Effective Theory of Warped Compactifications and the Implications for KKLT

Lisa Randall w/Severin Luest

CERN-CKC Worksop Feb 2022

Conference

- Fantastic conference
- Great detailed look at many phenomena and measurements
- But this also a great time to think about big questions
  - Not stupidly (I hope)
  - But with focus on specific questions, apparent puzzles or inconsistencies

This Talk:

- Notoriously difficult to find string theory backgrounds with positive cosmological parameter
  - After 20 years not entirely clear
  - Key question for string theory, and de Sitter space theory
  - Very likely related to needing supersymmetry for tractable solutions
  - Can be deeply in nonperturbative (noncalculable) regime
- Other question: role of extra dimensions and warped geometry: new phenomena?
- Final question: How to construct (in more detail) low energy EFT when warped compactification

KKLT to address first

- An important question whether there are tractable de Sitter solutions
- KKLT paradigm proposed construction
  - Questions because of "many moving parts"
  - No one has full Lagrangian
  - Hard to explicitly construct the 10d model of de Sitter space
- Yet exist compelling probe approximation and effective **4d** theory arguments

Kachru, Kallosh, Linde, Trivedi 2003

Moreover (and theme of new work and last questions)

- Warped compactifications are intrinsically interesting
- Exhibit new phenomena that have not yet been fully understood
- Here we will see construction of EFT is subtle
- And exhibits new phenomena that might well have important implications for BSM physics

New Work

- This talk (based on recent work with S Lust)
  - Effective potentials in warped compactifications more subtle
  - Need to take account of constraints
  - Significant change in IR of throat
  - Related to light KK modes in IR, even of the stabilized Kahler moduli
- Turns out effective theories for warped compactifications much more subtle
- Alternatively need to account for KK modes that are light owing to warping
- Low energy potential construction requires understanding full metric
- Qualitative change of potential behavior
- IN IR!

Big Lesson

- Kahler moduli appearing in string construction stabilized
  - Was big point of KKLT construction
- But in warped geometries their KK modes are still light
- Comparable in mass to conifold deformation parameter
- Runaway behavior (that I will review) goes away

Outline

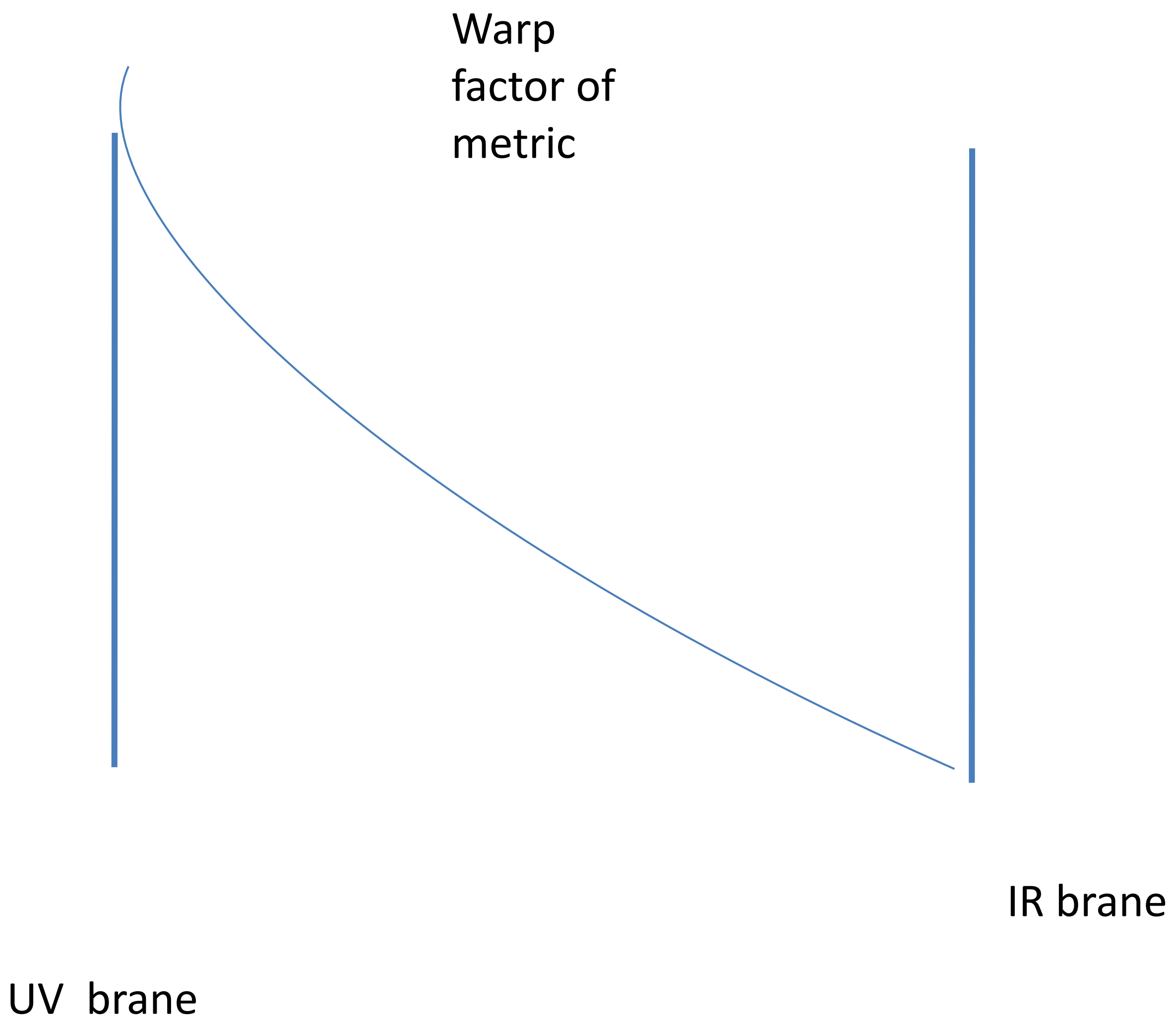
- Introduce KKLT: way of finding perturbative/manageable loophole for dS construction
- Review potential instabiilty
  - "conifold destabilization"
  - 5d EFT radion IS conifold deformation parameter
  - Seemed uplift destabilizes conifold (radion) if *M (flux)* too small--Really a runaway radion
- Show why IR EFT must be modified
  - And how it resolves issue

KKLT: Construction of de Sitter

$$V = \frac{aAe^{-a\sigma}}{2\sigma^2}\left(\frac{1}{3}\sigma a A e^{-a\sigma} + W_0 + Ae^{-a\sigma}\right) + \frac{D}{\sigma^3}$$

- 10d Calabi-Yau /F-theory construction
  - Fluxes stabilize all complex structure moduli
  - But Kahler (volume) modulus σ remains undetermined
- KKLT resolution
  - Step 1: Break no-scale structure with nonperturbative gauge contributions to stabilize Kahler modulus at large volume
    - Yields AdS4 as low-energy theory
- Uplift energy
  - Anti D3 brane; but in warped geometry (KS throat)
    - Suppresses uplift
    - Warped geometry gives smaller energy density to match AdS

RS Refresher



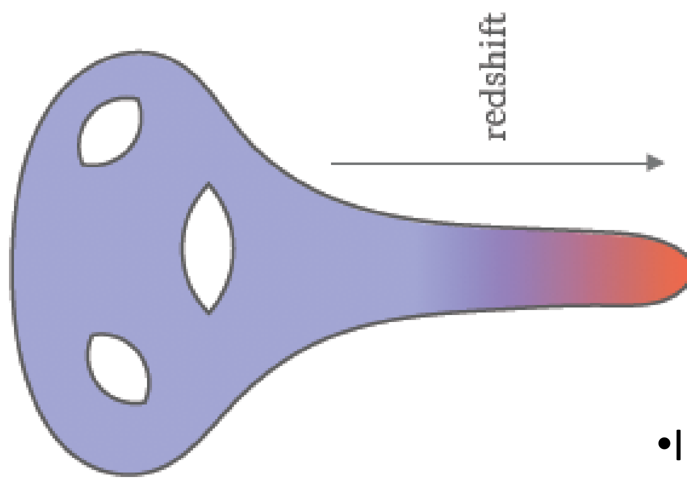
Warped Geometry (String Theory)

(Kachru, Polchinski, Verlinde)

- Cartoon: RS warped AdS throat glued onto CY
- CY **compactification** acts as UV brane
- But Klebanov-Strassler AdS space
  - Constantly changing (increasing) AdS curvature
  - AdS_5 but with "running N_{eff}"
    - N_{UV}=MK; N_{IR}= M; hierarchy from $e^{-2 \Pi K/Mg_s}$
- Caps off at a critical length
- Conifold deformation region is "IR brane"
- KPV paper: 4d Mink space as low-energy EFT

Klebanov-Strassler Solution

•K, M set by fluxes internal to compact geometry
•Total flux runs from KM to M
•K/M sets the hierarchy
•Note that role of GW in this warped geometry played by geometry itself
•Reshift $1/S^{1/3}$



Calabi-Yau/UV

•IR set by S
•Conifold deformation parameter

Goldberger-Wise Potential for Radion

- Guarantees radion moves so that
- Both junction conditions are satisfied
  - And entire bulk can be consistently sliced
- Radion is localized in IR
- Responding to mismatch in boundary conditions
- Clearly any stabilized geometry needs analog field
- GW bulk field, and radion

Can Identify Radion in KKLT!

- *S*: Conifold deformation parameter

$$\sum_{a=1}^{4} \omega_a^4 = S \, . \tag{3.10}$$

The deformation parameter S is the complex structure modulus whose absolute value corresponds to the size of the 3-sphere at the tip of the cone.

$$\int_A \Omega_3 = S \ , \tag{3.11}$$

Potential for *S*

The supersymmetric potential for this field induced by the Klebanov-Strassler geometry is

$$V_{KS} = \frac{\pi^{3/2}}{\kappa_{10}} \frac{g_s}{(Im\rho)^3} \left[c \log \frac{\Lambda_0^3}{|S|} + c' \frac{g_s(\alpha' M)^2}{|S|^{4/3}} \right]^{-1} \left| \frac{M}{2\pi i} \log \frac{\Lambda_0^3}{S} + i \frac{K}{g_s} \right|^2 , \qquad (3.12)$$

where g_s is the stabilized vev of the dilaton, $Im\rho = (\mathrm{Vol}_6)^{3/2}$, c as we argue below is not relevant here (and is in any case suppressed in the small S region), whereas the constant c', multiplying the term coming solely from the warp factor, denotes an order one coefficient, whose approximate numerical value was determined in [46] to be $c' \approx 1.18$.

Add potential from antibrane:

The antibrane contributes a perturbation

$$V_{\overline{D3}} = \frac{\pi^{1/2}}{\kappa_{10}} \frac{1}{(Im\rho)^3} \frac{2^{1/3}}{I(\tau)} \frac{|S|^{4/3}}{g_s(\alpha' M)^2} . \qquad (3.18)$$

We follow [56] and define $c'' = \frac{2^{1/3}}{I(0)} \approx 1.75$. For p anti-D3 branes the potential is multiplied by p, and this is taken care by simply replacing $c'' \to c''p$.

“Conifold” instability

The general form of the potential (we factor out $\lambda_1^2 \pi g_s / c'$) is

$$V = S^{4/3} \left(1 + \epsilon \log \frac{S}{\Lambda_0^3}\right)^2 + \delta S^{4/3} \tag{3.28}$$

The barrier disappears when $\delta/\epsilon^2 = 9/16$.

We see that the perturbation from the antibrane (yielding the δ type perturbation above) yields the potential proportional to the above with $\delta = c''c' g_s / \pi K^2$ and $|\epsilon| = M g_s / 2\pi K$. By writing it this way we keep ϵ and δ as small parameters. This gives precisely the stability condition found in [59], namely

$$\sqrt{g_s} M > M_{\text{min}} \qquad \text{with} \qquad M_{\text{min}} = \frac{8}{3}\sqrt{\pi c' c''} \approx 6.8\sqrt{p}\,. \tag{3.29}$$

] I. Bena, E. Dudas, M. Graña and S. Lüst, “Uplifting I
(2019) no.1-2, 1800100 [arXiv:1809.06861 [hep-th]].

L. Randall, “The Boundaries of KKLT,” Fortsch. Phys. **68** (2020) no.3-4, 1900105 [arXiv:1912.06693 [hep-th]].

S Potential

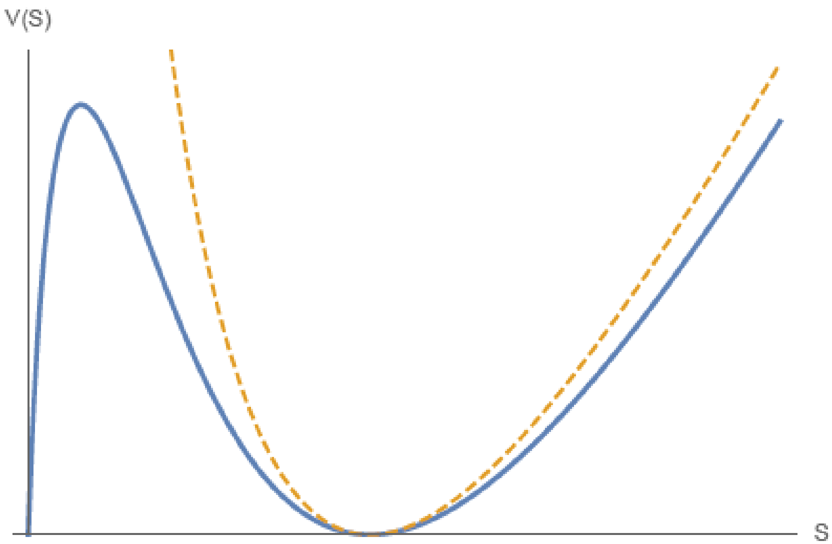


Figure 1: The potential V_{KS} of [16] for the complex structure modulus S of the Klebanov-Strassler throat given in (2.17). The solid blue line corresponds to the full potential, while the dotted orange line does shows the naïve potential that does not take into account the effects of warping ($c' = 0$). Both potentials have the same supersymmetric minimum but differ drastically at small S.

The potential (2.17) has a supersymmetric minimum, corresponding to $\partial_S W = 0$, which, for $S \ll \Lambda_0^3$, is at

$$s_{\mathrm{KS}} \simeq \Lambda_0^3 \exp\left(-\frac{2\pi K}{g_s M}\right) . \tag{2.19}$$

With Uplift

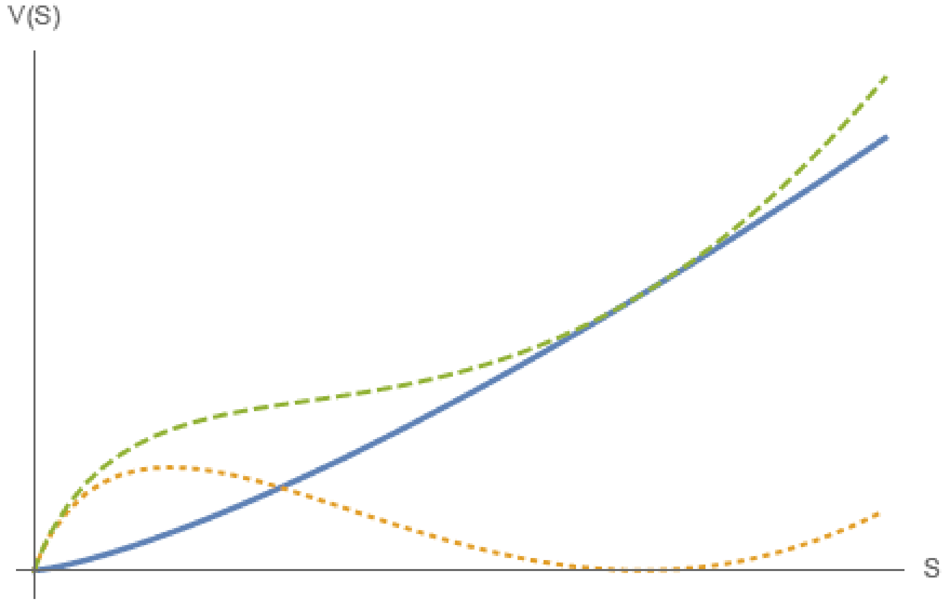


Figure 2: The contribution $V_{\overline{D3}}$ (solid blue line) of an $\overline{D3}$-brane placed in the Klebanov-Strassler throat to the potential for S. The two other lines represent the original potential V_{KS} (dotted orange line) for the specific value $\sqrt{g_s}M = 6$ as well as the superposition $V_{KS} + V_{\overline{D3}}$ (dashed green line).

Runaway radion if too big a perturbation

The general form of the potential (we factor out $\lambda_1^2 \pi g_s / c'$ is

$$V = S^{4/3}\left(1 + \epsilon \log \frac{S}{\Lambda_0^3}\right) + \delta S^{4/3} \tag{3.19}$$

The barrier disappears when $\delta/\epsilon^2 = 9/16$.

We see that the perturbation from the antibrane (yielding the δ type perturbation above) yields the potential proportional to the above with $\delta = c"c'/g_s\pi\lambda_1^2$ and $|\epsilon| = M g_s/2\pi K$. By writing it this way we keep ϵ and δ as small parameters. This gives precisely the stability condition found in [56], namely

$$\sqrt{g_s}M > M_{\rm min} \qquad \text{with} \qquad M_{\rm min} = \frac{8}{3}\sqrt{\pi c' c''} \approx 6.8\sqrt{p}\,. \tag{3.20}$$

Real potential instability

- Need largish $g_s M^2$
- But hard to satisfy
- Hierarchy problematic
  - $K/Mg_s \sim KM/M^2 g_s$
  - KM bounded in a given geometry
- Another potential problem
  - Cosmological phase transition for RS like geometries
    - Cremenelli, Nicolis, Rattazzi//Hassanain, March-Russell, Schellvinger
  - High temperature AdS/Schwarschild
  - Cosmological phase transition won't complete
  - Need to evolve to RS
  - Upper bound on $M^2 \sim 21$ for this geometry
- Caveat: We are assuming supergravity solution applies even for small M
  - However if it doesn't we still have to work out solution to have example

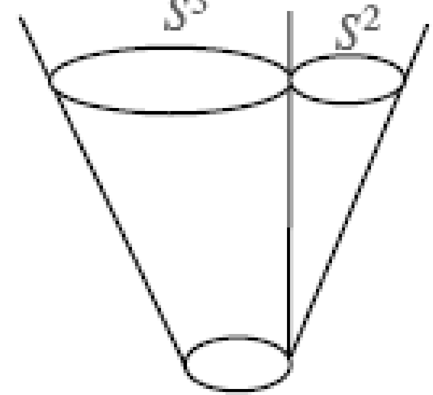
Solution !!
Warped Conifold Potential

- Turns out the assumed S potential is not correct
  - In IR!
  - Off-shell
- Need to impose various constraints
- Let’s get a taste of how this wokrs

A CLOSER LOOK AT THE CONIFOLD POTENTIAL

- deformed conifold: $$\sum_{i=1}^{4} z_i^2 = S$$

  $\rightarrow$ S = complex structure modulus



- Superpotential:

$$W \sim \int G_3 \wedge \Omega = \frac{M}{2\pi i} S\left(\log \frac{\Lambda_0^3}{S} + 1\right) + \frac{i}{g_s} KS$$

- Kähler potential requires knowledge of warp factor: Klebanov-Strassler solution:

$$e^{-4A(\tau)} \sim \frac{g_s(\alpha' M)^2}{|S|^{\frac{4}{3}}} I(\tau)$$

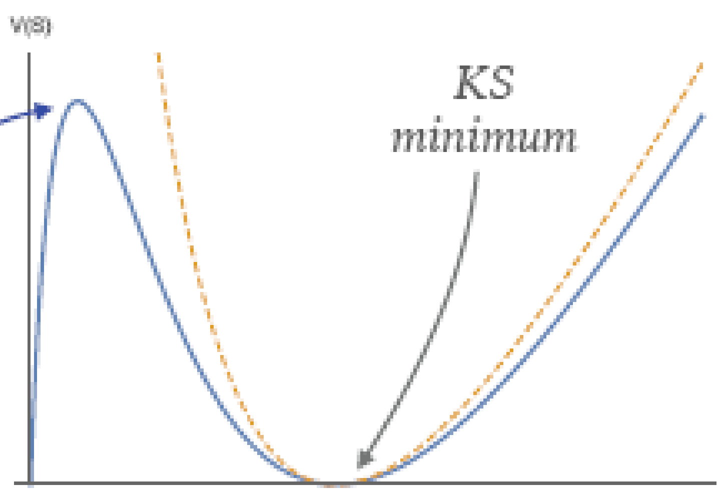
A CLOSER LOOK AT THE CONIFOLD POTENTIAL

- Kähler metric: [Douglas, Shelton, Torroba, '07, '08]

$$G_{S\bar{S}} = \partial_S \partial_{\bar{S}} K \sim \int e^{-4A} \chi_S \wedge \bar{\chi}_{\bar{S}} \approx e^{-4A(\tau=0)} \sim \frac{g_s(\alpha' M)^2}{|S|^{\frac{4}{3}}}$$

- Anti-brane instability:
  Requires knowledge of the (off-shell) potential away from the minimum!

can we use the KS warp factor here?!

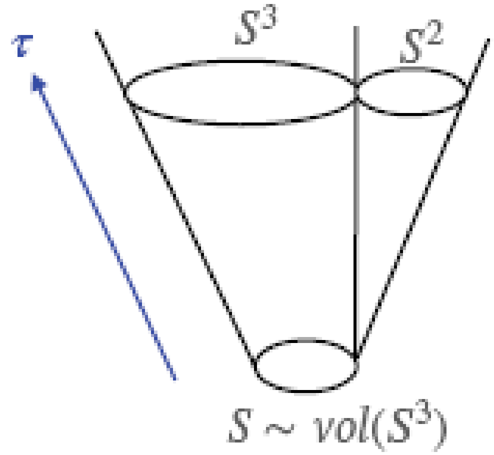


COMPLEX STRUCTURE OF THE DEFORMED CONIFOLD

- Deformed conifold in $\mathbb{C}^4$:

$$\sum_{i=1}^{4} z_i^2 = S$$

S = complex structure modulus



- metric on the deformed conifold:

$$ds^2_{DC} = \frac{S^{2/3}}{2} K(\tau) \left[\frac{1}{3K^3(\tau)} \left[d\tau^2 + (g^5)^2\right] + \cosh^2\left(\frac{\tau}{2}\right)\left[(g^3)^2 + (g^4)^2\right] + \sinh^2\left(\frac{\tau}{2}\right)\left[(g^1)^2 + (g^2)^2\right] \right]$$

complex structure just a conformal factor?!

Answer: did not fix gauge (coordinates) yet!

COMPLEX STRUCTURE OF THE DEFORMED CONIFOLD

- First: understand gauge fixing without warping:

$$ds_{10}^2 = ds_4^2 + ds_{DC}^2$$

- Gauge fixing of Calabi-Yau deformations:

$$g_{ij} \to g_{ij} + \delta g_{ij}$$

[Candelas, de la Ossa '91]

$\Rightarrow$ $g^{ij}\delta g_{ij} = 0$ *(traceless)* $\nabla^i \delta g_{ij} = 0$ *(harmonic)*

(will get modified in the presence of warping!) [Giddings, Maharana '05], [Shiu et al. '08], [Douglas, Torroba '08]

- Deformed conifold:

$$\delta g_{ij} = \partial_S g_{ij} \sim \frac{1}{S} g_{ij}$$

harmonic but not traceless!

With warp factor no longer traceless!

[redacted]

COMPLEX STRUCTURE OF THE DEFORMED CONIFOLD

- Add compensating diffeomorphism:

$$\delta g_{ij} = \partial_S g_{ij} + 2\nabla_{(i}\eta_{j)}$$

Ansatz:

$$\eta = \left(\eta^\tau(\tau),0,0,0,0,0\right)$$

Solution:

$$\eta^\tau(\tau) = -\frac{1}{2S}\frac{\sinh(2\tau)-2\tau}{\sinh^2\tau}$$

- Interpretation:

Replace τ with "new" S-dependent radial variable: $\tau \to T(\tau, S)$

$$\frac{dT}{dS} = \eta^\tau\left(T(\tau,S),S\right) \longrightarrow$$

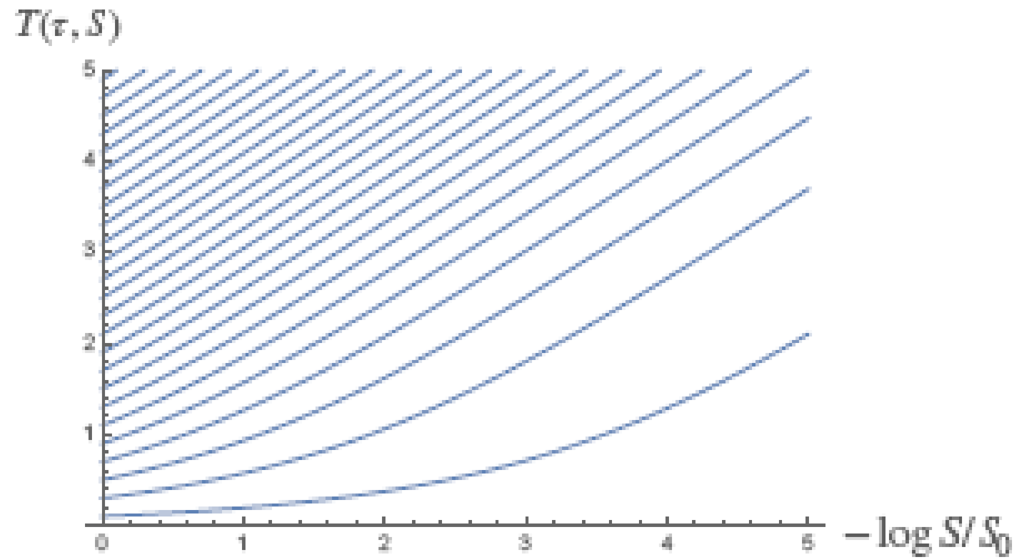
Analytic solution:

$$T(\tau,S) = F\left[F^{-1}(\tau) - \frac{1}{4}\log\frac{S}{S_0}\right]$$

with $F(x) = \frac{1}{2}\log\left[\sinh(2x) - 2x\right]$

COMPLEX STRUCTURE OF THE DEFORMED CONIFOLD

- The radial coordinate as a function of S:



- UV behavior $(\tau \to \infty)$: $T(\tau, S) \to \tau - \log S/S_0$
- Compare with UV expansion of the metric:

$$ds^2_{DC}(\tau \to \infty) \to S^2 e^{2T/3}\left(\frac{1}{9}dT^2 + \frac{1}{6}ds^2_{T^{1,1}}\right) = S_0^2 e^{2\tau/3}\left(\frac{1}{9}d\tau^2 + \frac{1}{6}ds^2_{T^{1,1}}\right)$$

Deformation acts only in the IR!

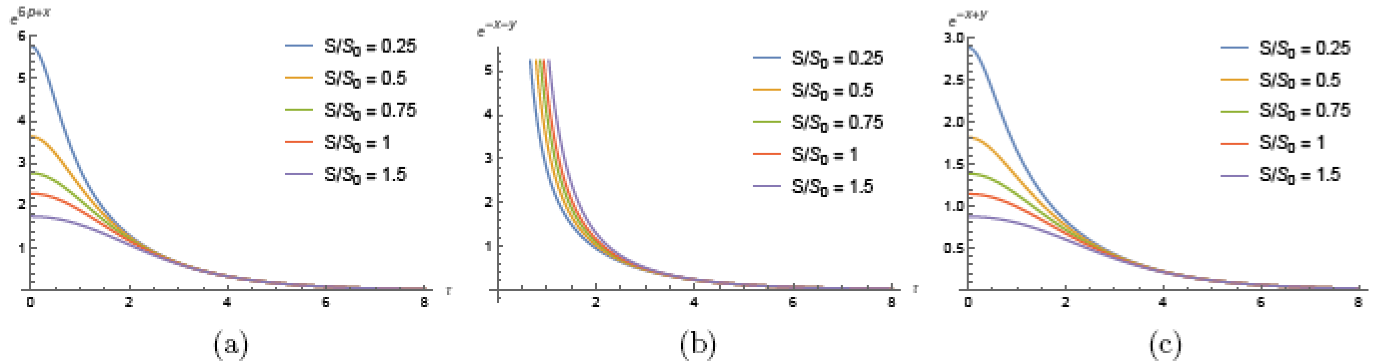


Figure 1: The three independent components of the inverse metric on the deformed conifold for different values of S/S_0, taking into account the effect of the S-dependent diffeomorphism (4.22) and (4.25). (a) $g^{\tau\tau} = g^{55} = e^{6p+x}$, (b) $g^{11} = g^{22} = e^{-x+y}$, (c) $g^{33} = g^{44} = e^{-x-y}$.

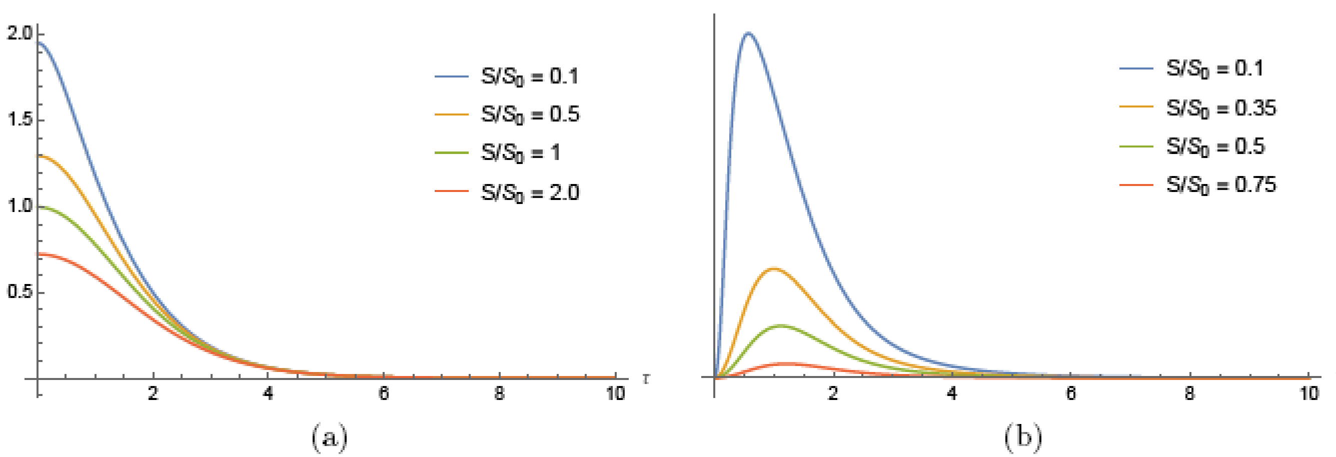


Figure 3: (a) The inverse warp factor e^{-4A} as a function of the radial coordinate τ for different values of S/S_0. (b) The integrand of the potential (5.23) for different values of S/S_0. In both figures the IR-tip of the throat geometry is located at $\tau = 0$ and $\tau \to \infty$ corresponds to its UV.

Deformations of warped geometries

The most general form of a background which preserves all isometries of a four-dimensional maximally-symmetric spacetime takes the form

$$ds_{10}^2 = e^{2A(y)} g_{\mu\nu} dx^\mu dx^\nu + e^{-2A(y)} \tilde{g}_{mn}(y) dy^m dy^n \,, \tag{4.1}$$

We need to consider both the variations of the warp factor δA and the variations of the internal metric δg_{mn}. It was found in [9, 10] that these are not independent but related by

$$\delta A = -\frac{1}{4} g^{mn} \delta g_{mn} \,. \tag{4.3}$$

This can be understood as an extension of the traceless condition (2.3) to the warped case. The harmonic gauge condition (2.2) also needs to be modified accordingly and becomes

$$\nabla^m (e^{2A} \delta\pi_{mn}) = 0 \,, \tag{4.4}$$

Solve using diffeomorpism

To find solutions δA and δg_{mn} satisfying these conditions, one can start from a deformation δg^0_{mn} and add an infinitesimal diffeomorphism which acts as a compensating gauge transformation,

$$\delta g_{mn} = \delta g^0_{mn} + \nabla_m \eta_n + \nabla_n \eta_m \,. \tag{4.6}$$

With this ansatz the modified harmonic gauge condition (4.4) becomes a set of second order differential equations on η_m which have to be solved to find δg_{mn}. Subsequently, one can use (4.3) to determine δA.

Even More General

Here, $u^I(x^i)$ denotes a set of four-dimensional scalar fields, parametrizing moduli or also massive excitations of the background solution. It seems to be natural to introduce a similar x^i dependence for the warp factor as $A\big[\tau, u^I(x^i)\big]$. However, as we will see this notation has to be taken with a grain of salt.

The general space-time dependent ansatz for the metric now reads

$$ds^2 = e^{2A\left(\tau, u^I(x)\right)} g_{\mu\nu}(x) dx^\mu dx^\nu + \left[e^{f\left(\tau, u^I(x)\right)} d\tau + K_\mu(\tau, x) dx^\mu\right]^2 , \qquad (8.2)$$

where g_{ij} denotes the components of an a-priori undetermined four-dimensional metric and we also allow for possible off-diagonal components K_i.

$$3D_\tau^2 A + 6(D_\tau A)^2 + g_{ab} D_\tau \phi^a D_\tau \phi^b + 2V(\phi) - \frac{1}{4} e^{-2A} R^{(4)} = 0 .$$

Satisfy higher d eq of motion

above as the off-diagonal E_{i5} component (8.6) of the five-dimensional Einstein equations. We recall that it takes the form

$$3D_I D_\tau A + 2g_{ab} D_I \phi^a D_\tau \phi^b = 0 , \qquad (8.15)$$

No off-diagonal

constraint arises from the traceless part of the five-dimensional Einstein equatior vanishing four-dimensional momenta, $\partial_\mu u^I = 0$, and a symmetric background spa $R_{\mu\nu} = \frac{1}{4} g_{\mu\nu} R$, it reduces to

$$\left(\nabla_\mu \partial_\nu u^I - \tfrac{1}{4} g_{\mu\nu} \Box u^I\right) \Big[2D_I A + D_I f\Big] = 0 .$$

Traceless EE

The constraint therefore reads

$$2D_I A + D_I f = 0 .$$

Solve

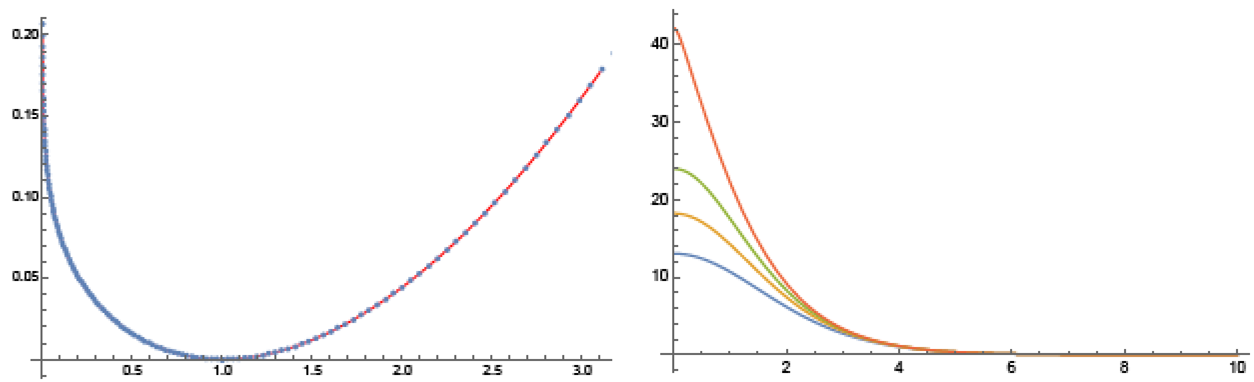


Figure 2: Left: The potential as a function of S/S_0. Right: The warp factor $e^{-4A(\tau)}$ for $S/S_0 = 2.0, 1.0, 0.5, 0.1$ Both plots are created using the differential constraints (8.15) and (8.17) but ignoring the Hamiltonian constrain (8.13).

No Second Minimum

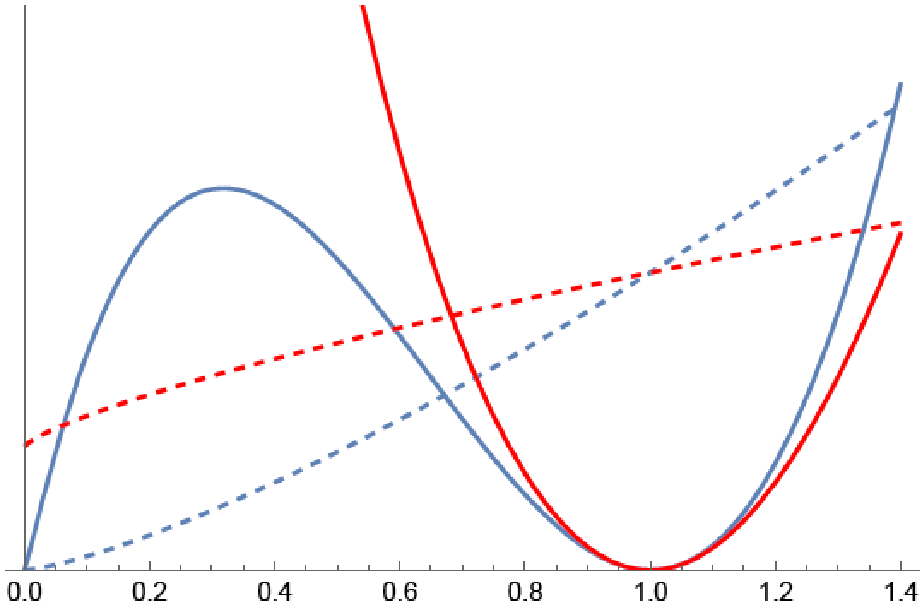


Figure 3: Comparison of the potential computed by [10] (blue) and our potential (red). The solid line is the potential for the conifold modulus S and the dashed line the contribution from the antibrane. Their superposition is illustrated in Figure 4.

Punchline

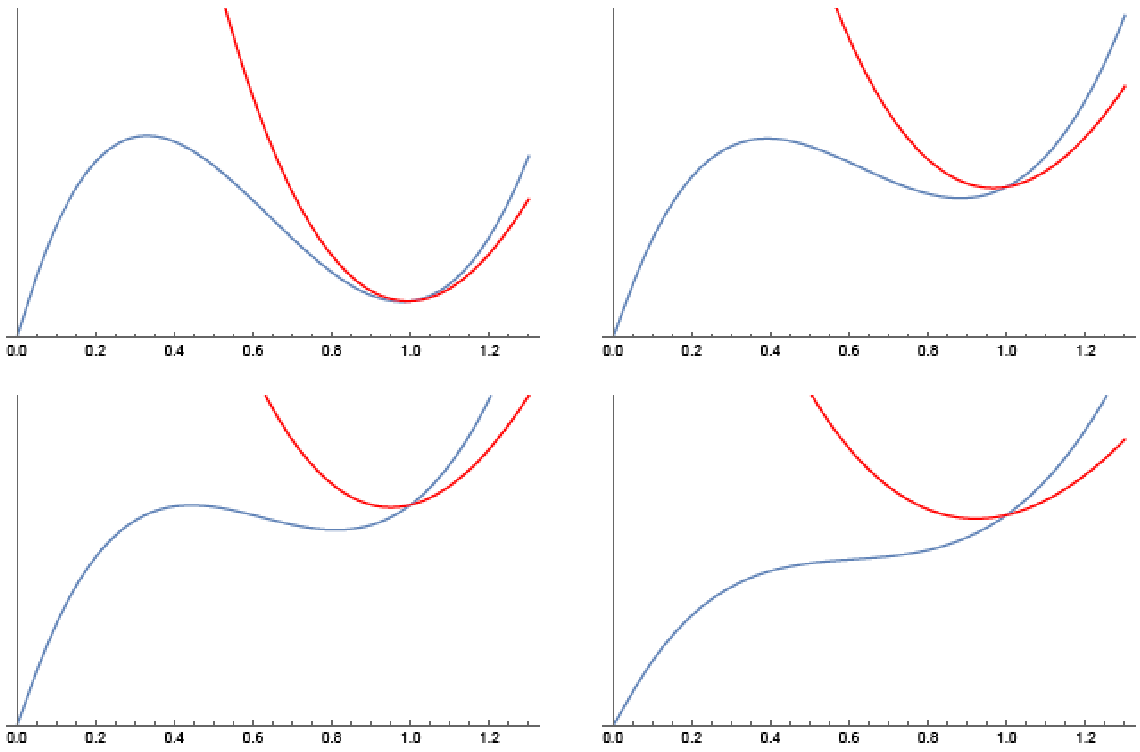


Figure 4: Superposition of the potential for S and the antibrane potential for different values of M^2 (from large to small). The "old" potential is in blue, for small values of M its minimum disappears. The red potential, which was computed using the constraint (8.15), always has a minimum, irrespective of the value of M.

Putting it Togehter

- Low energy effective theory nontrivial in context of warped compactifications
- Solving Einstein's Equations consistently even off-shell leads to qualitative change in form of potential
- Here related to fact that warped compactification shape can change in the IR
- Not determined solely by the UV stabilization
- Essentially allows for KK modes of volume moduli
- Though not yet explicit in our formalism
- Resolves the mysterious and now-seen-to-be spurious instability

Conclusions

- General Lesson is that warped compactifications are subtle
- Low energy effective theory still valid
  - But requires including KK modes of all fields
- Even heavy structure moduli
  - KK modes of heavy fields in warped geometries can be light
- Can perhaps have important implications for potentials, hierarchies in the future