Atomic clocks and Axion dark matter

Hyungjin Kim (DESY)

Consider

$$\mathcal{L} = \theta G\widetilde{G}$$

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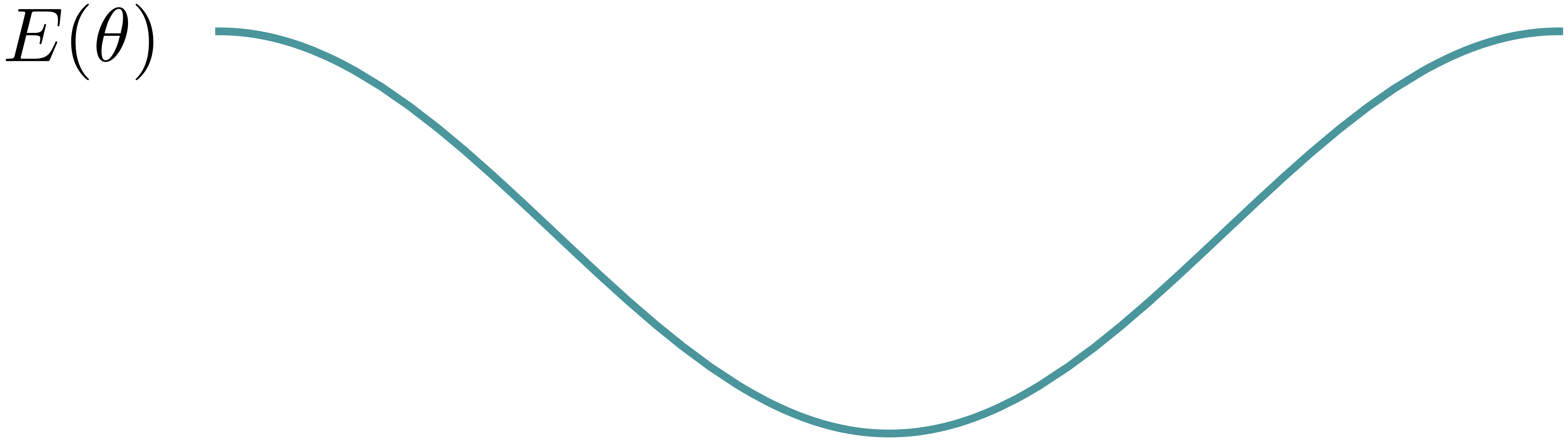
θ is physical

QCD vacuum energy depends on θ



Consider

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θ is physical

pion mass also depends on θ

$$m_\pi^2(\theta) = m_\pi^2(0)\sqrt{1 - \frac{4m_u m_d}{(m_u + m_d)^2}\sin^2(\theta/2)}$$

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Different choice of θ leads to different spectrum

Consider

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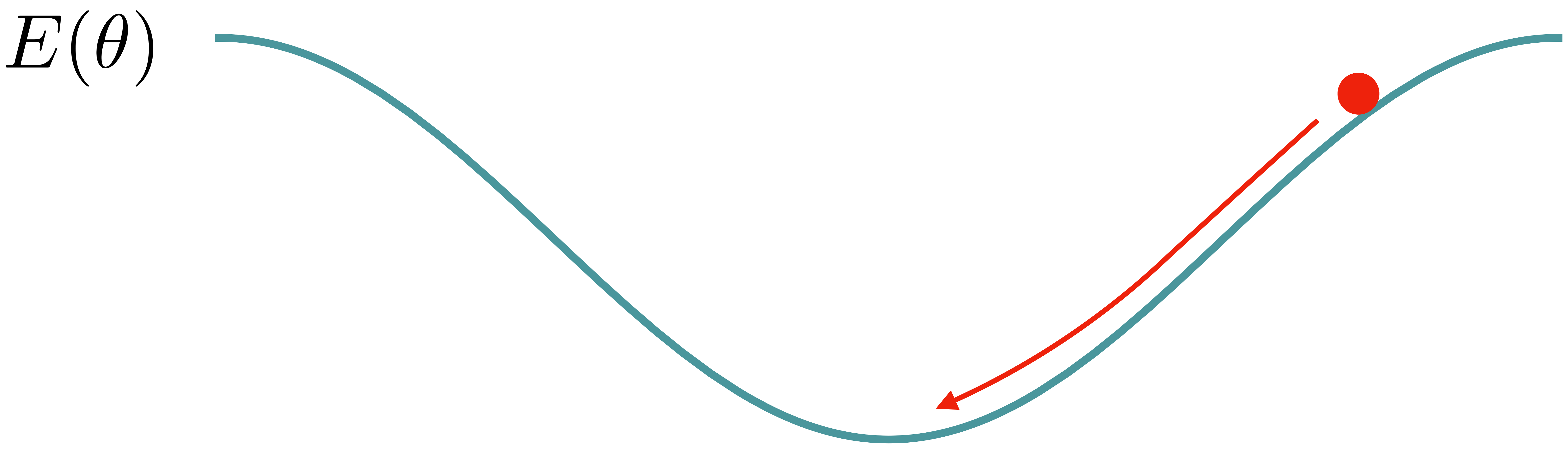
Now imagine θ=a/f as dynamical degrees of freedom



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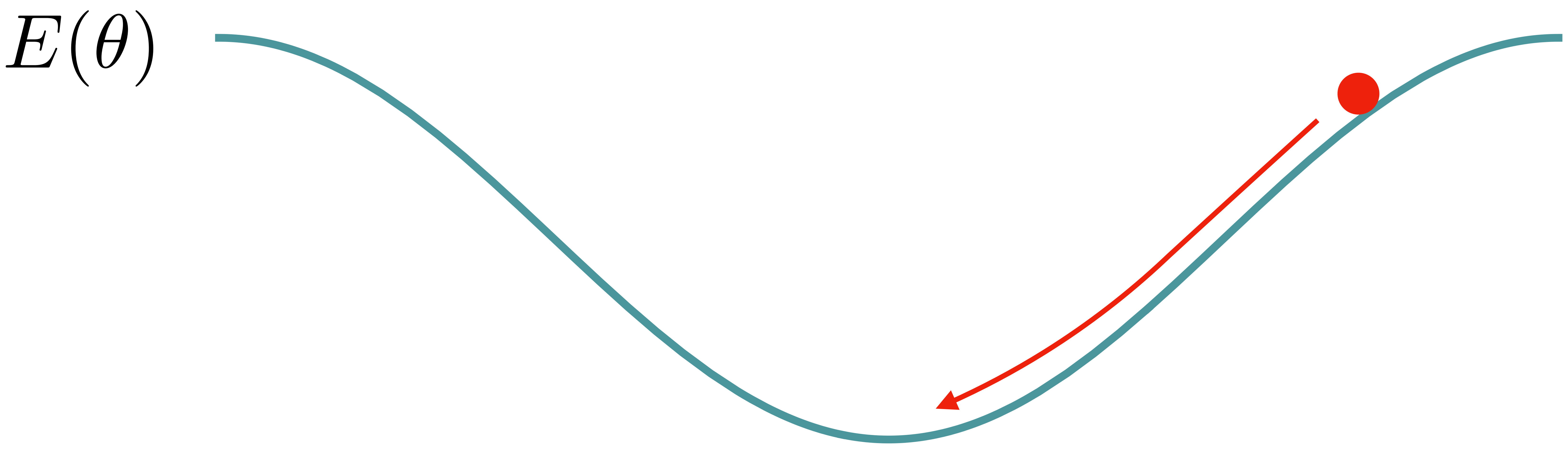
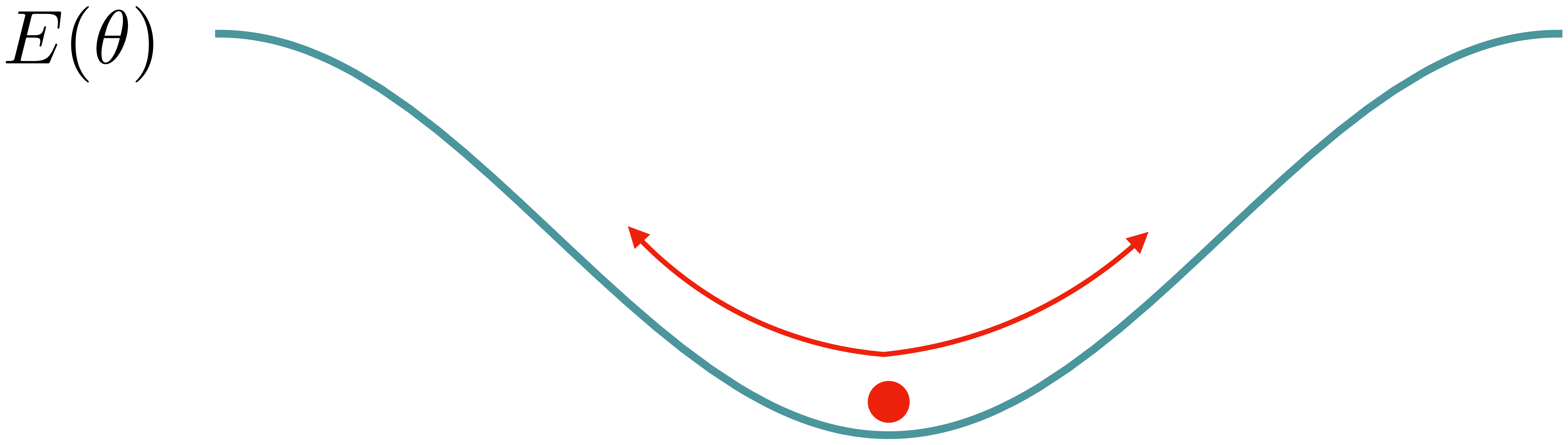


The strong CP angle dynamically relaxes to CP-conserving vacuum

solving the strong CP dynamically

This *axion* could oscillate around minimum



Such a oscillating field behaves like matter

Axion oscillation around its minimum comprise ***DM*** *in the present universe*

What are the implications?

Since QCD spectrum depends on θ

the presence of axion DM naturally imply

oscillations of nuclear quantities

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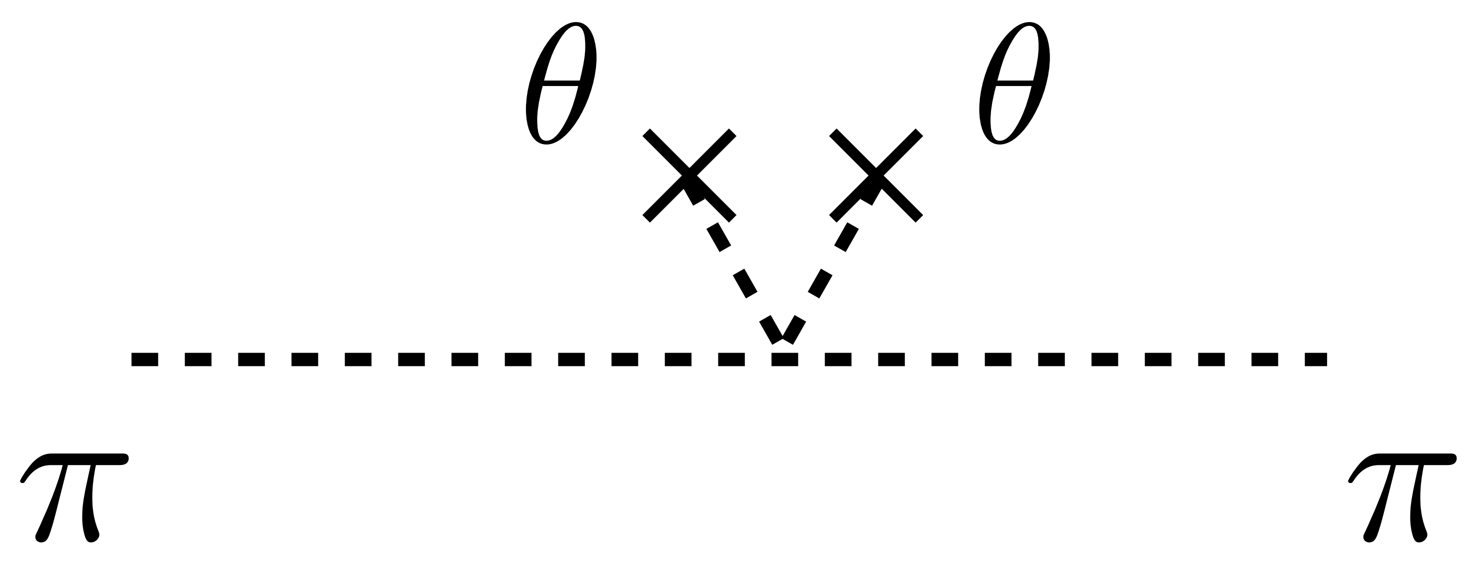
$$m_\pi^2(\theta)$$



$$\frac{\delta m_\pi^2}{m_\pi^2} = -\frac{m_u m_d}{2(m_u + m_d)^2}\theta^2$$

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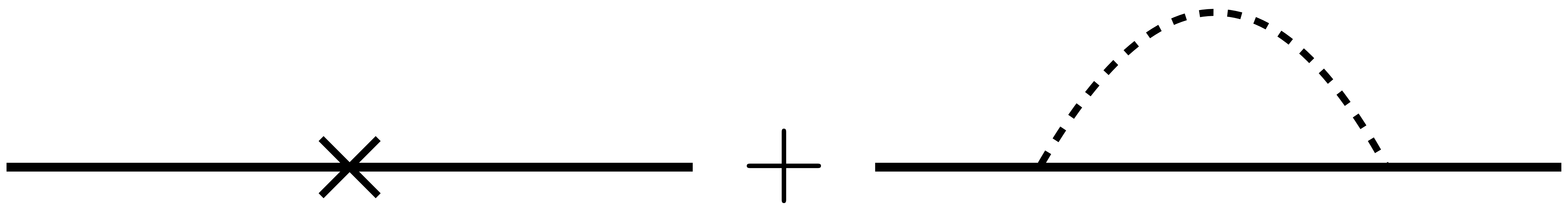
$$m_\pi^2(\theta) \qquad m_N(\theta)$$



$$\Delta m_N \simeq -4c_1 m_\pi^2(\theta) - \frac{3g_A^2 m_\pi^3(\theta)}{32\pi f_\pi^2}$$

What are the implications?

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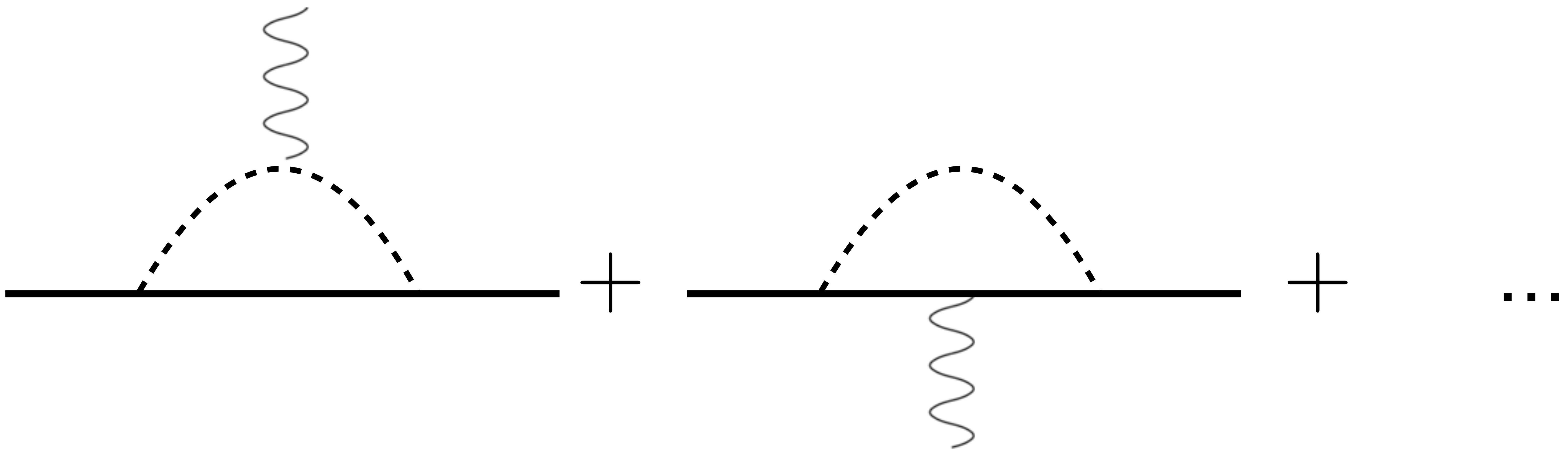
$$m_\pi^2(\theta) \qquad m_N(\theta) \qquad g_{n,p}(\theta)$$



$$\Delta g_{n,p}(\theta) \simeq \pm \frac{g_A^2 m_N m_\pi(\theta)}{4\pi f_\pi^2}$$

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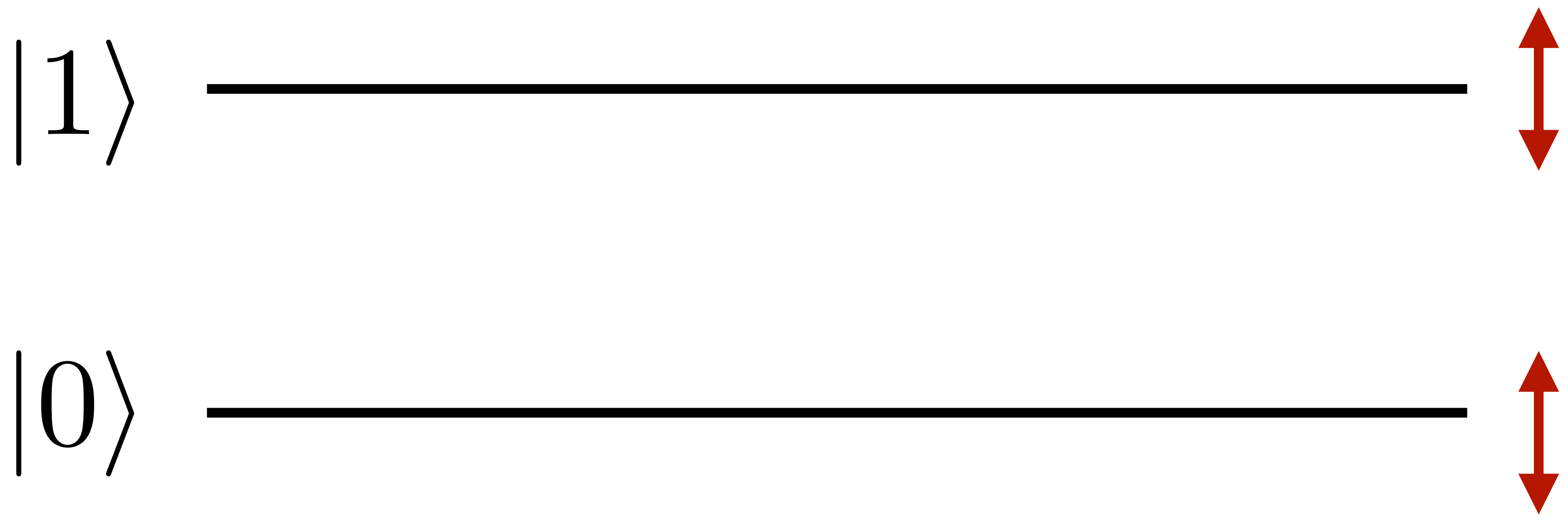
$$m_\pi^2(\theta) \qquad m_N(\theta) \qquad g_{n,p}(\theta)$$

The atomic energy level oscillates accordingly!



Are oscillations of atomic energy level observable in a lab?

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Imagine a situation where we interrogate atoms periodically

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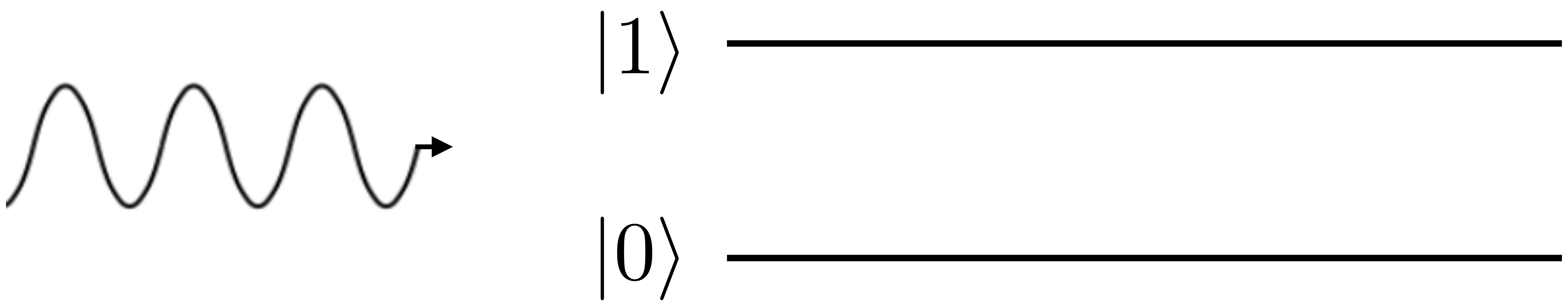

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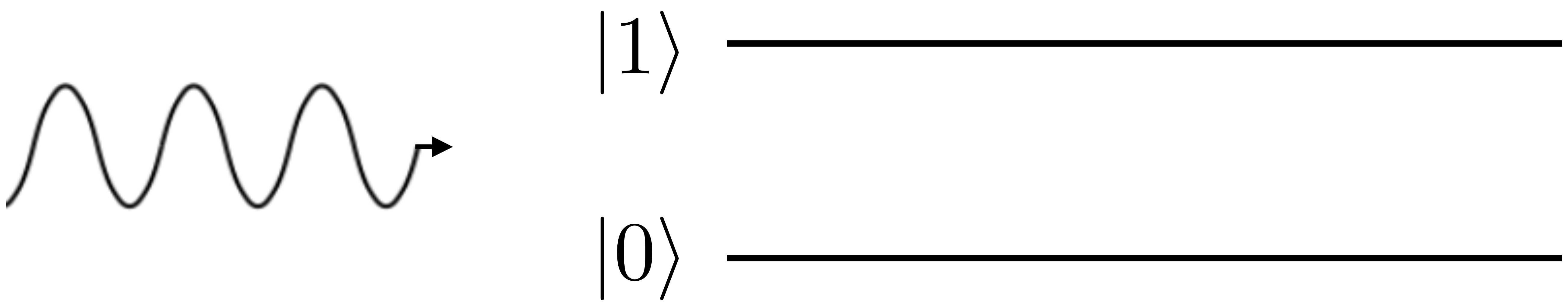
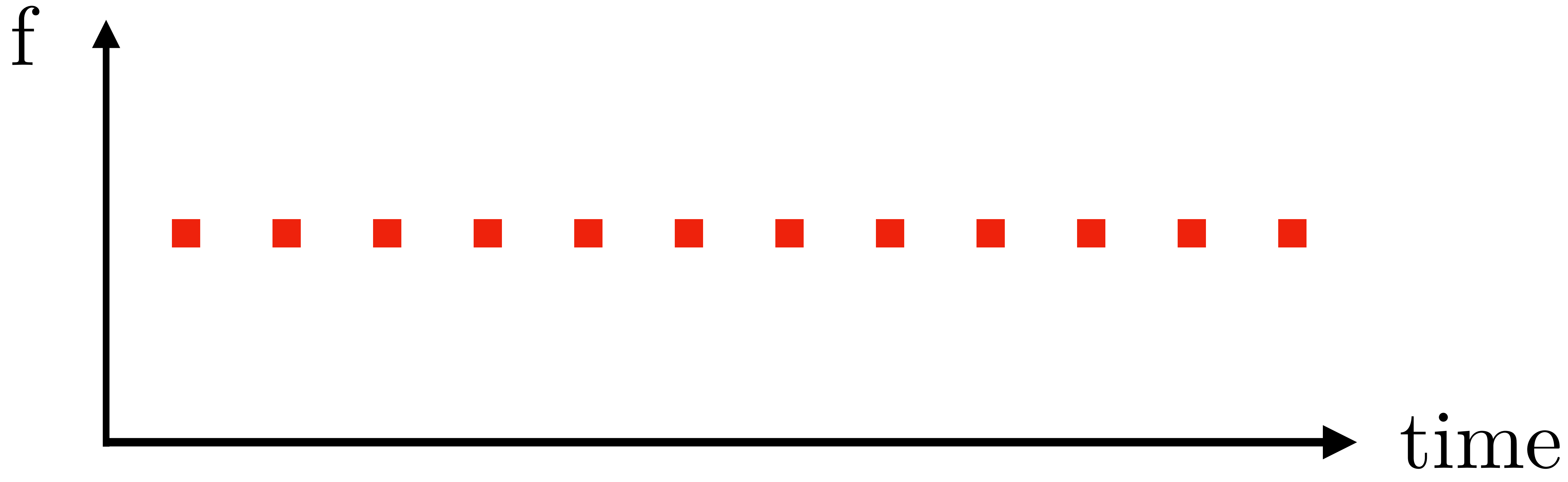
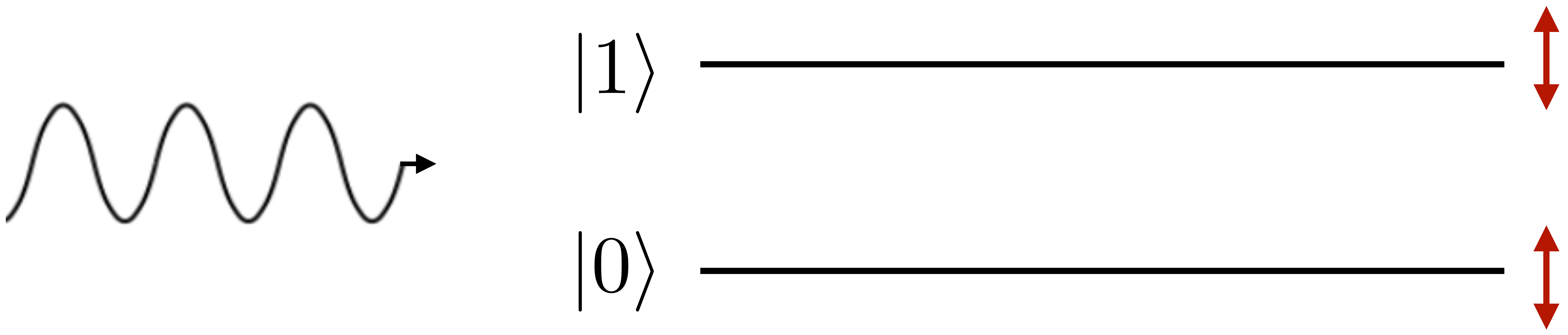


In an *ideal world* without axion, this measurement would give us



Now imagine the same exp. but with axion



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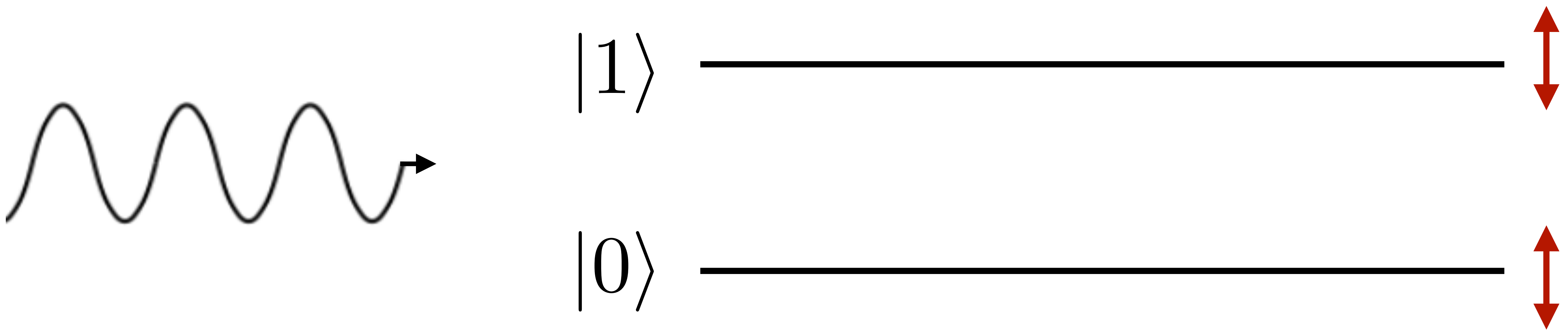

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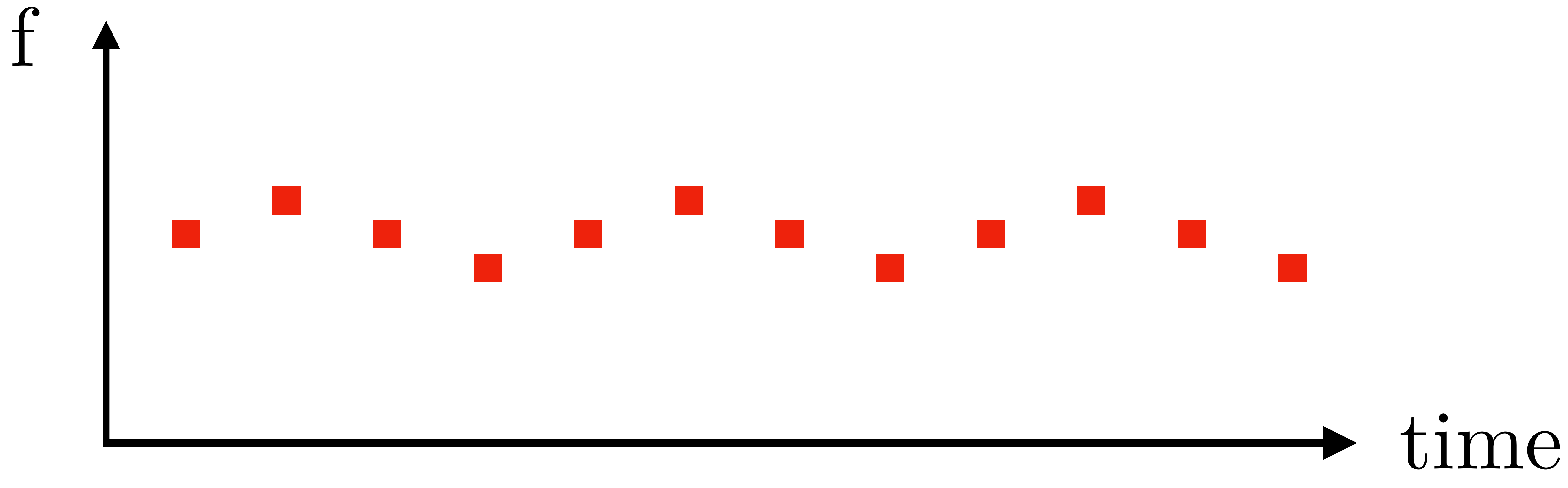

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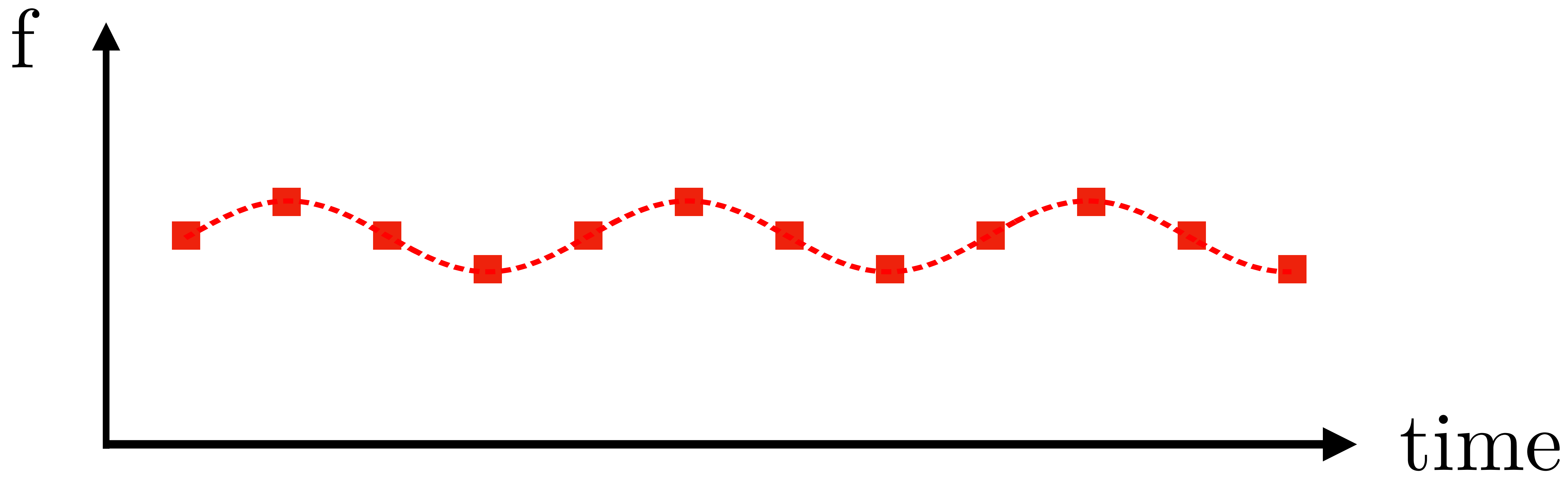

by looking for a harmonic signal
oscillations of energy level can be probed

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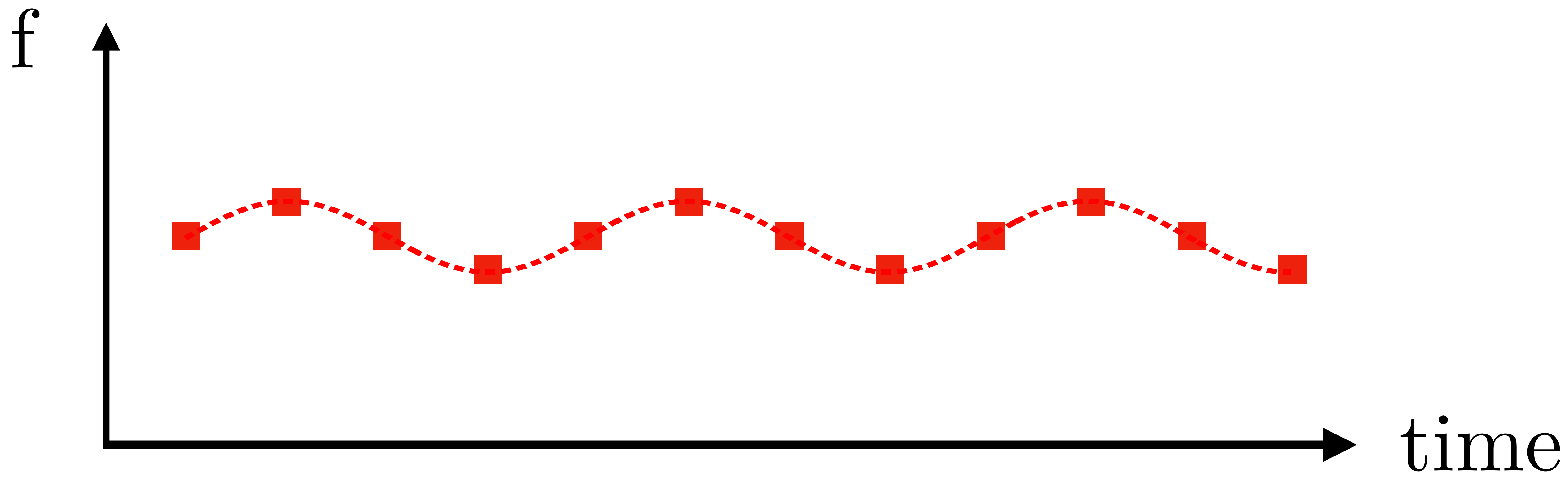

by looking for a harmonic signal
oscillations of energy level can be probed

for dilaton-like (or scalar) DM searches

$$\mathcal{L} = \phi\bar{\psi}\psi + \phi FF + \phi GG$$

atomic clocks provide useful tools

[Arvanitaki, van Tilburg, Huang 14]

We claim that the same principle can be used for

$$\mathcal{L} = \frac{g_s^2}{32\pi^2}\frac{a}{f}G\widetilde{G}$$

because nuclear quantities oscillate according to axion DM

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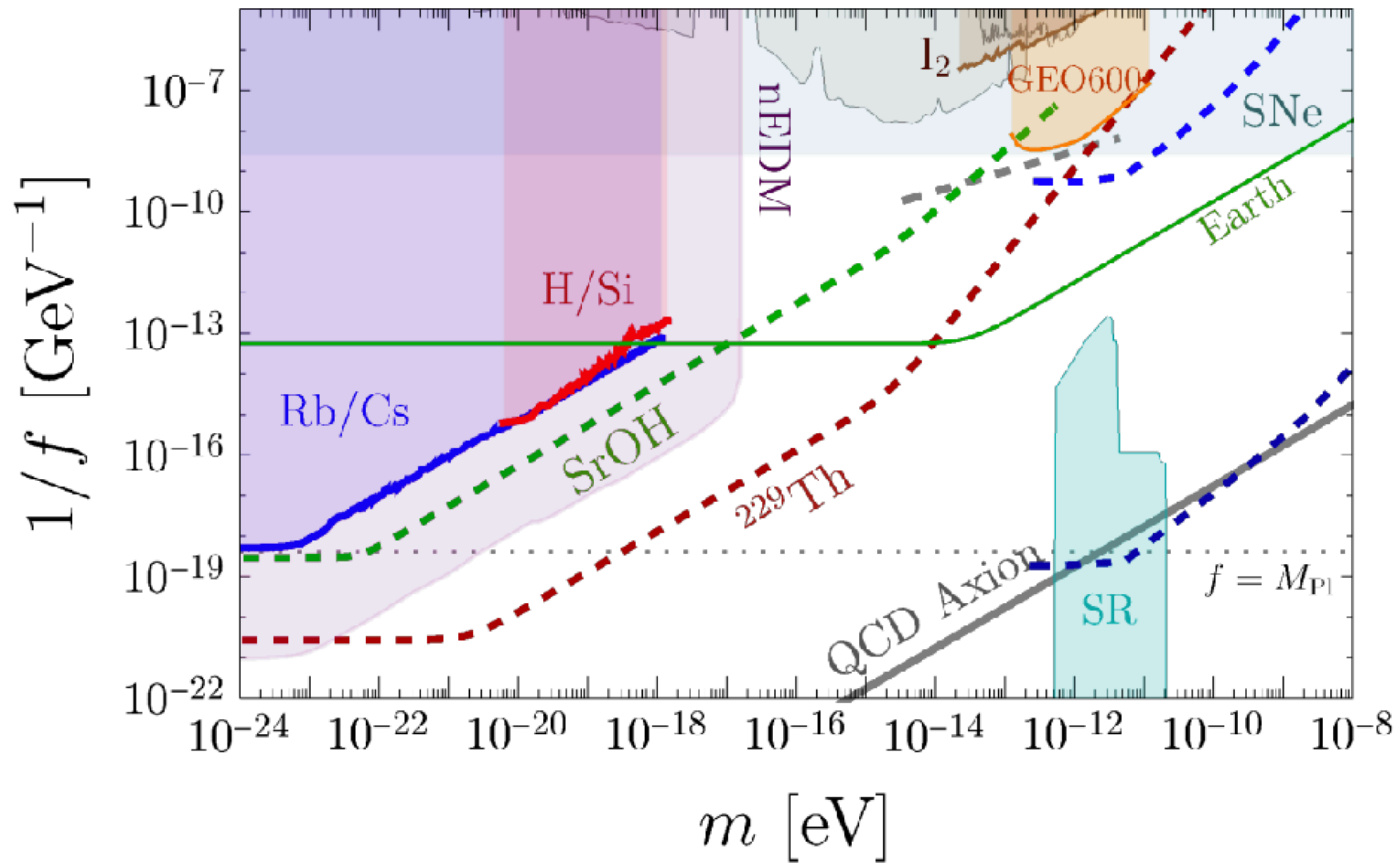
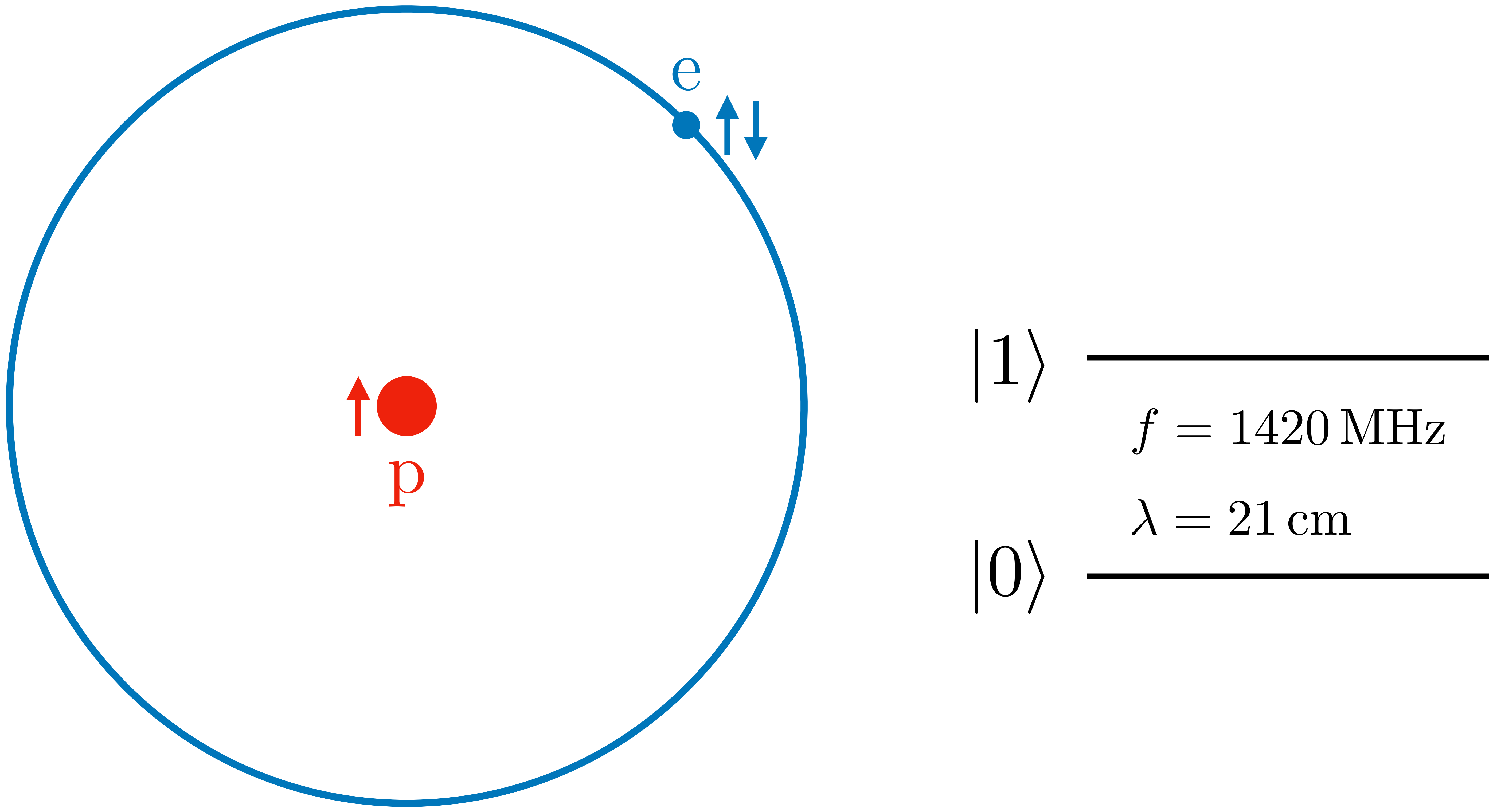
[**HK** and Gilad Perez, 22]



To see how it works

let us consider *hyperfine splitting in hydrogen atom*



$$H = -\vec{\mu}_e \cdot \vec{B}_p$$

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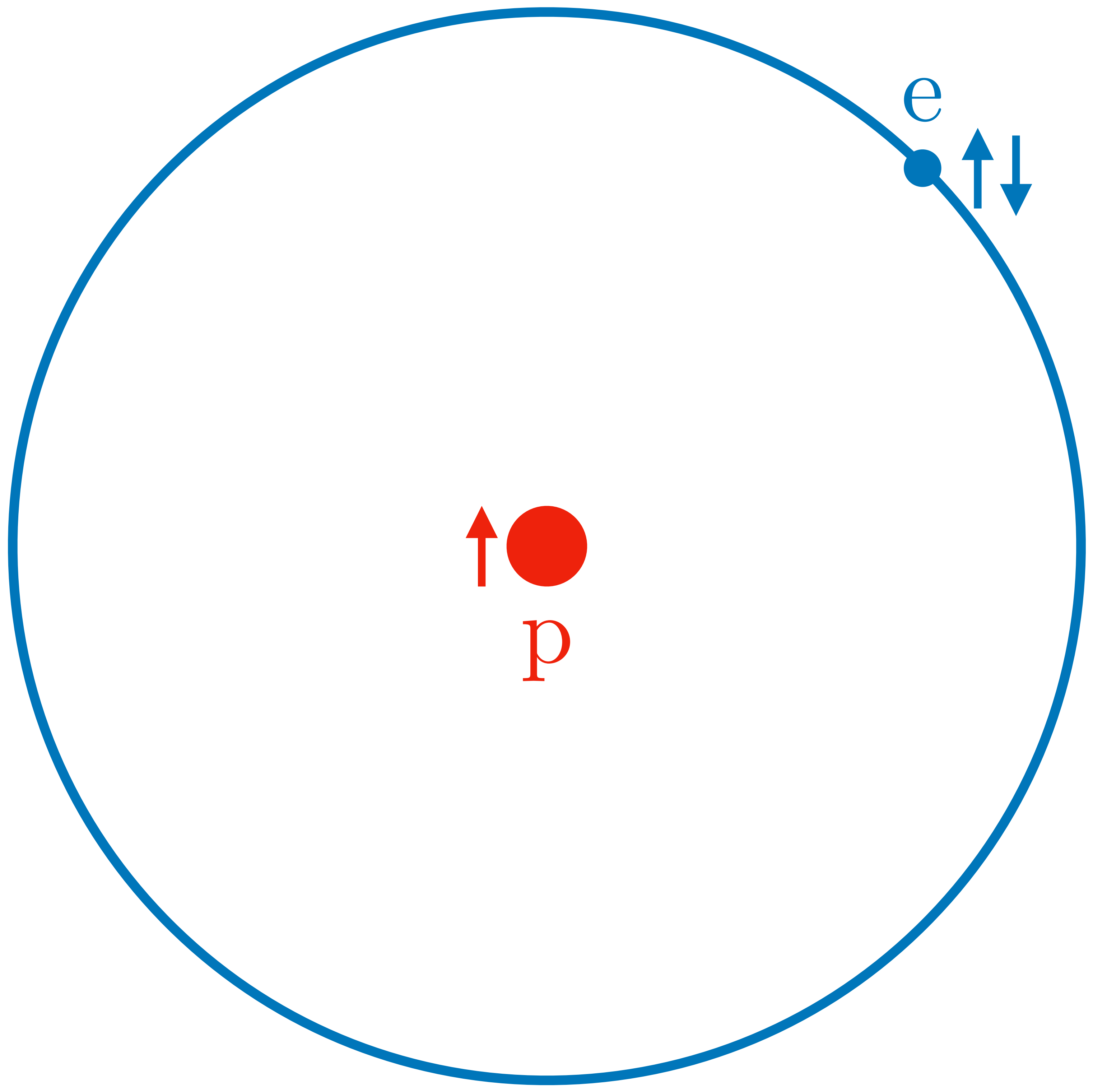

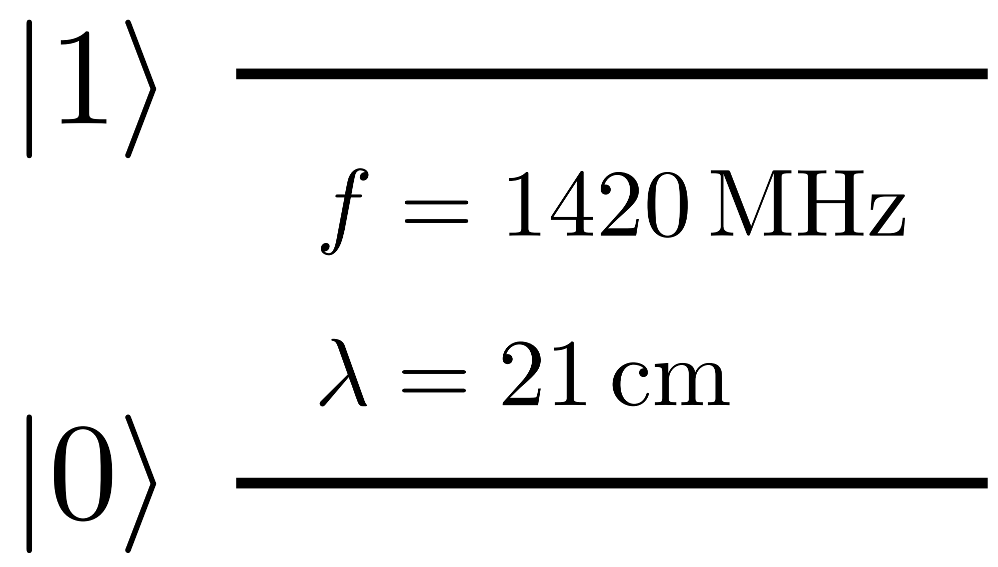

$$E = \frac{4}{3}(m_e^2\alpha^4)\frac{g_p}{m_p}$$

In the presence of axion DM

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energy level slightly changes as

$$\frac{\delta E}{E} = \frac{\delta g_p}{g_p} - \frac{\delta m_p}{m_p}$$

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$$= \left(\frac{\partial \ln g_p}{\partial \ln m_\pi^2} - \frac{\partial \ln m_p}{\partial \ln m_\pi^2}\right)\frac{\delta m_\pi^2}{m_\pi^2}$$

$$\frac{\delta m_\pi^2}{m_\pi^2} = -\frac{m_u m_d}{2(m_u + m_d)^2}\theta^2$$

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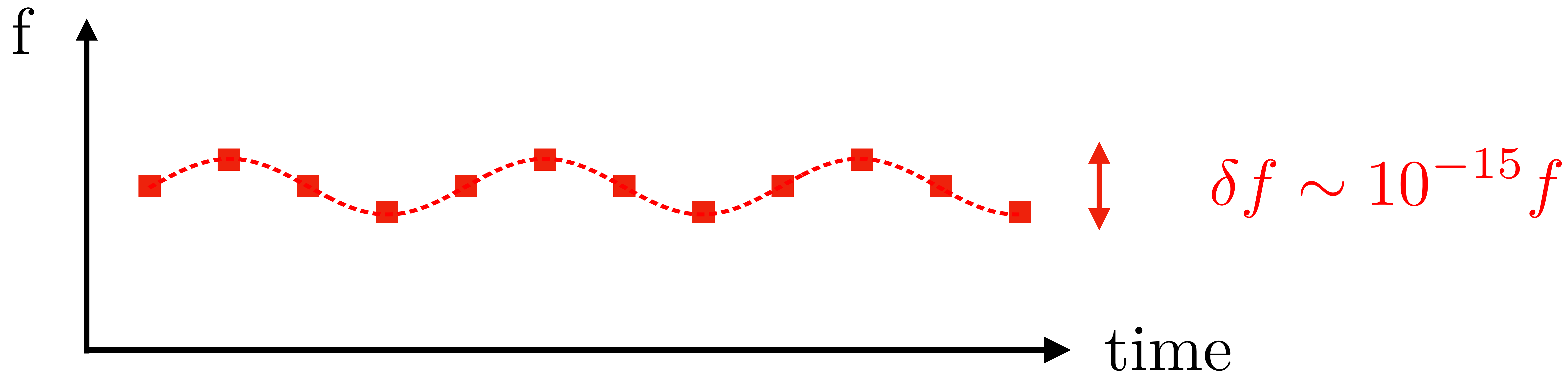
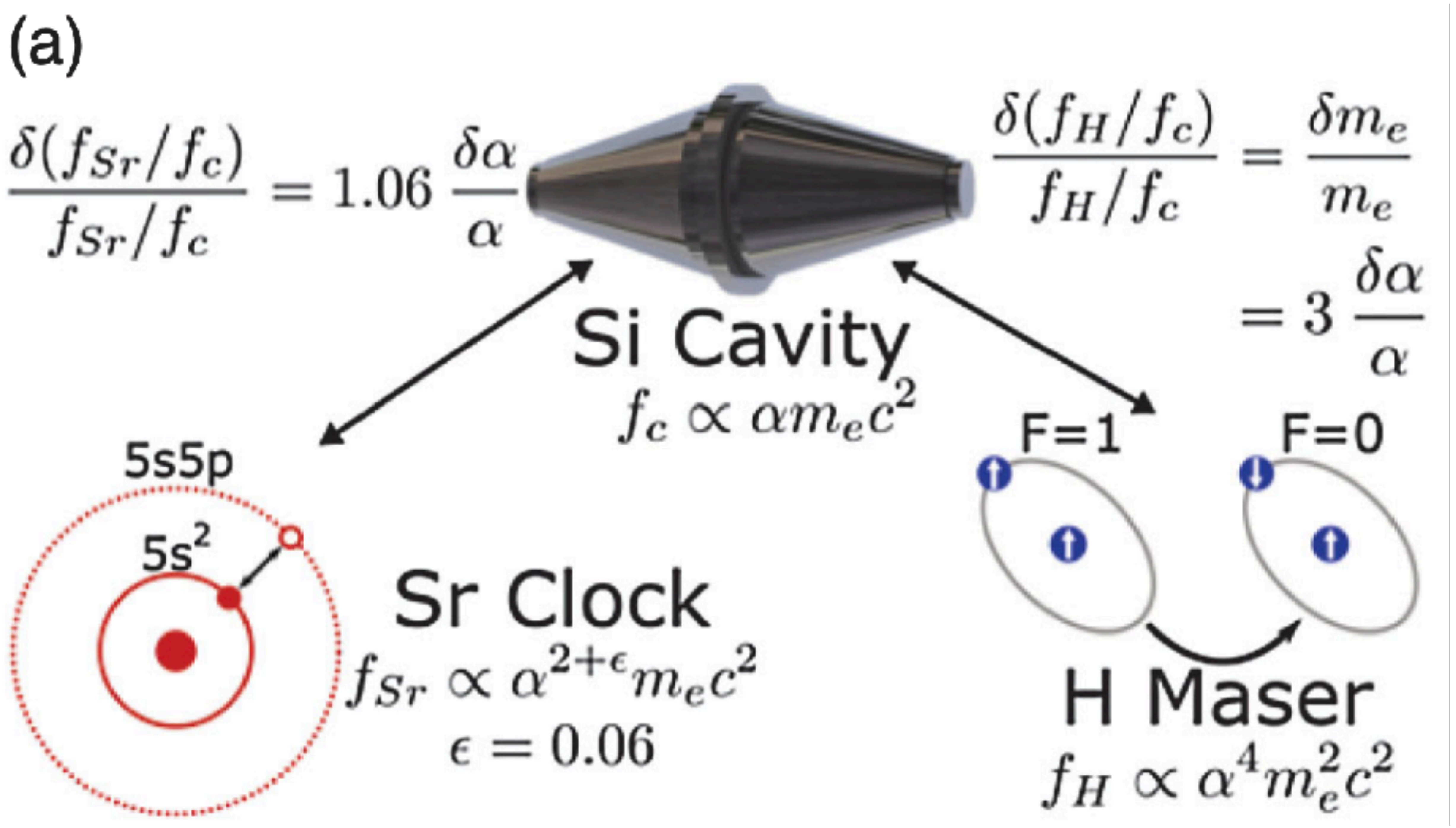
$$\sim 10^{-15}\frac{1}{m_{15}^2 f_{10}^2}[1 + \cos(2mt)]$$

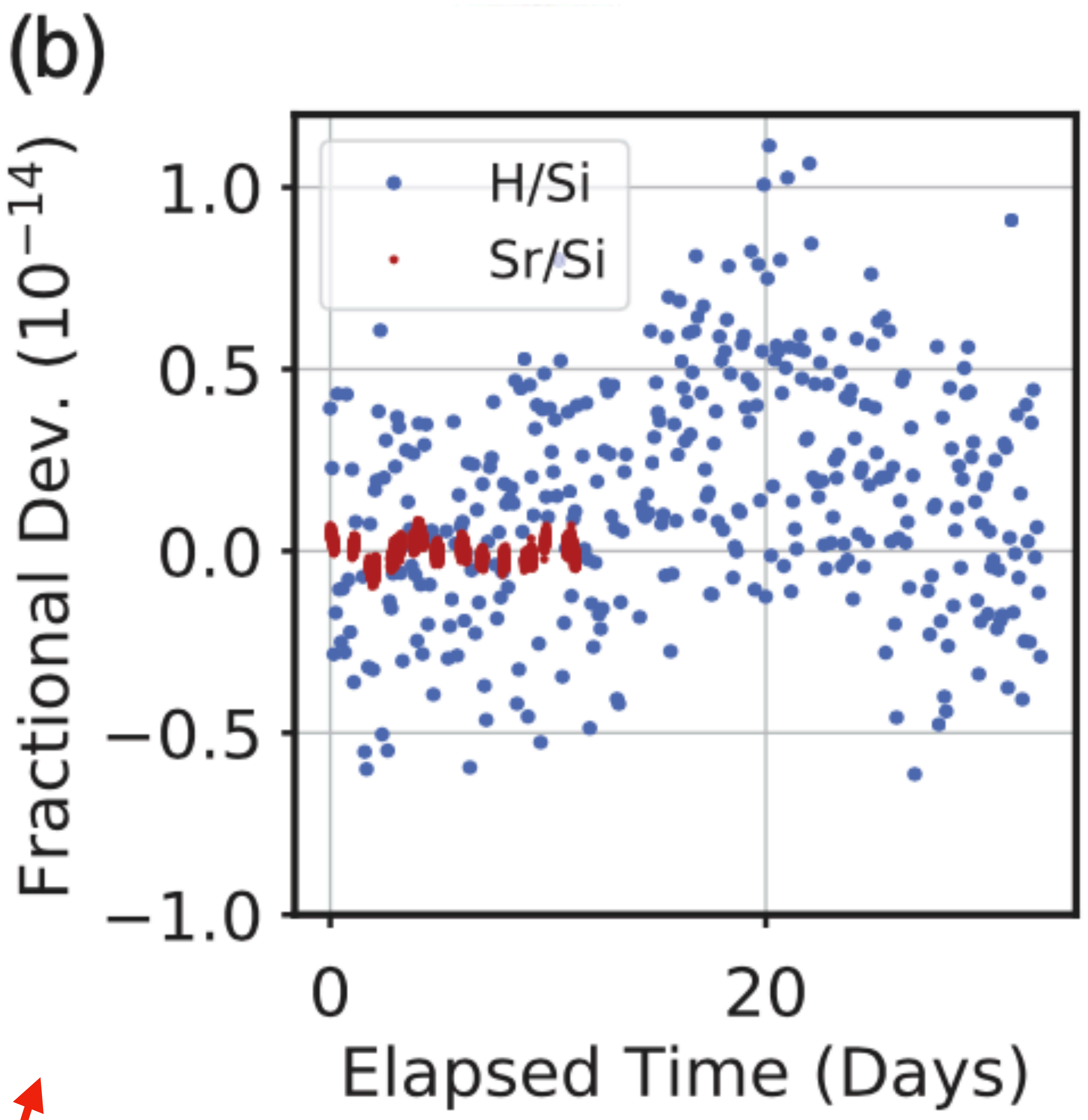
harmonic signal @ $\omega = 2m$



Is it a realistic system or just a toy model?

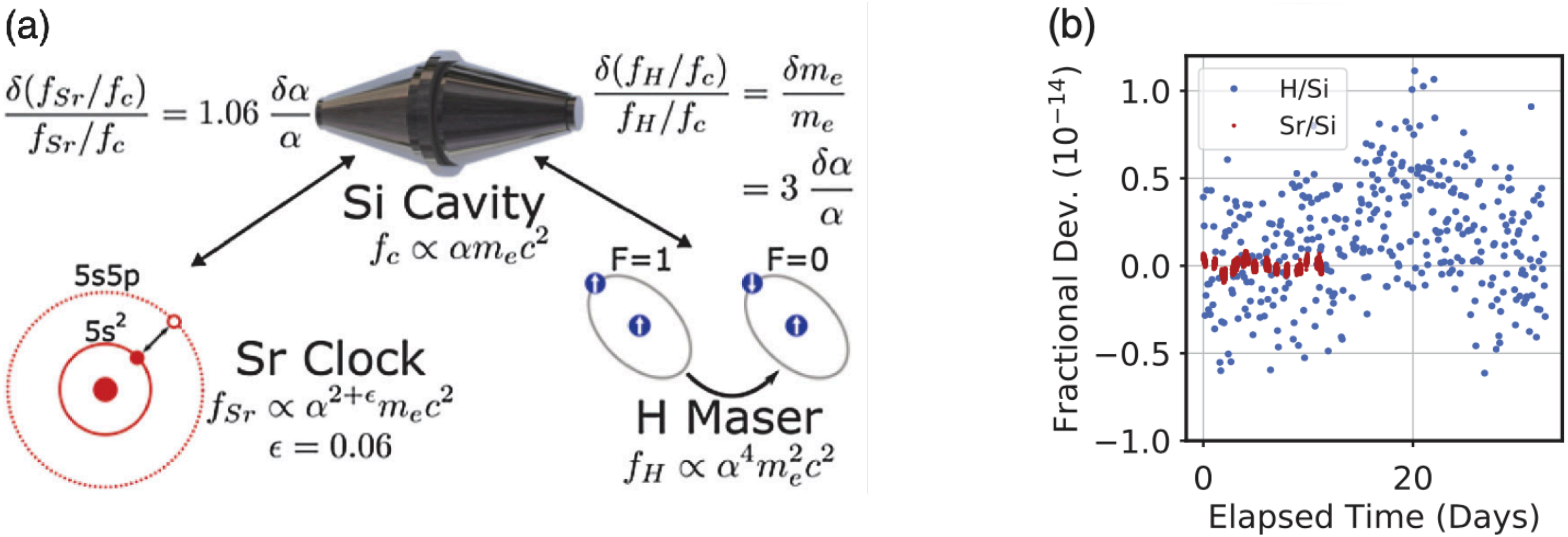
Is this observable?





$$\frac{\delta(f_H/f_{\rm Si})}{(f_H/f_{\rm Si})} \approx \frac{\delta f_H}{f_H}$$

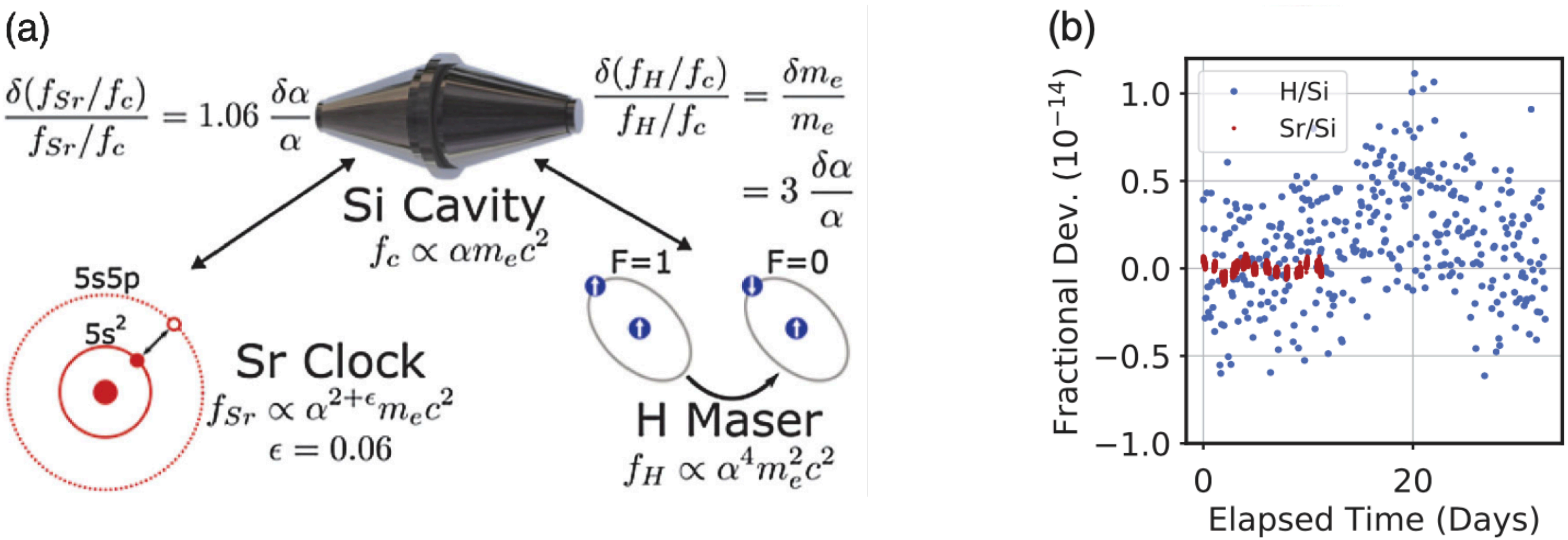
[Kennedy et al 20]



With 33 days of observation

$$\delta f/f \sim 10^{-16}$$

[Kennedy et al 20]

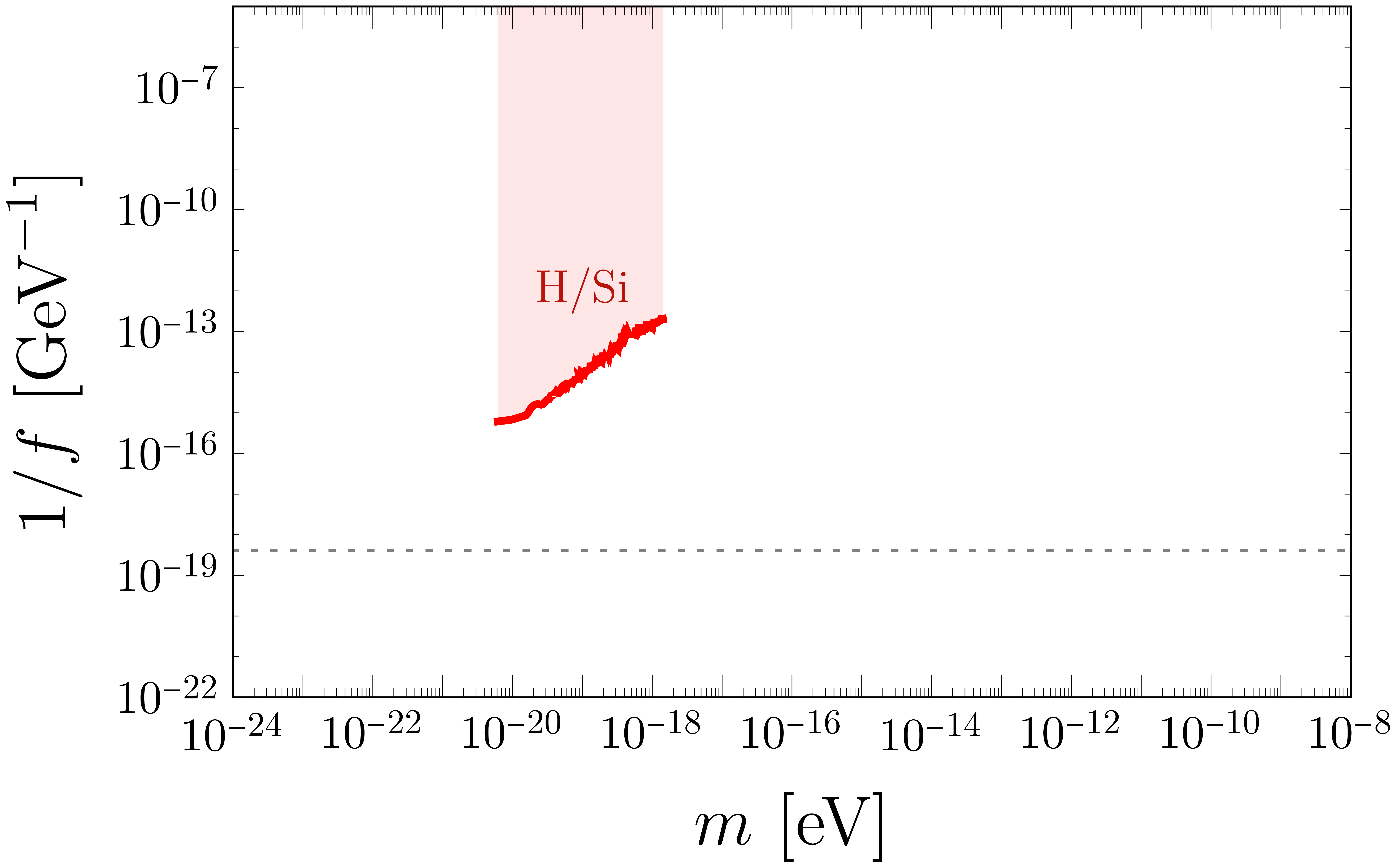


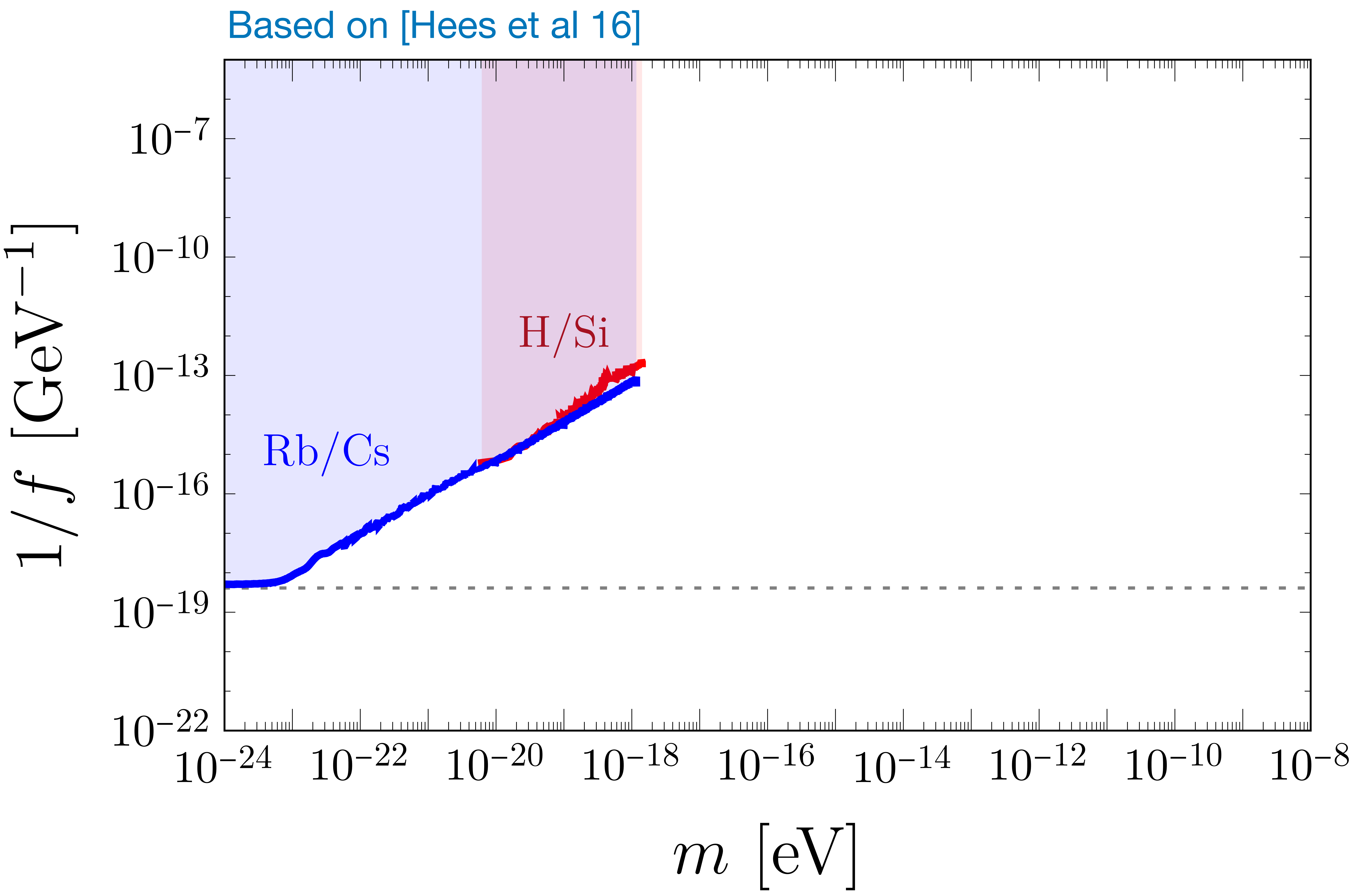
With 33 days of observation

$$\delta f/f \sim 10^{-16}$$

$$(\delta f/f) \sim 10^{-15} \frac{1}{m_{15}^2 f_{10}^2} [1 + \cos(2mt)]$$

Based on [Kennedy et al 20]





Based on [Hees et al 16]
H/Si
Rb/Cs
$1/f$ [GeV^{-1}]
10^{-7}
10^{-10}
10^{-13}
10^{-16}
10^{-19}
10^{-22}
10^{-24} 10^{-22} 10^{-20} 10^{-18} 10^{-16} 10^{-14} 10^{-12} 10^{-10} 10^{-8}
m [eV]

any frequency standards

where their frequency depends on nuclear quantities

can be used for axion DM search

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can be used for axion DM search

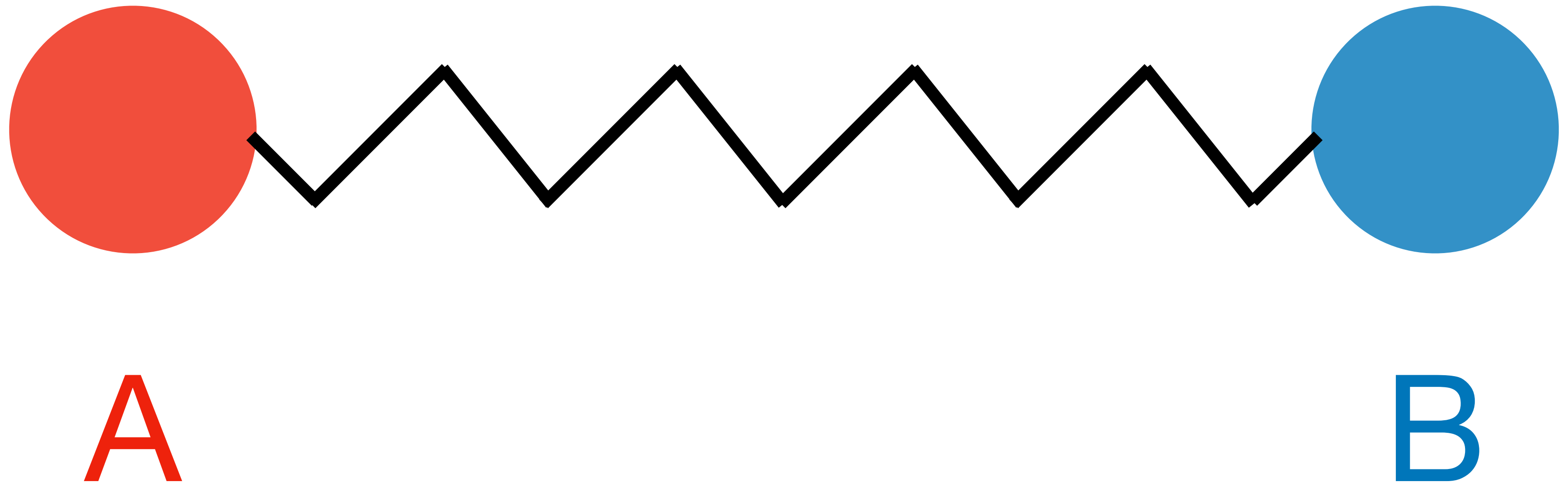
e.g.

nuclear transition

molecular transition

and others …

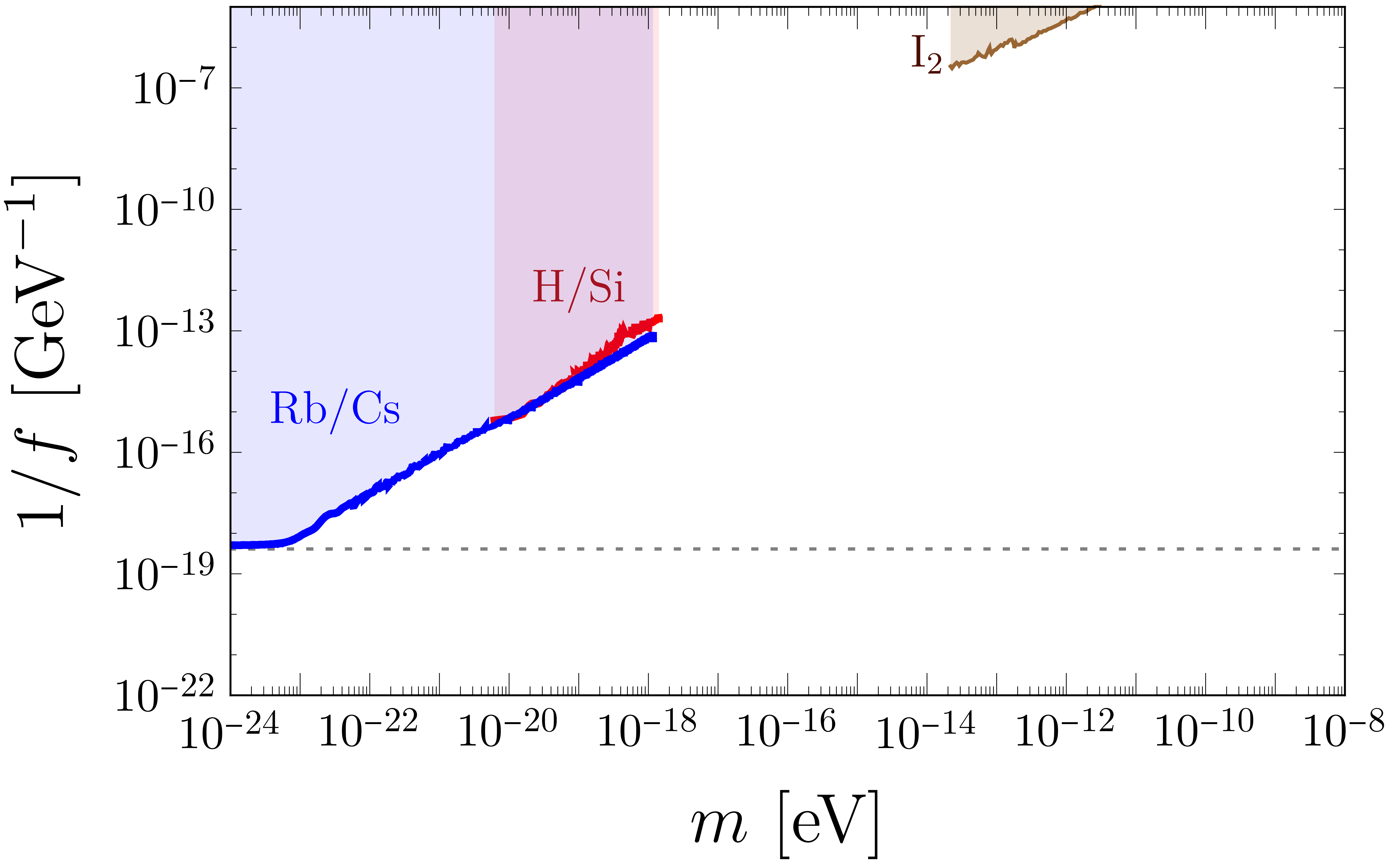
Vibrational excitation



$$M\ddot{x} + kx \simeq 0$$

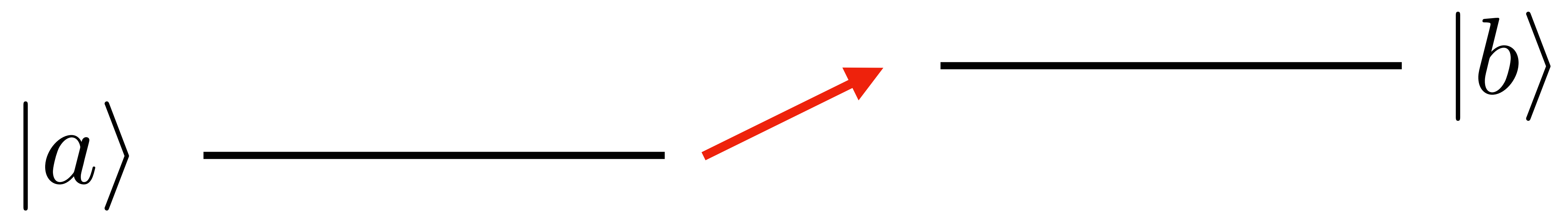
$$\omega \propto \sqrt{\frac{k}{M}}$$

Based on [Oswald et al 21]


I2
H/Si
Rb/Cs
10−7
10−10
10−13
10−16
10−19
10−22
1/f [GeV−1]
10−24
10−22
10−20
10−18
10−16
10−14
10−12
10−10
10−8
m [eV]

Can we do better than this?

Maybe, with almost degenerate energy levels



$$f = f_b - f_a$$

$$\frac{\delta f}{f} = \frac{1}{f}\left(f_b \frac{\delta f_b}{f_b} - f_a \frac{\delta f_a}{f_a}\right) \qquad f \ll f_a \simeq f_b$$

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$$\approx \frac{f_b}{f}\left(\frac{\delta f_b}{f_b} - \frac{\delta f_a}{f_a}\right)$$

$$\gg 1$$

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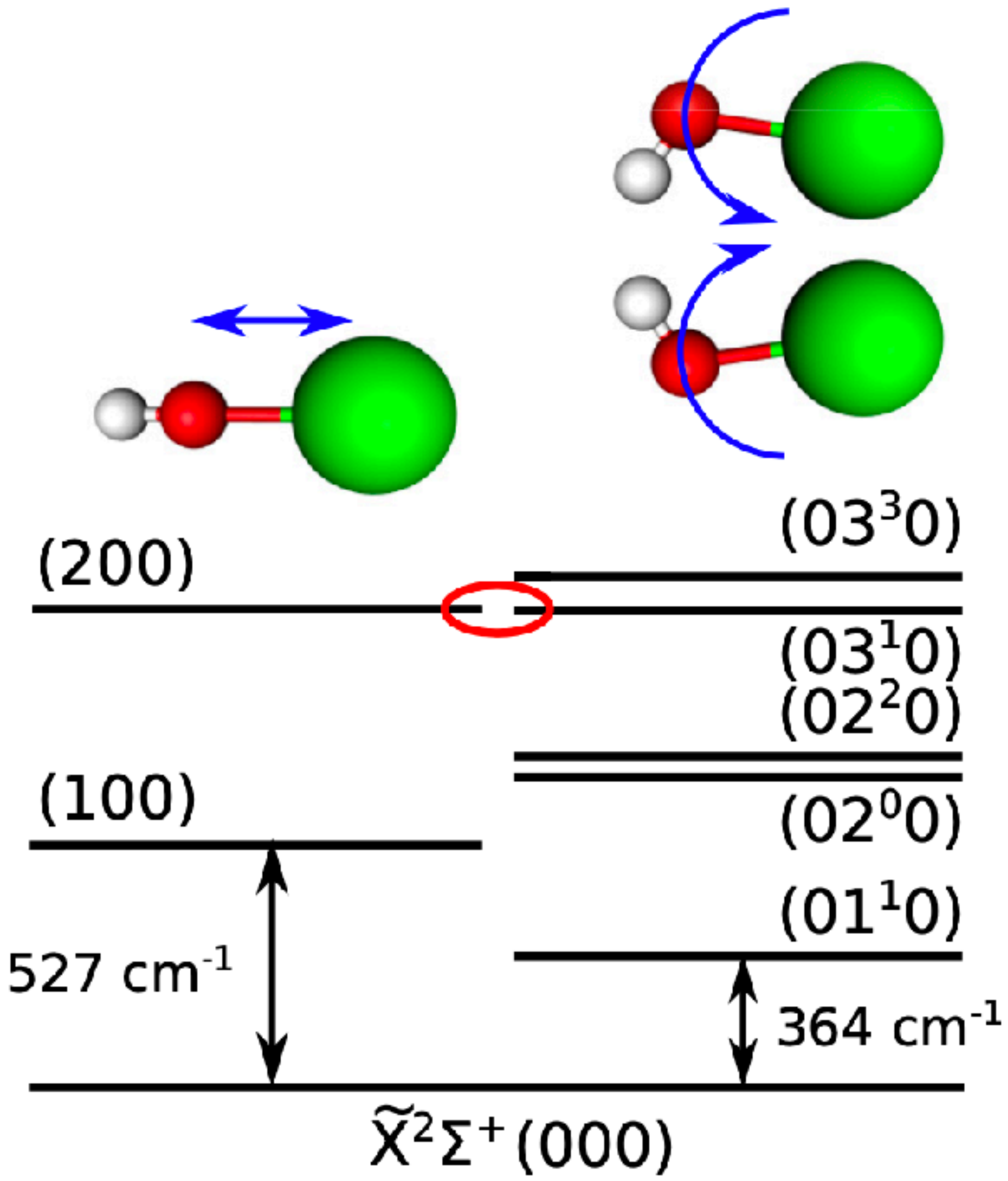
$$\approx \frac{f_b}{f}\left(\frac{\delta f_b}{f_b} - \frac{\delta f_a}{f_a}\right) = \frac{f_b}{f}\left(\frac{\partial \ln f_b}{\partial \ln m_\pi^2} - \frac{\partial \ln f_a}{\partial \ln m_\pi^2}\right)\frac{\delta m_\pi^2}{m_\pi^2}$$

$$\frac{f_b}{f} \gg 1$$

Example (1): SrOH

[Kozyryev et al 21]



$$\frac{\delta f}{f} \sim -600 \frac{\delta m_p}{m_p}$$

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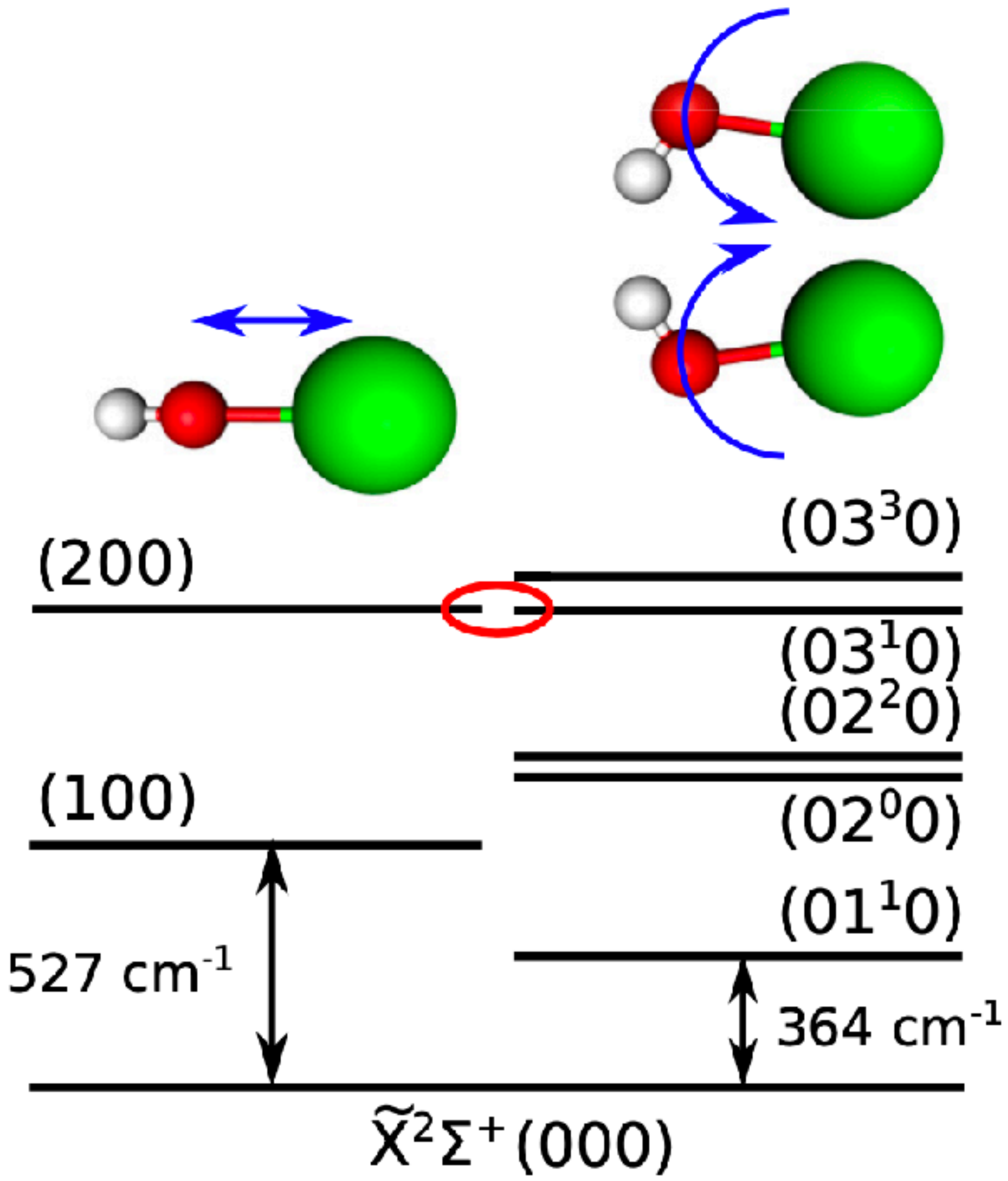

$$\frac{\delta f}{f} \sim -600 \frac{\delta m_p}{m_p}$$

H hyperfine

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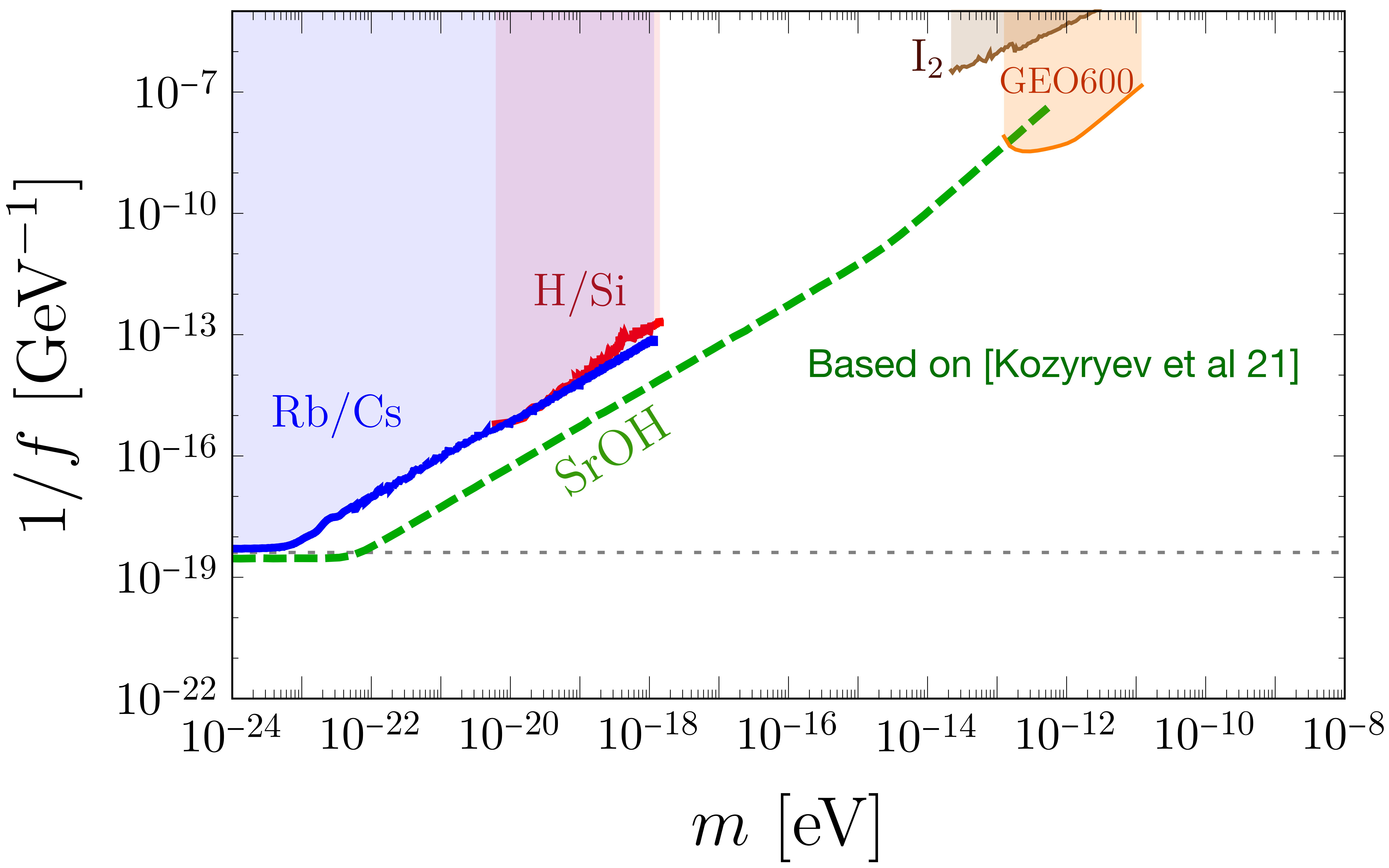
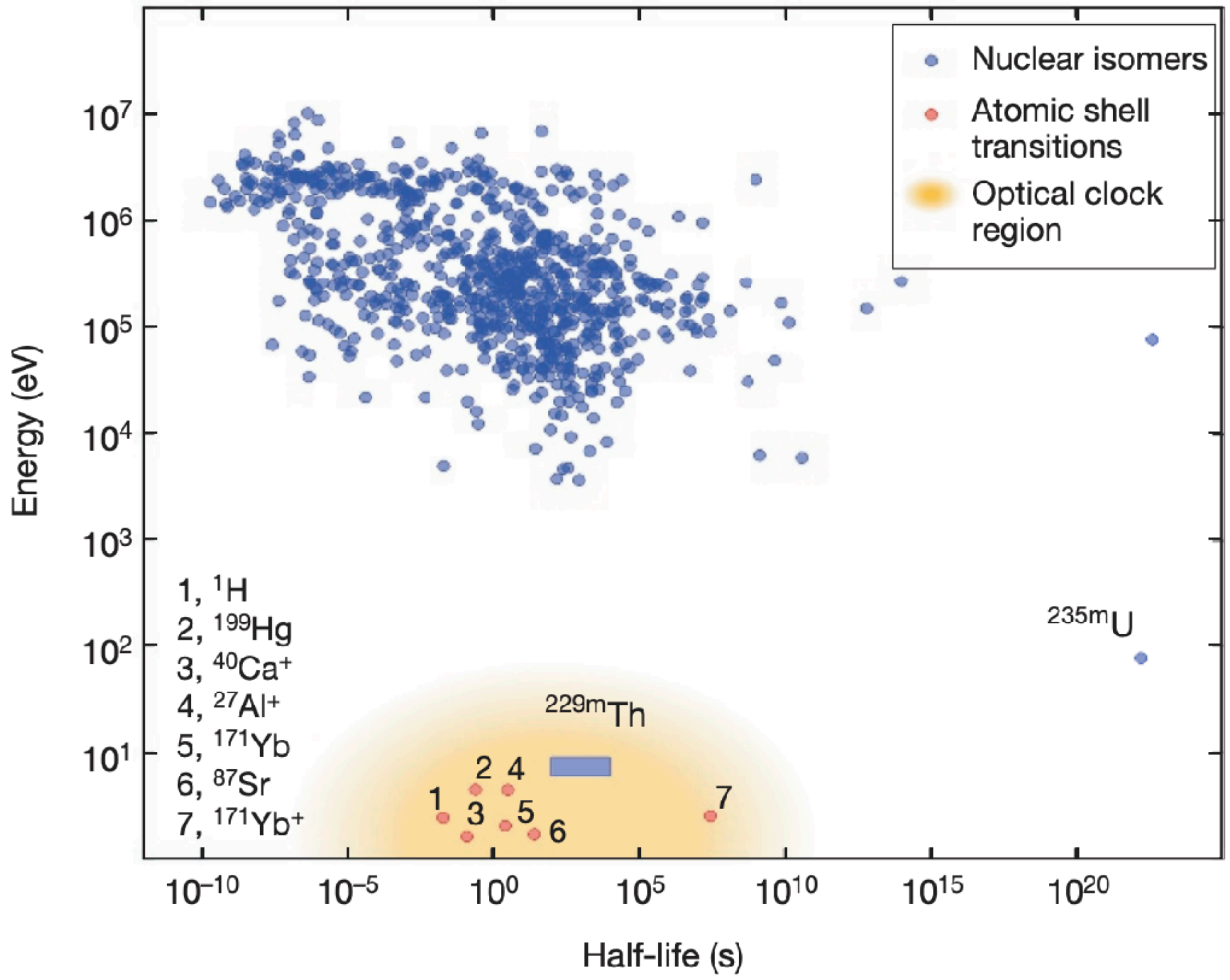
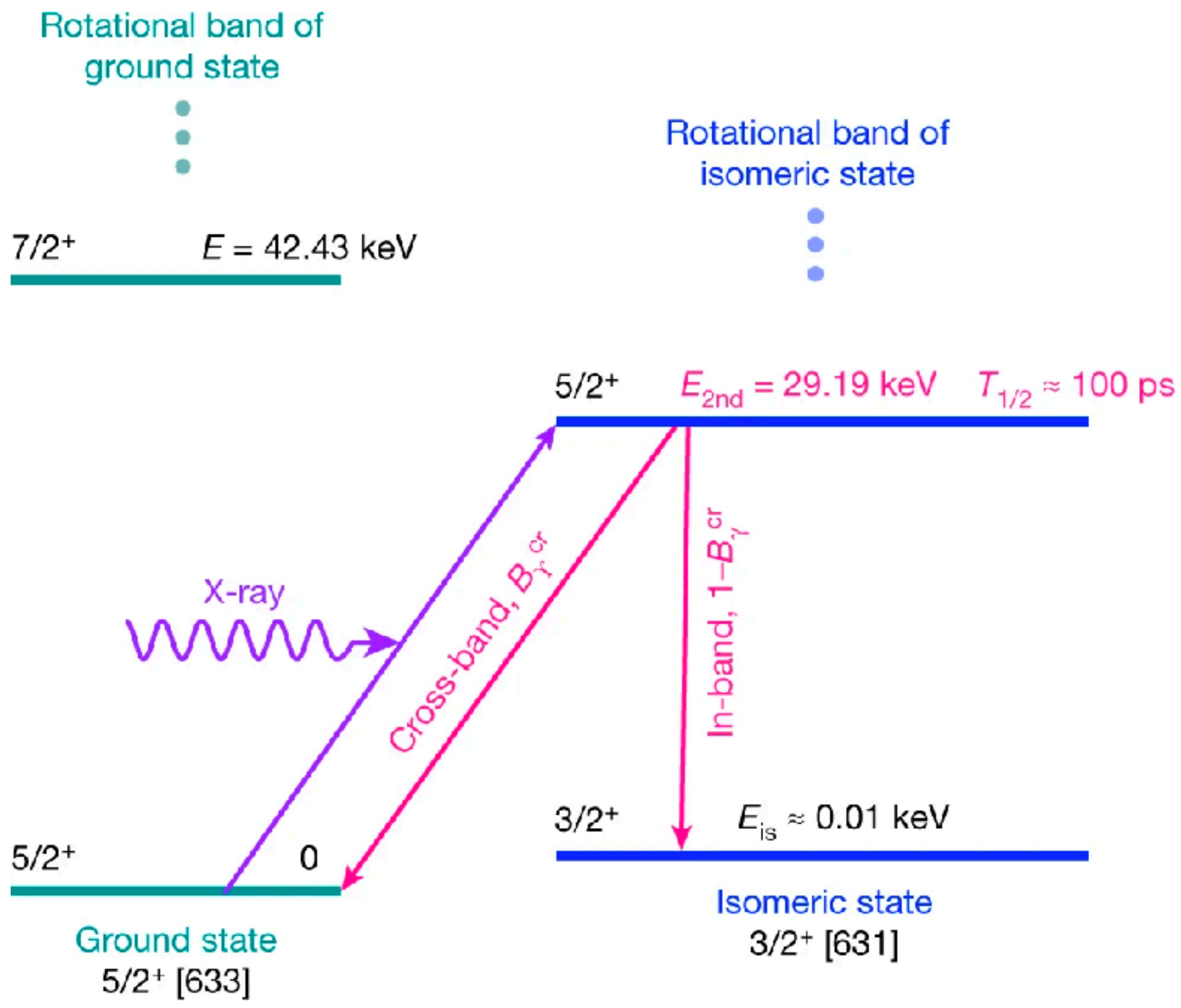


$1/f$ [GeV^{-1}]
10^{-7}
10^{-10}
10^{-13}
10^{-16}
10^{-19}
10^{-22}
10^{-24}
10^{-22}
10^{-20}
10^{-18}
10^{-16}
10^{-14}
10^{-12}
10^{-10}
10^{-8}
m [eV]
Rb/Cs
H/Si
SrOH
I_2
GEO600
Based on [Kozyryev et al 21]

Example (2): Nuclear isomer transition

[von der Wense et al 16]





[Masuda et al 19]

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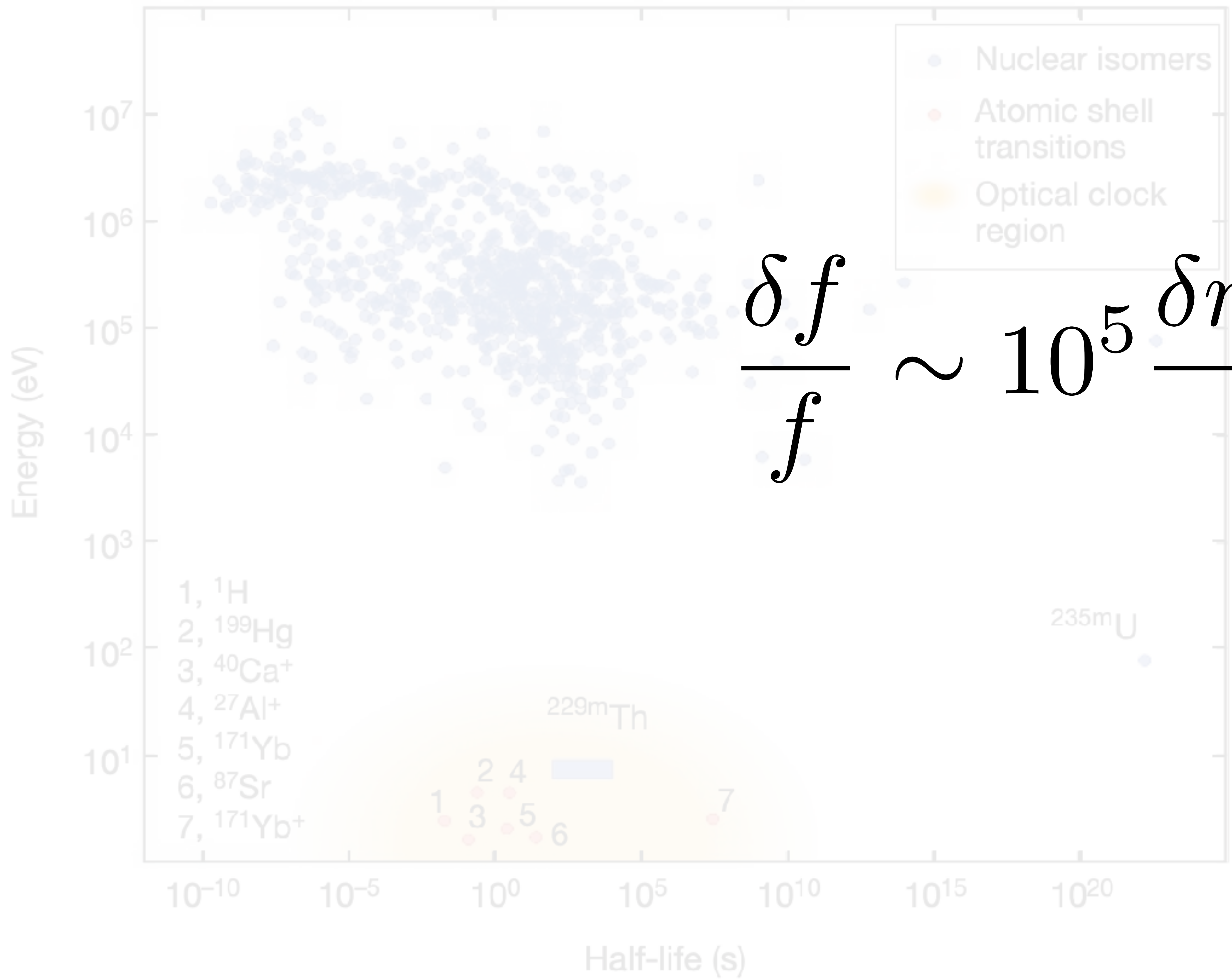
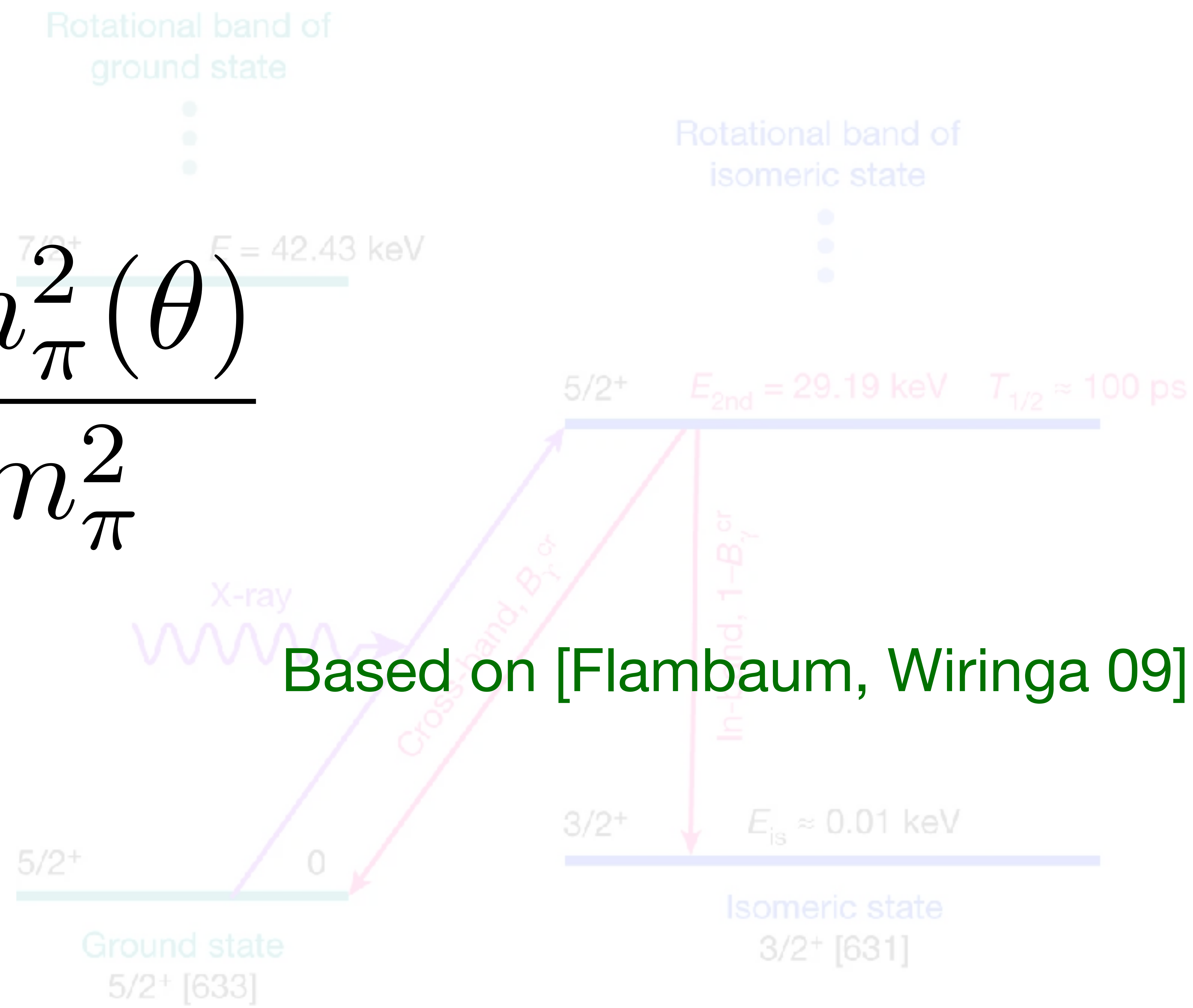
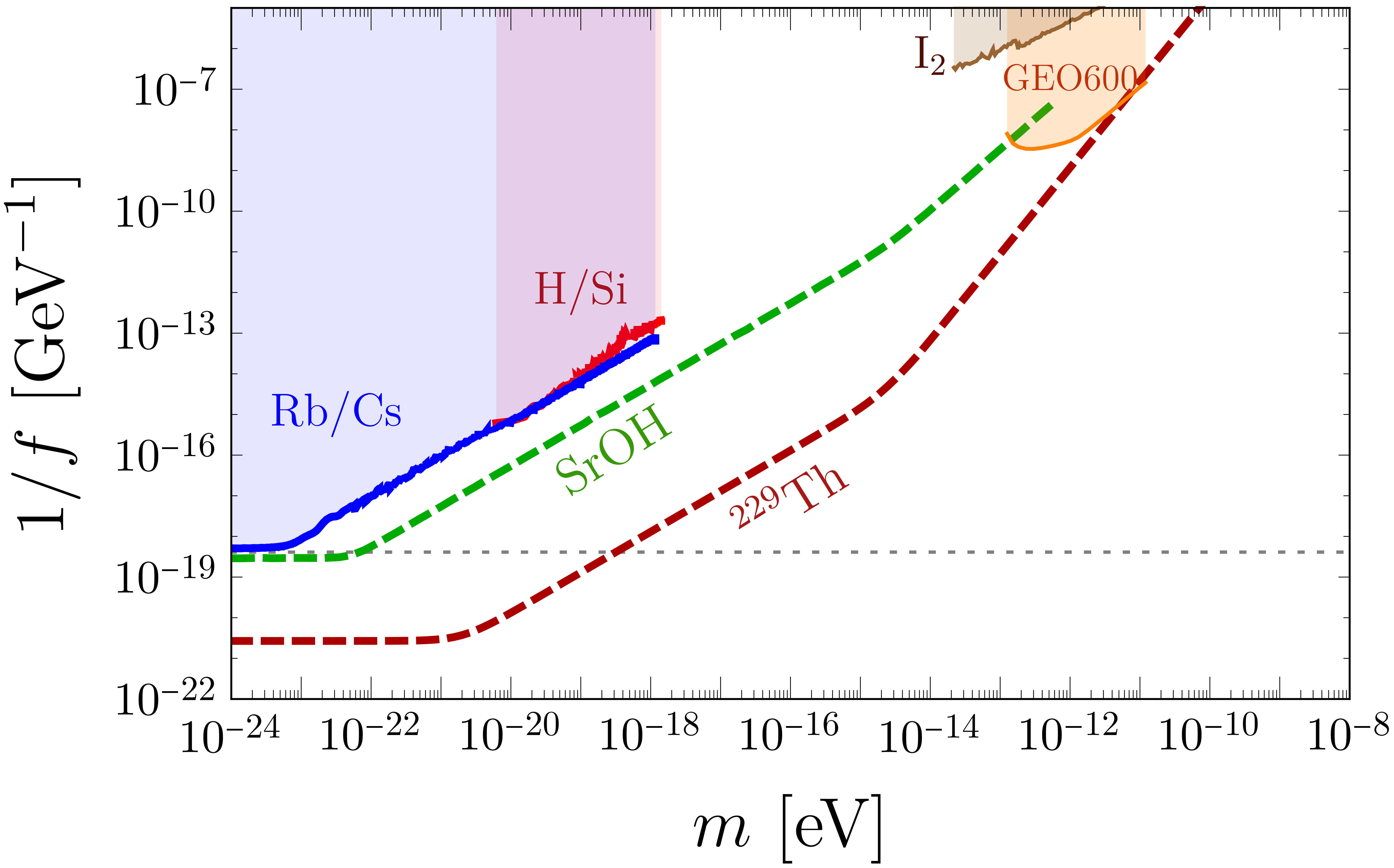


$$\frac{\delta f}{f} \sim 10^5 \frac{\delta m_\pi^2(\theta)}{m_\pi^2}$$



Based on [Flambaum, Wiringa 09]

[Masuda et al 19]



$1/f$ [GeV^{-1}]
m [eV]
10^{-7}
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Rb/Cs
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^{229}Th
I_2
GEO600

Comparison to other works

& limitations

$$\mathcal{L} = \frac{g_s^2}{32\pi^2}\frac{a}{f}G\widetilde{G}$$

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↓ Below QCD scale

$$\mathcal{L} = -\frac{ieC_{\rm edm}}{2}\theta\bar{N}\sigma^{\mu\nu}\gamma_5 N F_{\mu\nu} + \frac{C_f}{2}\partial_\mu\theta\bar{f}\gamma^\mu\gamma_5 f$$

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oscillating EDM

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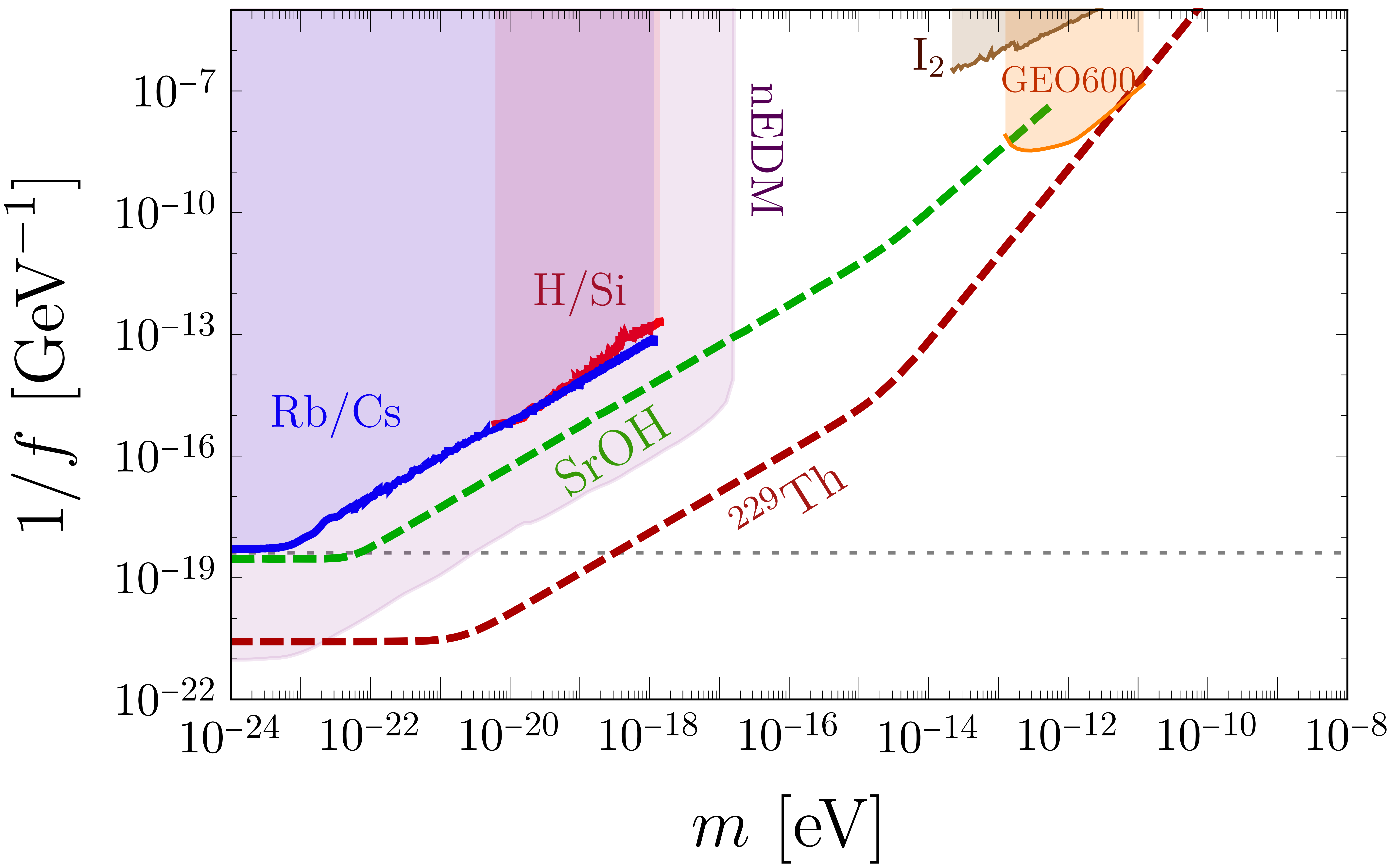
oscillating EDM

magnetometer

[Abel et al 17]



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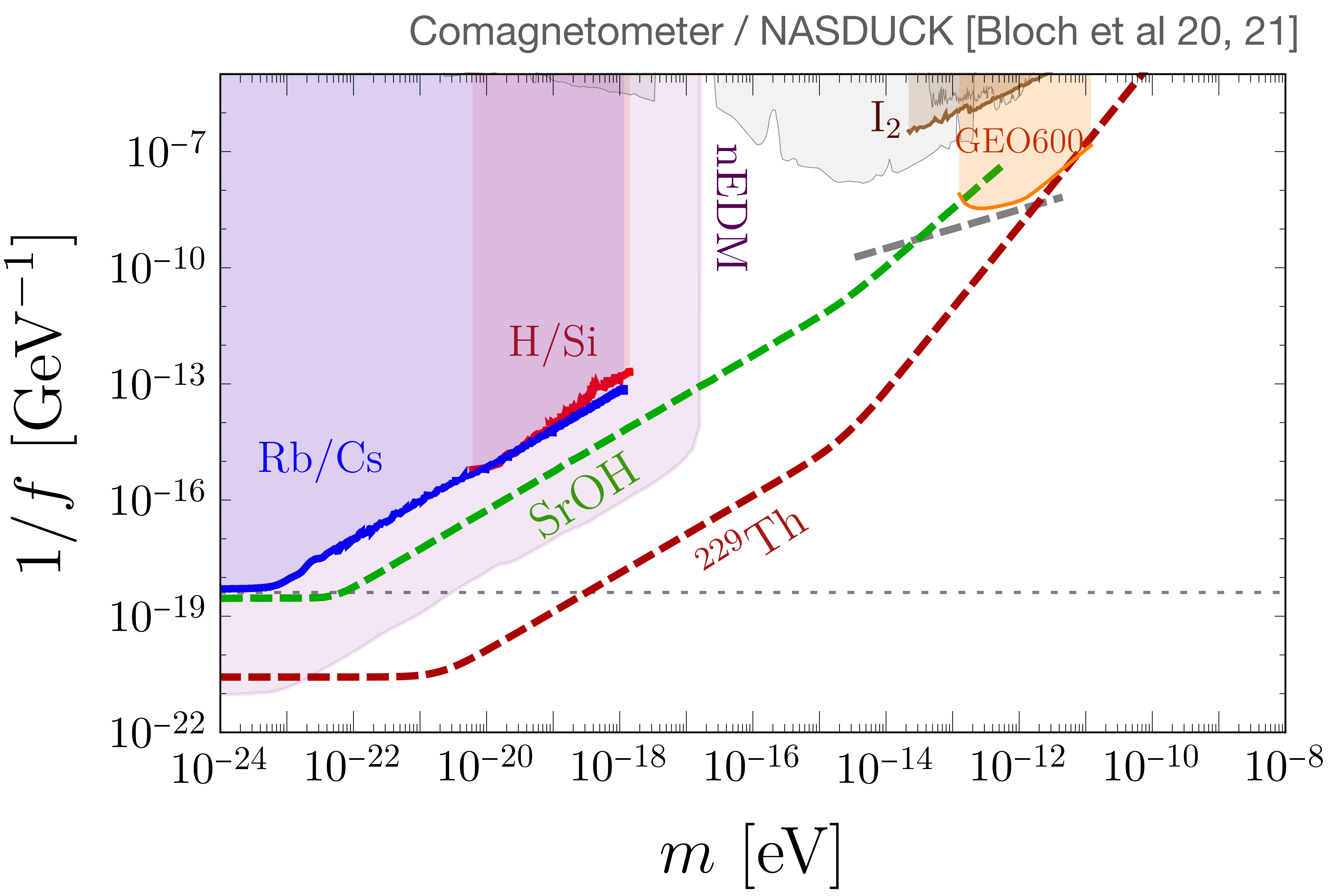

Comagnetometer / NASDUCK [Bloch et al 20, 21]
1/f [GeV−1]
10−7
10−10
10−13
10−16
10−19
10−22
m [eV]
10−24
10−22
10−20
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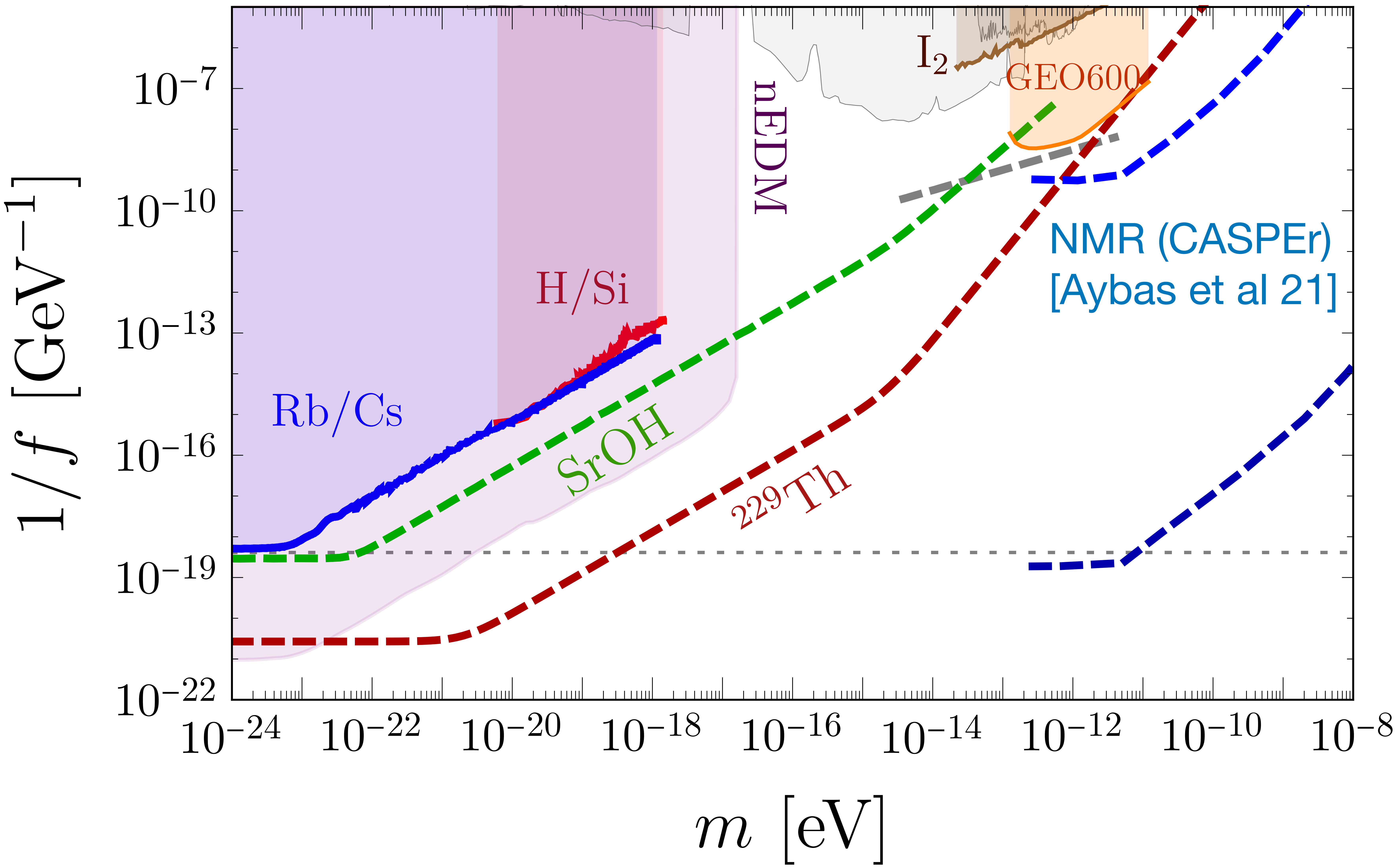

$1/f$ [GeV^{-1}]
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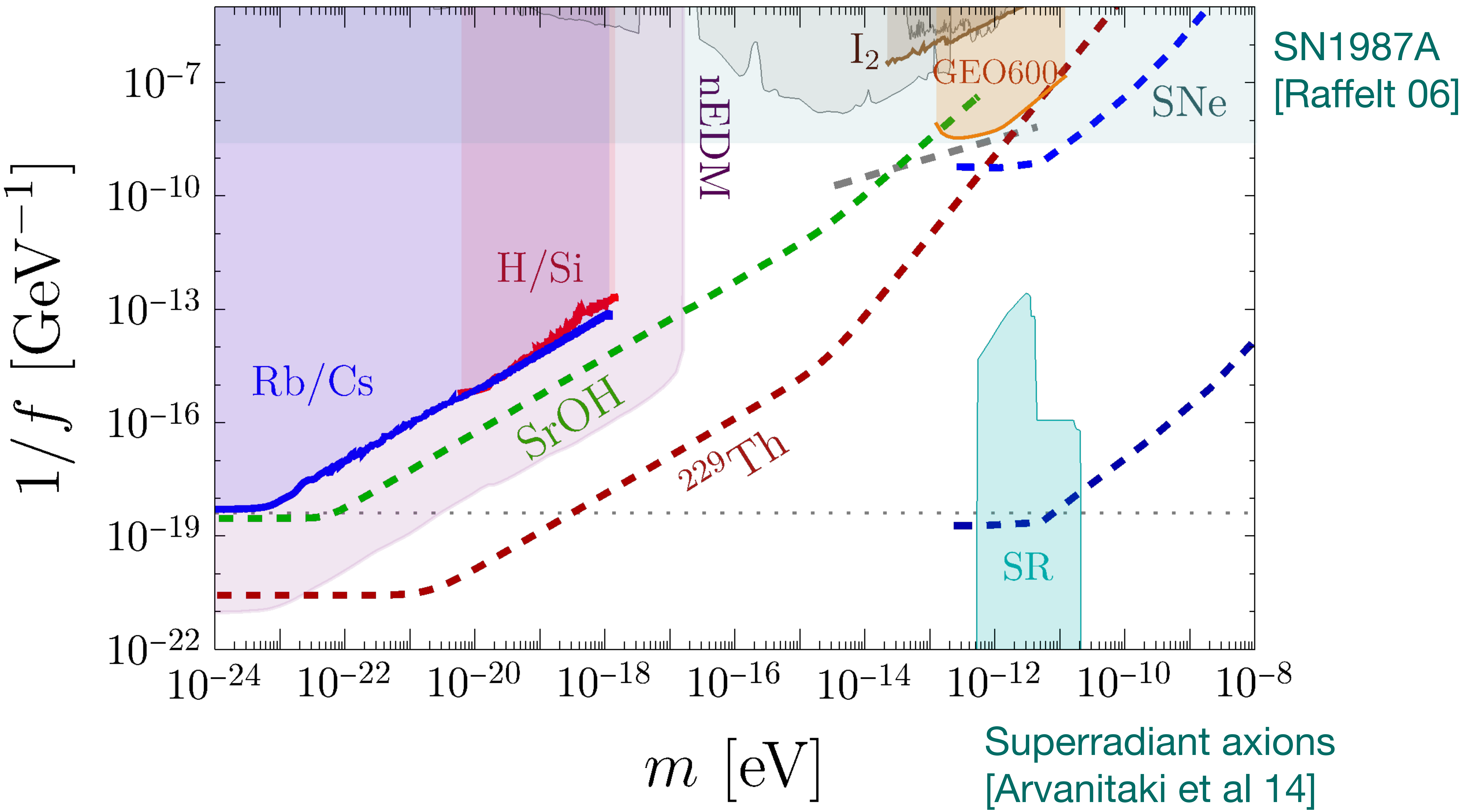
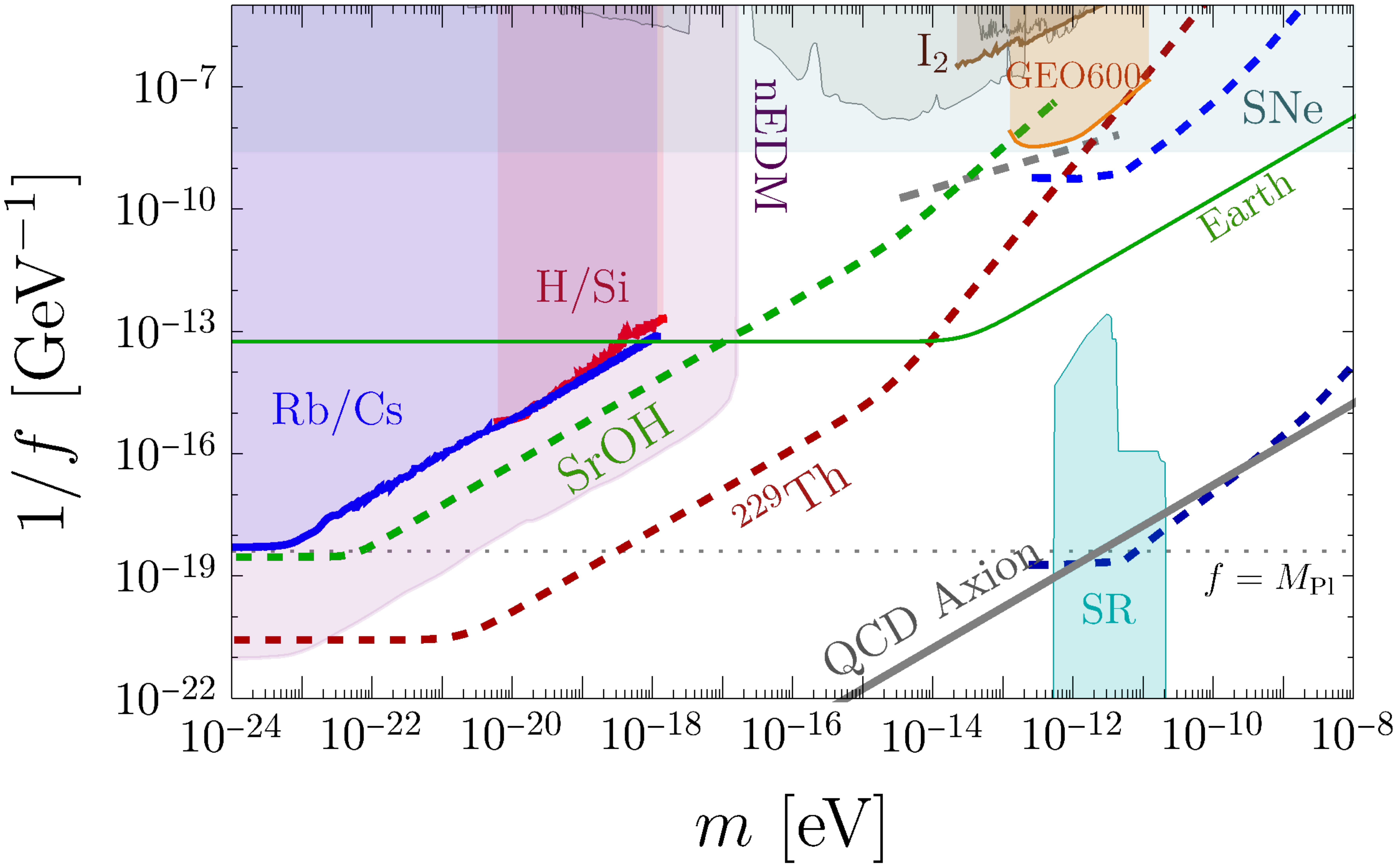
10^{-24}
10^{-22}
10^{-20}
10^{-18}
10^{-16}
10^{-14}
10^{-12}
10^{-10}
10^{-8}
Rb/Cs
H/Si
SrOH
nEDM
I_2
GEO600
^{229}Th
NMR (CASPEr)
[Aybas et al 21]


SN1987A
[Raffelt 06]
SNe
GEO600
I_2
nEDM
H/Si
Rb/Cs
SrOH
^{229}Th
SR
$1/f$ [GeV^{-1}]
m [eV]
Superradiant axions
[Arvanitaki et al 14]



$1/f$ [GeV^{-1}]
m [eV]
10^{-7}
10^{-10}
10^{-13}
10^{-16}
10^{-19}
10^{-22}
10^{-24}
10^{-22}
10^{-20}
10^{-18}
10^{-16}
10^{-14}
10^{-12}
10^{-10}
10^{-8}
Rb/Cs
H/Si
nEDM
I_2
GEO600
SNe
Earth
SrOH
^{229}Th
QCD Axion
SR
$f = M_{Pl}$

Back up

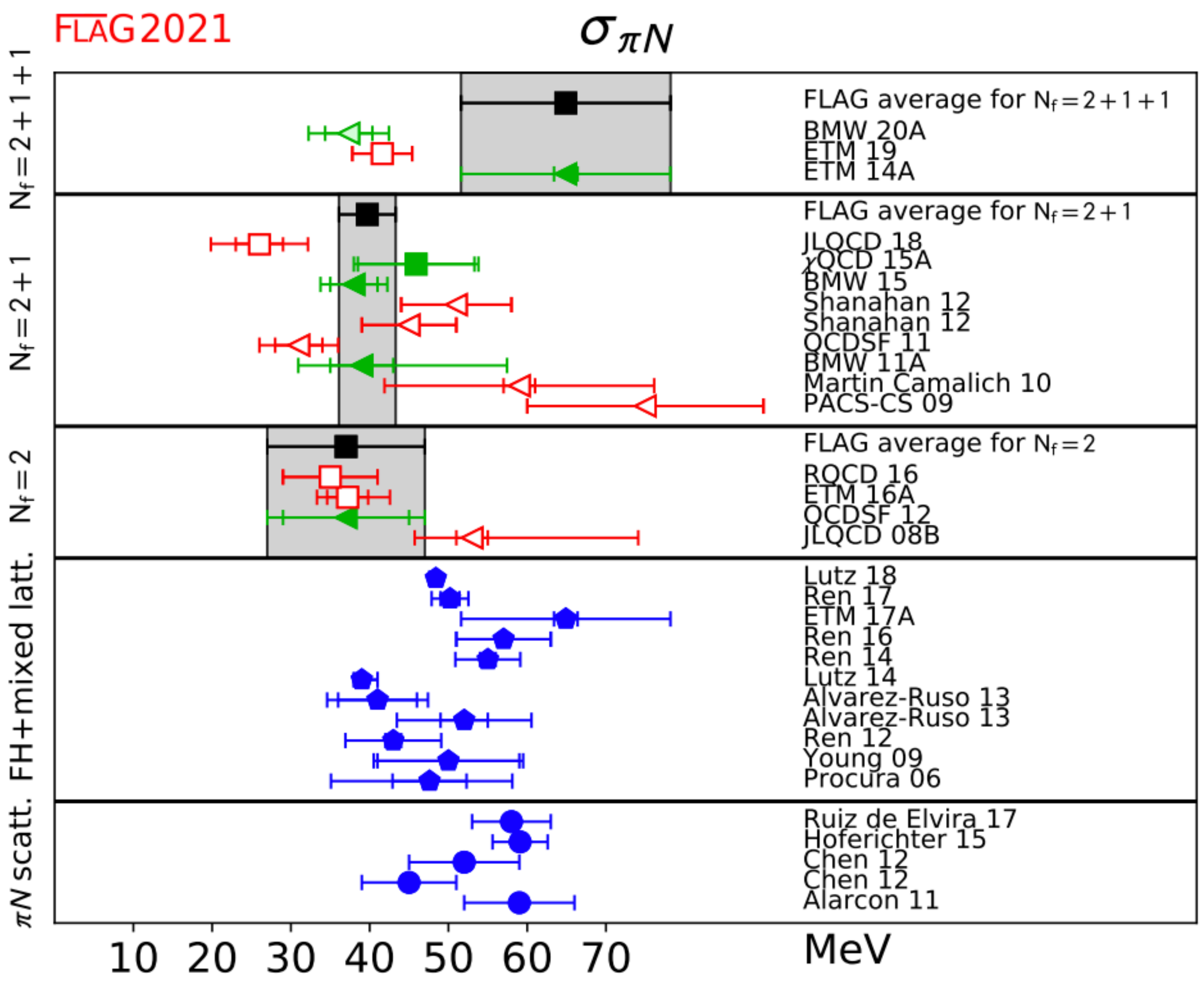
Sensitivity coefficients

Nucleon sigma term

$$m_N(\theta) = m_0 - 4c_1 m_\pi^2(\theta) - \frac{3g_A^2 m_\pi^3(\theta)}{32\pi f_\pi^2}$$

$$\frac{\partial \ln m_N}{\partial \ln m_\pi^2} \simeq 0.06$$ (ChPT)

$$\frac{\partial \ln m_N}{\partial \ln m_\pi^2} = \frac{\sigma_{\pi N}}{m_N} = 0.03 - 0.08$$



[FLAG 21]

BBN

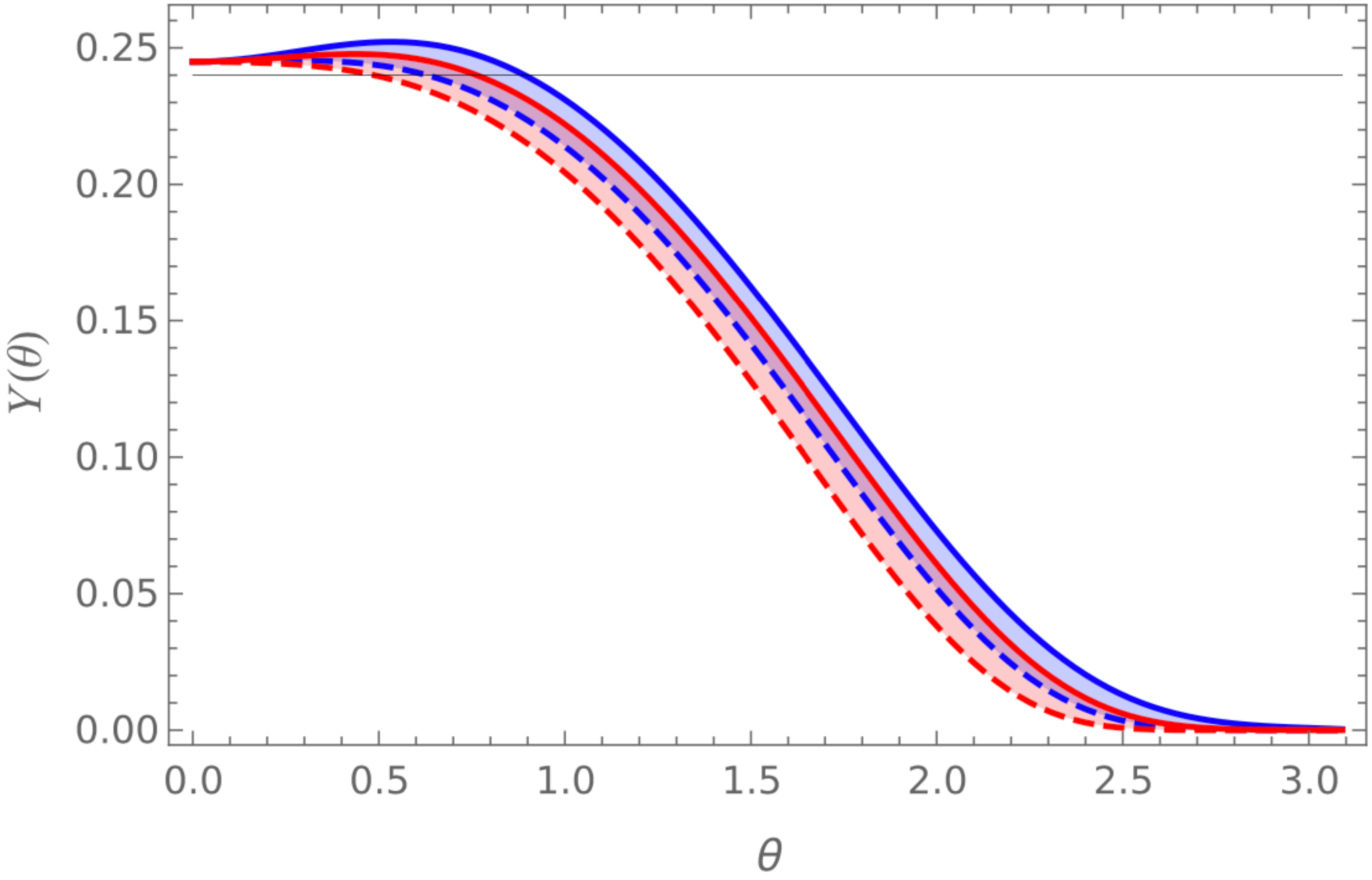
[Blum et al 14]

$$\mathcal{L} \supset -\frac{1}{2}\frac{f_\pi^2 m_\pi^2 m_u m_d}{(m_u+m_d)^2}\left(\frac{a}{f_a}\right)^2 - \bar{N}\pi\cdot\sigma\left(i\gamma^5 g_{\pi NN} - 2\,\bar{g}_{\pi NN}\frac{a}{f_a}\right)N + \frac{f_\pi\,\bar{g}_{\pi NN}}{2}\frac{m_d-m_u}{m_d+m_u}\left(\frac{a}{f_a}\right)^2 \bar{N}\sigma^3 N\,.$$

$$\left(\frac{n}{p}\right)_{\text{freeze-out}} \approx e^{-Q_{\mathrm{F}}/T_{\mathrm{F}}}\,,$$

Helium 4 mass fraction changes accordingly

[Lee et al 20]

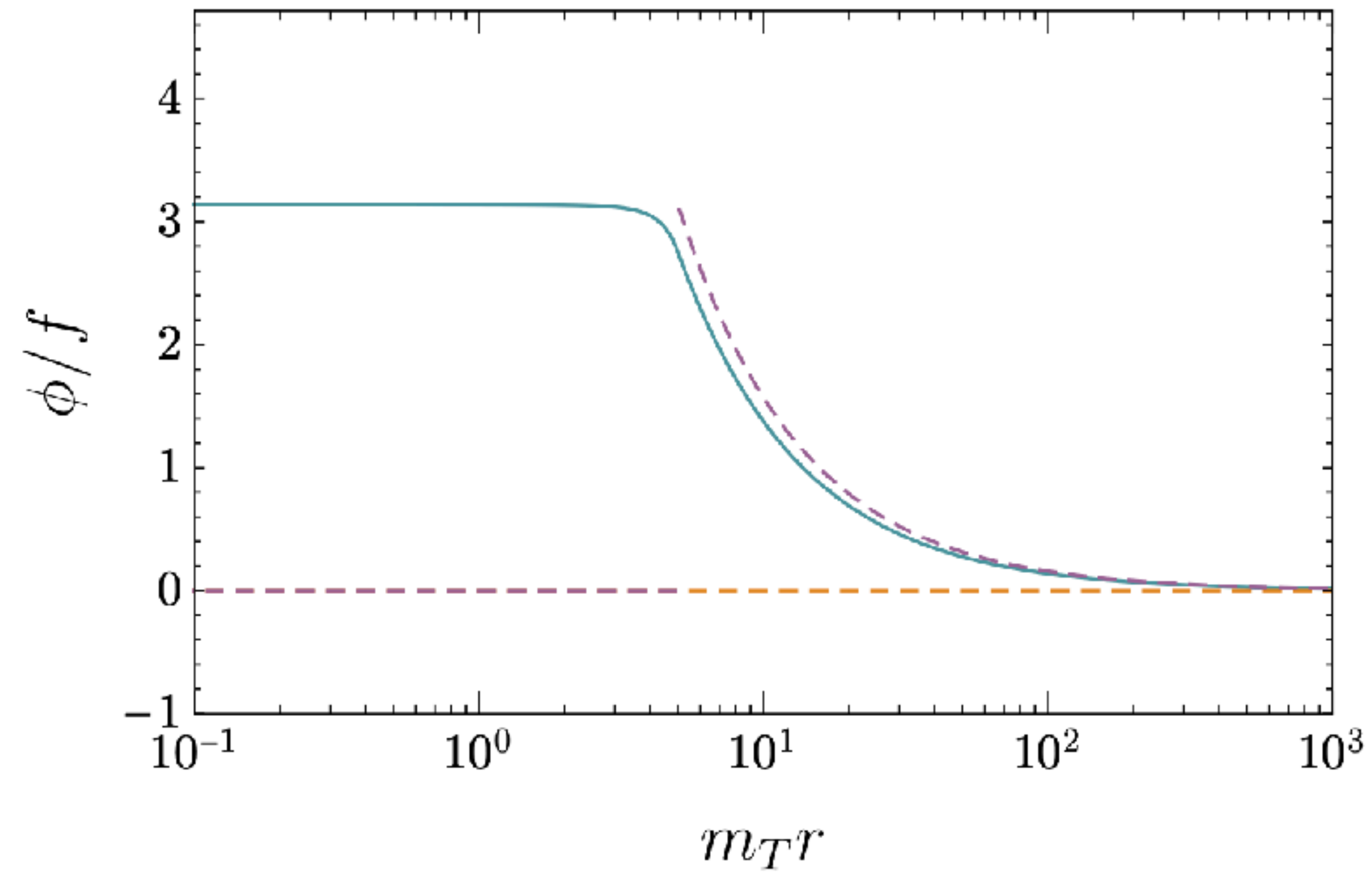


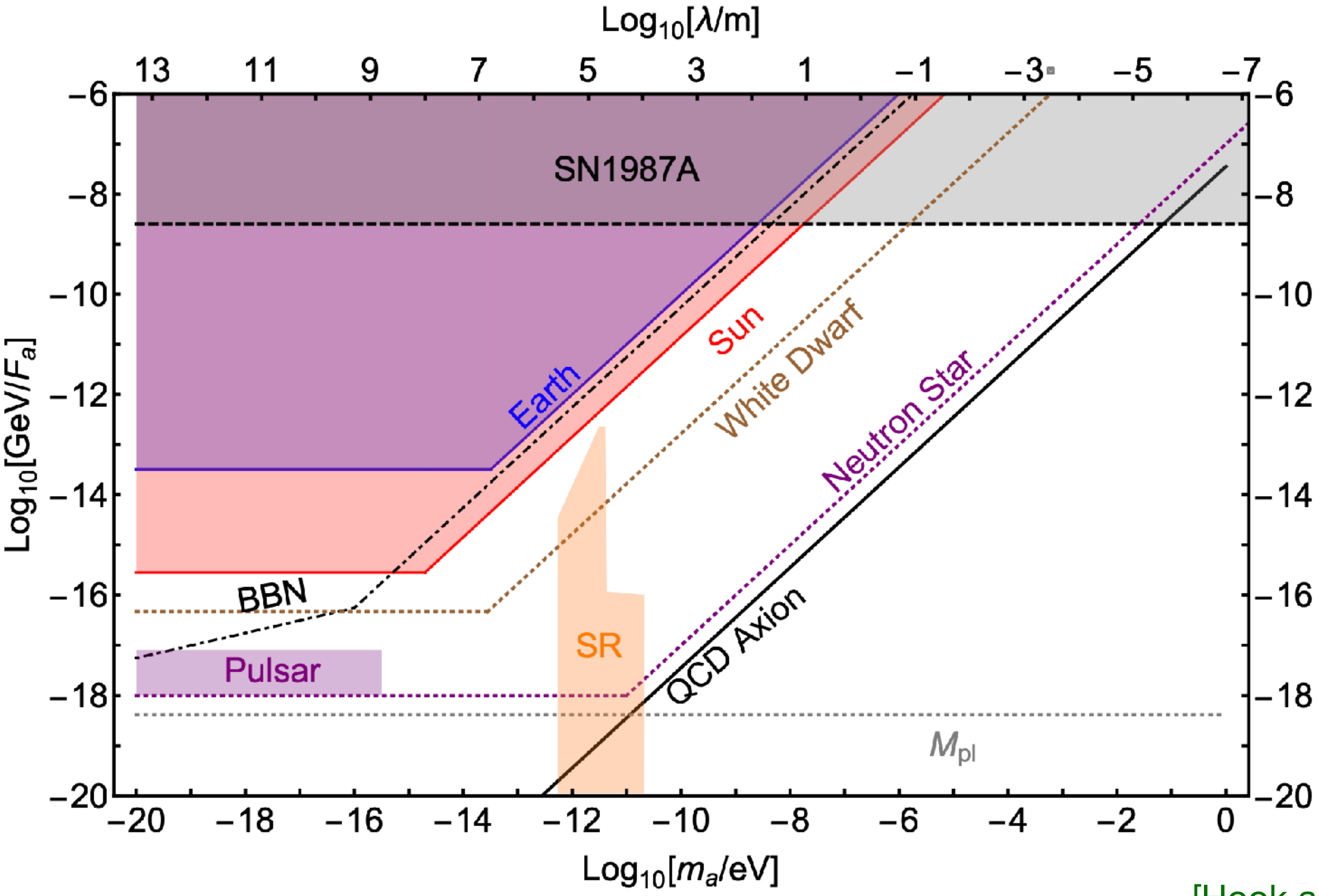
Finite density corrections

$$V = -m_\pi^2 f_\pi^2 \left\{ \left(\epsilon - \frac{\sigma_N n_N}{m_\pi^2 f_\pi^2} \right) \left| \cos\left(\frac{a}{2 f_a} \right) \right| + \mathcal{O}\left(\left(\frac{\sigma_N n_N}{m_\pi^2 f_\pi^2} \right)^2 \right) \right\},$$

Finite density might flip the potential inside stars

[Hook and Huang 17]





[Hook and Huang 17]

Alternative axion models

$$\mathcal{L} = \sum_k \left(\frac{a}{f} + \frac{2\pi k}{N} + \theta \right) G_k \tilde{G}_k .$$

$$V(a) = -\, m_\pi^2 f_\pi^2 \sum_k \sqrt{1 - 4 \frac{m_u m_d}{(m_u + m_d)^2} \sin^2 \left(\frac{a}{2f} + \frac{\pi k}{N} \right)}$$

$$\frac{m_a(N)}{m_a(N=1)} \sim \frac{4}{2^{N/2}}$$

[Hook 18]

[Di Luzio et al 21]