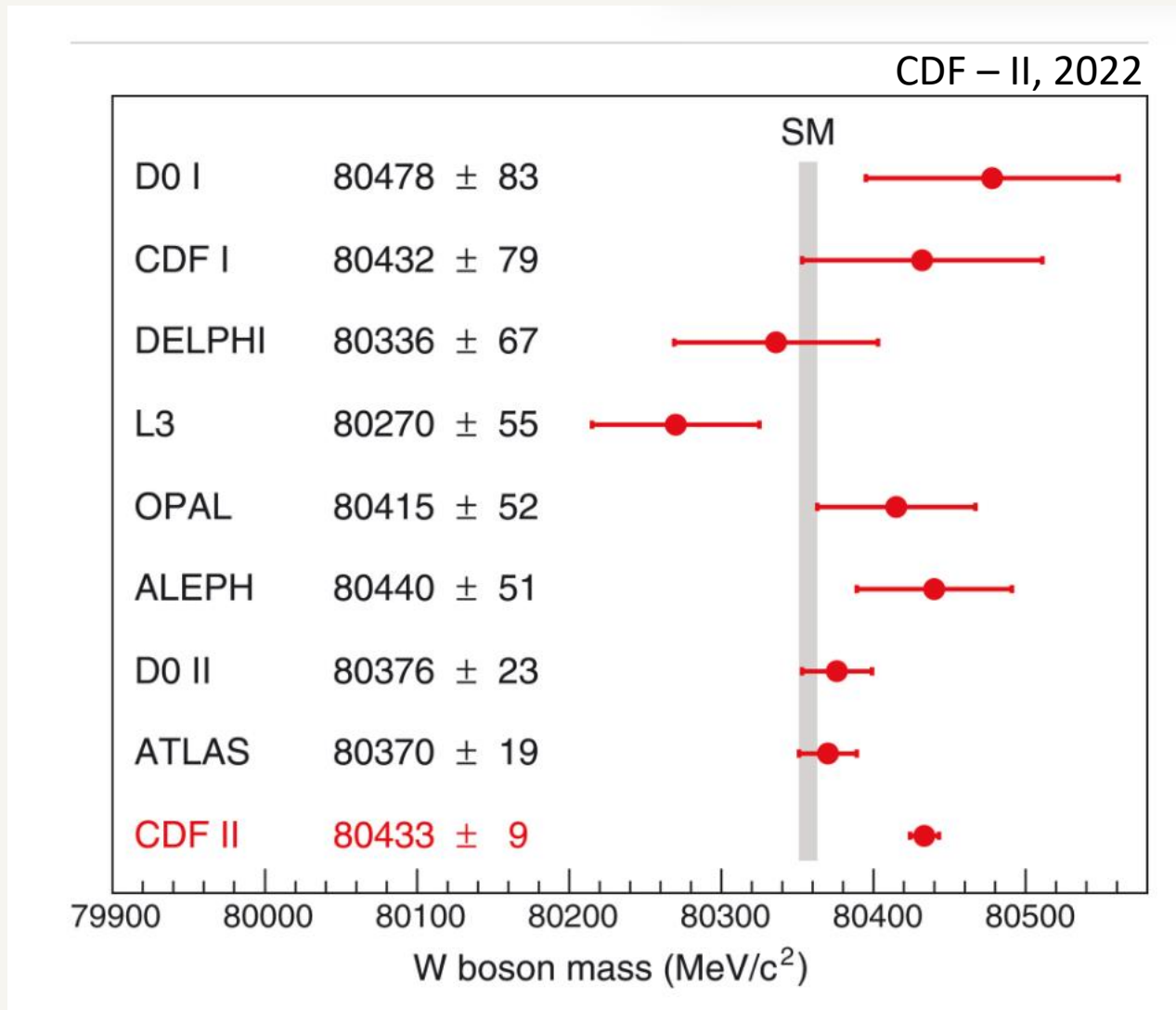
$Z_6 v^2$ controls mixing

$m_A \sim$ mass of heavy Higgs

$$\mathcal{M}_H^2 = \begin{pmatrix} Z_1 v^2 & Z_6 v^2 \\ Z_6 v^2 & m_A^2 + Z_5 v^2 \end{pmatrix}$$

THE W MASS

Recent CDF result: 7- σ (!!!) discrepancy with the SM



$$\rho \equiv \frac{G_F^{NC}}{G_F} = \frac{M_W^2}{M_Z^2 c_W^2} = 1 + \delta\rho.$$

Difference in G_F between neutral and charged processes

SM contributions (including h_{125}) calculated to 4-5 loops

Eg SM diagrams (Erlar & Schott, 2019).



$$M_W = 80,357 \pm 4_{\text{inputs}} \pm 4_{\text{theory}} \text{ MeV}$$

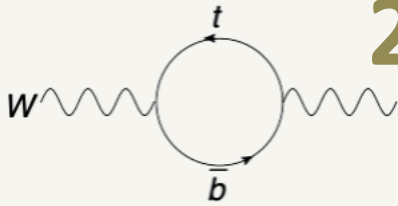
SM from PDG

New CDF – II value:

$$M_W = 80,433.5 \pm 6.4_{\text{stat}} \pm 6.9_{\text{syst}} = 80,433.5 \pm 9.4 \text{ MeV}$$

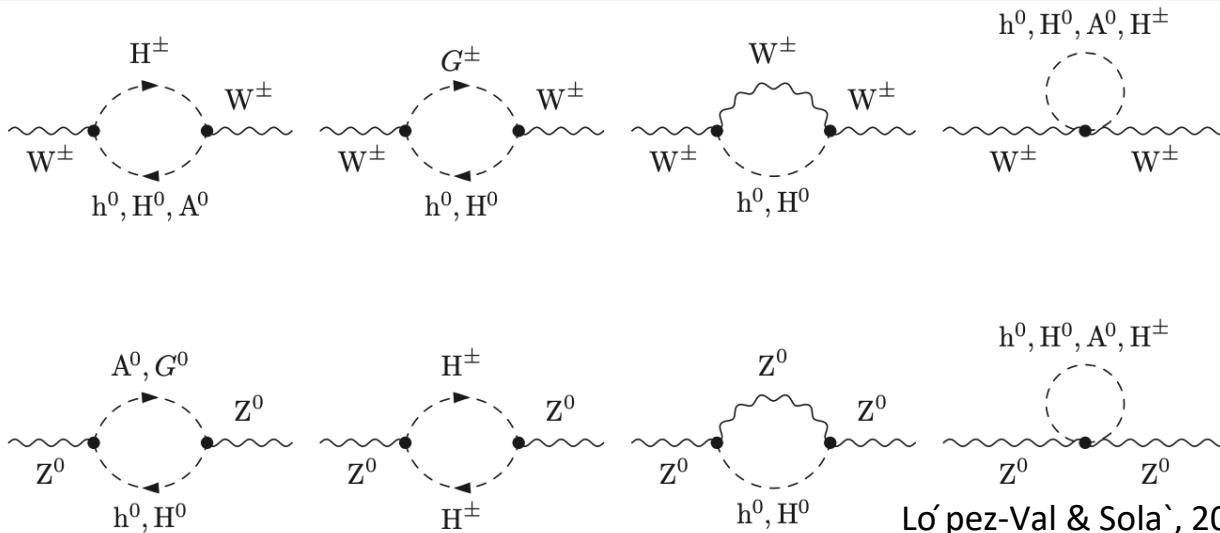


2HDM Custodial Symmetry Breaking?



$$\delta\rho_{2\text{HDM}} = \frac{\alpha}{16\pi s_w^2 M_W^2} \left\{ \cos^2(\beta - \alpha) [F(M_{h^0}^2, M_{H^\pm}^2) - F(M_{h^0}^2, M_{A^0}^2)] \right. \\ \left. + \sin^2(\beta - \alpha) [F(M_{H^0}^2, M_{H^\pm}^2) - F(M_{H^0}^2, M_{A^0}^2)] + F(M_{A^0}^2, M_{H^\pm}^2) \right. \\ \left. - 3 \cos^2(\beta - \alpha) [F(M_{H^0}^2, M_W^2) - F(M_{h^0}^2, M_Z^2) - F(M_{H^0}^2, M_Z^2) - F(M_{h^0}^2, M_W^2)] \right\},$$

$$F(x, y) = F(y, x) = \begin{cases} \frac{x+y}{2} - \frac{xy}{x-y} \log\left(\frac{x}{y}\right) & x \neq y \\ 0 & x = y \end{cases}$$



López-Val & Sola, 2013

$\Delta\rho$ in the Alignment Limit

$$\delta\rho_{2\text{HDM}} = \frac{G_F}{8\sqrt{2}\pi^2} \left\{ F(M_{H^0}^2, M_{H^\pm}^2) - F(M_{H^0}^2, M_{A^0}^2) + F(M_{A^0}^2, M_{H^\pm}^2) \right\}.$$

$$\Delta M_{H^\pm H^0}^2 = M_{H^0}^2 - M_{H^\pm}^2 = (Z_4 + Z_5) v^2 / 2$$

$$\Delta M_{H^\pm A^0}^2 = M_{A^0}^2 - M_{H^\pm}^2 = (Z_4 - Z_5) v^2 / 2$$

$$= \frac{G_F}{8\sqrt{2}\pi^2} \frac{(Z_4^2 - Z_5^2) v^4}{12M_{H^\pm}^2} \left(1 - \frac{Z_4 v^2}{4M_{H^\pm}^2} + \frac{(Z_4^2 - Z_5^2) v^4}{40M_{H^\pm}^4} \right).$$

$$\mathcal{V} = Y_1 H_1^\dagger H_1 + Y_2 H_2^\dagger H_2 + \left[Y_3 H_1^\dagger H_2 + h.c. \right]$$

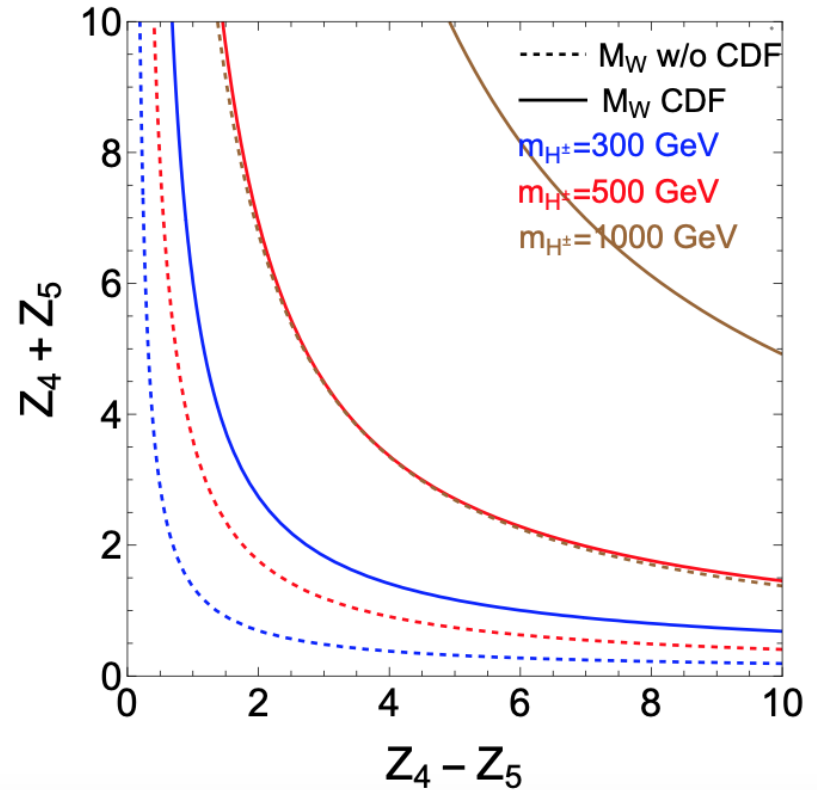
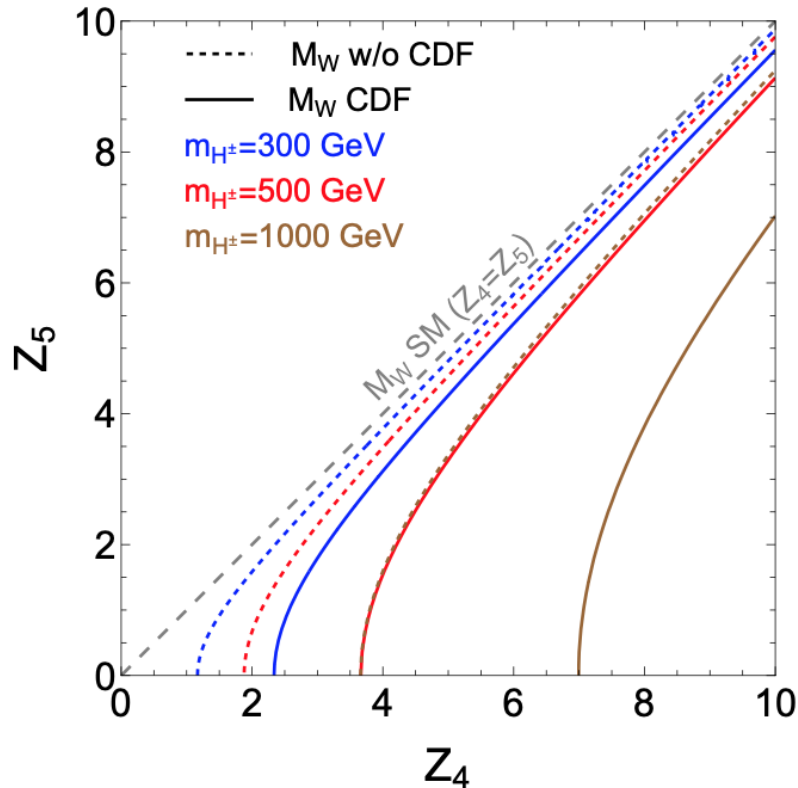
(125 GeV)²

$$+ \left[\frac{Z_1}{2} (H_1^\dagger H_1)^2 + \frac{Z_2}{2} (H_2^\dagger H_2)^2 + Z_3 (H_1^\dagger H_1) (H_2^\dagger H_2) + \underline{Z_4 (H_1^\dagger H_2) (H_2^\dagger H_1)} \right. \\ \left. + \left[\underline{\frac{Z_5}{2} (H_1^\dagger H_2)^2} - \underline{Z_6 (H_1^\dagger H_1) (H_1^\dagger H_2)} + Z_7 (H_2^\dagger H_2) (H_1^\dagger H_2) + h.c. \right] \right].$$

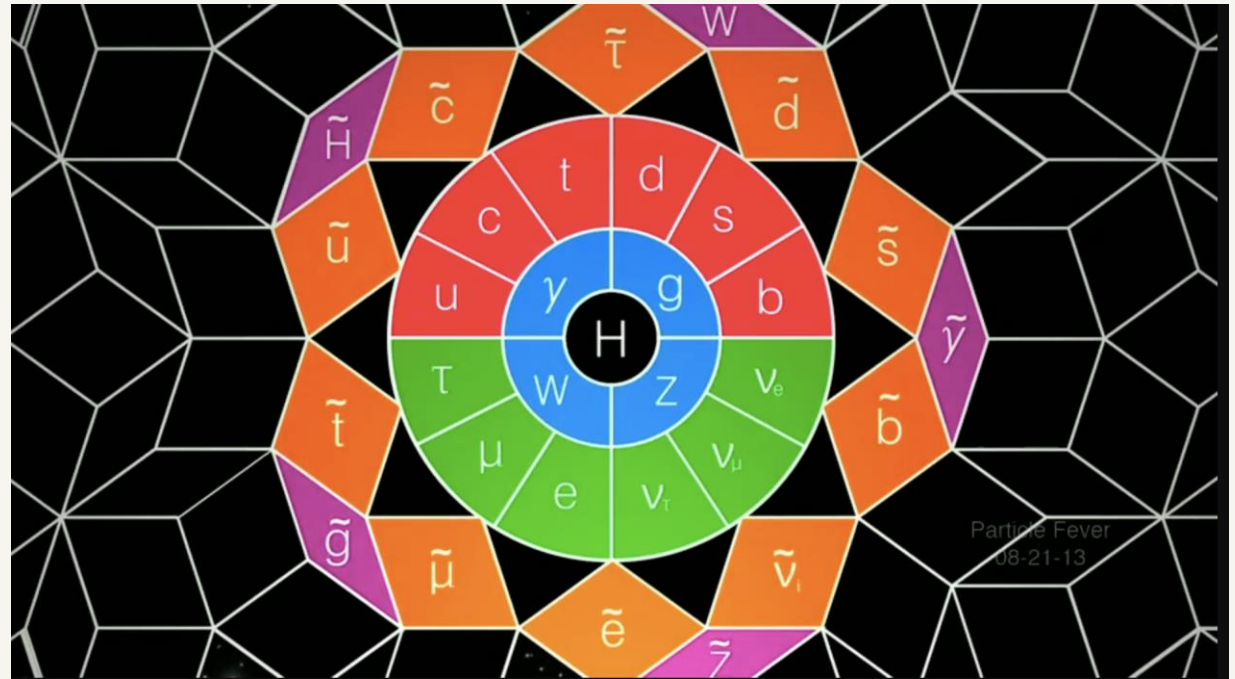
0 = Alignment

Parameters of our Ignorance

Preliminary, M. Carena, I. Low, NRS, C. Wagner, X. Wang



H_{125} mass and couplings **independent** of Heavy Higgs mass and W mass contributions.



Particle Fever
08-21-13

NOTORIOUS

SUPERSYMMETRY

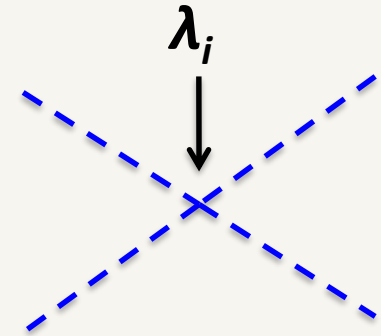
Higgs Mass = 125 GeV.

$$V = m_{ij}^2 \Phi_i^\dagger \Phi_j + \lambda_i \Phi_j^\dagger \Phi_k \Phi_l^\dagger \Phi_m$$

H. Haber and J. Gunion, '03

Quartics without quantum corrections related only to SM couplings.

Higgs mass bounded by m_Z at tree-level.



91 \neq 125

MSSM

Need large radiative corrections.
...Stops...?

$$Z_1 v^2 = m_Z^2 c_{2\beta}^2 + \frac{3v^2 s_\beta^4 h_t^4}{8\pi^2} \left[\ln \left(\frac{M_S^2}{m_t^2} \right) + \frac{X_t^2}{M_S^2} \left(1 - \frac{X_t^2}{12M_S^2} \right) \right],$$

$$Z_2 v^2 = m_Z^2 c_{2\beta}^2 + \frac{3v^2 c_\beta^4 h_t^4}{8\pi^2} \left[\ln \left(\frac{M_S^2}{m_t^2} \right) + \frac{Y_t^2}{M_S^2} \left(1 - \frac{Y_t^2}{12M_S^2} \right) \right],$$

$$Z_3 v^2 = \frac{1}{4}(g^2 - g'^2)v^2 + s_{2\beta}^2 \left\{ m_Z^2 + \frac{3v^2 h_t^4}{32\pi^2} \left[\ln \left(\frac{M_S^2}{m_t^2} \right) + \frac{(X_t + Y_t)^2}{4M_S^2} - \frac{X_t^2 Y_t^2}{12M_S^4} \right] \right\},$$

$$Z_4 v^2 = \frac{1}{2}g^2 v^2 + s_{2\beta}^2 \left\{ m_Z^2 + \frac{3v^2 h_t^4}{32\pi^2} \left[\ln \left(\frac{M_S^2}{m_t^2} \right) + \frac{(X_t + Y_t)^2}{4M_S^2} - \frac{X_t^2 Y_t^2}{12M_S^4} \right] \right\},$$

$$Z_5 v^2 = s_{2\beta}^2 \left\{ m_Z^2 + \frac{3v^2 h_t^4}{32\pi^2} \left[\ln \left(\frac{M_S^2}{m_t^2} \right) + \frac{X_t Y_t}{M_S^2} \left(1 - \frac{X_t Y_t}{12M_S^2} \right) \right] \right\},$$

$$Z_6 v^2 = -s_{2\beta} \left\{ m_Z^2 c_{2\beta} - \frac{3v^2 s_\beta^2 h_t^4}{16\pi^2} \left[\ln \left(\frac{M_S^2}{m_t^2} \right) + \frac{X_t(X_t + Y_t)}{2M_S^2} - \frac{X_t^3 Y_t}{12M_S^4} \right] \right\},$$

$$Z_7 v^2 = s_{2\beta} \left\{ m_Z^2 c_{2\beta} + \frac{3v^2 c_\beta^2 h_t^4}{16\pi^2} \left[\ln \left(\frac{M_S^2}{m_t^2} \right) + \frac{Y_t(X_t + Y_t)}{2M_S^2} - \frac{X_t Y_t^3}{12M_S^4} \right] \right\}.$$

Leading order contributions

Preliminary, M. Carena, I. Low, NRS, C. Wagner, X. Wang

Relevant for W mass (too small!!):

$$(Z_4 - Z_5)v^2 = \frac{1}{2}g^2 v^2 + s_{2\beta}^2 \frac{3v^2 h_t^4}{32\pi^2} \frac{(X_t - Y_t)^2}{4M_S^2} = \frac{1}{2}g^2 v^2 + \frac{3v^2 h_t^4}{32\pi^2} \frac{\mu^2}{M_S^2}$$

Direct Stops and W Mass

Preliminary, M. Carena, I. Low, NRS, C. Wagner, X. Wang

$$m_{\tilde{t}_{1,2}}^2 = M_S^2 + m_t^2 \pm m_t X_t.$$

$$\delta\rho_{\tilde{t}} = \frac{3G_F}{4\sqrt{2}\pi^2} \left(c_t^2 F(M_{\tilde{t}_1}^2, M_{\tilde{b}_1}^2) + s_t^2 F(M_{\tilde{t}_2}^2, m_{\tilde{b}_1}^2) - s_t^2 c_t^2 F(M_{\tilde{t}_1}^2, M_{\tilde{t}_2}^2) \right)$$

$$(\delta M_W)_{\tilde{t}} = \frac{G_F}{8\sqrt{2}\pi^2} \frac{m_t^4}{M_S^2} \frac{c_w^2}{c_w^2 - s_w^2} \left(1 - \frac{X_t^2}{2M_S^2} + \frac{X_t^4}{10M_S^4} \right) M_W.$$

Ignoring bottom mass dependent corrections to sbottom sector and degenerate soft masses

$$(\delta M_W)_{\tilde{t}} \sim 9 \text{ MeV} \frac{1}{M_S^2 [\text{TeV}^2]} \left(1 - \frac{X_t^2}{2M_S^2} + \frac{X_t^4}{10M_S^4} \right)$$

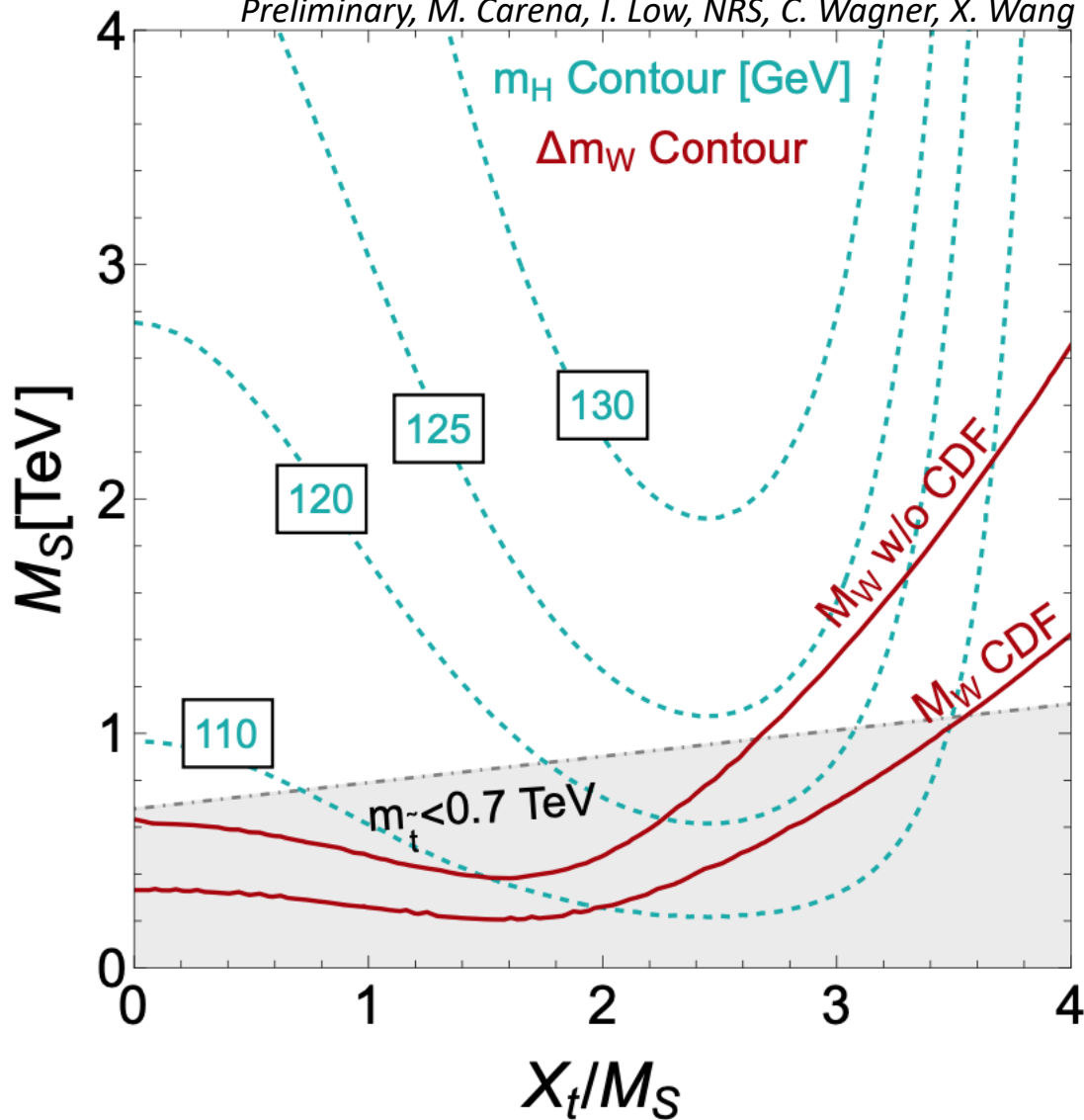
$$M_W = 80,357 \pm 4_{\text{inputs}} \pm 4_{\text{theory}} \text{ MeV}$$

SM from PDG

~ 76 MeV

$$M_W = 80,433.5 \pm 6.4_{\text{stat}} \pm 6.9_{\text{syst}} = 80,433.5 \pm 9.4 \text{ MeV}$$

Preliminary, M. Carena, I. Low, NRS, C. Wagner, X. Wang



m_w inconsistent with $m_{h_{125}}$!

Larger $X_t \rightarrow$ Unstable Vacuum

Light stops LHC constraints

Light stops:
 h_{125} gluon fusion + $\gamma\gamma$ rate

$$(\delta M_W)_{\bar{t}} \lesssim 30 \text{ MeV.}$$

(~ 76 MeV)

SOMETHING ELSE ...

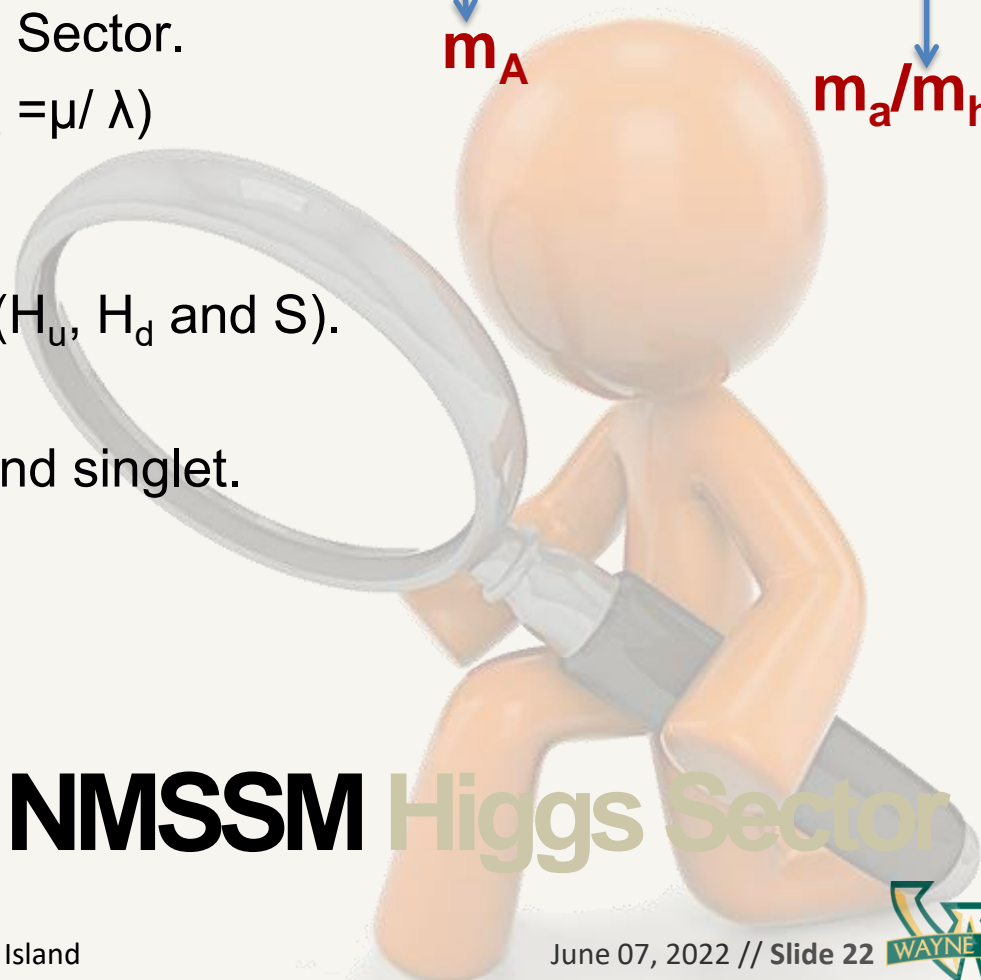
$$W = \lambda S H_u H_d + \frac{\kappa}{3} S^3$$

$$-\mathcal{L}_{\text{soft}} = \lambda A_\lambda S H_u H_d + \frac{1}{3} \kappa A_\kappa S^3$$

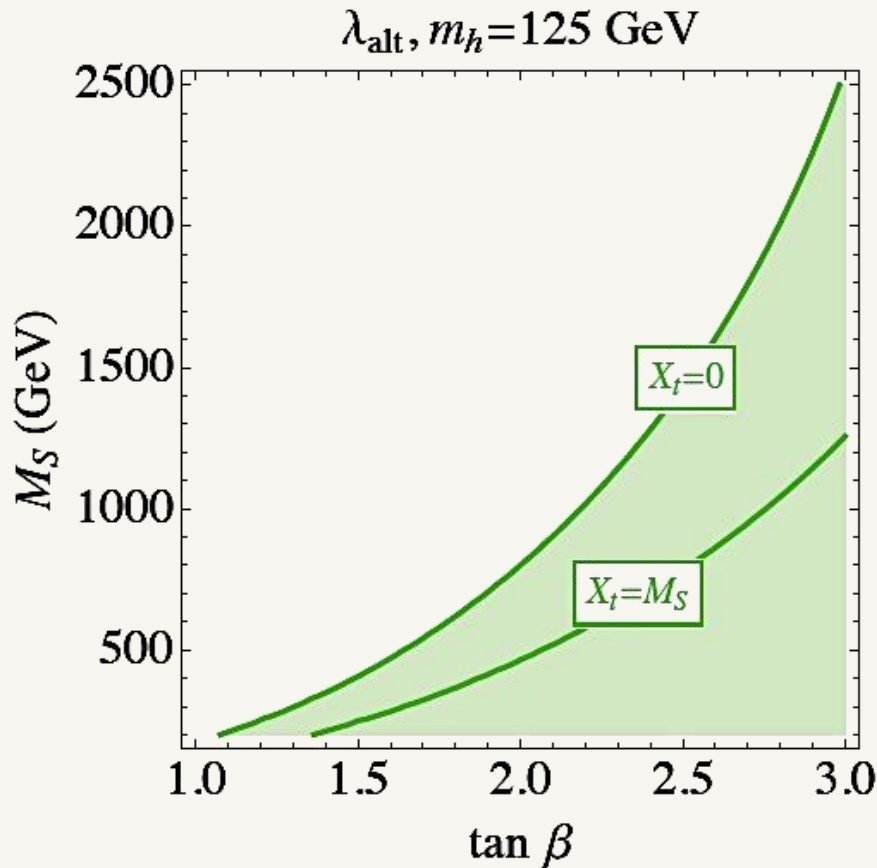
- **2 Doublets (H_u, H_d) + Singlet (S)**
- Singlet couples only to Higgs Sector.
- vevs: $(H_u, H_d, S) = (v_u, v_d, v_S = \mu/\lambda)$
- **3 CP-Even Higgs bosons:**
 - Mixing between all three (H_u, H_d and S).
- **2 CP-Odd Higgs bosons:**
 - Mixtures of “MSSM” m_A and singlet.
- **Charged Higgs bosons**
- **Singlino** mass: $2 \kappa \mu/\lambda$

m_A

m_a/m_{hs}



Something else: Z_3 NMSSM Higgs Sector



Alignment (No-Mixing):

$$m_h^2 \simeq \lambda^2 \frac{v^2}{2} \sin^2 2\beta + M_Z^2 \cos^2 2\beta + \Delta_{\tilde{t}}$$

$$\Delta_{\tilde{t}} = -\cos 2\beta (m_h^2 - M_Z^2)$$

Well Known

- 125 GeV Higgs
 - Tree-level contribution to Higgs mass from λ .
 - $\lambda \sim 0.65-0.7$
- Low $\tan \beta$
- Light Stops

NMSSM-

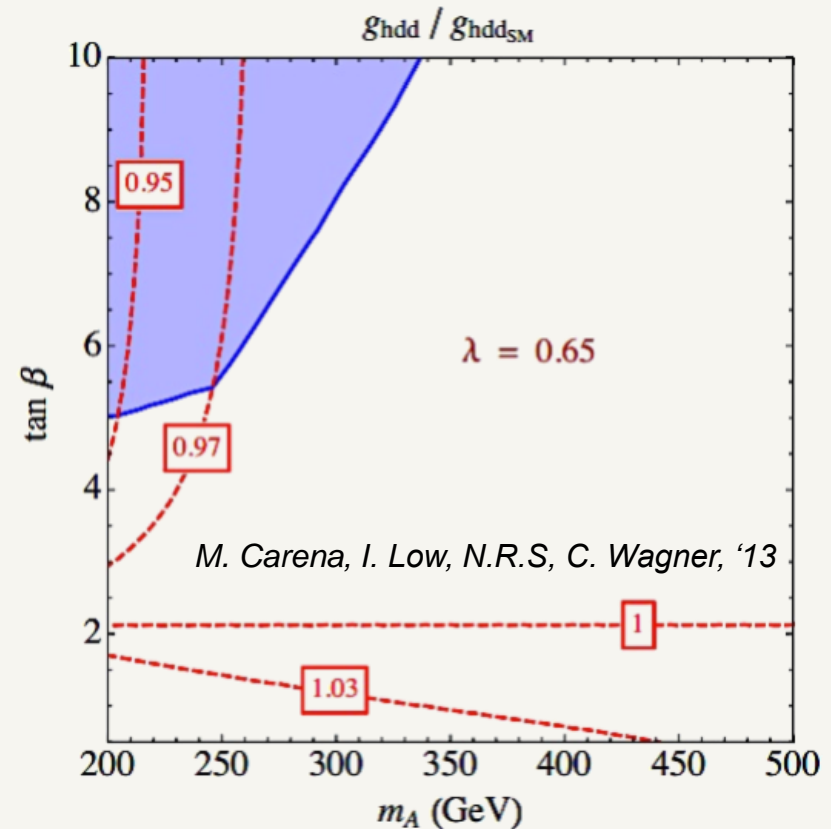
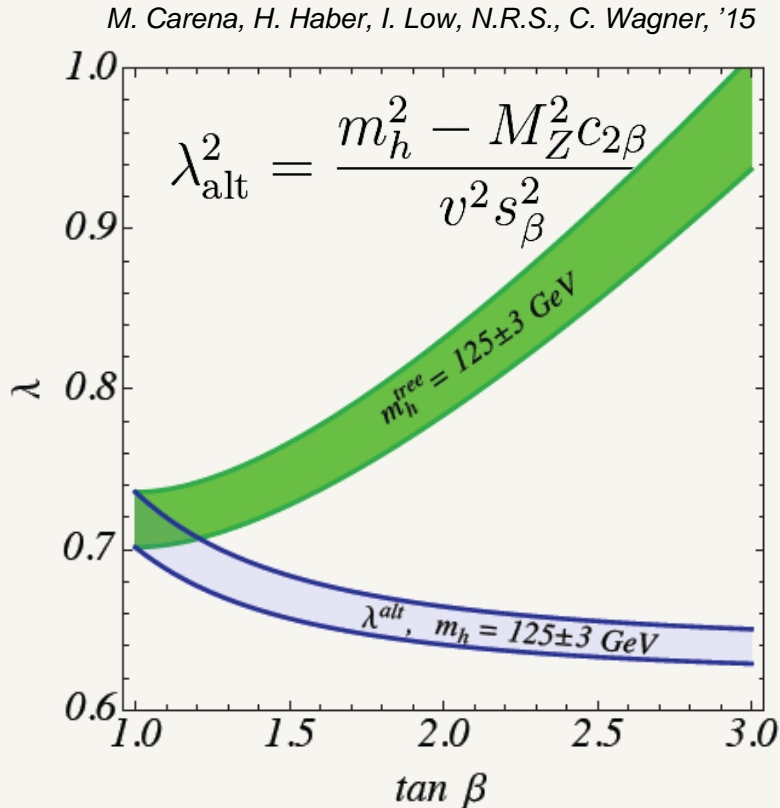
125 GeV Higgs Naturally!

- Perturbative up to GUT scale.

- $\lambda_{\max} \sim 0.7$, $\kappa_{\max} \sim \lambda/2$

Not so well known:

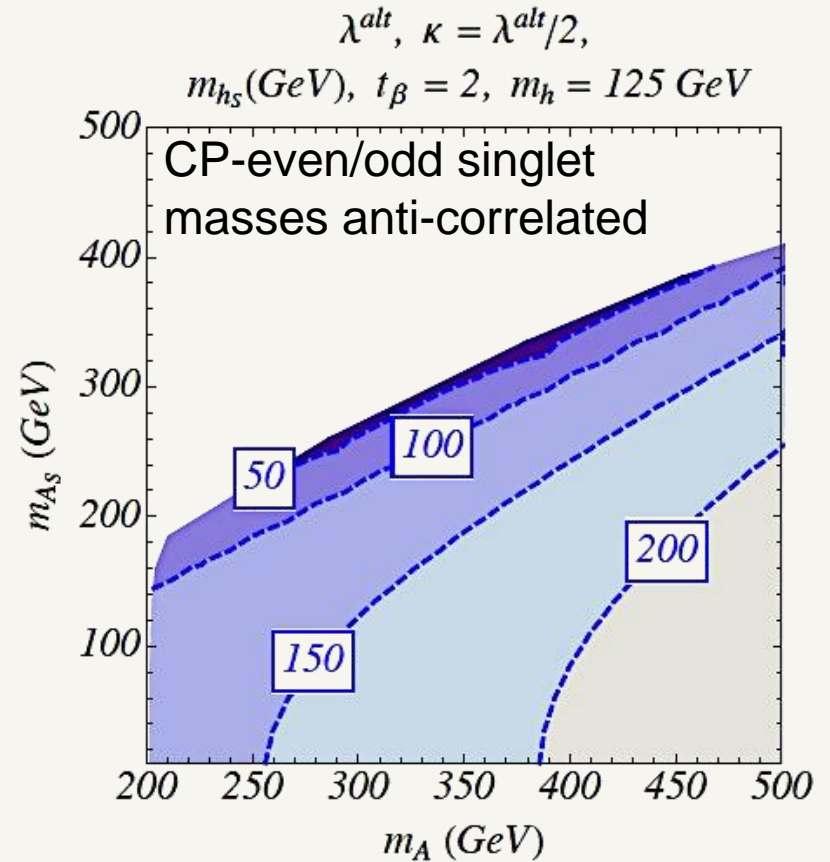
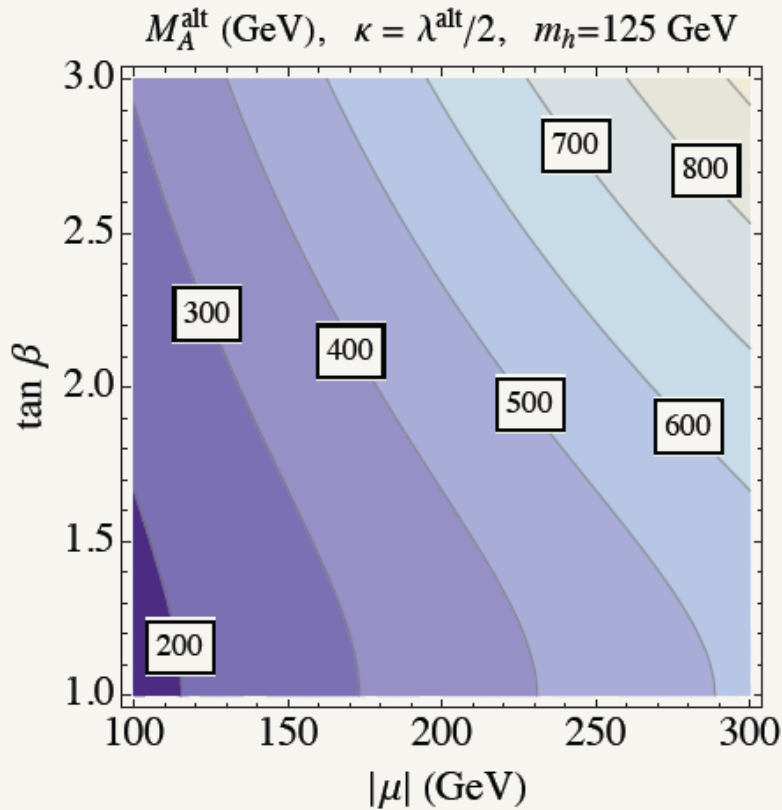
- Excellent Alignment (very little mixing)



Originate dynamically from Higgs compositeness at the GUT scale?

N. Coyle, C. Wagner, '19

SM-Like Higgs Naturally!



$$1 - \frac{m_A^2}{4\mu^2} s_{2\beta}^2 - \frac{\kappa}{2\lambda} s_{2\beta} = 0$$

$h_{125} = H_{\text{SM}}$

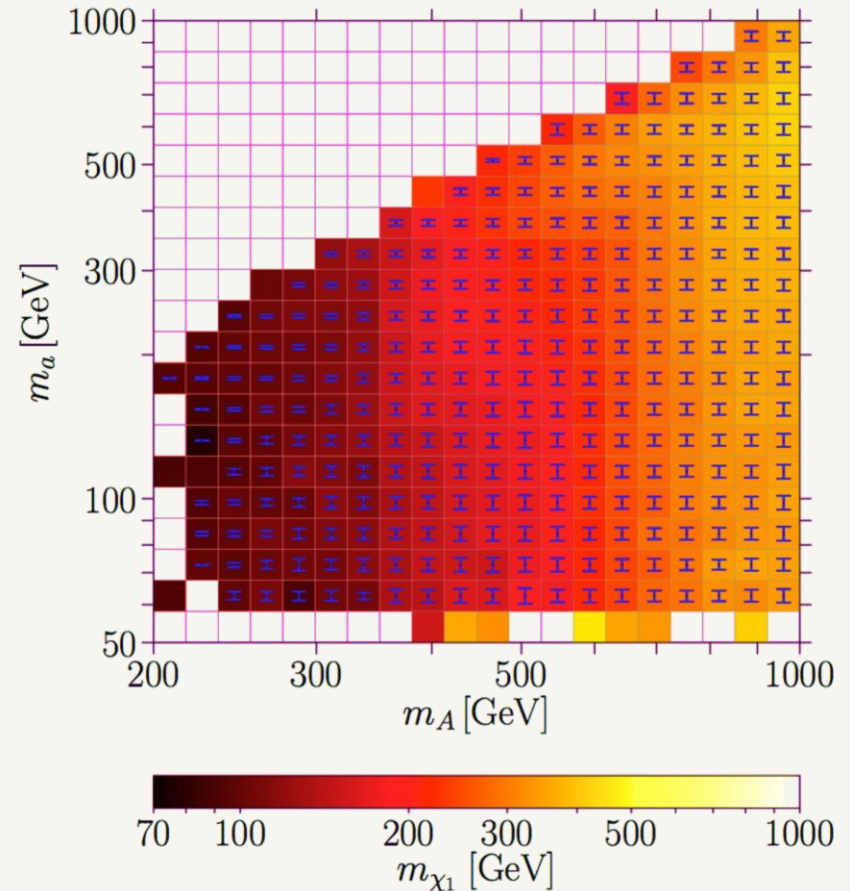
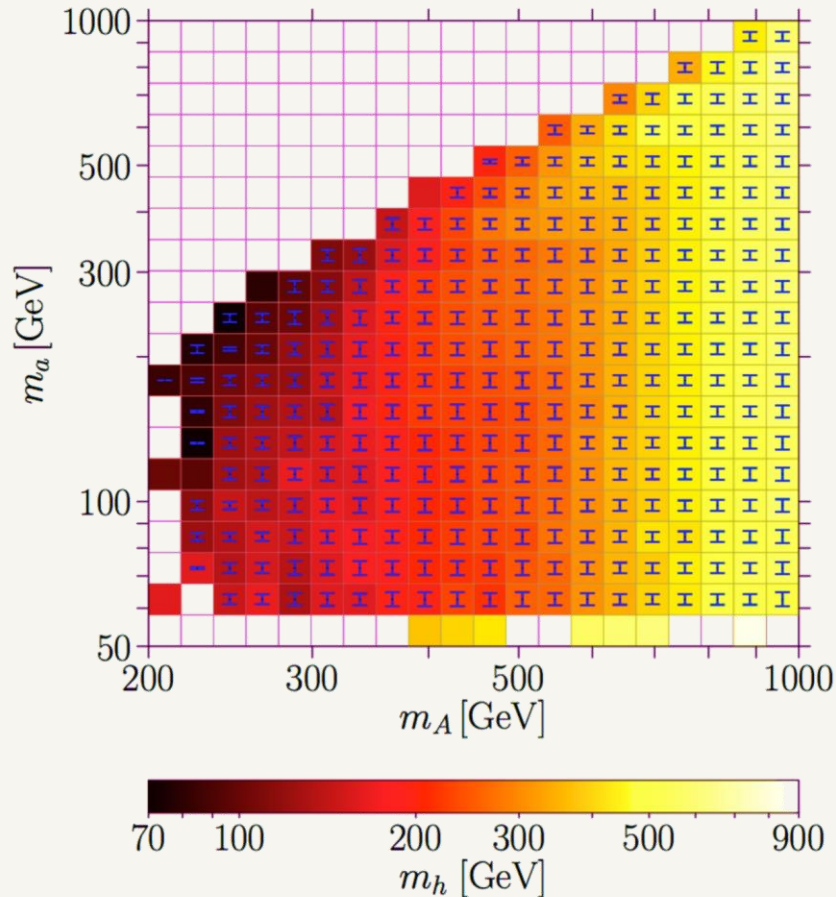
LIGHT SPECTRUM

Singlino: $2 \kappa \mu / \lambda \sim < \mu$

Singlet Alignment

SM - like h_{125} : Mass Correlations!

S. Baum, N.R.S, K. Freese, '19



NMSSMTools scan with consistent h_{125} pheno

W Mass and Alignment – ☹️

Preliminary, M. Carena, I. Low, NRS, C. Wagner, X. Wang

$$g_{hbb}/g_{hbb}^{SM} < 10\%$$

$$\lambda^2 < \frac{0.1m_H^2 + 2m_h^2 + 2M_Z^2}{v^2}$$

$$\Delta M_W \sim 1.2 \cdot 10^6 \frac{\lambda^4}{m_H^2 [\text{GeV}^2]} \text{ MeV.}$$

$$\lambda > \sim 1$$

$$m_H \sim \text{Few hundred GeV}$$

$$\Delta M_W < \underline{3.2} \cdot 10^{-6} m_H^2 [\text{GeV}^2] \left(1 + \frac{4.8 \times 10^5}{m_H^2 [\text{GeV}^2]} \right)^2 \text{ MeV}$$

$$m_H \gg m_{h125}$$

TOO SMALL!!

Singlet Decoupling – Threshold Effects

Preliminary, M. Carena, I. Low, NRS, C. Wagner, X. Wang

Tadpole terms for Singlet: in scalar potential (ξ_S) and Superpotential (ξ_F)

$$m_S^2 \simeq -\frac{\xi_S}{\langle S \rangle} = -\frac{\xi_S \lambda}{\mu}$$

$$m_{h_S}^2 = m_S^2 + 2\xi_F \kappa,$$

$$m_{A_S}^2 = m_S^2 - 2\xi_F \kappa,$$

Decoupling singlets: Replace singlet fields by their equation of motion:

Corrections to the effective quartic couplings at low energies

$$\delta Z_4 \simeq -\lambda^2 \left(\frac{A_\lambda^2}{2m_{h_S}^2} + \frac{A_\lambda^2}{2m_{A_S}^2} \right)$$

$$\delta Z_5 \simeq -\lambda^2 \left(\frac{A_\lambda^2}{2m_{h_S}^2} - \frac{A_\lambda^2}{2m_{A_S}^2} \right)$$

$$\delta Z_6 \simeq \lambda^2 \frac{A_\lambda^2}{t_\beta m_{h_S}^2} - \lambda^2 \frac{\mu A_\lambda}{m_{h_S}^2}$$

W Mass and Alignment – 😊

Preliminary, M. Carena, I. Low, NRS, C. Wagner, X. Wang

$$g_{hbb}/g_{hbb}^{\text{SM}} < 10\%$$

$$\lambda^2 < 0.1 \frac{m_H^2}{v^2} \left(1 + \frac{4.8 \times 10^5}{m_H^2 [\text{GeV}^2]}\right) \times \left(1 - \frac{A_\lambda^2 - A_\lambda \mu t_\beta}{m_{h_S}^2}\right)^{-1}$$

$$\left[\mu \left(A_\lambda + \frac{\kappa}{\lambda} \mu\right) + \lambda \xi_F\right] t_\beta \simeq M_A^2$$

Relationship of A_λ and m_A in the presence of tadpoles.

$$\Delta M_W < 3.2 \cdot 10^{-6} m_H^2 [\text{GeV}^2] \left(1 + \frac{4.8 \times 10^5}{m_H^2 [\text{GeV}^2]}\right)^2 \frac{\left(1 - \frac{A_\lambda^2}{m_{h_S}^2}\right) \left(1 - \frac{A_\lambda^2}{m_{A_S}^2}\right)}{\left(1 - \frac{A_\lambda^2 - A_\lambda \mu t_\beta}{m_{h_S}^2}\right)^2}$$

For eg:

$$m_H \sim 500 \text{ GeV}, \quad A_\lambda \sim \mu \text{ tb}, \quad A_\lambda/m_S \text{ large,} \\ + \text{ Stop contributions!}$$

Thank You!



SM works beautifully...

But **MANY** puzzles remain.

UV physics? **BREAK** the SM!
May be systematics, but may be hints for BSM physics!

What are the right questions?

???

Data + Theory:
Where to look next!

Absence of Evidence \neq Evidence of Absence

Data driven age: Collider + Precision + Astrophysical Probes

~~“May we live
in interesting times.”~~

BACK UP SLIDES



- Interaction basis: (H_u , H_d , S)
 - H_u : Couples only to up-type fermions
 - H_d : Couples only to down-type fermions
 - S : Only couples to Higgs

$$\langle H_u \rangle = v_u$$

$$\langle H_d \rangle = v_d$$

$$t_\beta = v_u/v_d$$

$$\langle S \rangle = \mu/\lambda$$

CP-Even Higgs Bases

- Interaction basis: (H_u, H_d, S)
 - H_u : Couples only to up-type fermions
 - H_d : Couples only to down-type fermions
 - S : Only couples to Higgs

$$\begin{aligned} \langle H_u \rangle &= v_u \\ \langle H_d \rangle &= v_d \\ t_\beta &= v_u/v_d \\ \langle S \rangle &= \mu/\lambda \end{aligned}$$

- “Extended” Higgs basis: (H_{NSM}, H_{SM}, S)
 - H_{NSM} : (down, up, V) = ($y_d t_\beta, y_u/t_\beta, 0$)
 - H_{SM} : (down, up, V) = (y_d, y_u, g_{hVV})

Only SM state couples to WW or ZZ!!

$$\begin{aligned} \langle H_{NSM} \rangle &= 0 \\ \langle H_{SM} \rangle &= v \end{aligned}$$

CP-Even Higgs Bases



- Interaction basis: (H_u, H_d, S)
 - H_u : Couples only to up-type fermions
 - H_d : Couples only to down-type fermions
 - S : Only couples to Higgs

$$\begin{aligned} \langle H_u \rangle &= v_u \\ \langle H_d \rangle &= v_d \\ t_\beta &= v_u/v_d \\ \langle S \rangle &= \mu/\lambda \end{aligned}$$

- “Extended” Higgs basis: (H_{NSM}, H_{SM}, S)
 - H_{NSM} : (down, up, V) = $(y_d t_\beta, y_u/t_\beta, 0)$
 - H_{SM} : (down, up, V) = (y_d, y_u, g_{hVV})

$$\begin{aligned} \langle H_{NSM} \rangle &= 0 \\ \langle H_{SM} \rangle &= v \end{aligned}$$

- Mass basis: (H^3, H^2, H^1)

$$H^i = S_{NSM}^i H_{NSM} + S_{SM}^i H_{SM} + S_S^i H_S$$

CP-Even Higgs Bases

- Interaction basis: (H_u, H_d, S)
 - H_u : Couples only to up-type fermions
 - H_d : Couples only to down-type fermions
 - S : Only couples to Higgs

$$\begin{aligned} \langle H_u \rangle &= v_u \\ \langle H_d \rangle &= v_d \\ t_\beta &= v_u/v_d \\ \langle S \rangle &= \mu/\lambda \end{aligned}$$

- “Extended” Higgs basis: (H_{NSM}, H_{SM}, S)
 - H_{NSM} : (down, up, V) = $(y_d t_\beta, y_u/t_\beta, 0)$
 - H_{SM} : (down, up, V) = (y_d, y_u, g_{hVV})

$$\begin{aligned} \langle H_{NSM} \rangle &= 0 \\ \langle H_{SM} \rangle &= v \end{aligned}$$

- Mass basis: $(H_3, H_2, H_1) \rightarrow (H, h_{125}, h)$

$$H_i = S_i^{NSM} H_{NSM} + S_i^{SM} H^{SM} + S_i^S H_S$$

CP-Even Higgs Bases

CP-odd mix similarly:

$$A_i = P_i^{NSM} A_{NSM} + P_i^S A_S$$

