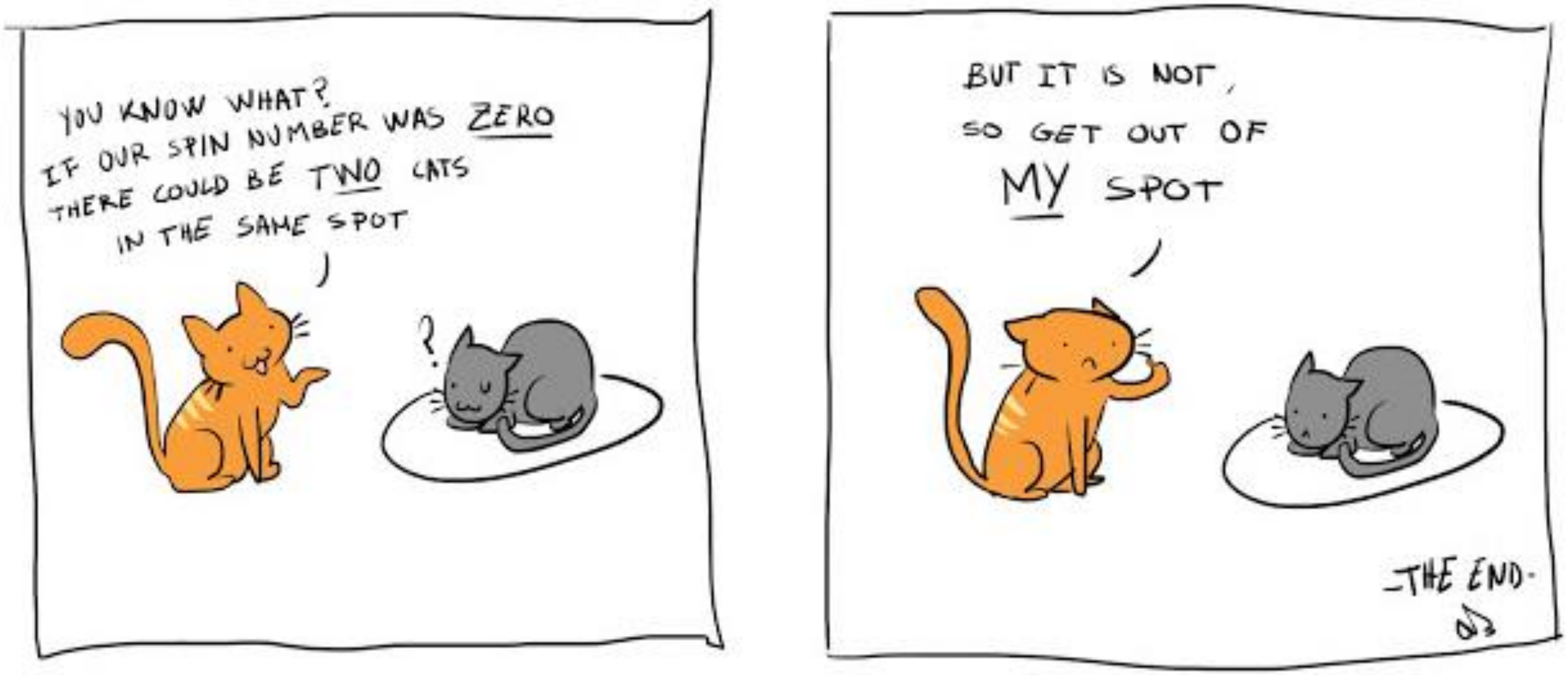
Dark Matter scattering in low threshold detectors

Simon Knapen

LBNL



http://dingercatadventures.blogspot.com/2012/08/

SK, J. Kozaczuk, T. Lin: arXiv 2104.12786, 2101.08275, 2011.09496
B. Campbell-Deem, SK, T. Lin, E. Villarama: 2205.02250

Light Dark matter direct detection

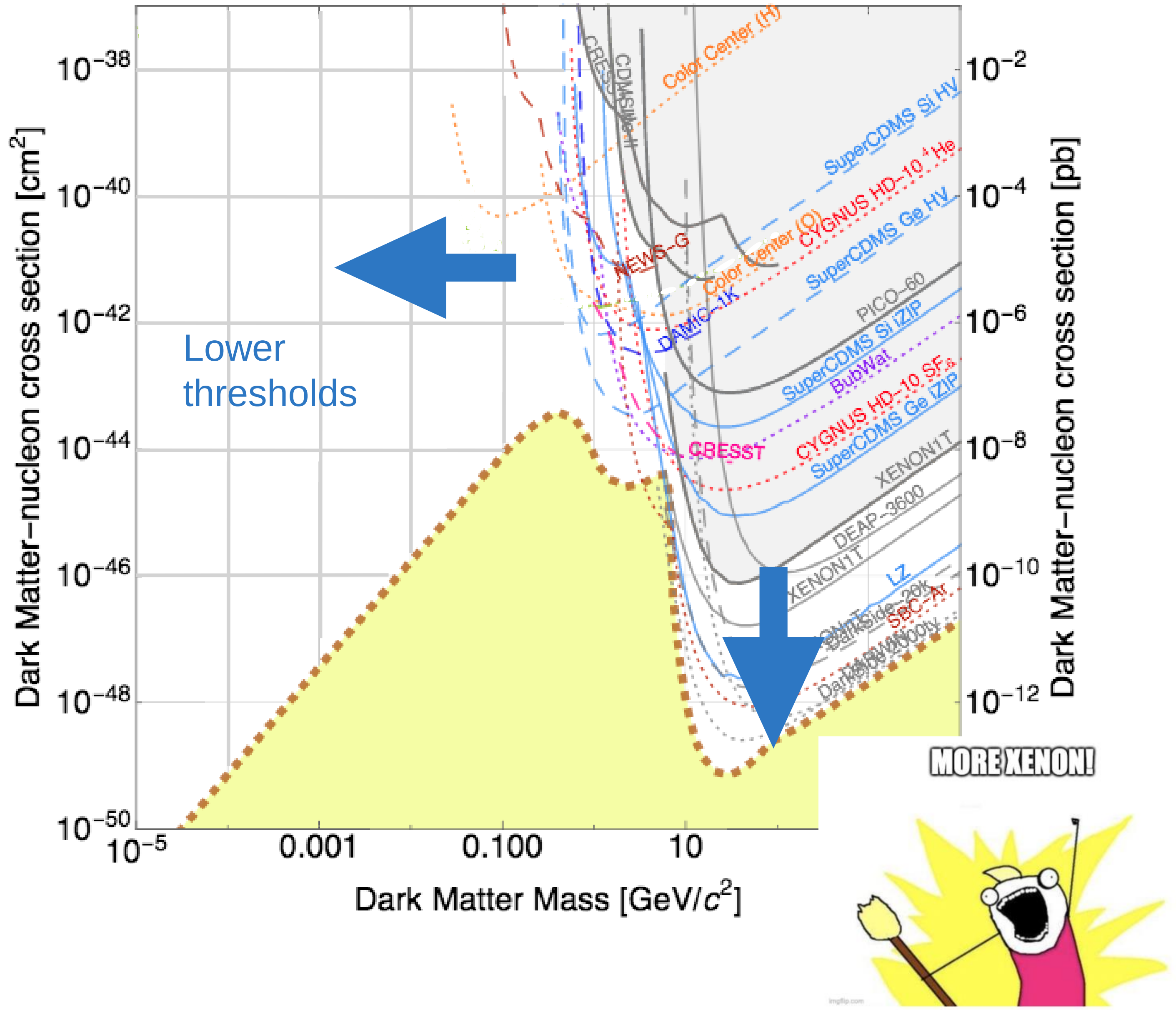
What do we need?

Experiment:

1. Ultra-low threshold calorimeters
2. Single electron detectors

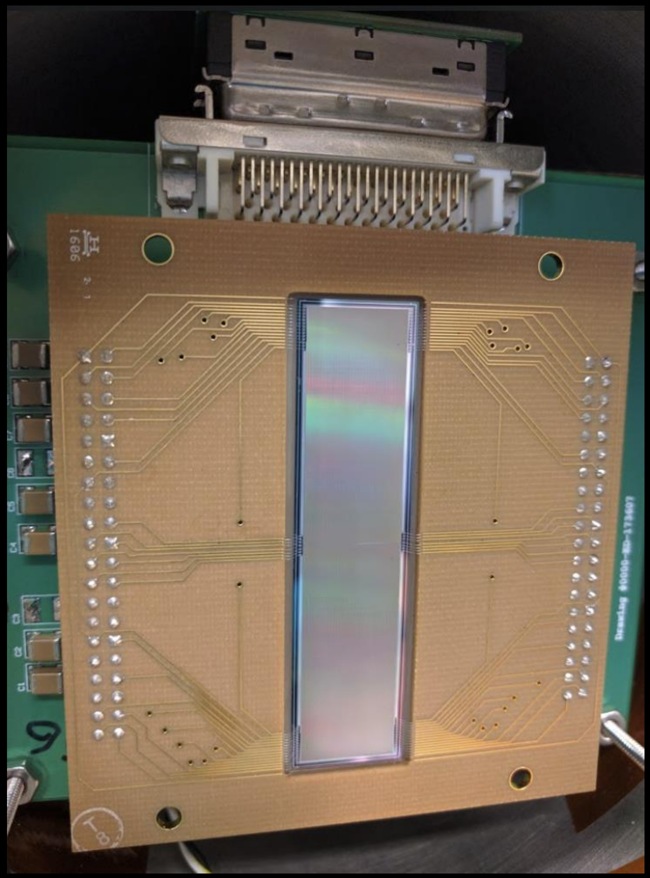
Theory:

1. Models
   (constraints are complicated)
2. Rate calculations
   (Collective effects important)
3. Background predictions

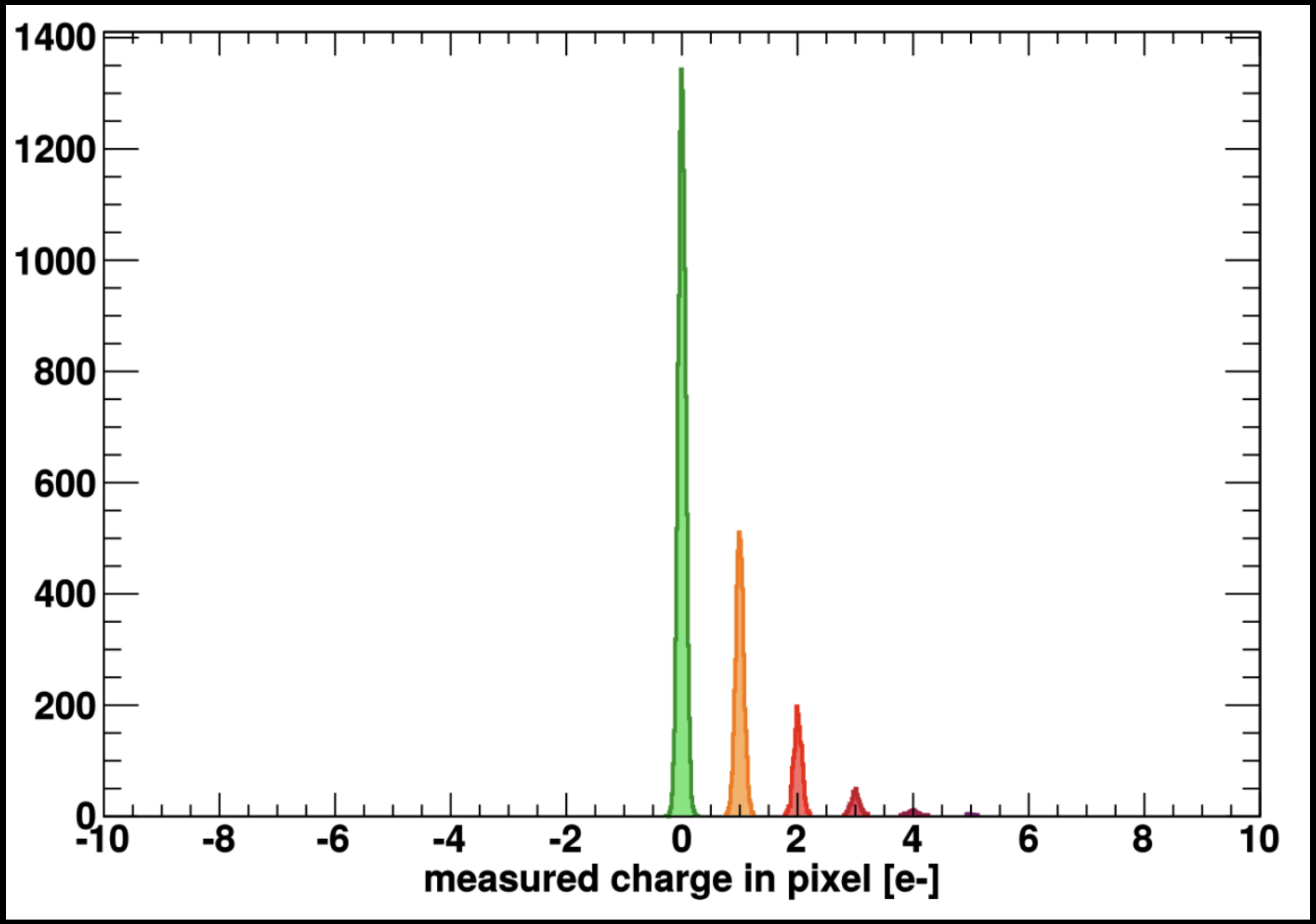


Electron detectors

Skipper CCD



Exquisite charge resolution



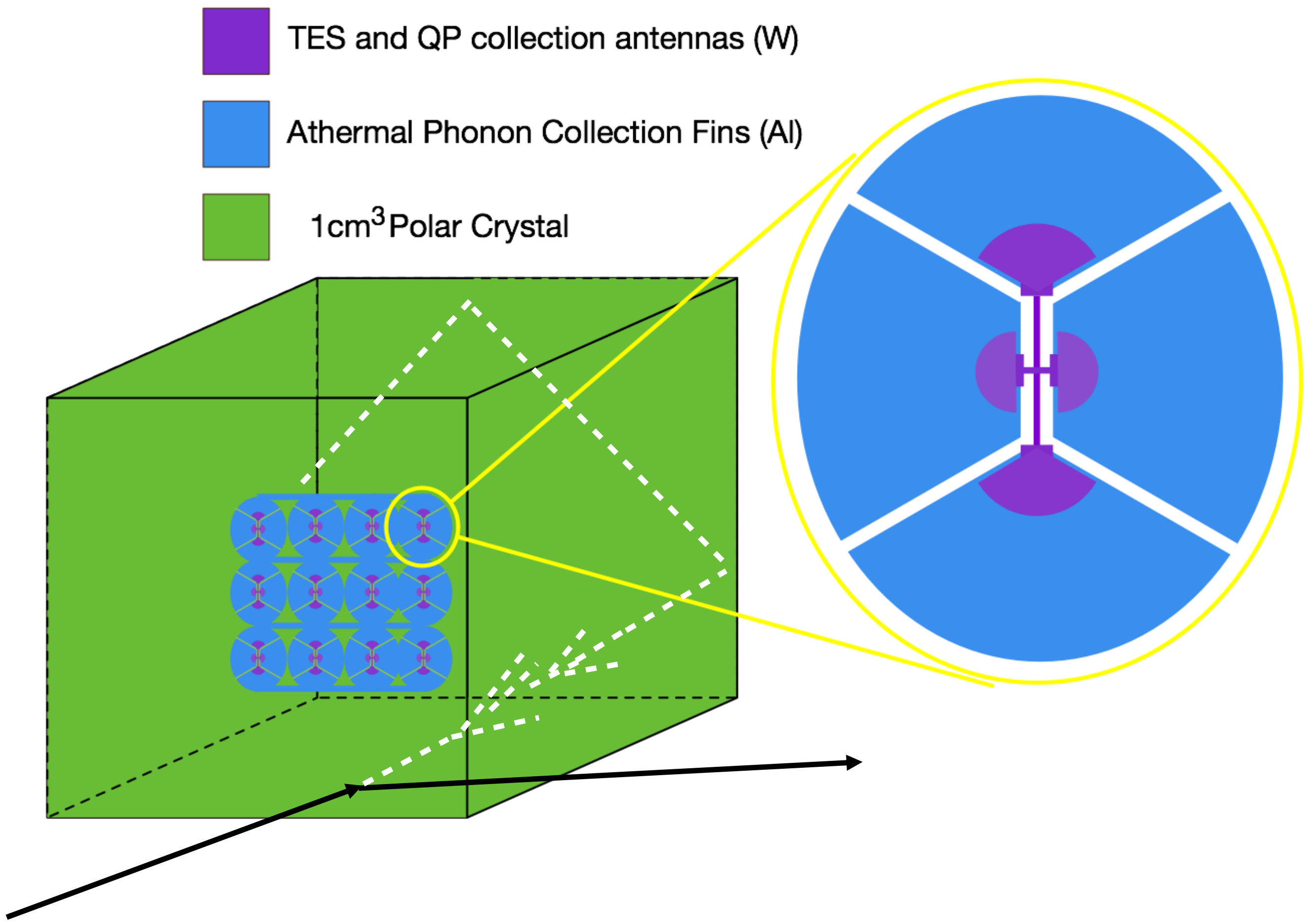
Sensitivity to single e- excitations has already been demonstrated

SENSEI already has 50g-day exposure in shallow underground site

See also DAMIC, superCDMS

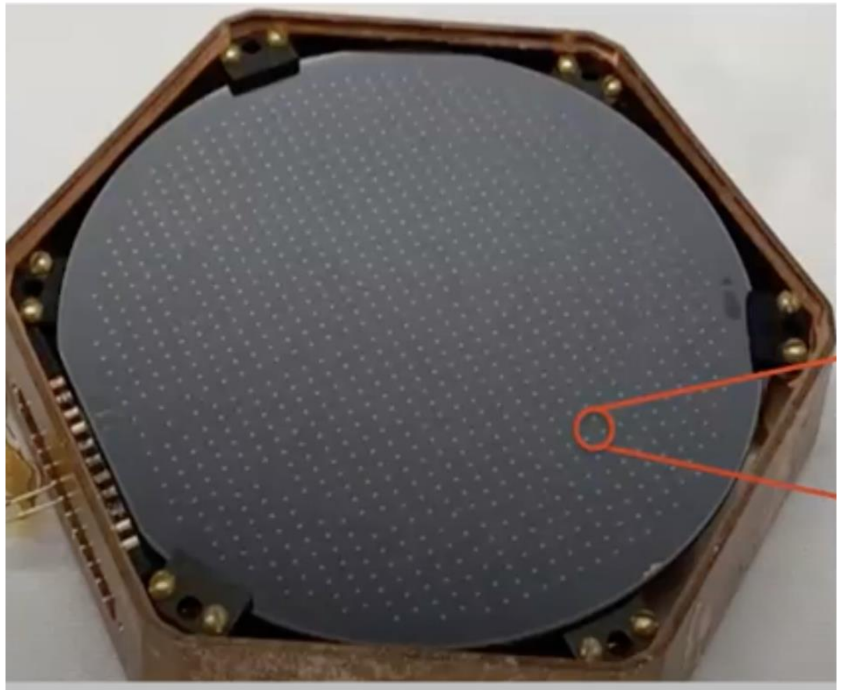
Phonon Detectors

SPICE conceptual design



SPICE / HeRALD

4 eV energy resolution



sub-eV sapphire detector being tested

Lead by Matt Pyle (UC Berkeley)

Wei will talk about Helium / HeRALD

The need for theory

Need to use the right EFT to calculate rates
& match it to the UV theory

m_X

M_{pl}

TeV

GeV

MeV

keV

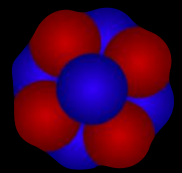
$q \sim v\,\mu$ ($v \sim 10^{-3}$)

10^2 MeV — 10^{-4} Å

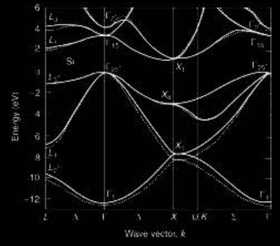
MeV — 10^{-2} Å

keV — 10 Å

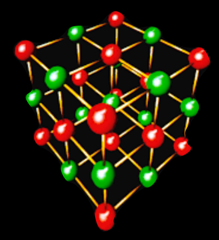
eV — 10^4 Å



Nuclear recoil



Electronic structure

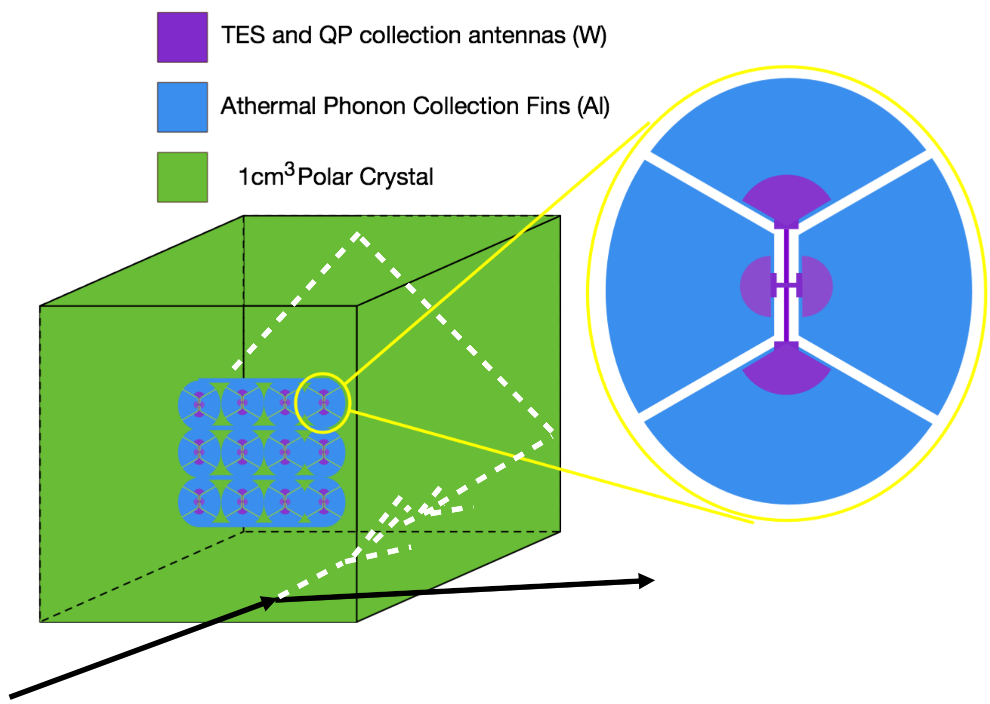


Phonon excitations

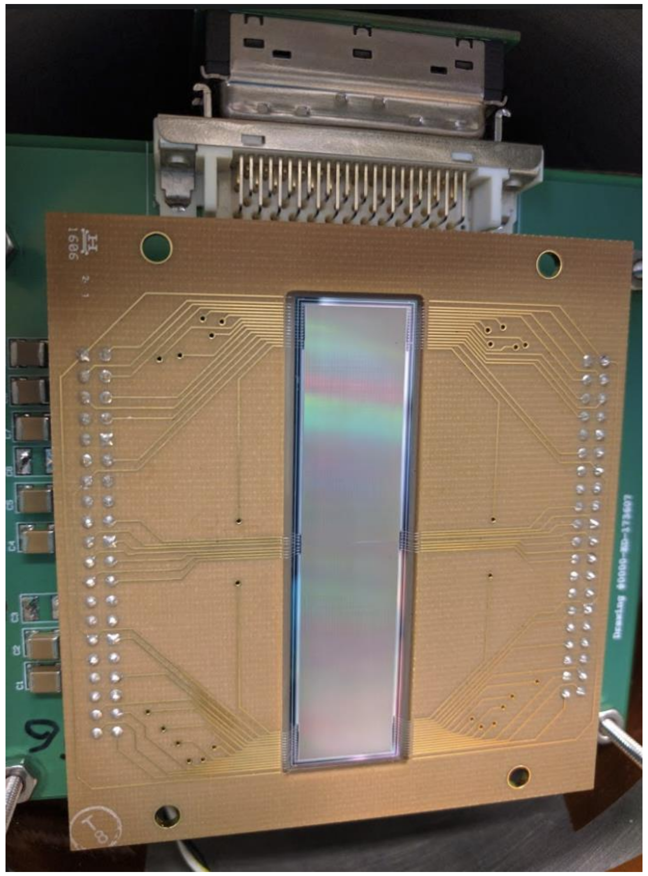
Long term goal: ~ 30% precision on all relevant processes

Calculations needed

Phonon signals



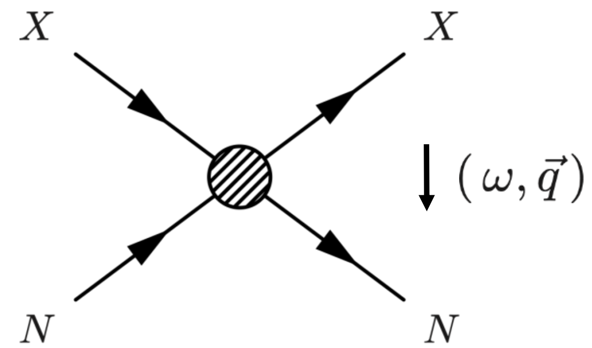
Electronic signals



Phonon EFT

Nuclear recoil

$$\omega = \frac{q^2}{2m_N}$$

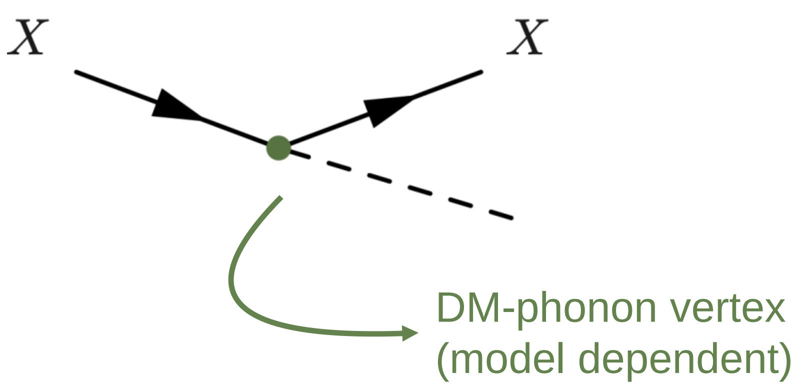


Phonon regime

$$q \ll \sqrt{2m_N\omega} \longrightarrow$$

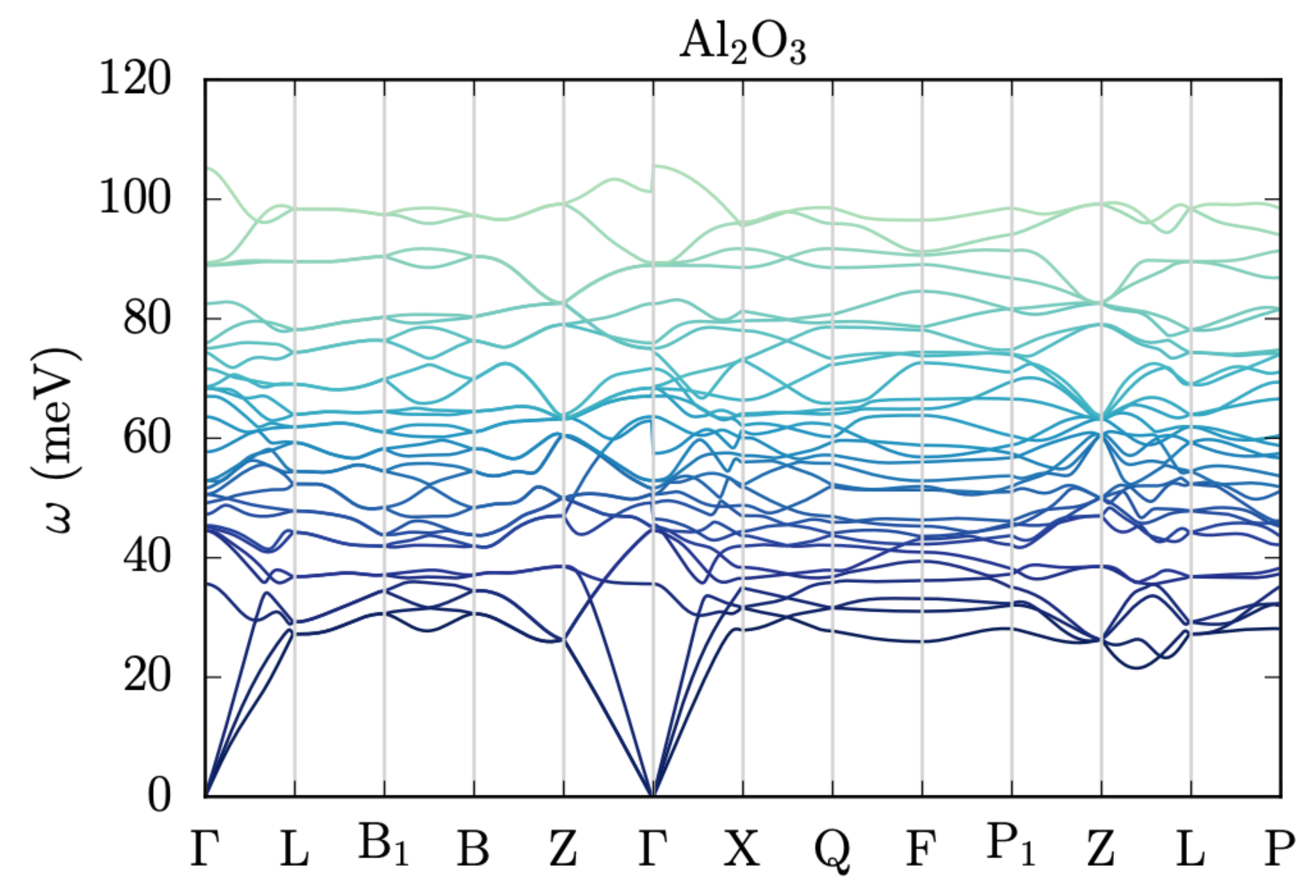
Momentum exchange is a good expansion parameter
(phonons are goldstones) similar to chiral perturbation theory)

LO



$$\mathcal{O}(q),\ \mathcal{O}(q^2) \text{ or } \mathcal{O}(q^4)$$

(Depends on DM model & phonon branch)

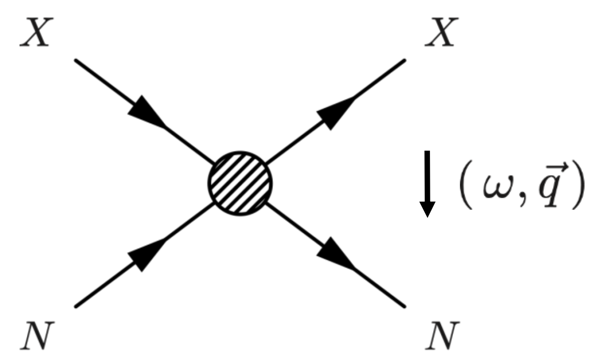


SK, T. Lin, M. Pyle, K. Zurek: 1712.06598
S. Griffin, SK, T. Lin, M. Pyle, K. Zurek: 1807.10291

Phonon EFT

Nuclear recoil

$$\omega = \frac{q^2}{2m_N}$$



Phonon regime

$$q \ll \sqrt{2m_N\omega}$$ →

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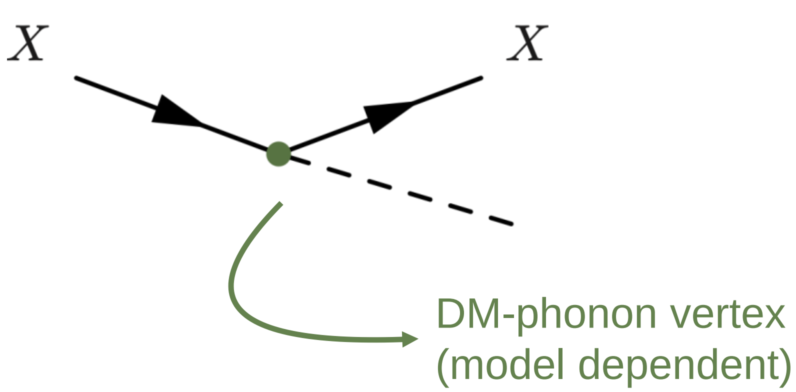

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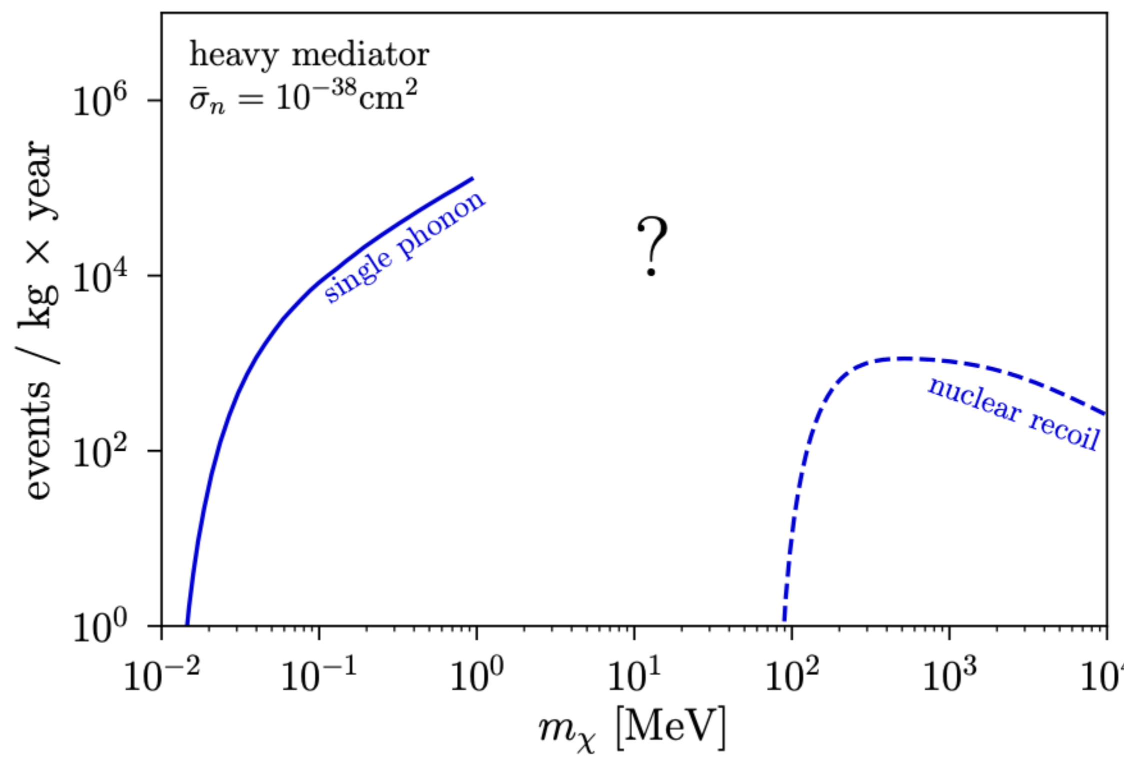
(Depends on DM model & phonon branch)

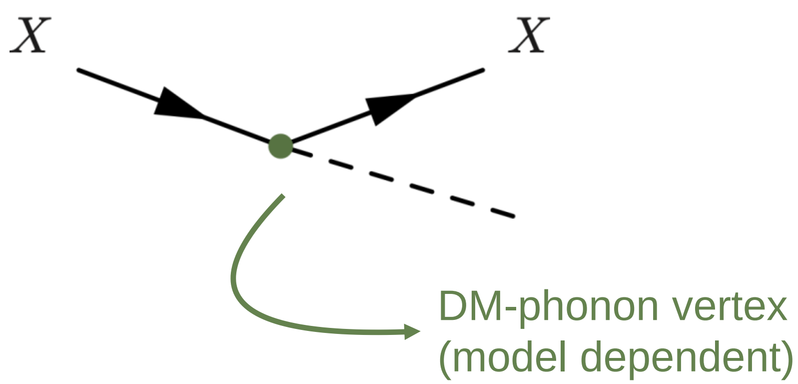


SK, T. Lin, M. Pyle, K. Zurek: 1712.06598
S. Griffin, SK, T. Lin, M. Pyle, K. Zurek: 1807.10291

LO insufficient for m_χ > 1 MeV!

Phonon EFT

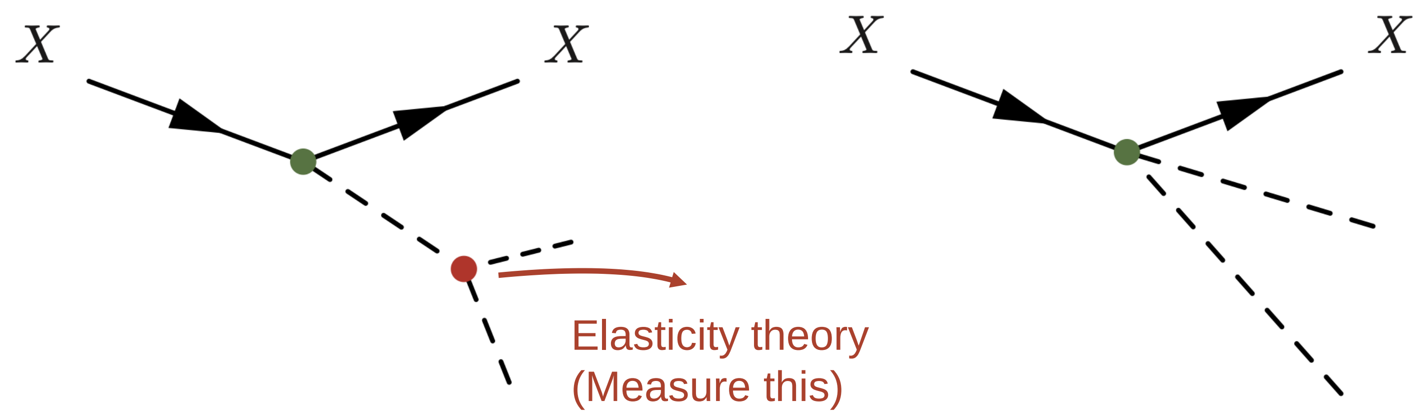
LO



$\mathcal{O}(q),\ \mathcal{O}(q^2)$ or $\mathcal{O}(q^4)$

(Depends on DM model & phonon branch)

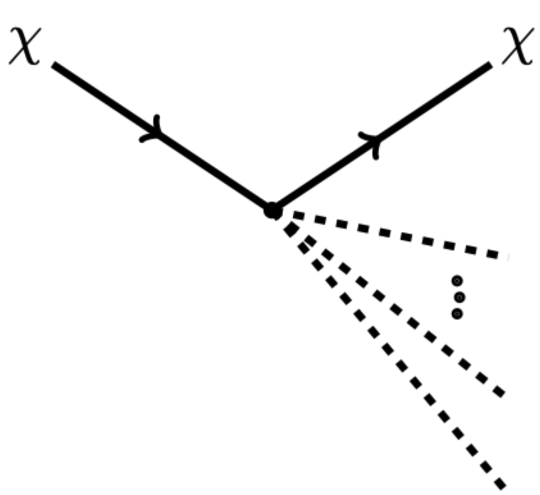
NLO



$\mathcal{O}(q^4)$

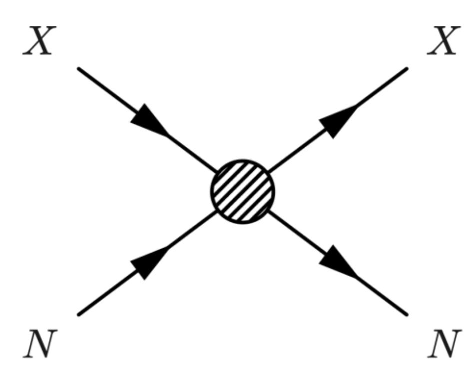
B. Campbell-Deem, P. Cox, SK, T. Lin, T. Melia : [1911.03482](1911.03482)

N^{n}LO



$\mathcal{O}(q^{2n})$

N$^\infty$LO = nuclear recoil



$$\sim \delta\left(\omega - \frac{q^2}{2m_N}\right)$$

Brian Campbell-Deem (UCSD)

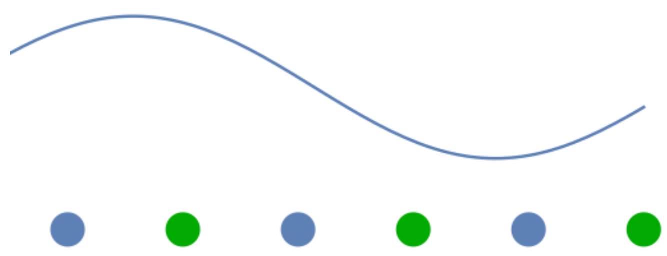
Ethan Villarama (UCSD)

B. Campbell-Deem, SK, T. Lin, E. Villarama: [2205.02250](2205.02250)

Approximations

1. Incoherent approximation

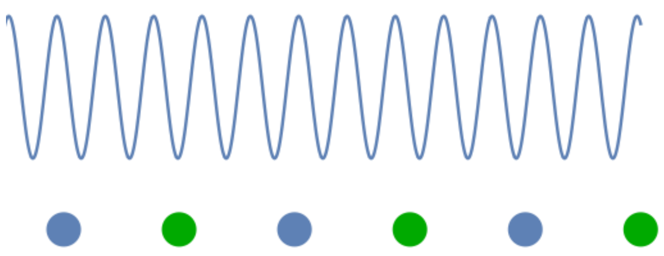
Low q



Interference is critical

(e.g. Bragg diffraction)

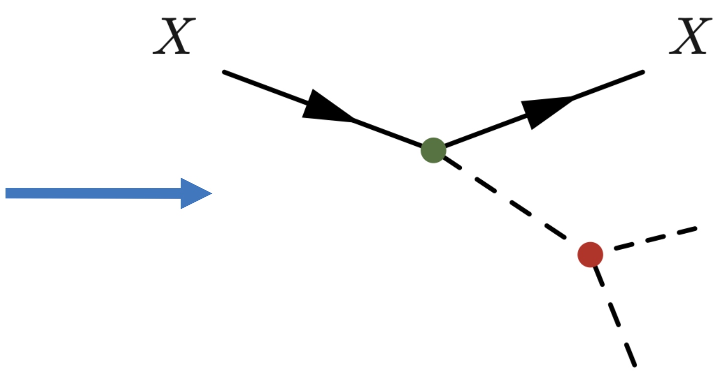
High q



Interference can be neglected

2. Anharmonic approximation

Neglect these beyond leading order



3. Isotropic approximation

Results

d: labels atoms (e.g. Ga and As)
n: number of phonons

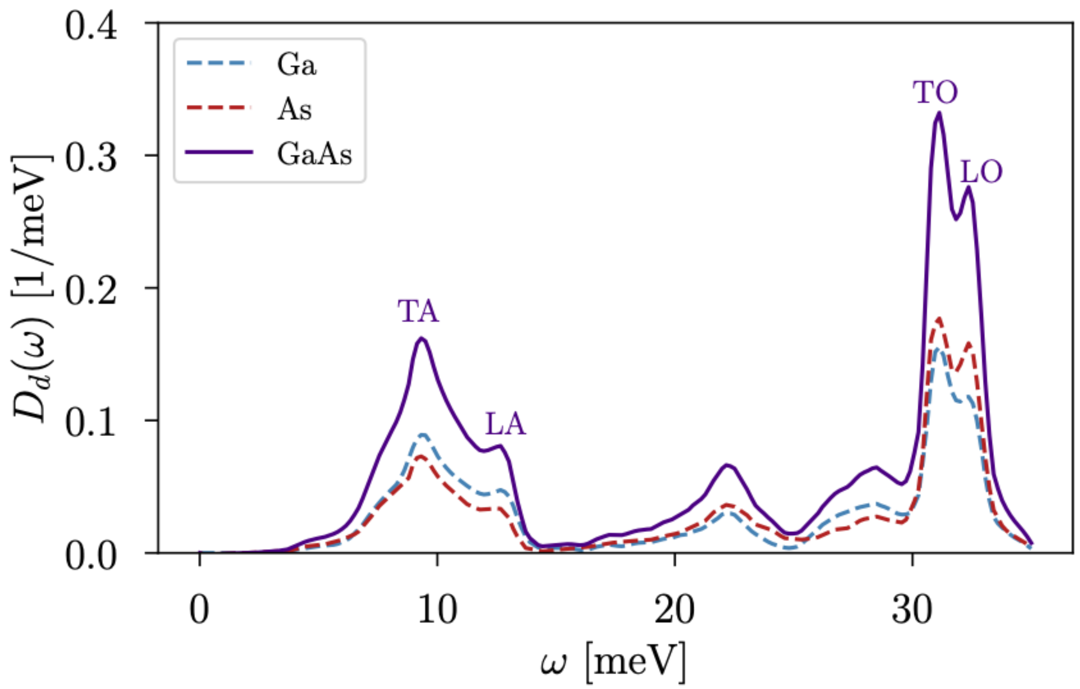
<u>All orders result</u>

$$\frac{d\sigma}{d^3\mathbf{q}d\omega} \sim \sum_d^{\mathrm{n}} A_d^2 e^{-2W_d(\mathbf{q})} \sum_n \left(\frac{q^2}{2m_d}\right)^n \frac{1}{n!} \left(\prod_{i=1}^n \int d\omega_i \frac{D_d(\omega_i)}{\omega_i}\right) \delta\left(\sum_j \omega_j - \omega\right).$$

Partial density of states

$q \gg \sqrt{2\omega m_d}$

$$\frac{d\sigma}{d^3\mathbf{q}d\omega} \sim \sum_d^{\mathrm{n}} A_d^2 \sqrt{\frac{2\pi}{\Delta_d^2}} \exp\left(-\frac{\left(\omega - \frac{q^2}{2m_d}\right)^2}{2\Delta_d^2}\right)$$



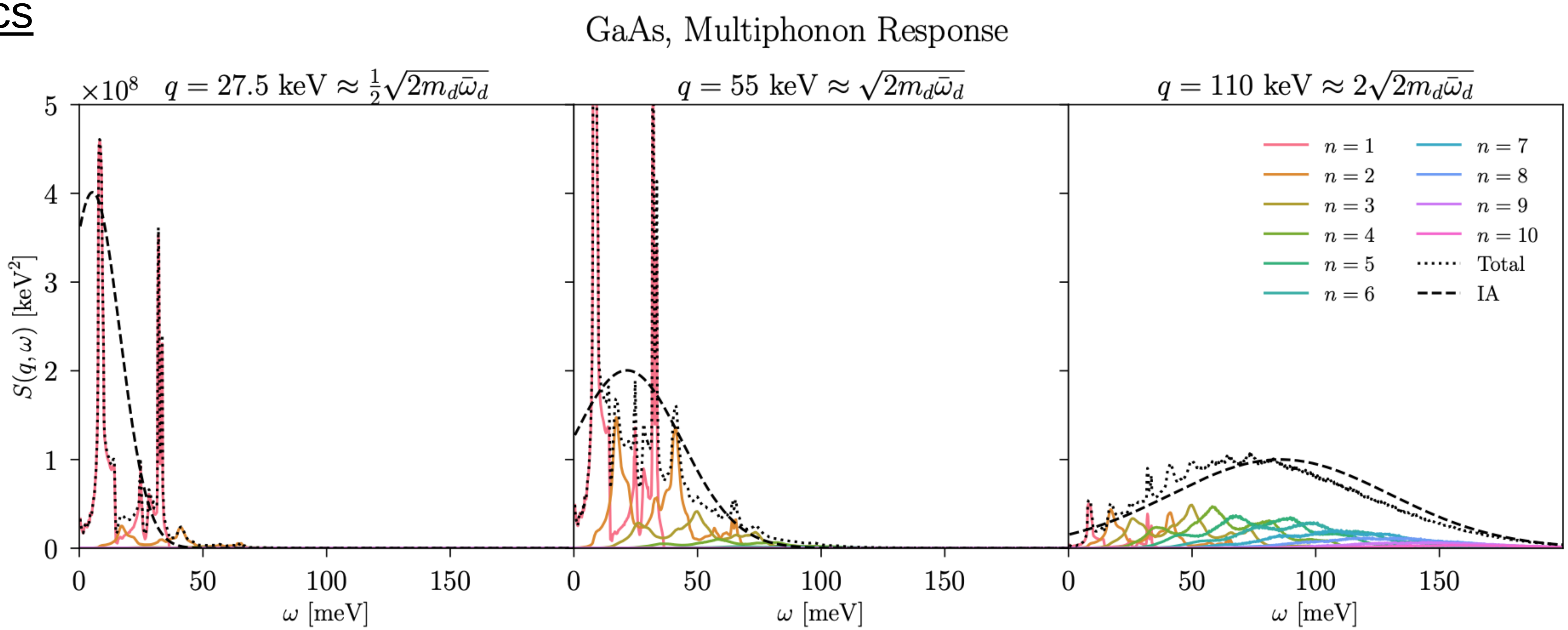
$q \ggg \sqrt{2\omega m_d}$

$$\frac{d\sigma}{d^3\mathbf{q}d\omega} \sim \sum_d^{\mathrm{n}} A_d^2 \times \delta\left(\omega - \frac{q^2}{2m_d}\right)$$

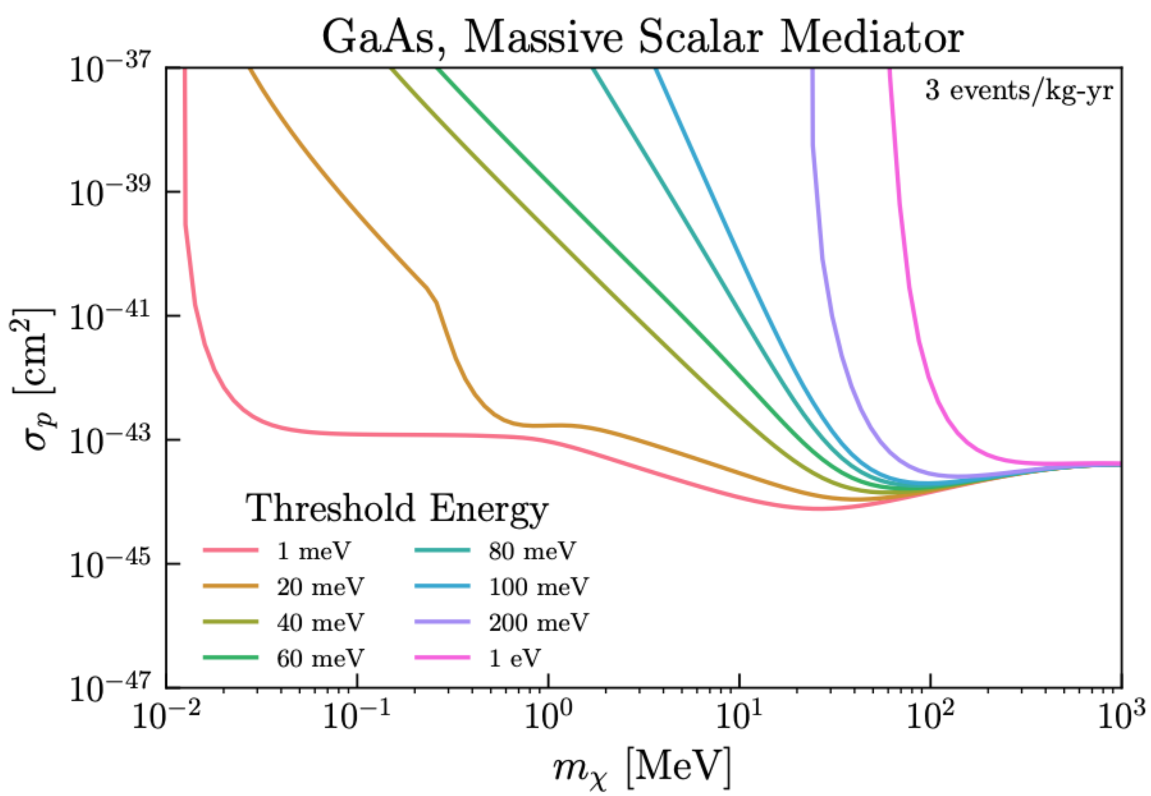
Free nuclear recoil limit

Results

Numerics



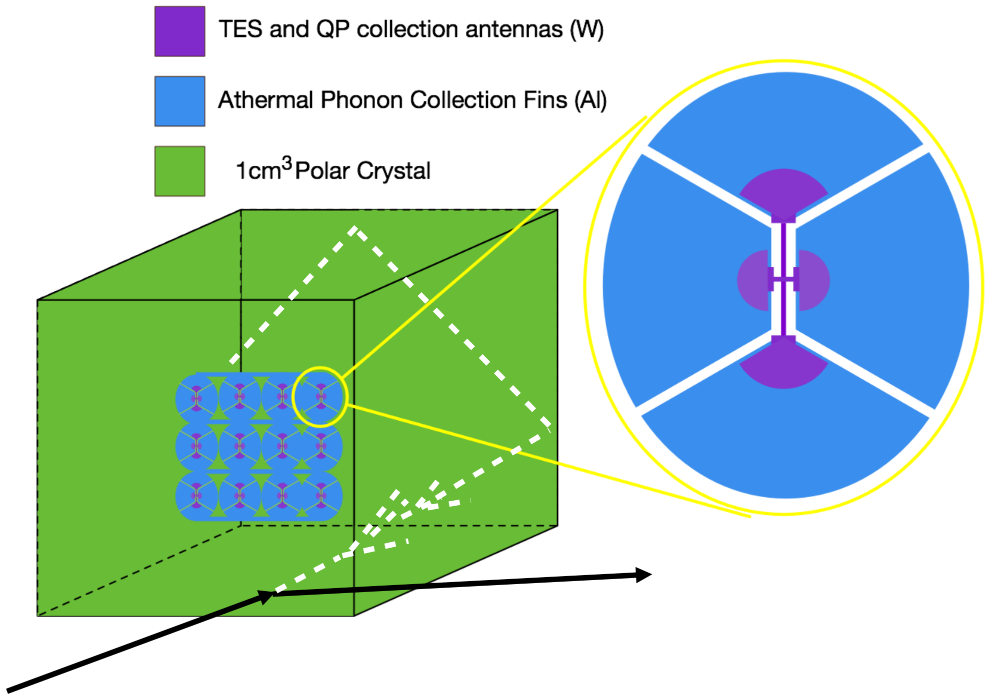
Cross section curves



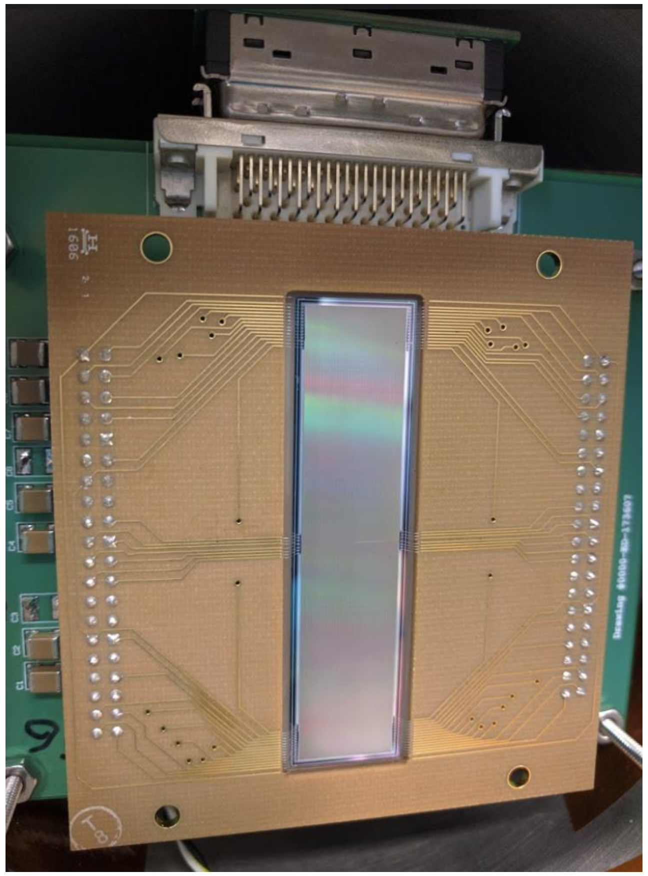
First full results for all m_χ and arbitrary thresholds!

Calculations needed

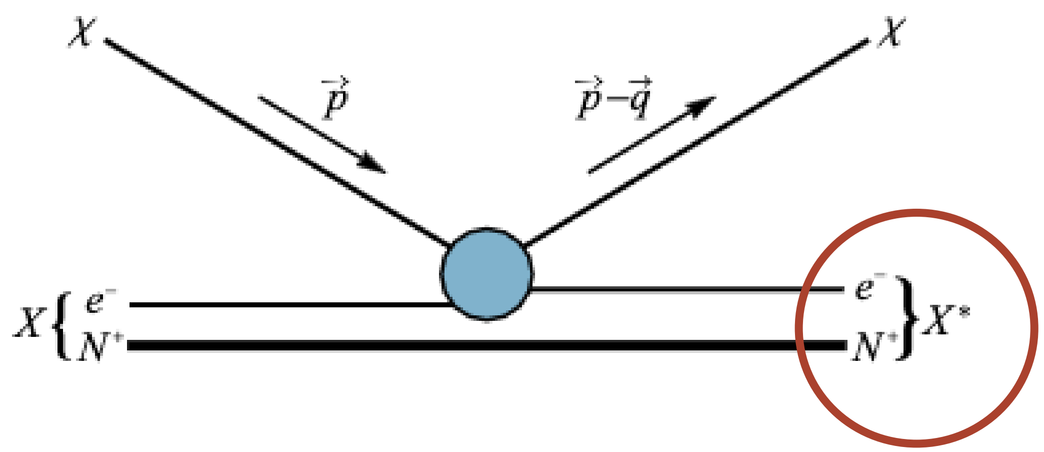
Phonon signals



Electronic signals



Electrons are underrated



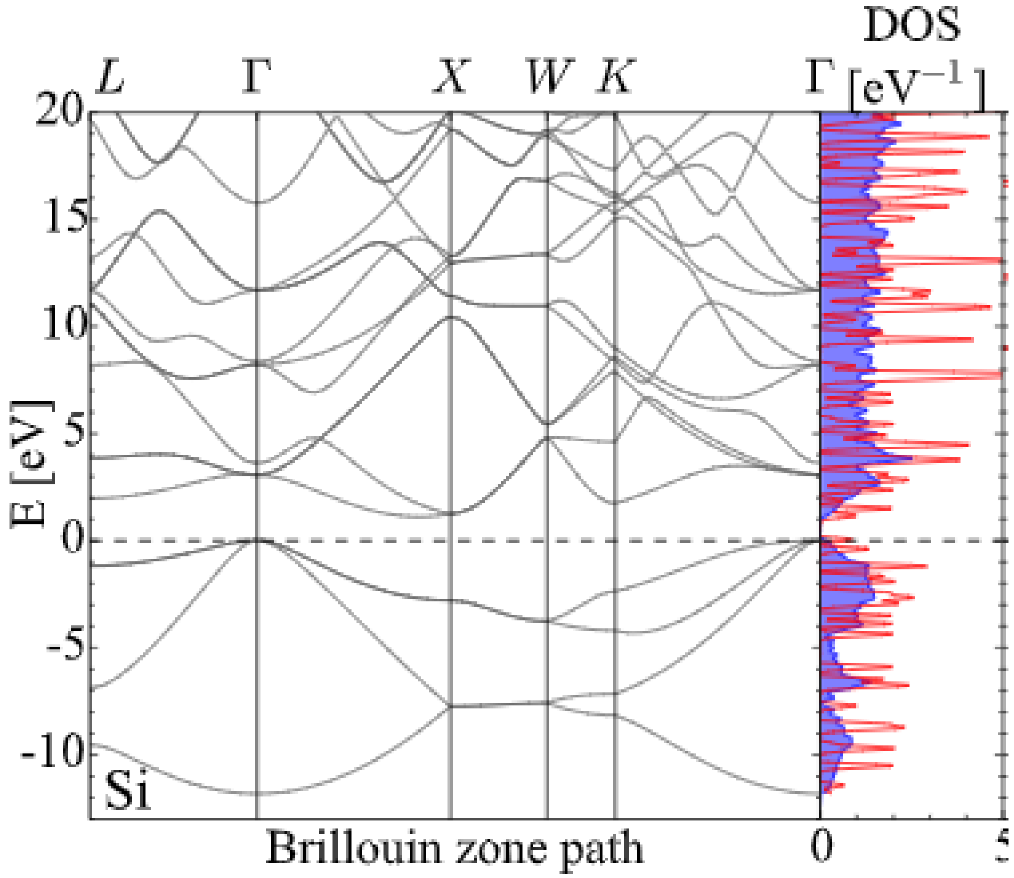
e⁻ are not free

e⁻ are not at rest

e⁻ are not localized

e⁻ are not alone

⟶ screening



Essig et. al. 1509.01598

Problem: Calculate wave functions & stick them into matrix element calculation

Essig et. al. 1509.01598

Equivalent problem: Calculate rate of energy dissipation in the crystal

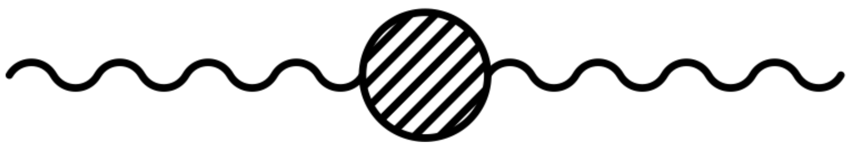
SK, J. Kozaczuk, T. Lin: arXiv 2101.08275
Y. Hochberg et. al. Arxiv: 2101.08263

Schematic argument

Coulomb potential in a dielectric:

$$H = eQ_\chi \int \frac{d^3\mathbf{k}}{(2\pi)^2} \frac{1}{\epsilon(\mathbf{k},\omega)} \frac{e^{i\mathbf{k}\cdot\mathbf{x}}}{k^2}$$

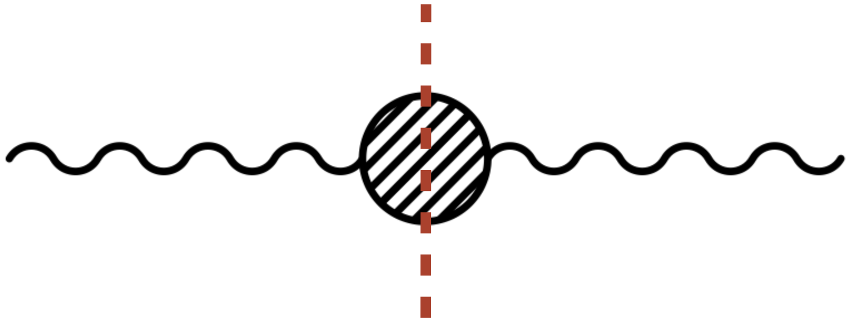
In QFT language:



$$\sim \frac{1}{\epsilon(\mathbf{k},\omega)} \frac{1}{k^2}$$

(Non-relativistic limit)

We are interested in energy dissipation:



$$\sim \mathrm{Im}\left[\frac{-1}{\epsilon(\mathbf{k},\omega)}\right]$$

“Energy Loss Function” (ELF)

(Exact derivation in the back-up slides)

DM-electron scattering rate

Full formula

$$R = \frac{1}{\rho_T}\frac{\rho_\chi}{m_\chi}\frac{\bar{\sigma}_e}{\mu_{\chi e}^2}\frac{\pi}{\alpha_{em}}\int d^3v\, f_\chi(v) \int \frac{d^3\mathbf{k}}{(2\pi)^3} k^2 |F_{DM}(k)|^2 \int \frac{d\omega}{2\pi} \frac{1}{1-e^{-\beta\omega}} \mathrm{Im}\left[\frac{-1}{\epsilon(\omega,\mathbf{k})}\right] \delta\left(\omega + \frac{k^2}{2m_\chi} - \mathbf{k}\cdot\mathbf{v}\right).$$

$f_\chi(v)$ → DM velocity distribution

$|F_{DM}(k)|^2$ → DM form factor

$\mathrm{Im}\left[\frac{-1}{\epsilon(\omega,\mathbf{k})}\right]$ → ELF

<u>Advantages of using the ELF:</u>

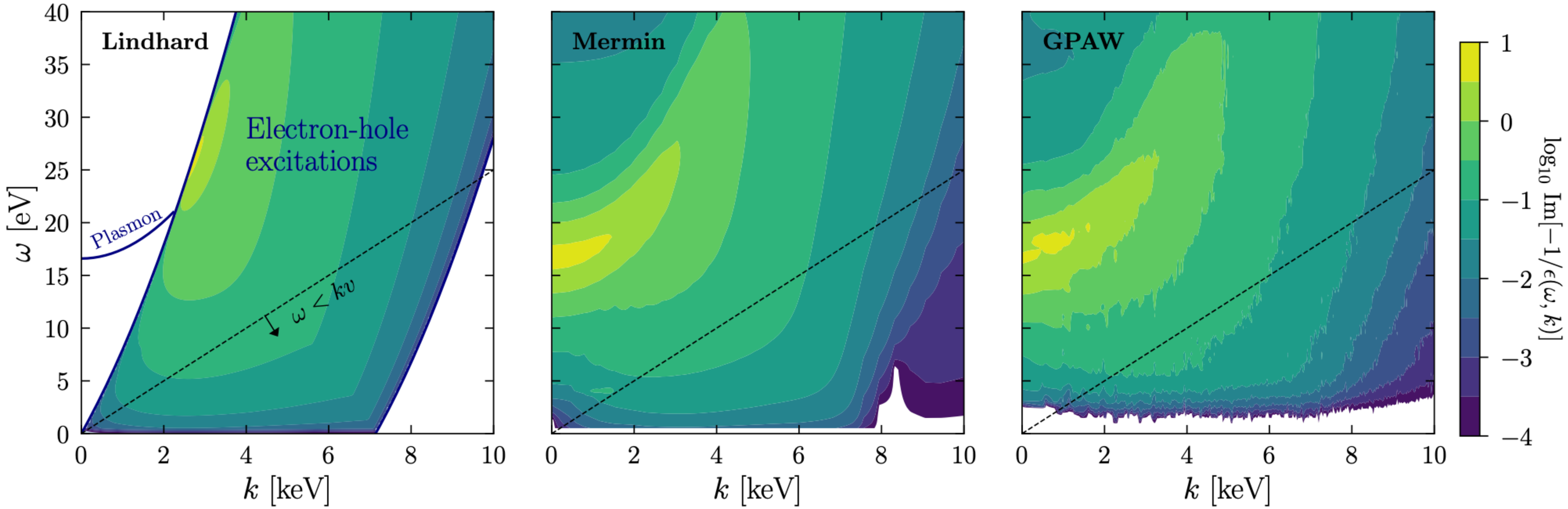
- Screening included automatically
- ELF has been measured and calculated extensively in the condensed matter literature

Applicable to *any* mediator that couples to e^- density
(e.g. scalar mediator and dark photon mediator yield *identical* scattering rate)

SK, J. Kozaczuk, T. Lin: arXiv 2101.08275, 2104.12786

Calculating the ELF

Simple   Sophisticated



Free electron gas approximation

Phenomenological model fit to data

First principles DFT calculation

100% analytic

semi-analytic

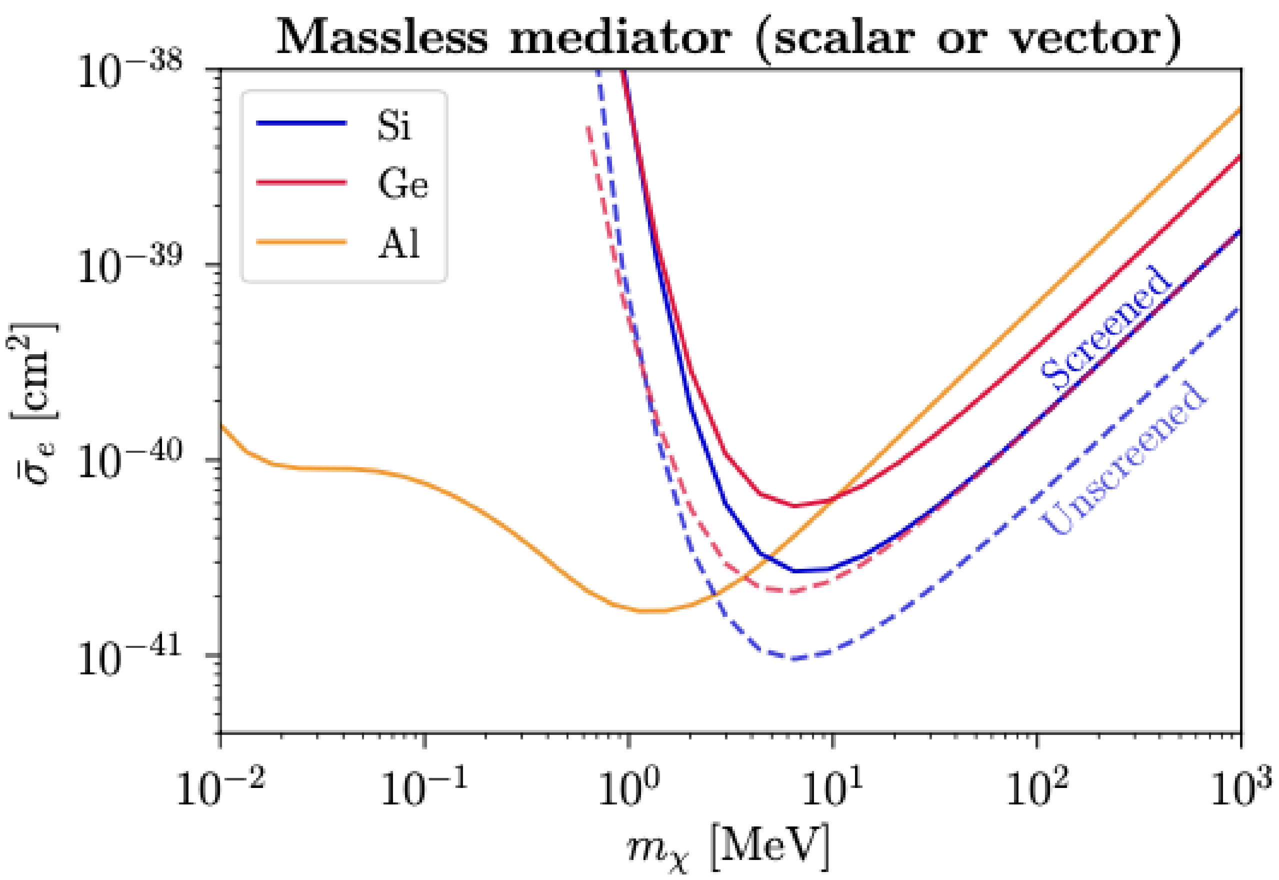
Fully numerical

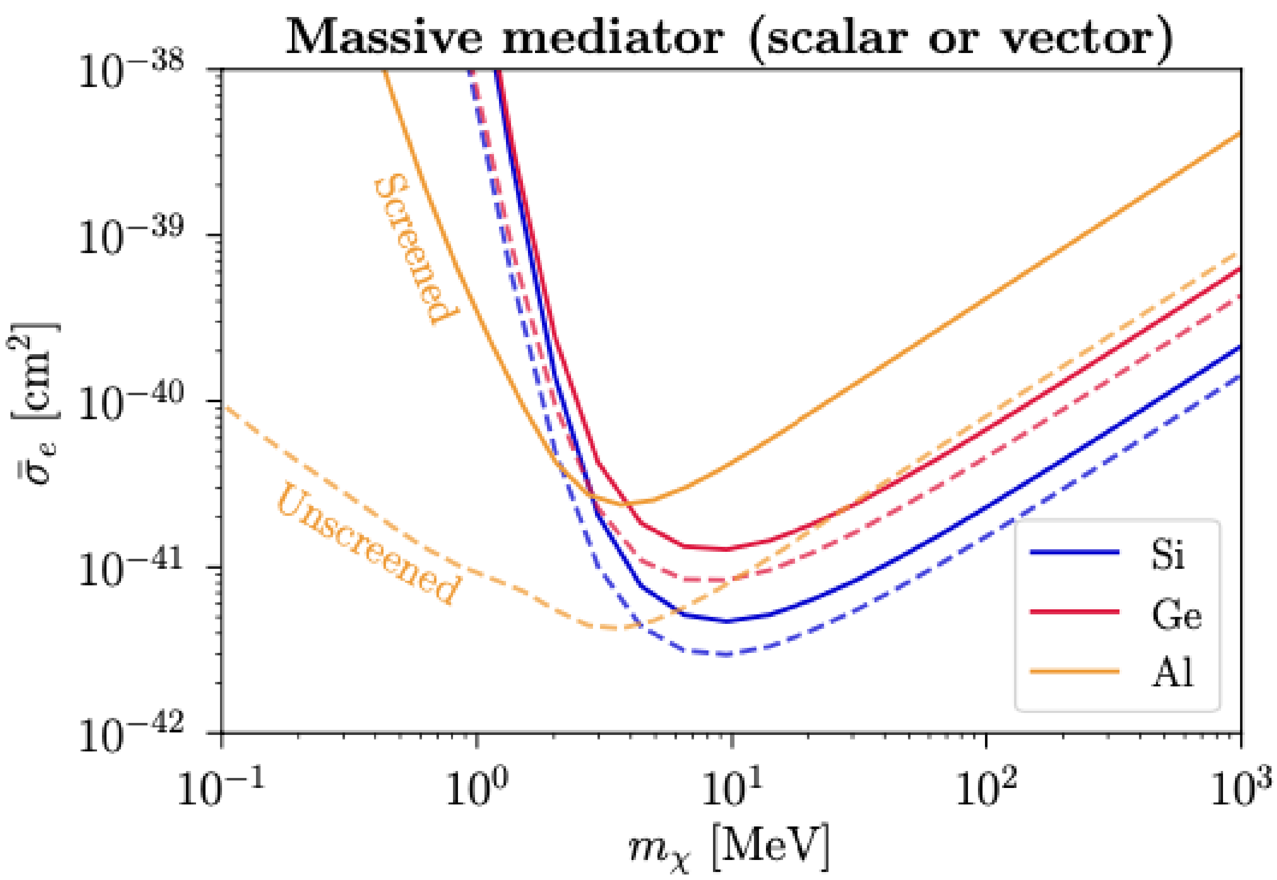
(Details in back-up slides)

SK, J. Kozaczuk, T. Lin: arXiv 2101.08275, 2104.12786

Results

Screening has O(1) effect on integrated rate



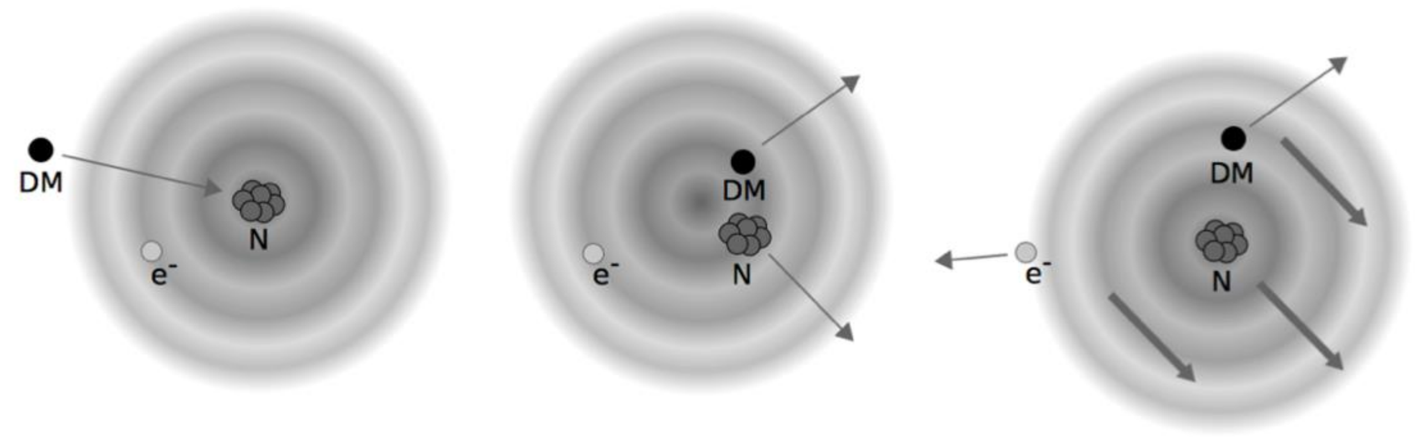


SK, J. Kozaczuk, T. Lin: arXiv 2101.08275, 2104.12786

Bonus: The Migdal effect

A hard nuclear recoil can cause some electrons to be ionized

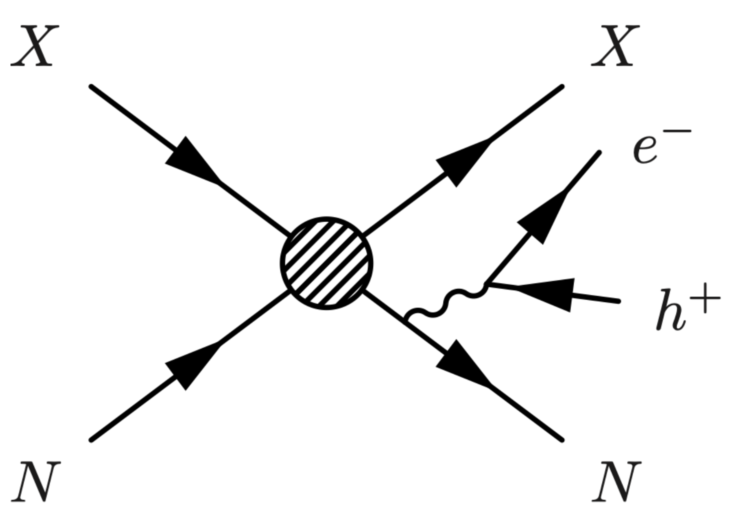
Opportunity for e.g. SENSEI, superCDMS



From 1711.09906 (Dolan et al.)

Confusing in (semi-)conductors, because electrons are highly delocalized!
→ Spectator ions cannot be ignored!

Perform perturbative calculation in the lab frame, rather than in the frame of the recoiling nucleus



SK, J. Kozaczuk, T. Lin: arXiv 2011.09496
Liang et.al. : arXiv 2011.13352

Bonus: The Migdal effect

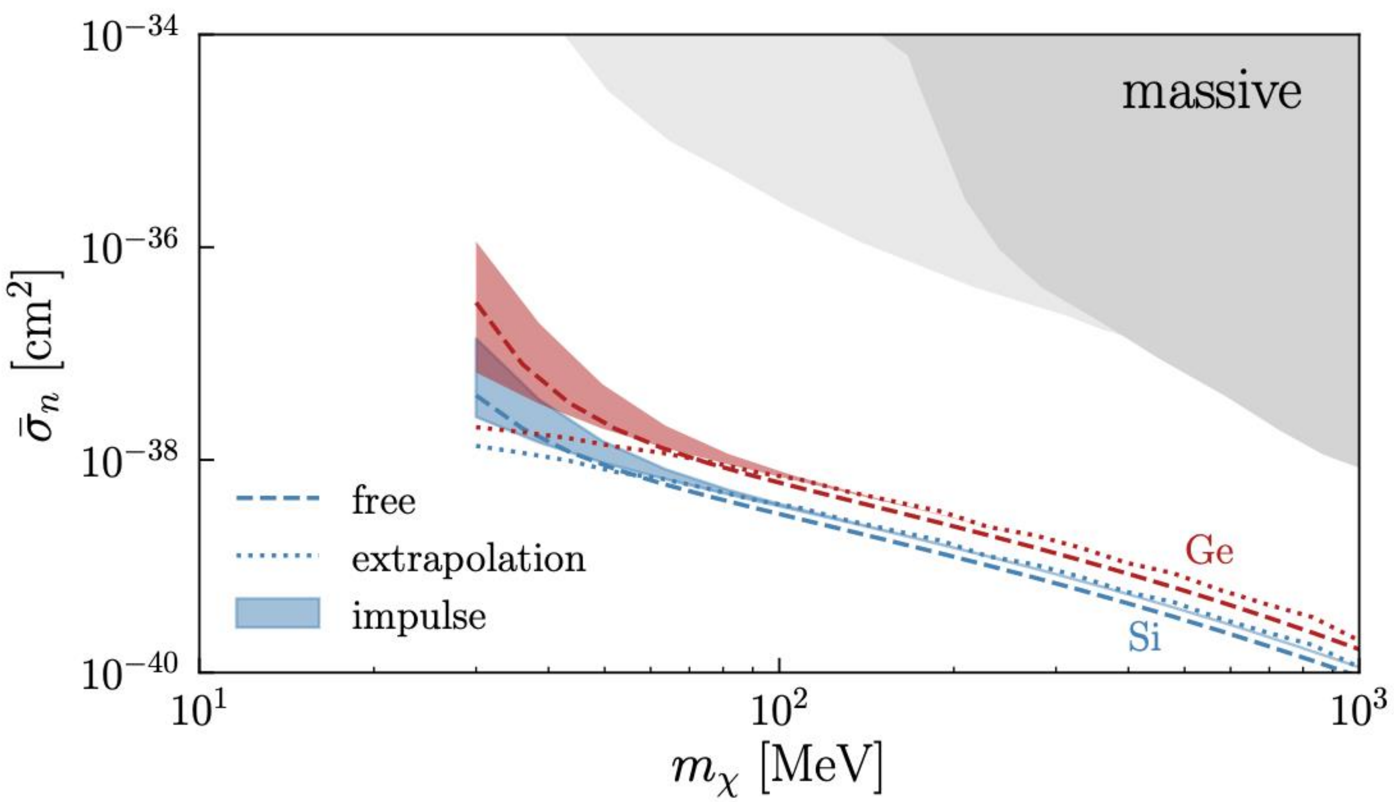
Result:

ELF — Nucleus propagator

$$R = \frac{8\pi^2 Z_{\text{ion}}^2 \alpha A^2 \rho_\chi \bar{\sigma}_n}{m_N m_\chi \mu_{\chi n}^2} \int d^3v f_\chi(v) \int d\omega \int \frac{d^3\mathbf{q}_N}{(2\pi)^3} \int \frac{d^3\mathbf{p}_f}{(2\pi)^3} \int \frac{d^3\mathbf{k}}{(2\pi)^3} \frac{1}{k^2} \text{Im}\left[\frac{-1}{\epsilon(\mathbf{k},\omega)}\right] \left[\frac{1}{\omega - \frac{\mathbf{q}_N \cdot \mathbf{k}}{m_N}} - \frac{1}{\omega}\right]^2$$
$$\times |F_{DM}(\mathbf{p}_i - \mathbf{p}_f)|^2 |F(\mathbf{p}_i - \mathbf{p}_f - \mathbf{q}_N - \mathbf{k})|^2 \delta\left(E_i - E_f - E_N - \omega\right).$$

DM form factor — Crystal form factor

Nucleus is not a free particle!



ͺK, J. Kozaczuk, T. Lin: arXiv 2011.09496
Liang et.al. : arXiv 2011.13352

DarkELF

You'd like to quickly calculate DM scattering rates, but don't want to learn Density Functional Theory?

Then DarkELF is the answer for you!

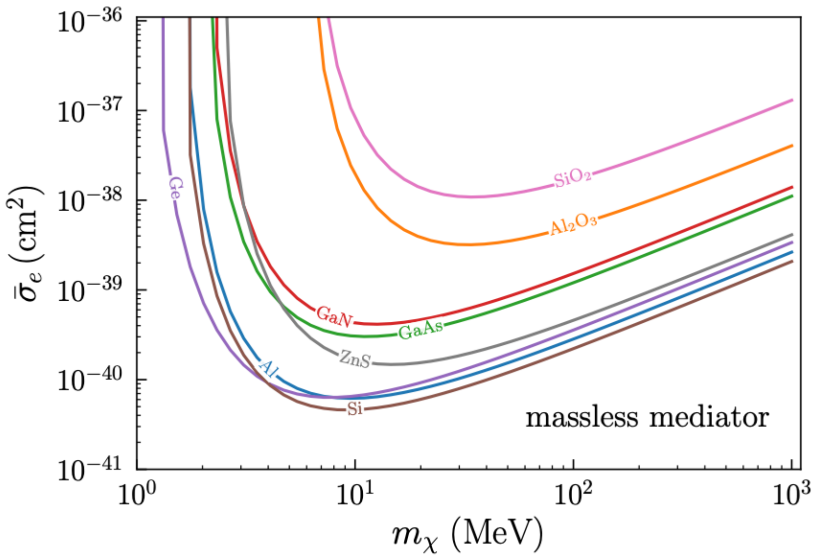
https://github.com/tongylin/DarkELF

- Python 3 package with lots of example Jupyter notebooks
- No dependencies other than numpy, scipy and Vegas
- All DFT results included as look-up tables, no DFT code necessary
- Includes most common materials (Si, Ge, GaAs, diamond, sapphire, etc etc)
- Can easily add user supplied look-up tables and/or materials or DM form factors

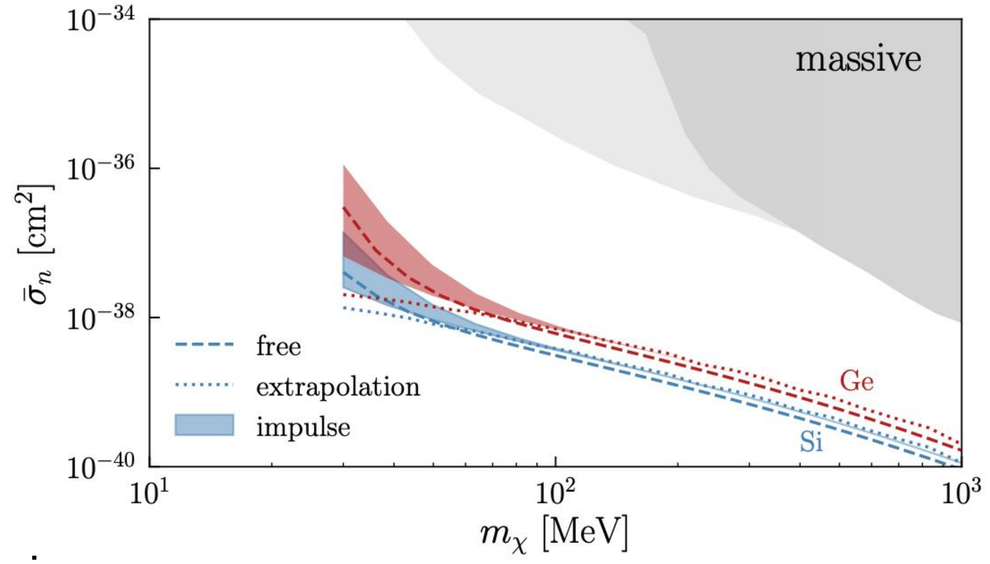
Happy to give you a quick tutorial if someone is interested

DarkELF processes

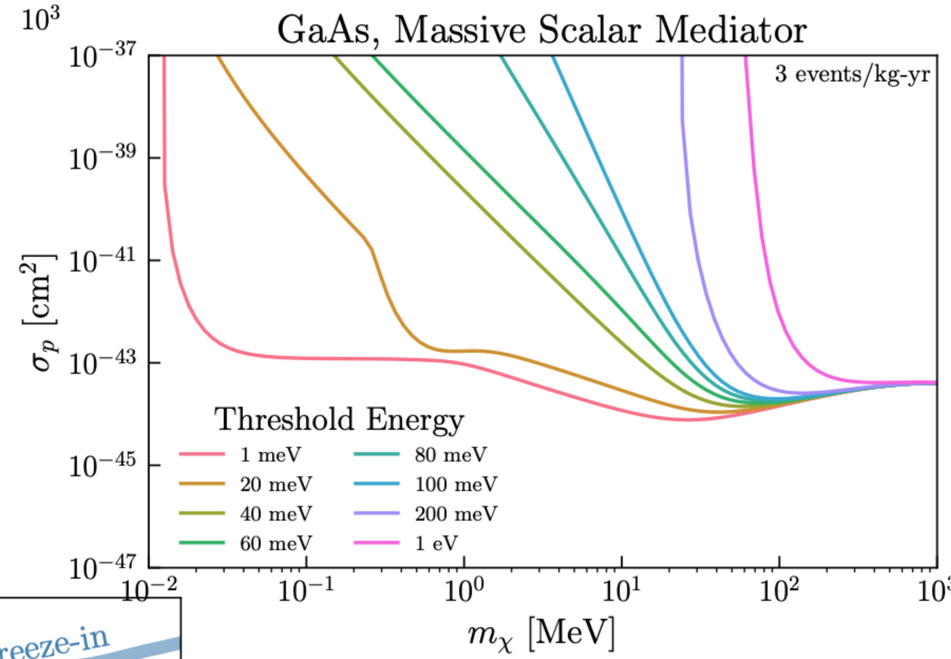
DM - electron scattering



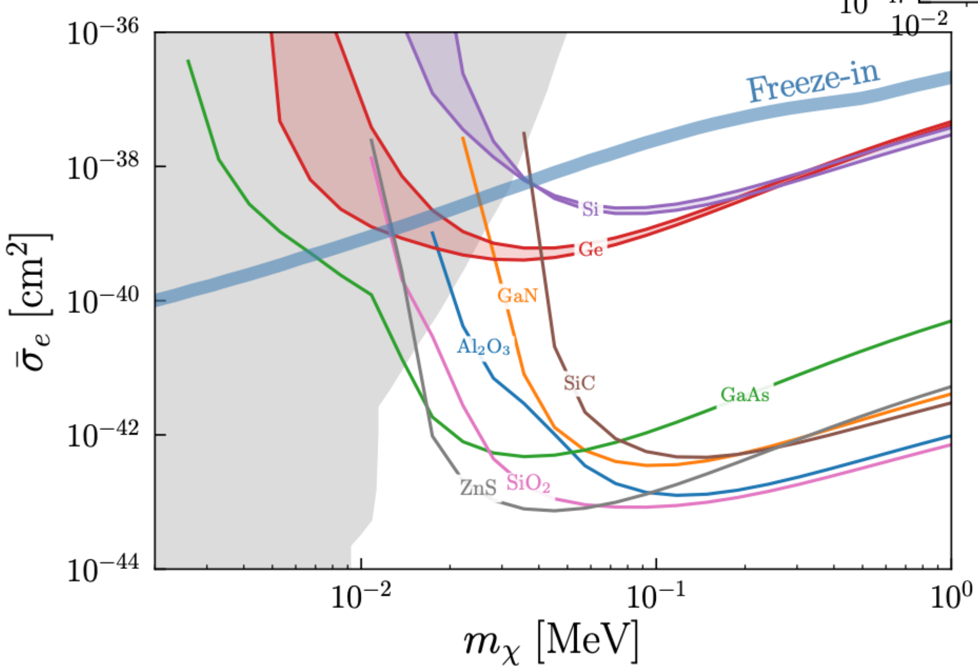
Migdal effect



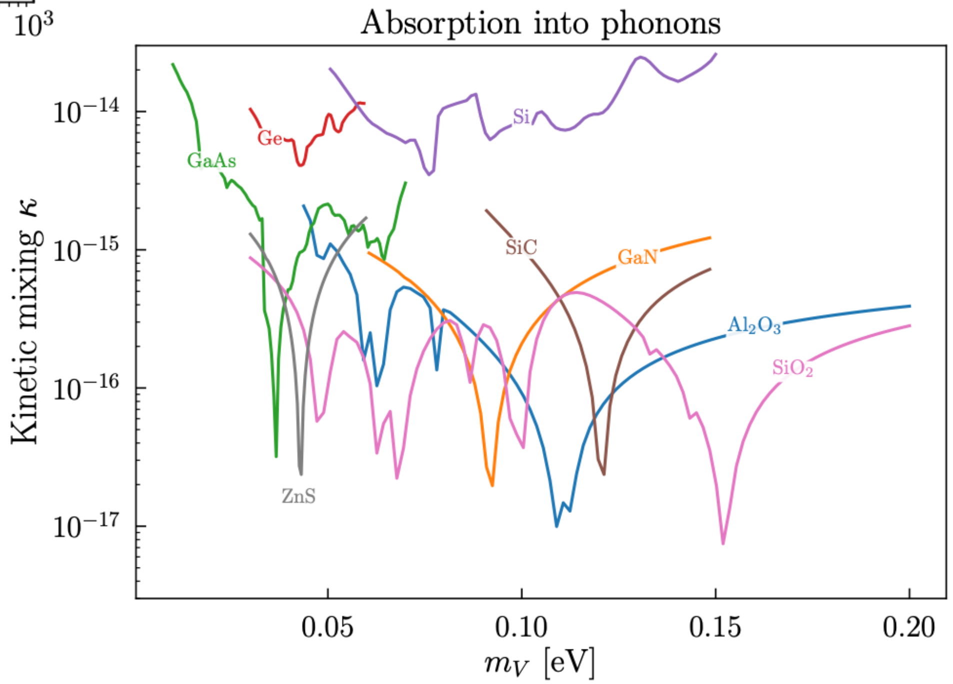
DM - multiphonon scattering



DM - phonon scattering



Dark photon absorption



Summary

We computed:

- DM-phonon scattering to all orders in multiphonon expansion
- DM-electron scattering, *including screening*
- The Migdal effect in semi-conductors

All calculations publicly available in DarkELF package

Some future / ongoing work:

- Background processes, e.g. Frenkel pair recombination
- Lindhard's theory for low energy recoils
- Going beyond the isotropic limit

Extra slides

ELF Derivation (I)

Need to use *linear response theory*, essentially non-relativistic QFT

Susceptibility: how does the crystal respond to a density perturbation?

$$\chi(\omega, \mathbf{k}) = -\frac{i}{V}\int_0^\infty dt\, e^{i\omega t}\langle[n_{\mathbf{k}}(t), n_{-\mathbf{k}}(0)]\rangle$$

V → Crystal volume

$n_{\mathbf{k}}$ → Electron number density operator

This is the non-relativistic, retarded Green's function (fully dressed)

Now we use the fluctuation-dissipation theorem

$$\mathrm{Im}\chi(\omega, \mathbf{k}) = -\frac{1}{2}(1 - e^{-\beta\omega})S(\omega, \mathbf{k}) \qquad \beta \equiv \frac{1}{k_B T}$$

With the dynamical structure factor defined as

$$S(\omega, \mathbf{k}) \equiv \frac{2\pi}{V}\sum_{i,f}\frac{e^{-\beta E_i}}{Z}|\langle f|n_{-\mathbf{k}}|i\rangle|^2\delta(\omega + E_i - E_f)$$

Fermi's golden rule

P. Nozières, D. Pines (1958)

ELF Derivation (II)

Now consider the response to an external electromagnetic perturbation.
The induced electron number density is

$$\begin{aligned}\langle \delta n(\mathbf{k},\omega)\rangle &= \langle n(\mathbf{k},\omega) H_{coul}\rangle \\ &= -\frac{e}{k^2}\chi(\mathbf{k},\omega)\rho_{ext}(\mathbf{k},\omega)\end{aligned}$$

with

$$H_{coul} = -e\int \frac{d^3\mathbf{k}}{(2\pi)^2}\frac{e^{i\mathbf{k}\cdot\mathbf{x}}}{k^2} n(-\mathbf{k},\omega)\rho_{ext}(\mathbf{k},\omega)$$

Using Maxwell's equations

$$\begin{aligned} i\mathbf{k}\cdot\mathbf{D}(\mathbf{k},\omega) &= 4\pi\rho_{ext}(\mathbf{k},\omega) \\ i\mathbf{k}\cdot\mathbf{E}(\mathbf{k},\omega) &= 4\pi\rho_{ext}(\mathbf{k},\omega) - 4\pi e\langle\delta n(\mathbf{k},\omega)\rangle\end{aligned}$$

with

$$\mathbf{D}(\mathbf{k},\omega) = \epsilon(\mathbf{k},\omega)\mathbf{E}(\mathbf{k},\omega)$$

Which results in the relation

$$\frac{1}{\epsilon(\omega,\mathbf{k})} = 1 + \frac{4\pi\alpha_{em}}{k^2}\chi(\omega,\mathbf{k}),$$

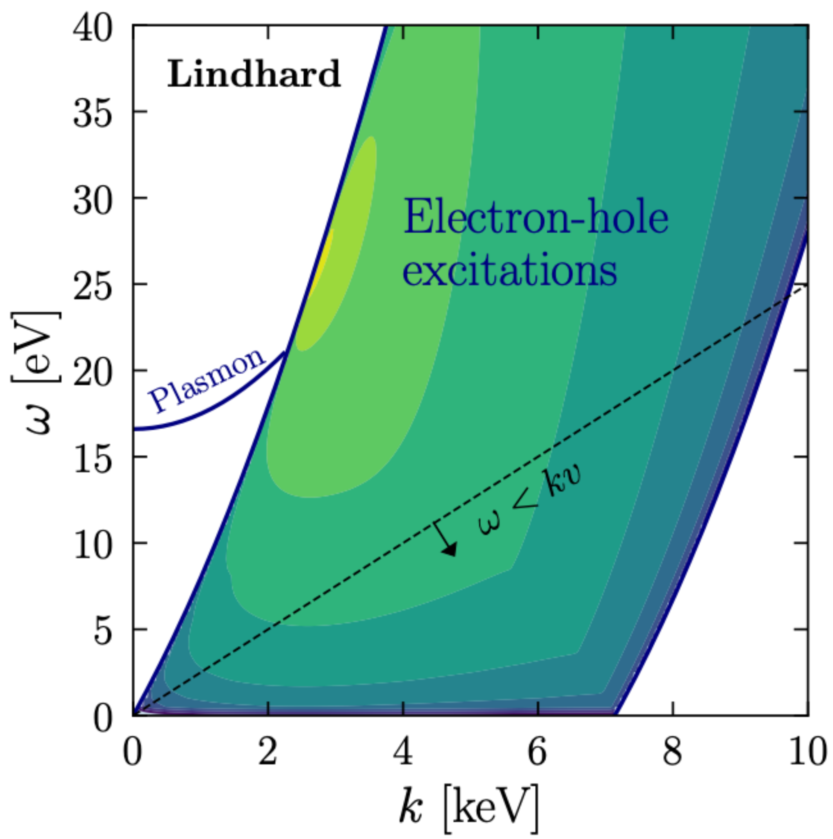
Now plugging this into the fluctuation-dissipation theorem

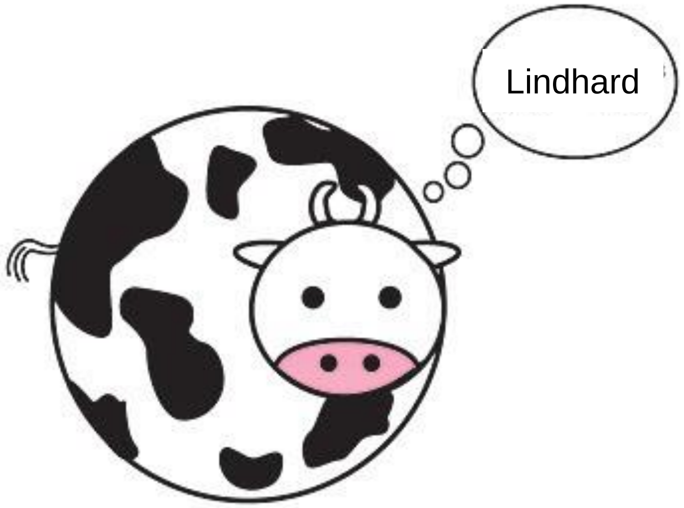
Energy Loss Function (ELF)

$$S(\omega,\mathbf{k}) = \frac{k^2}{2\pi\alpha_{em}}\frac{1}{1-e^{-\beta\omega}}\,\mathrm{Im}\left[\frac{-1}{\epsilon(\omega,\mathbf{k})}\right]$$

P. Nozières, D. Pines (1958)

Lindhard model





Homogenous, free electron gas:

$$\epsilon_{\text{Lin}}(\omega, k) = 1 + \frac{3\omega_p^2}{k^2 v_F^2} \lim_{\eta \to 0} \left[f\left(\frac{\omega + i\eta}{k v_F}, \frac{k}{2 m_e v_F} \right) \right]$$

with

$$v_F = \left(\frac{3\pi \omega_p^2}{4\alpha m_e^2} \right)^{1/3}$$

Plasmon frequency

$$f(u, z) = \frac{1}{2} + \frac{1}{8z} \left[g(z - u) + g(z + u) \right]$$

$$g(x) = (1 - x^2) \log \left(\frac{1 + x}{1 - x} \right)$$

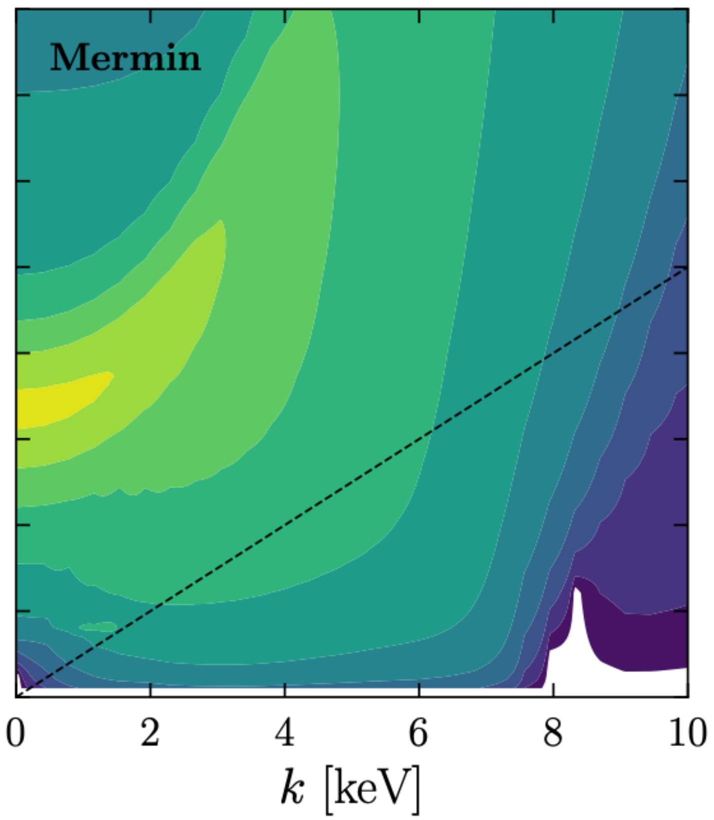
<u>Features:</u>

- ☑ Pauli blocking
- ☑ e-h pair continuum
- ☐ Plasmon width
- ☐ Low k region
- ☐ Bandgap

Mermin model

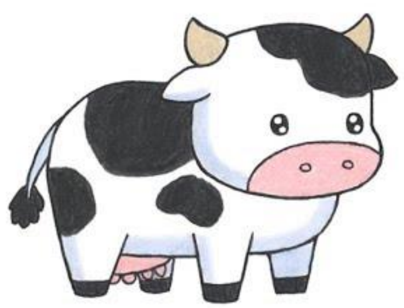
Homogenous, free electron gas with dissipation (Γ)

$$\epsilon_{\text{Mer}}(\omega, k) = 1 + \frac{(1 + i\frac{\Gamma}{\omega})(\epsilon_{\text{Lin}}(\omega + i\Gamma, k) - 1)}{1 + (i\frac{\Gamma}{\omega})\frac{\epsilon_{\text{Lin}}(\omega+i\Gamma,k)-1}{\epsilon_{\text{Lin}}(0,k)-1}}.$$



Fit a linear combination of Mermin oscillators to optical data:

$$\text{Im}\left[\frac{-1}{\epsilon(\omega, k)}\right] = \sum_i A_i(k)\text{Im}\left[\frac{-1}{\epsilon_{\text{Mer}}(\omega, k; \omega_{p,i}, \Gamma_i)}\right]$$

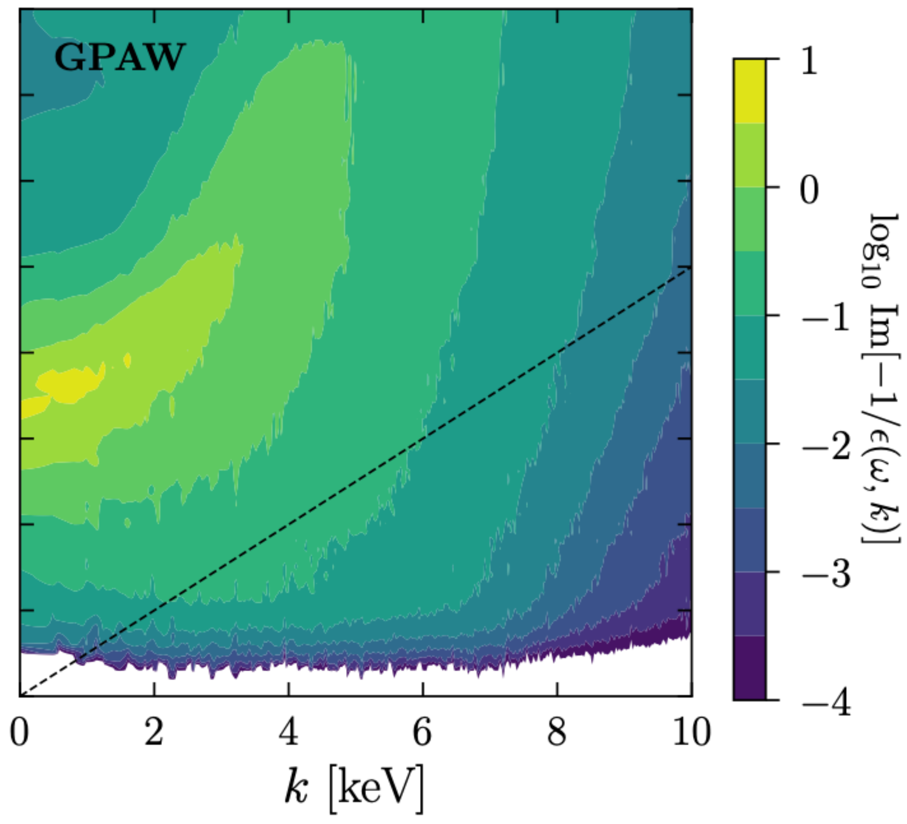


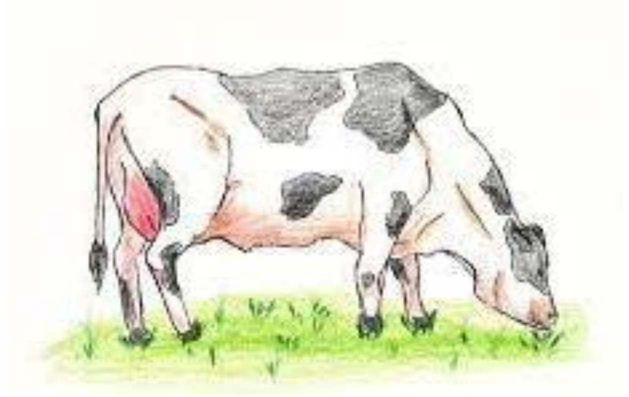
Features:

- ☑ Pauli blocking
- ☑ e-h pair continuum
- ☑ Plasmon width
- ☑ Low k region
- ☐ Bandgap

M. Vos, P. Grande: chapidif package
Data from Y. Sun et. al. Chinese Journal of Chemical Physics 9, 663 (2016)

GPAW method





GPAW: https://wiki.fysik.dtu.dk/gpaw/

Compute the ELF from first principles with time-dependent Density Functional Theory methods (TD-DFT)

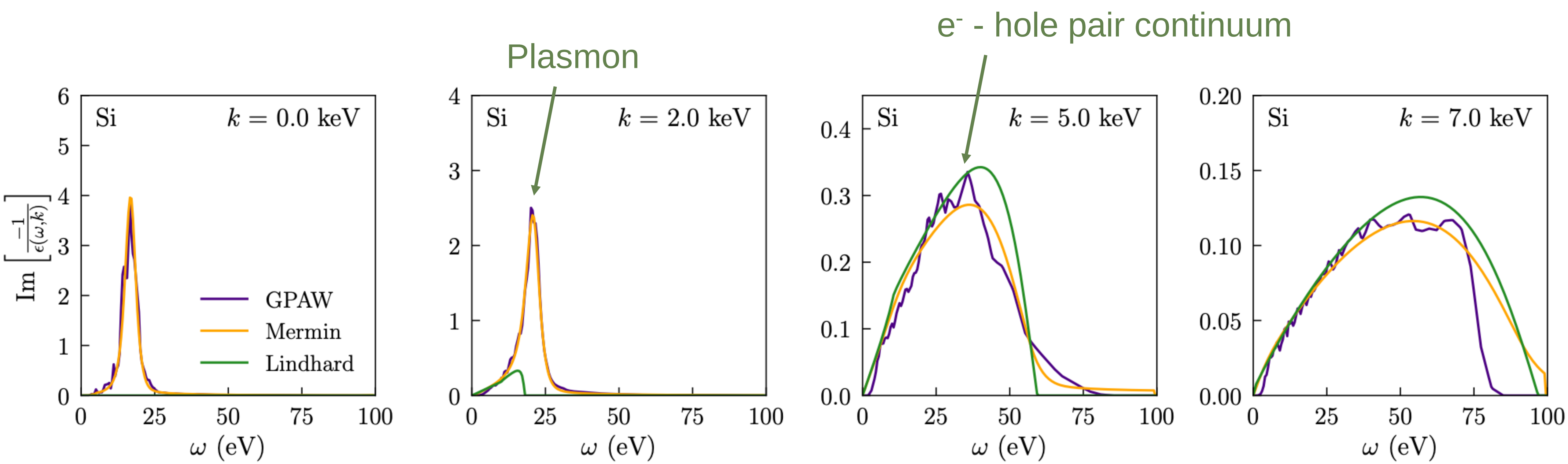
Puts atoms on periodic lattice and model interacting e^- as non-interacting e^- + effective external potential (Kohn-Sham method)

Inner shell e^- are treated as part of the ion (frozen core approximation)

Features:

- ☑ Pauli blocking
- ☑ e-h pair continuum
- ☑ Plasmon width
- ☑ Low k region
- ☑ Bandgap

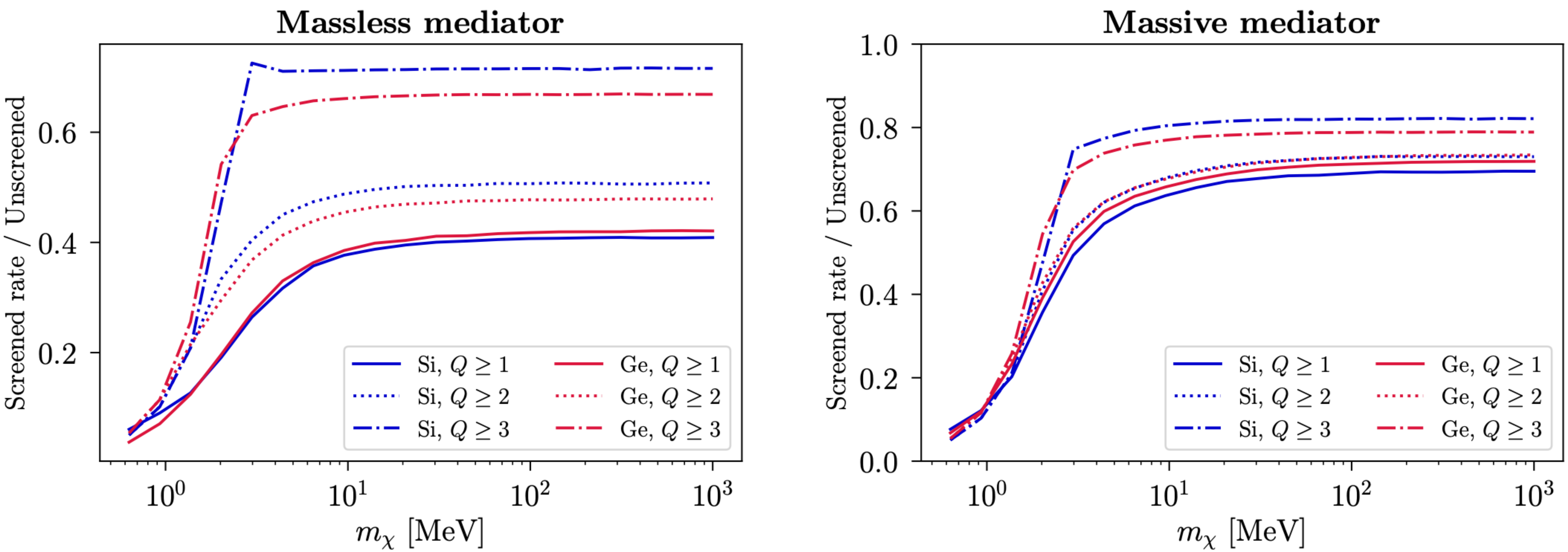
Comparing all three methods



To be understood further

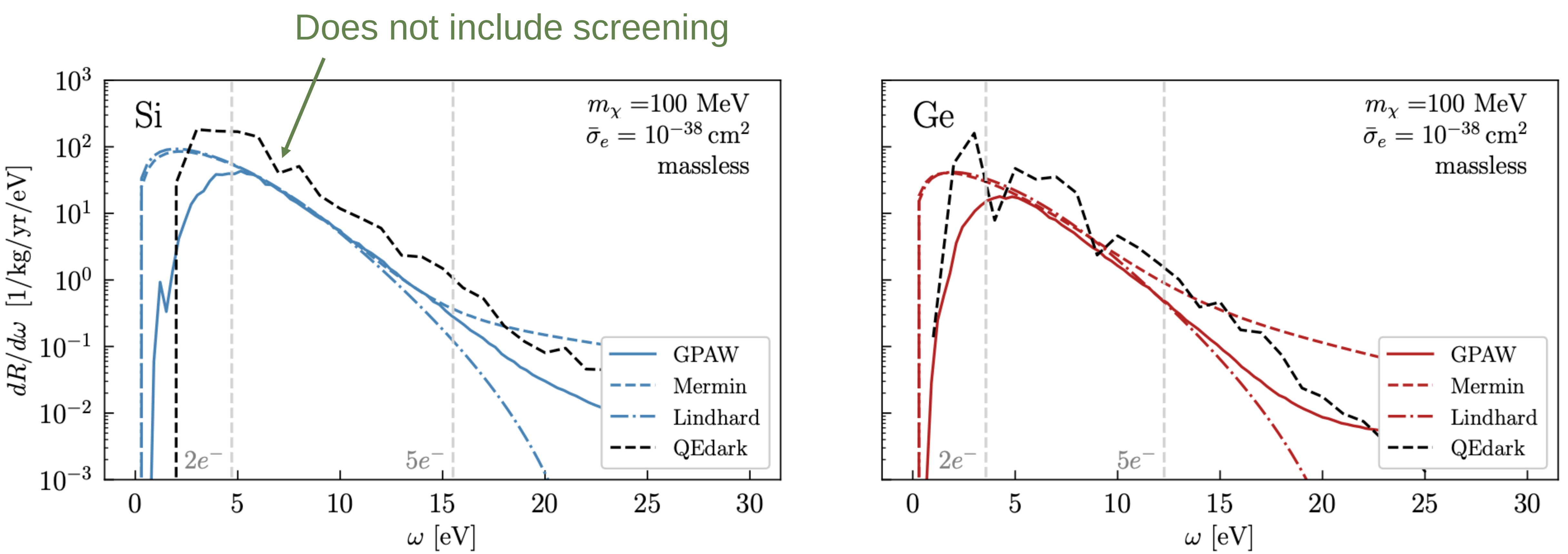
Generally very good agreement, especially between Mermin and GPAW!

Threshold dependence



The screening is the strongest for energies near the bandgap, so the higher the threshold the less important it becomes

Differential rate

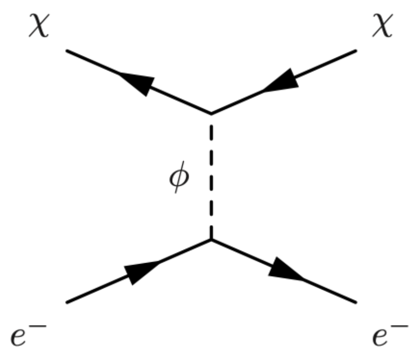


Mermin & GPAW in very good agreement except:

- Single ionization e- region (background dominated)
- High energy region (subdominant)

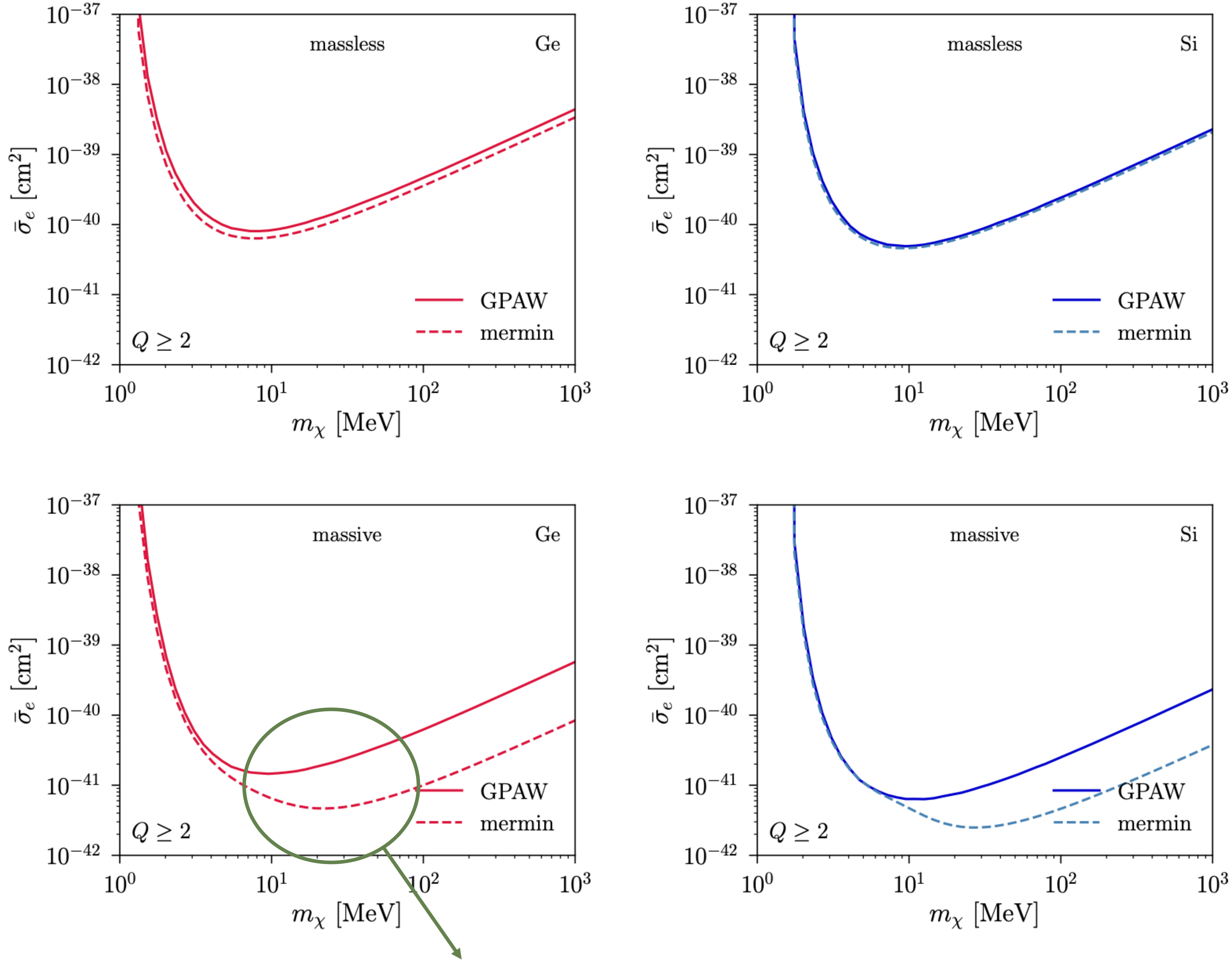
(Agreement is less good in massive mediator case; work in progress)

Integrated rate: Mermin vs GPAW



$m_\phi \ll \alpha m_e$

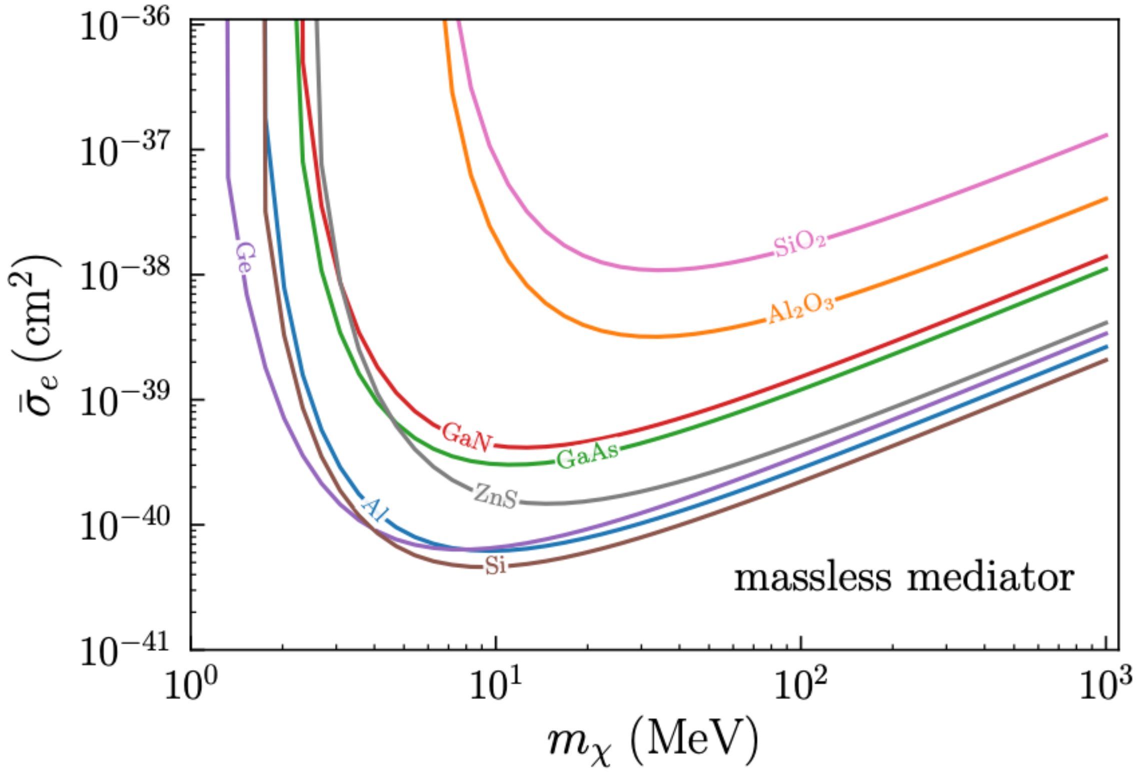
$m_\phi \gg \alpha m_e$

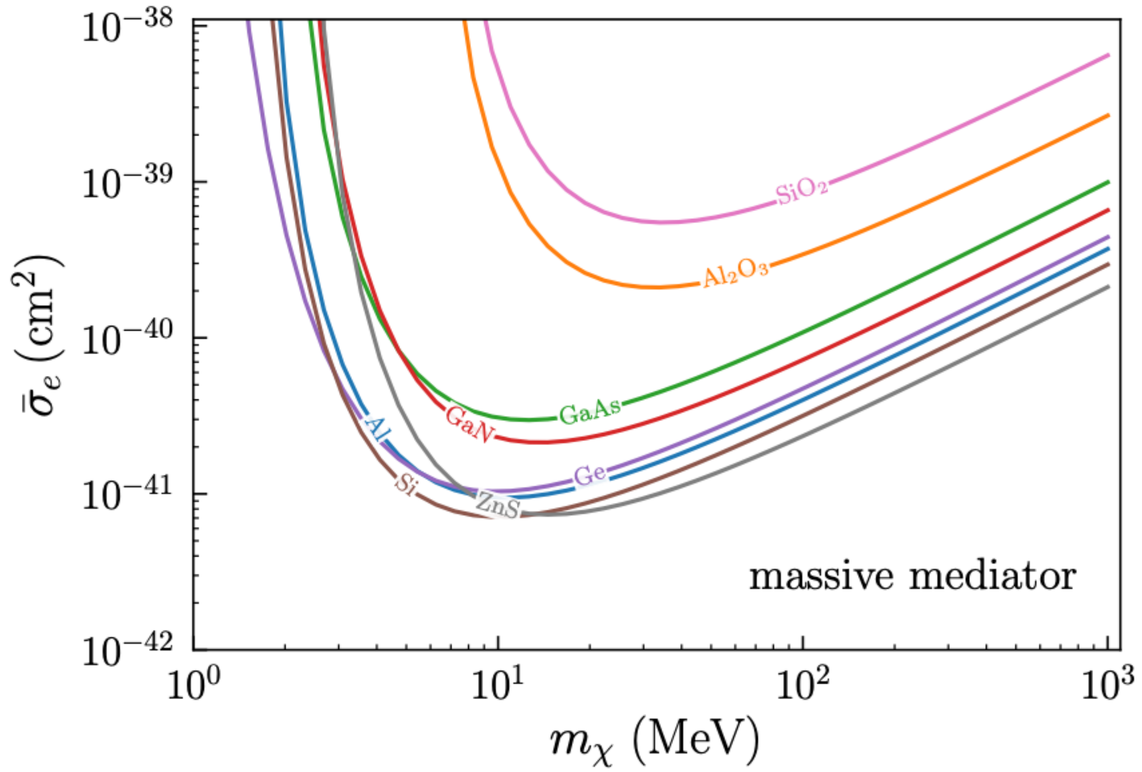


High k region of the ELF (work in progress)

Integrated rate: Other materials

Using the *Mermin* method we can easily scan over many possible targets:

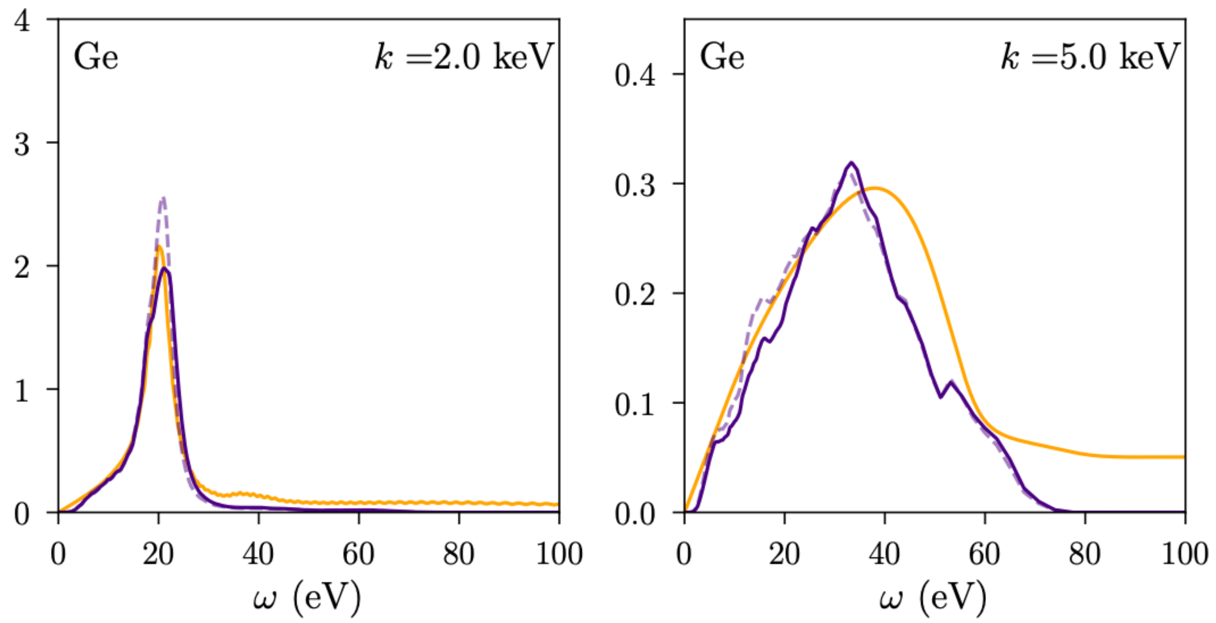




So far only *GPAW* results for Ge and Si, other materials are work in progress

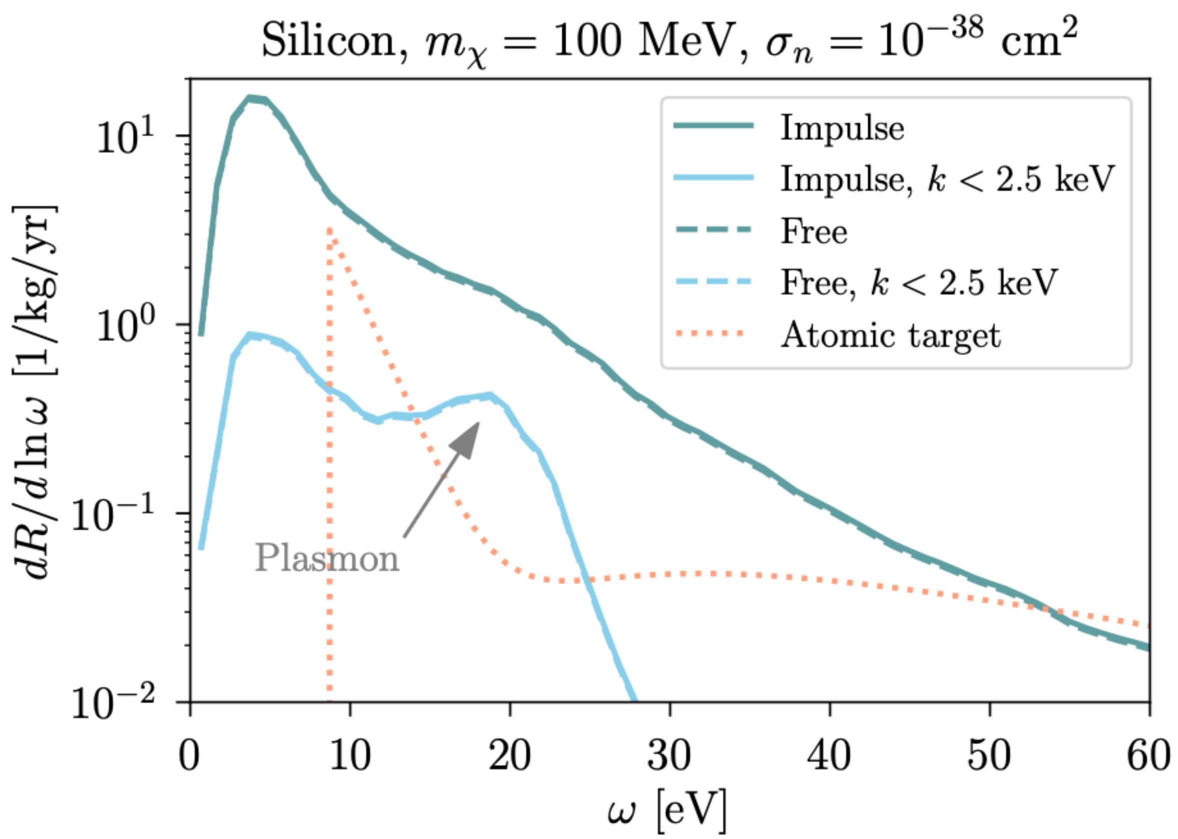
Plasmons

<u>In electron recoils</u>



DM-e- scattering: $k \gtrsim 4\,\mathrm{keV} \times \omega/(10\,\mathrm{eV})$

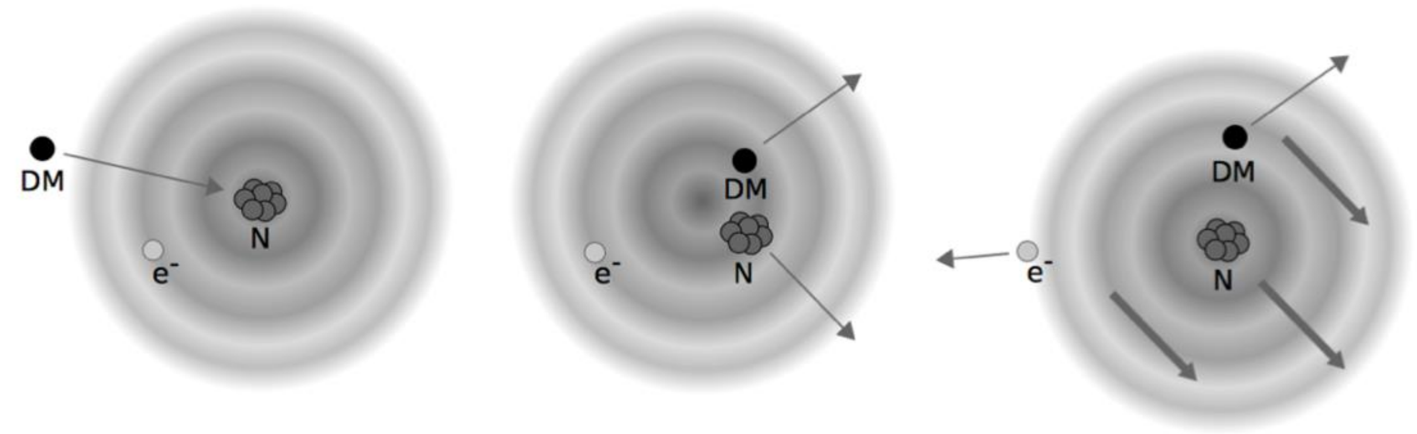
<u>In nuclear recoils</u>



Plasmon production is not relevant in normal materials, for a standard DM velocity profile

Migdal effect in atoms

A hard nuclear recoil can cause some electrons to be ionized



From 1711.09906 (Dolan et al.)

Studied in detail for atoms (e.g. Xe)

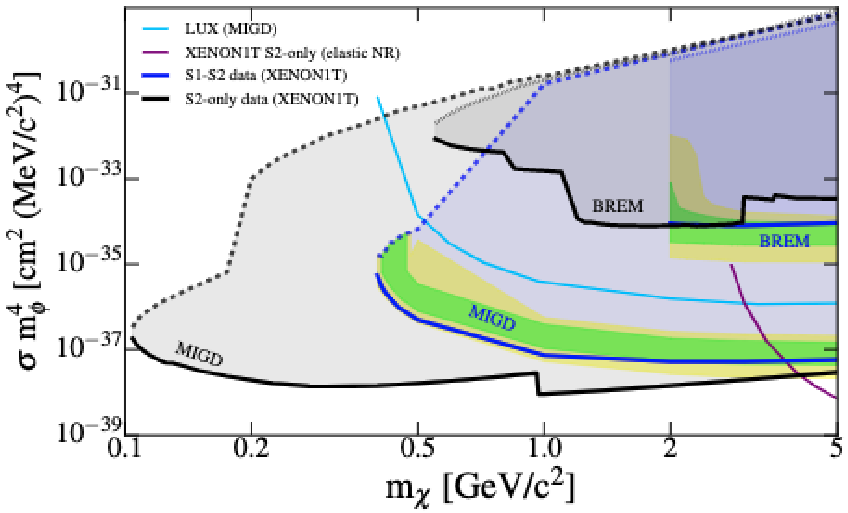
Step 1: boost to the rest frame of recoiling nucleus

$$|i\rangle \to e^{im_e \mathbf{v}_N \cdot \sum_\beta \mathbf{r}_\beta} |i\rangle$$

Step 2: Compute the overlap with the excited wave functions |f>

$$\mathcal{M}_{if} = \langle f | e^{im_e \mathbf{v}_N \cdot \sum_\beta \mathbf{r}_\beta} |i\rangle \approx im_e \langle f | \mathbf{v}_N \cdot \textstyle\sum_\beta \mathbf{r}_\beta |i\rangle$$

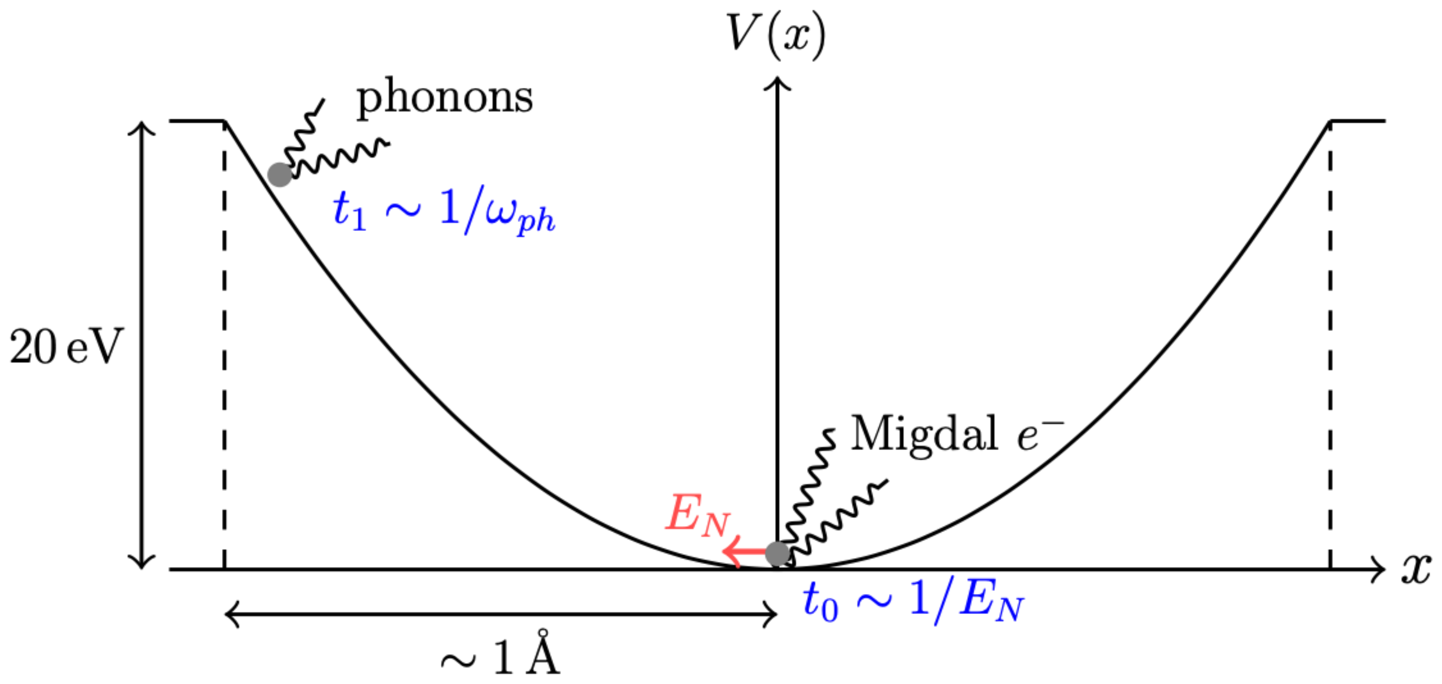
Transition dipole moment



Xenon1T arXiv: 1907.12771

A. Migdal (1939)
M. Ibe et.al. arXiv: 1707.07258

The impulse approximation



If the DM is heavy enough, most collisions take place at an energy well above the typical phonon energy (~ 30 meV)

If this is the case, the nucleus doesn't feel the crystal potential during the initial hard recoil

We can treat the *outgoing nucleus* as plane wave on the time scale of the DM collision (The *initial state nucleus* is however still treated as bound in the crystal potential)

This is known as the adiabatic approximation or the *impulse approximation*

When it is valid we can factorize the long distance physics (phonons) from the short distance physics (Migdal effect).

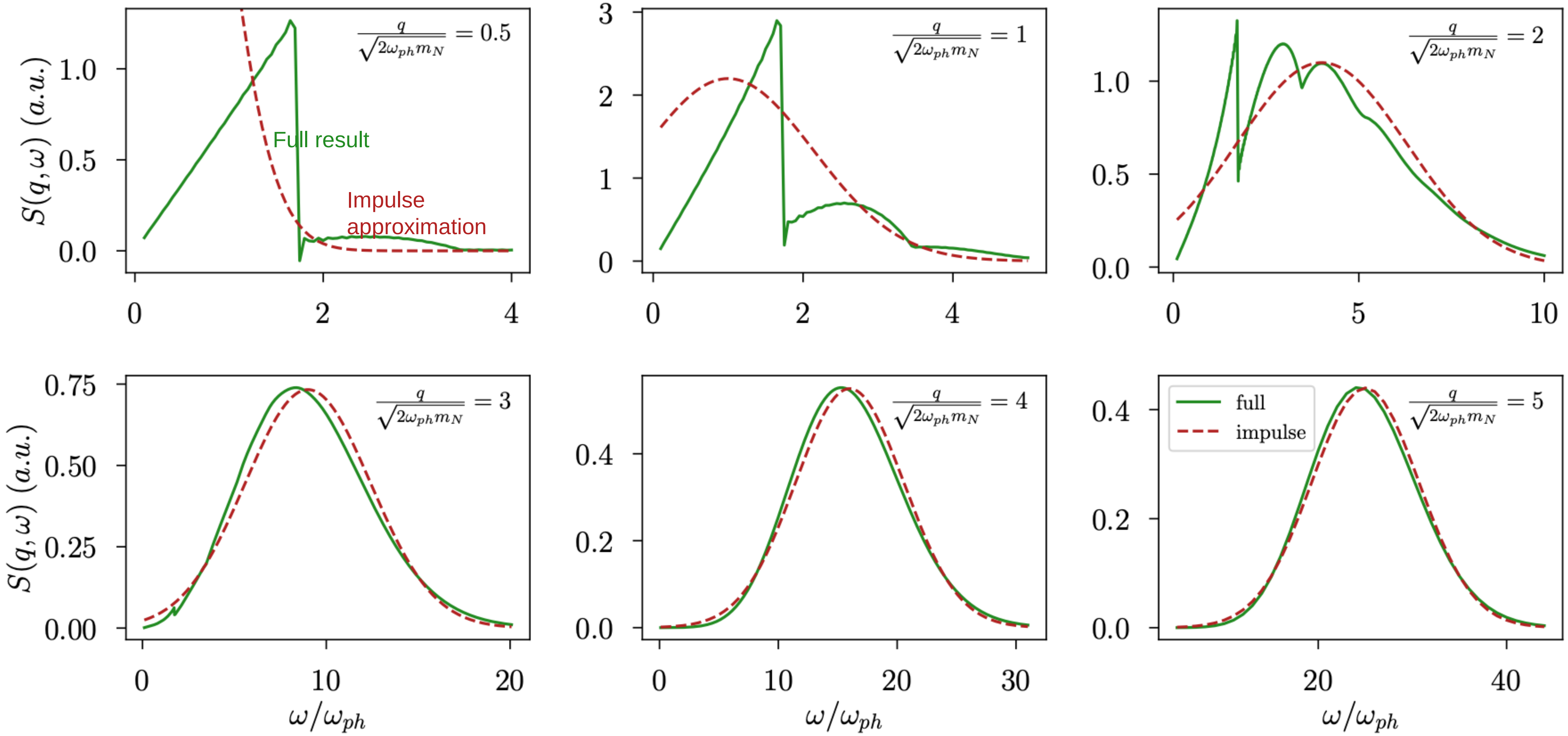
SK, J. Kozaczuk, T. Lin: arXiv 2011.09496

Crystal form factor

How important is the presence of the lattice for the kinematics of the recoiling nucleus?

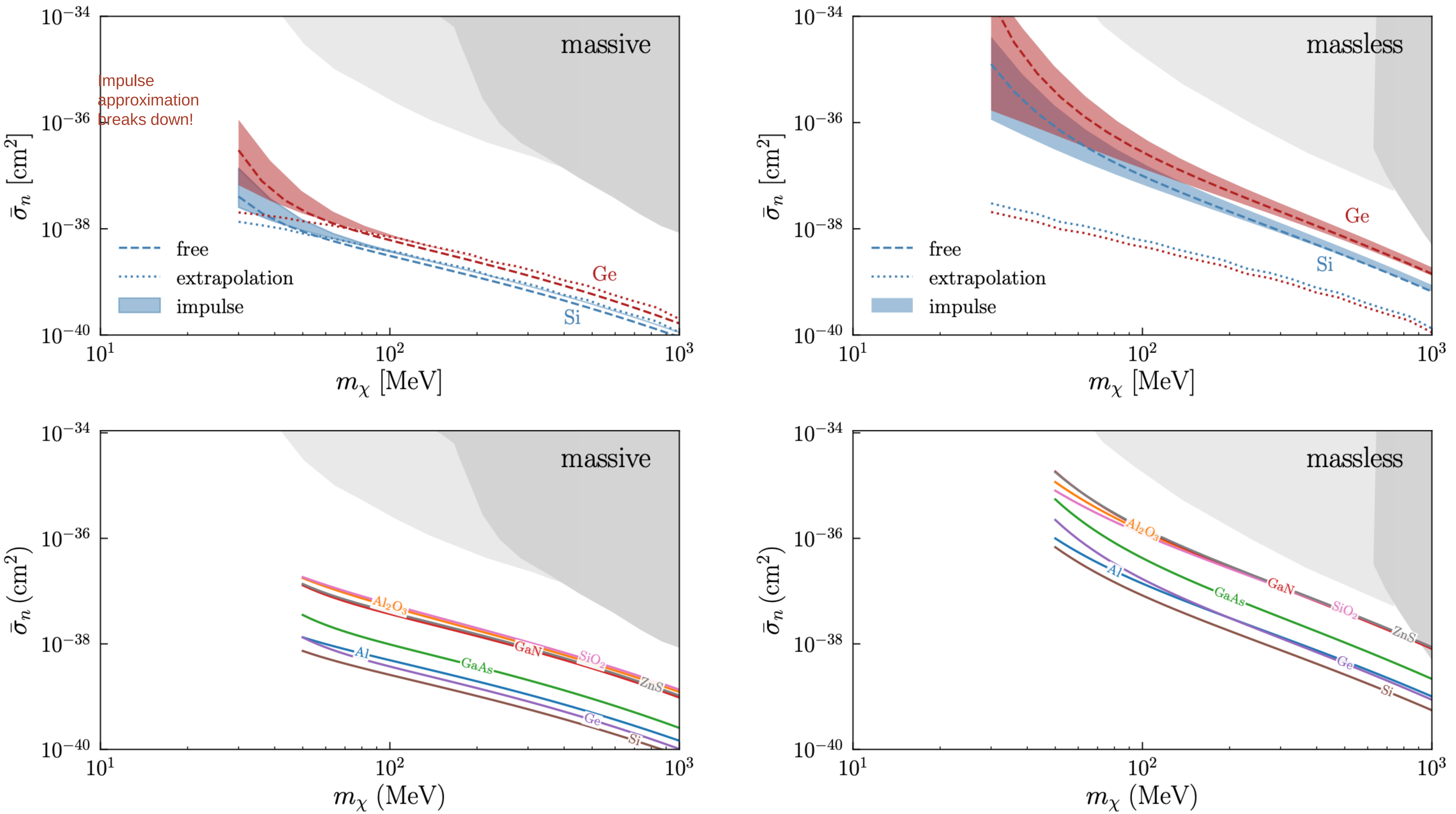
Let's analyse a simplified model of a harmonic crystal with a Debeye density of states:

Structure function for regular nuclear recoil (no Migdal):



The impulse approximation fails badly for $q^2 \lesssim m_N \omega_{ph}$

Migdal effect results



We believe the electronic response is on solid ground

Nuclear recoil (impulse approximation) is main source of uncertainty