# Adopting axion dark matter ideas to detect high-frequency gravitational waves

**Camilo García Cely** 

### **RENCONTRES DE BLOIS, May 24**



Based on 2202.00695 with Domcke, CGC, Rodd and PRL (2021) 2, 021104 with Domcke, CGC



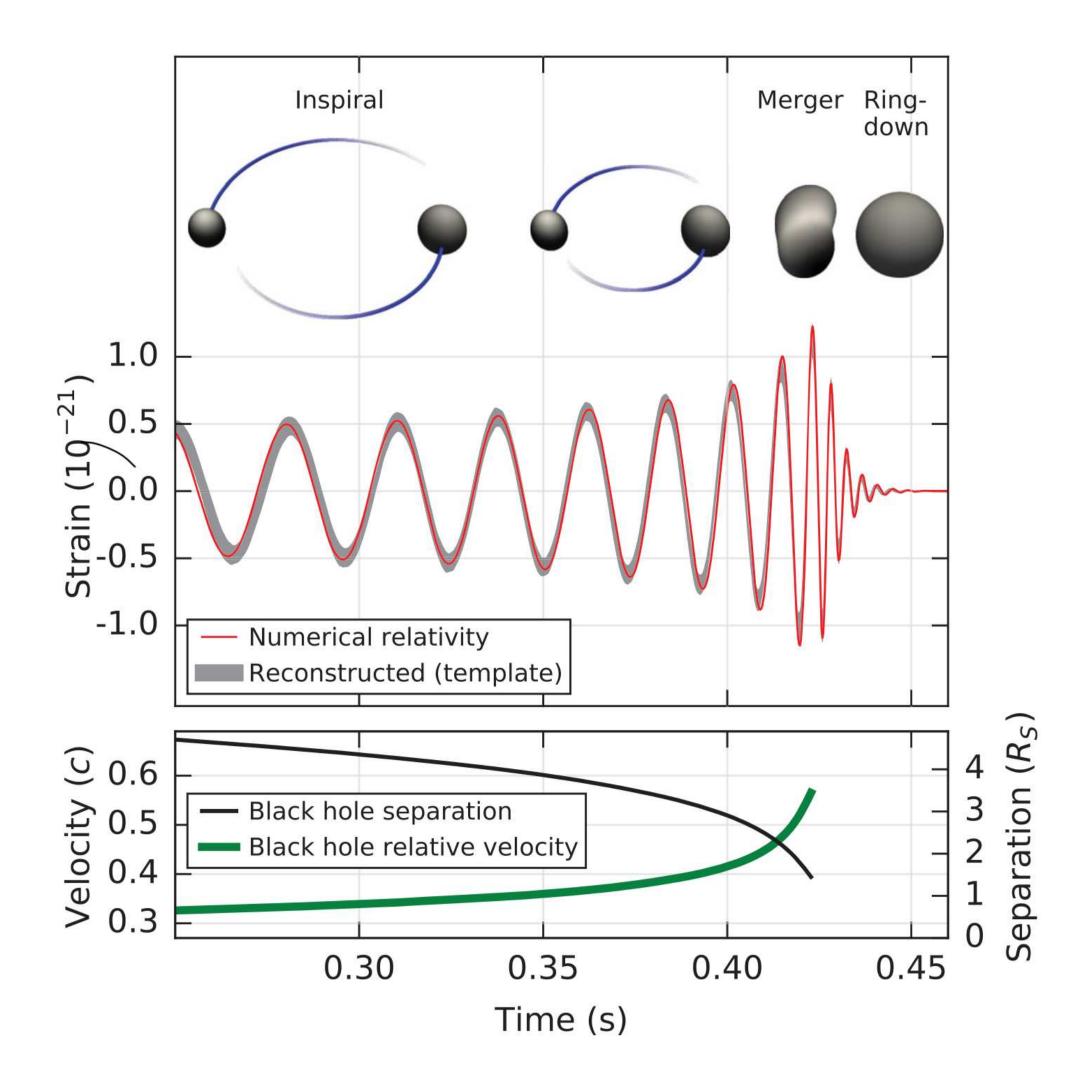
Ramón y Cajal Researcher







# High-frequency gravitational waves



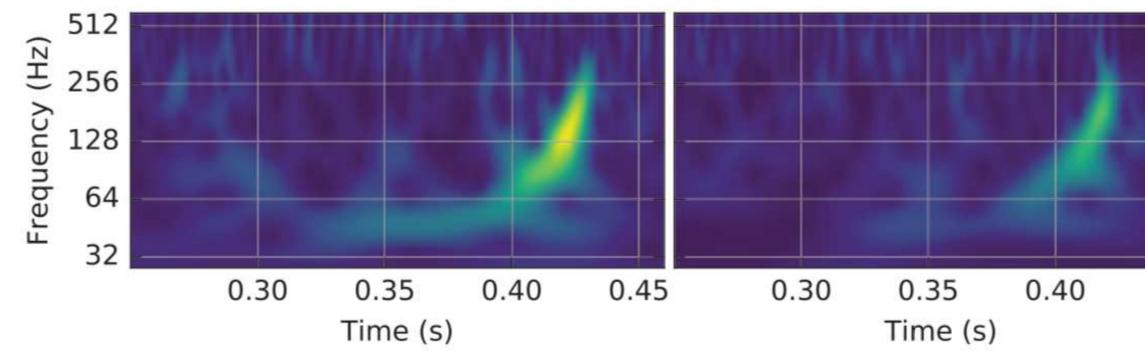
PRL 116, 061102 (2016)

PHYSICAL REVIEW LETTERS

S

**Observation of Gravitational Waves from a Binary Black Hole Merger** 

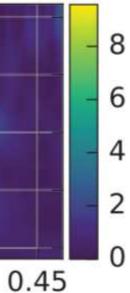
B. P. Abbott et al.\* (LIGO Scientific Collaboration and Virgo Collaboration)



 $f \ll 10 \,\mathrm{kHz}$ 

There are no known astrophysical objects that are small and dense enough to produce gravitational waves beyond 10 kHz





# High-frequency gravitational waves

### Part of a collection:

Gravitational Waves

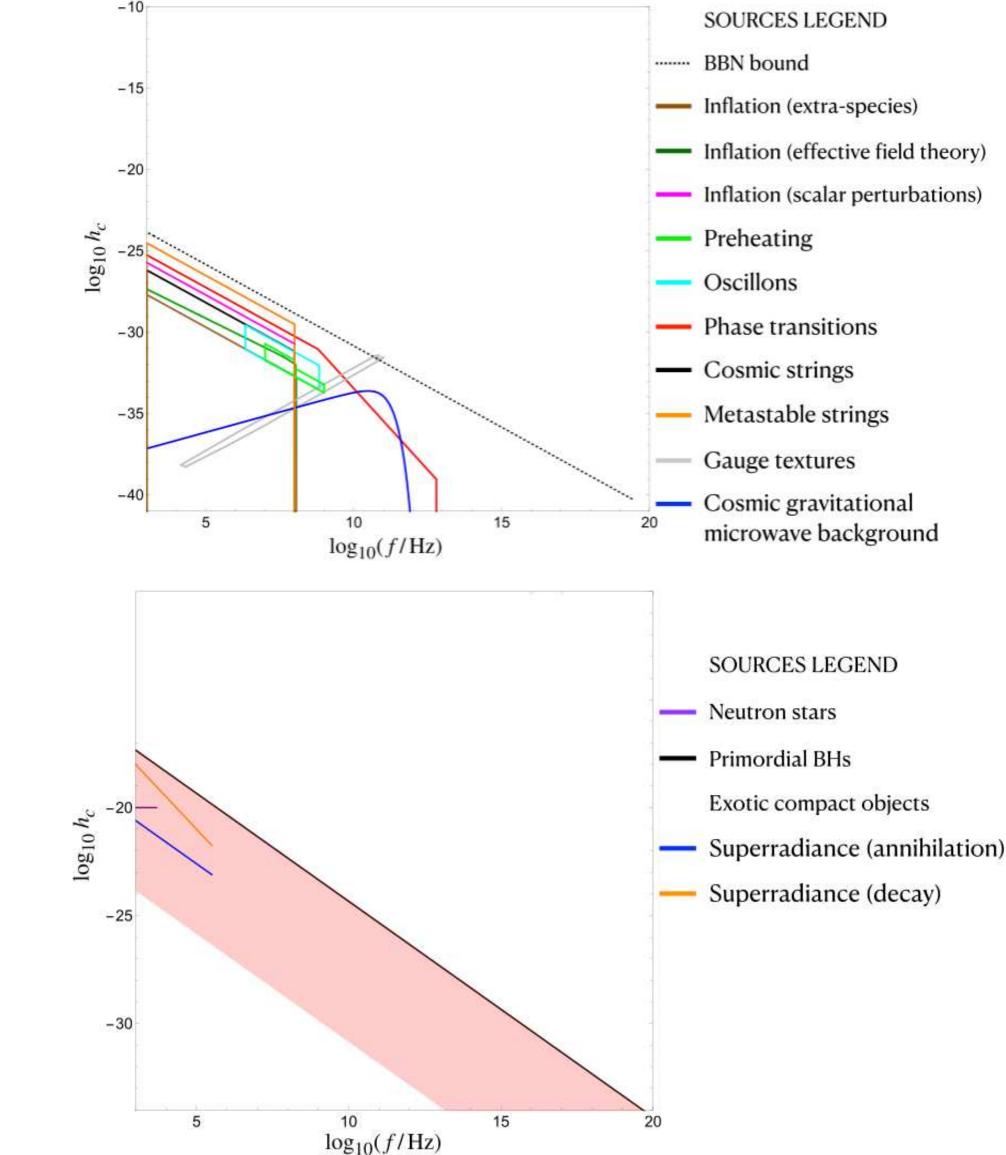
Review Article | Open Access | Published: 06 December 2021

### Challenges and opportunities of gravitational-wave searches at MHz to GHz frequencies

Nancy Aggarwal  $\boxdot$ , Odylio D. Aguiar, Andreas Bauswein, Giancarlo Cella, Sebastian Clesse, Adrian Michael Cruise, Valerie Domcke  $\boxdot$ , Daniel G. Figueroa, Andrew Geraci, Maxim Goryachev, Hartmut Grote, Mark Hindmarsh, Francesco Muia  $\boxdot$ , Nikhil Mukund, David Ottaway, Marco Peloso, Fernando Quevedo  $\boxdot$ , Angelo Ricciardone, Jessica Steinlechner  $\boxdot$ , Sebastian Steinlechner  $\boxdot$ , Sichun Sun, Michael E. Tobar, Francisco Torrenti, Caner Ünal & Graham White

*Living Reviews in Relativity* **24**, Article number: 4 (2021) Cite this article

### A growing community is seriously considering the search of high frequency gravitational waves





• Solution to the strong CP problem

• Excellent dark matter candidate

Peccei, Quinn 1977

Weinberg, Wilczek 1978

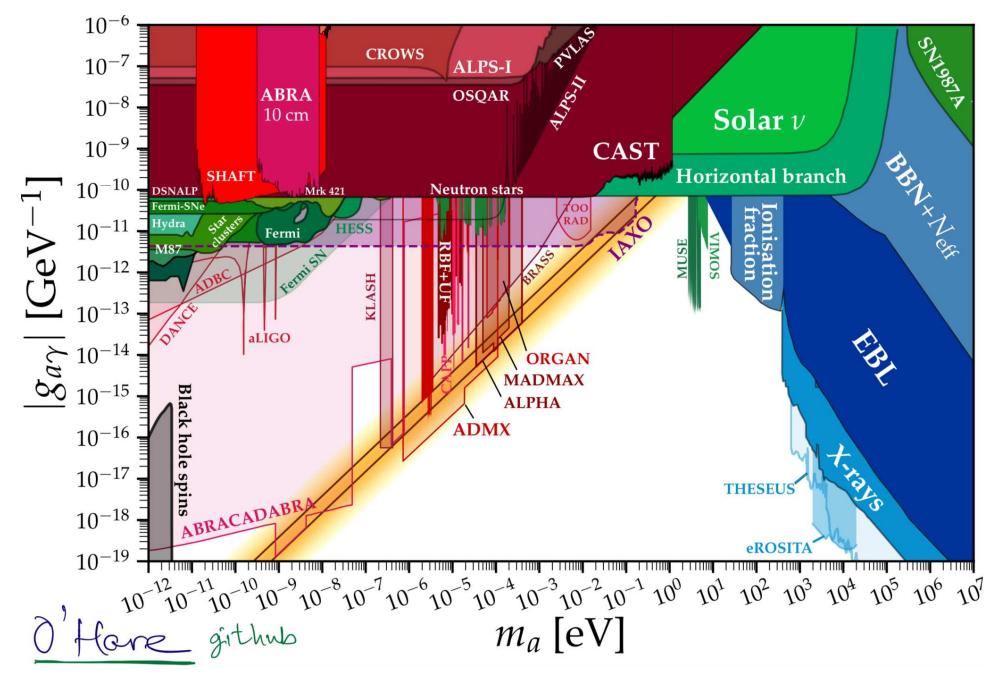


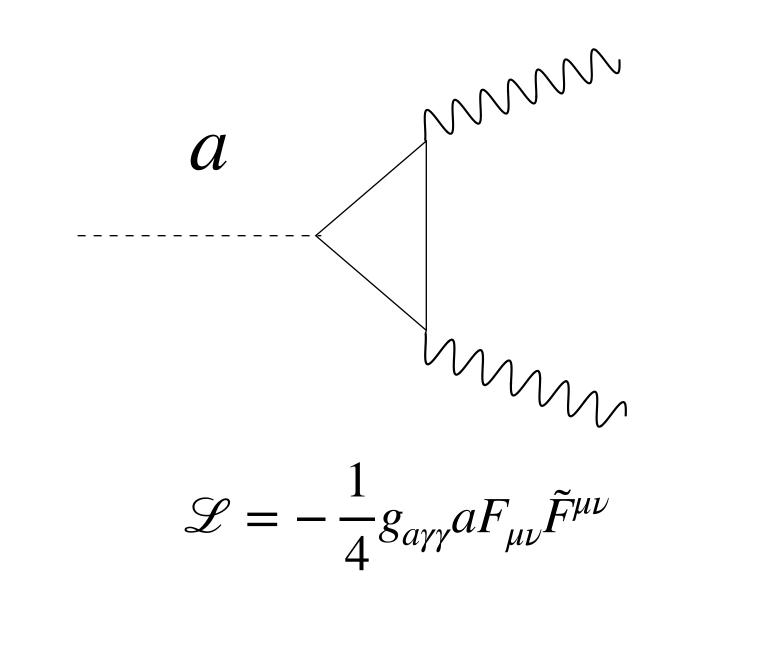
• Solution to the strong CP problem

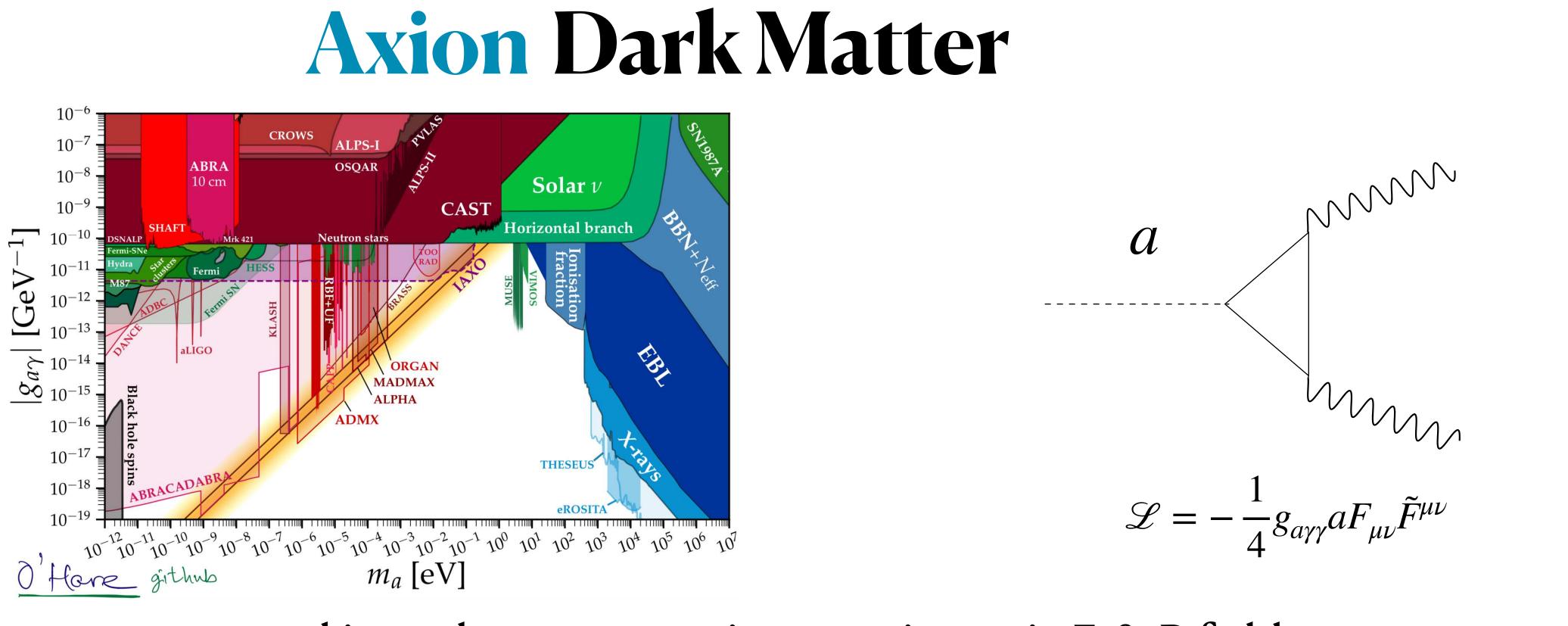
• Excellent dark matter candidate

Peccei, Quinn 1977  $\mathcal{A}$  $\mathscr{L} = -\frac{1}{4}g_{a\gamma\gamma}aF_{\mu\nu}\tilde{F}^{\mu\nu}$ Weinberg, Wilczek 1978

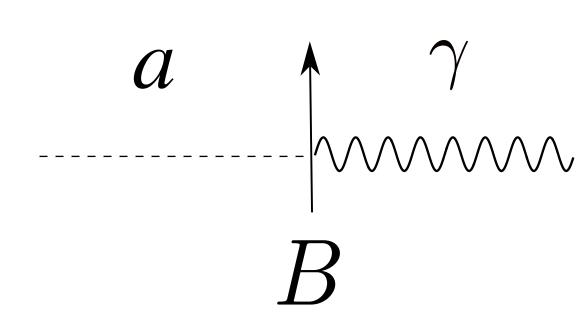
### Axion Dark Matter

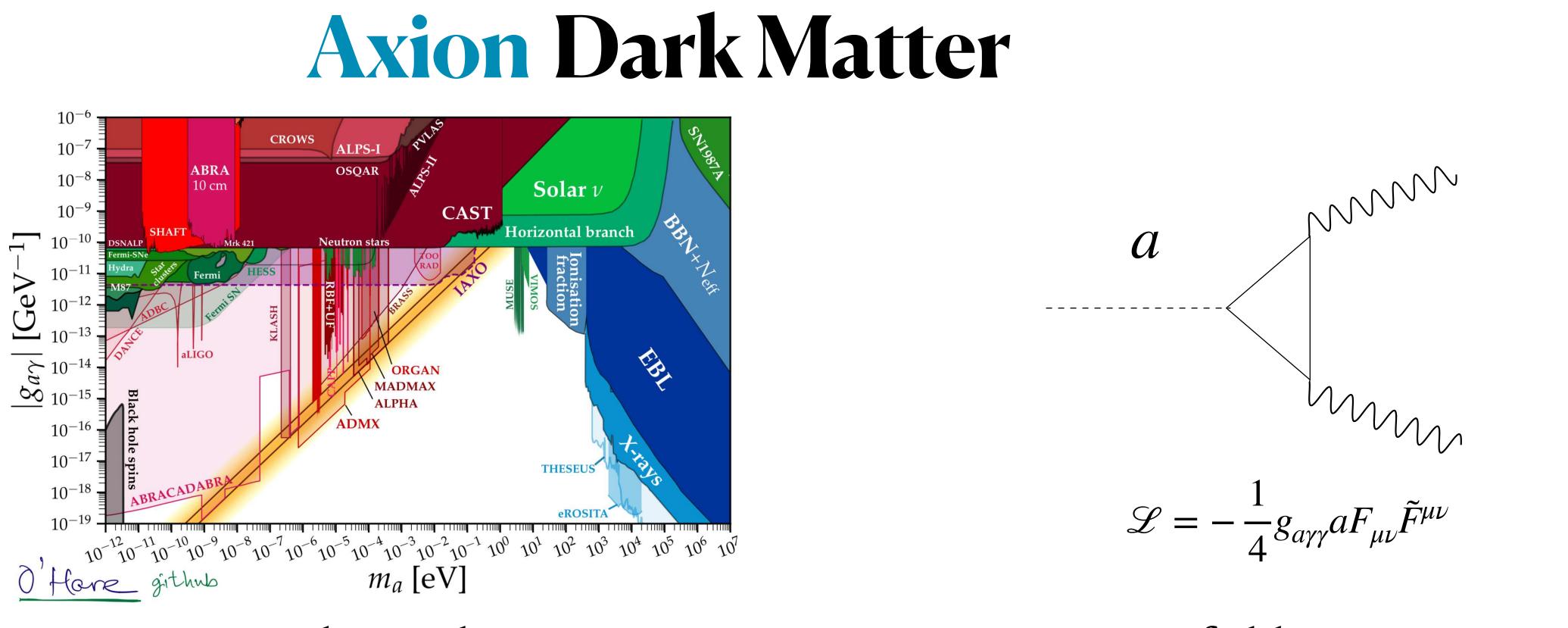






• Axions are converted into electromagnetic waves in static E & B fields





### • Axions are converted into electromagnetic waves in static E & B fields

 $h_{\mu
u}$  $\mathcal{A}$ 

### Ideas and techniques developed for axions can be adapted to gravitational waves Raffelt, Stodolski'89



# The Gertsenshtein Effect

# The (inverse) Gertsenhstein Effect

SOVIET PHYSICS JETP

VOLUME 14, NUMBER 1

JANUARY, 1962

### WAVE RESONANCE OF LIGHT AND GRAVITIONAL WAVES

M. E. GERTSENSHTEĬN

Submitted to JETP editor July 29, 1960

J. Exptl. Theoret. Phys. (U.S.S.R.) 41, 113-114 (July, 1961)

The energy of gravitational waves excited during the propagation of light in a constant magnetic or electric field is estimated.

### $\Box h_{\mathsf{X}} = 16\pi GB \,\partial_{\ell} A_{\mathsf{X}} \qquad \Box A_{\mathsf{X}} = -B\partial_{\ell} h_{\mathsf{X}}$

(The same equation holds for + instead of × polarization)

### The (inverse) Gertsenhstein Effect

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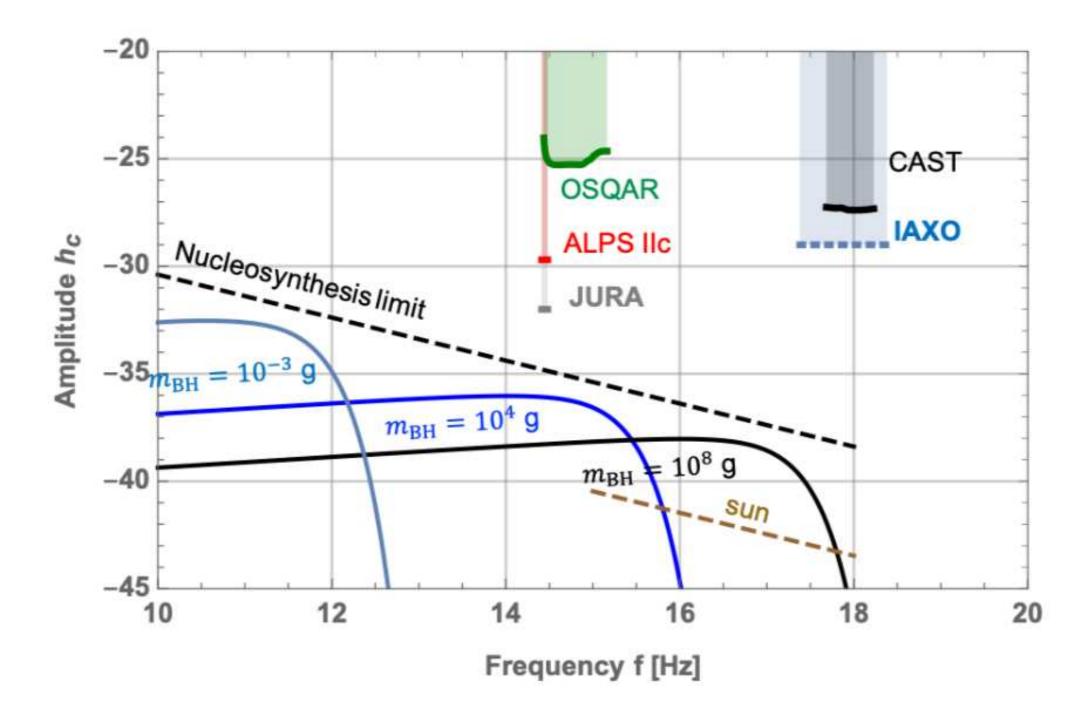
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(The same equation holds for + instead of  $\times$  polarization)

Upper limits on the amplitude of ultra-high-frequency gravitational waves from graviton to photon conversion

A. Ejlli 🖂, D. Ejlli, A. M. Cruise, G. Pisano & H. Grote

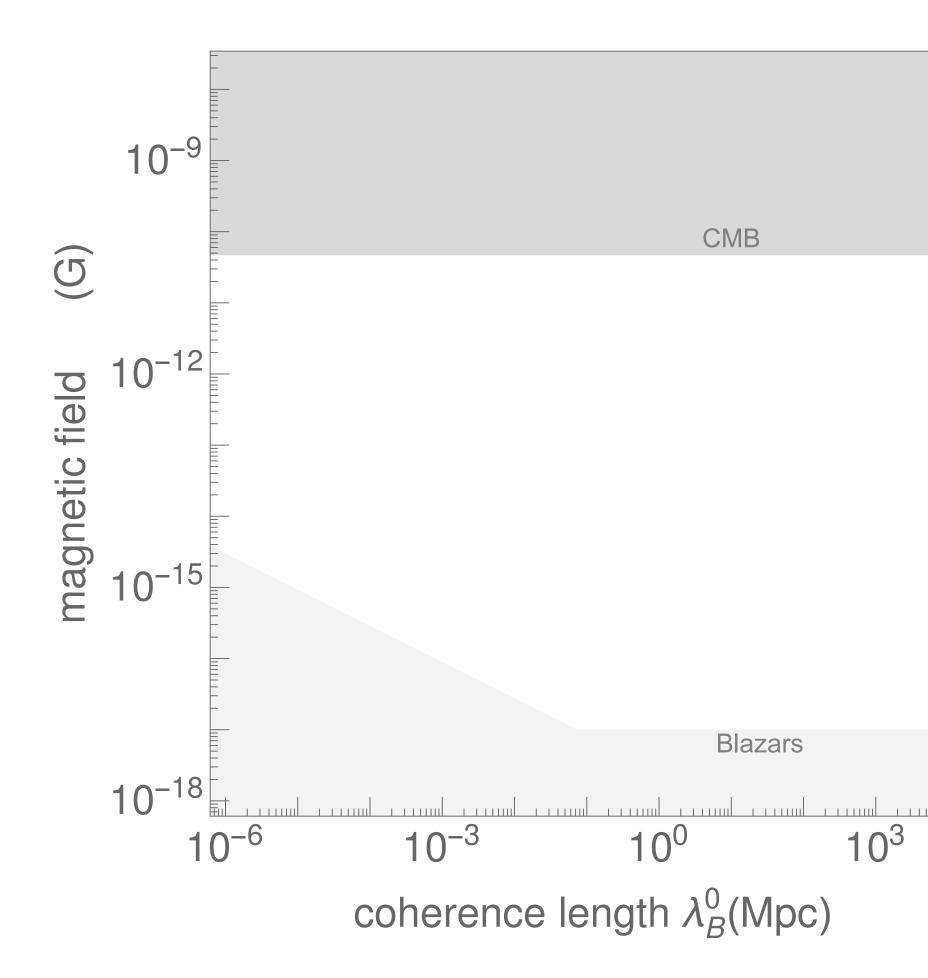
The European Physical Journal C 79, Article number: 1032 (2019) Cite this article



### $\wedge$ Ideas and techniques developed for axions can be adapted to gravitational waves Raffelt, Stodolski'89



# Cosmic magnetic fields



PHYSICAL REVIEW LETTERS 123, 021301 (2019)

### Stringent Limit on Primordial Magnetic Fields from the Cosmic Microwave Background Radiation

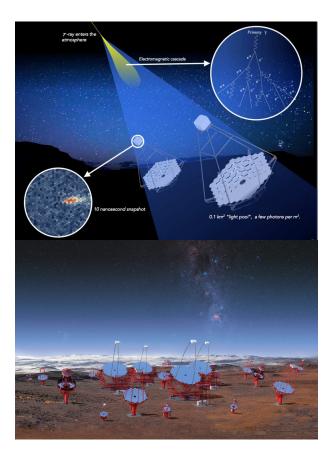
Karsten Jedamzik<sup>1,\*</sup> and Andrey Saveliev<sup>2,3,†</sup>

<sup>1</sup>Laboratoire Univers et Particules de Montpellier, UMR5299-CNRS, Université de Montpellier, 34095 Montpellier, France <sup>2</sup>Institute of Physics, Mathematics and Information Technology, Immanuel Kant Baltic Federal University, 236016 Kaliningrad, Russia <sup>3</sup>Faculty of Computational Mathematics and Cybernetics, Lomonosov Moscow State University, 119991 Moscow, Russia

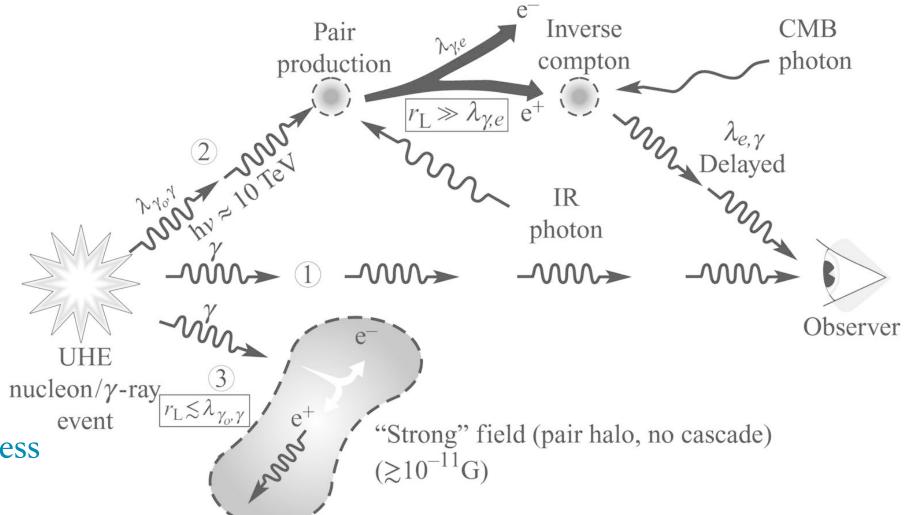
(Received 8 May 2018; revised manuscript received 13 September 2018; published 10 July 2019)



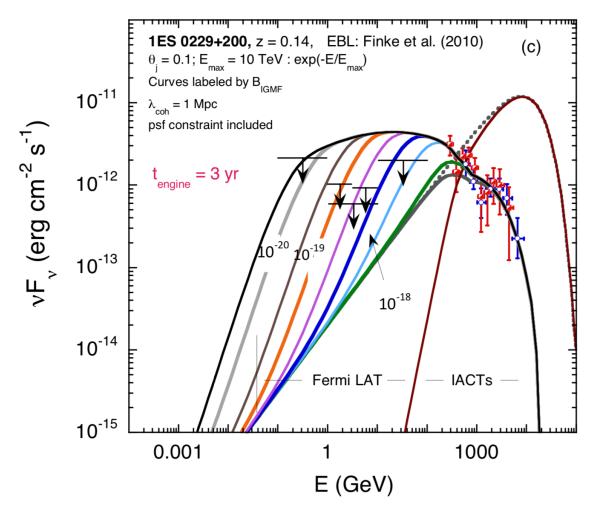
# **Evidence from TeV Blazars**

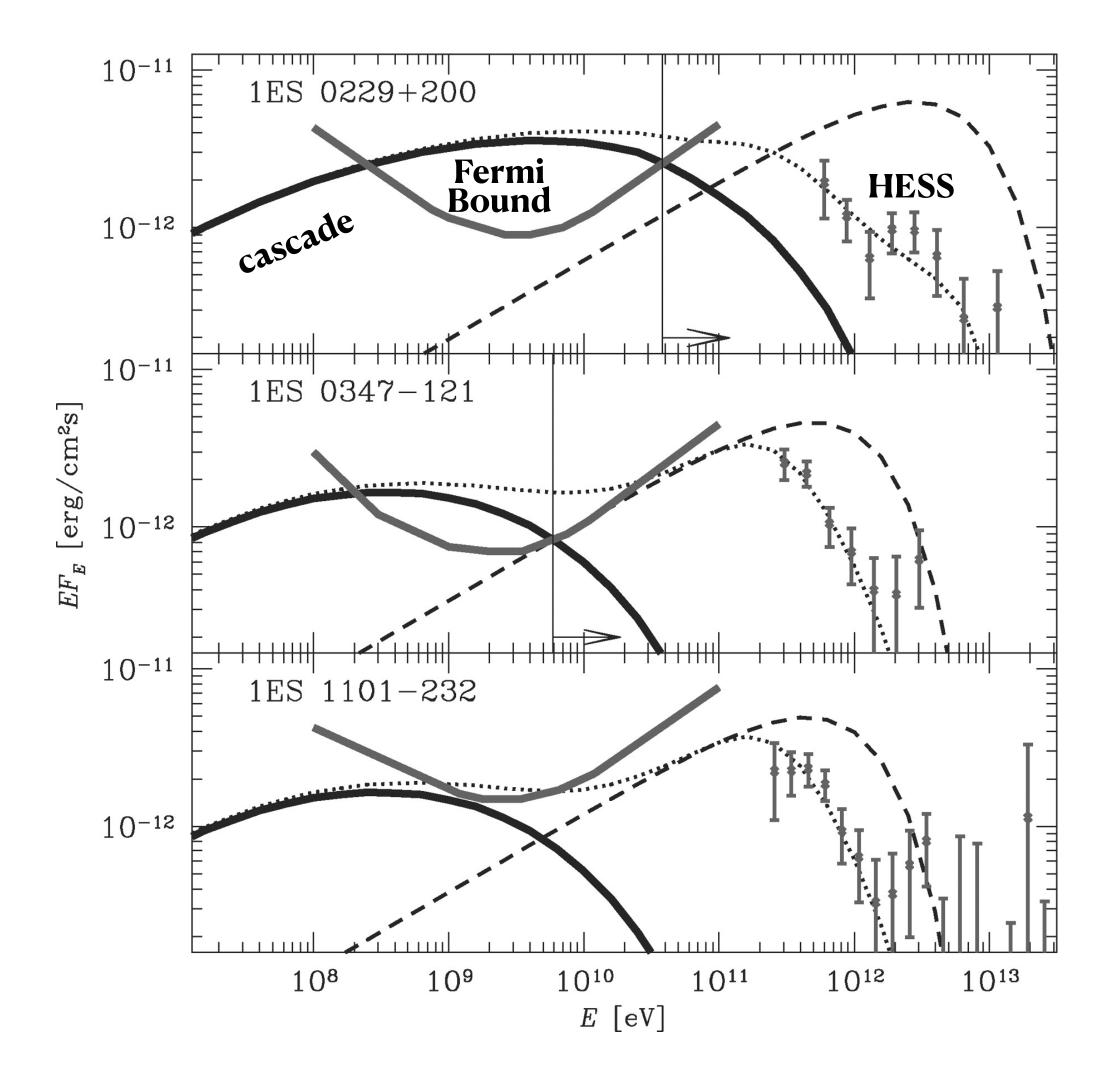


Kronberg, 2016 Cambridge University Press High energy  $hv - e^+e^-$  cascades in the intergalactic medium

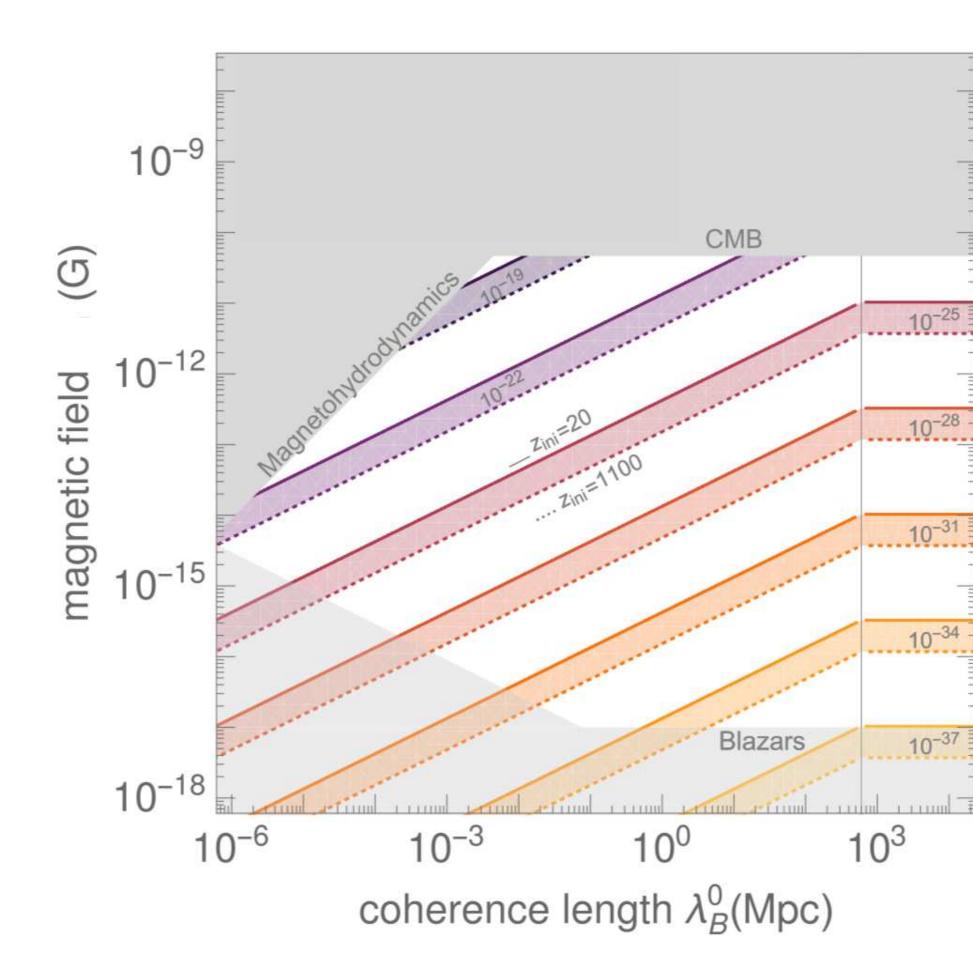


### CTA consortium 2017 Dermer et al





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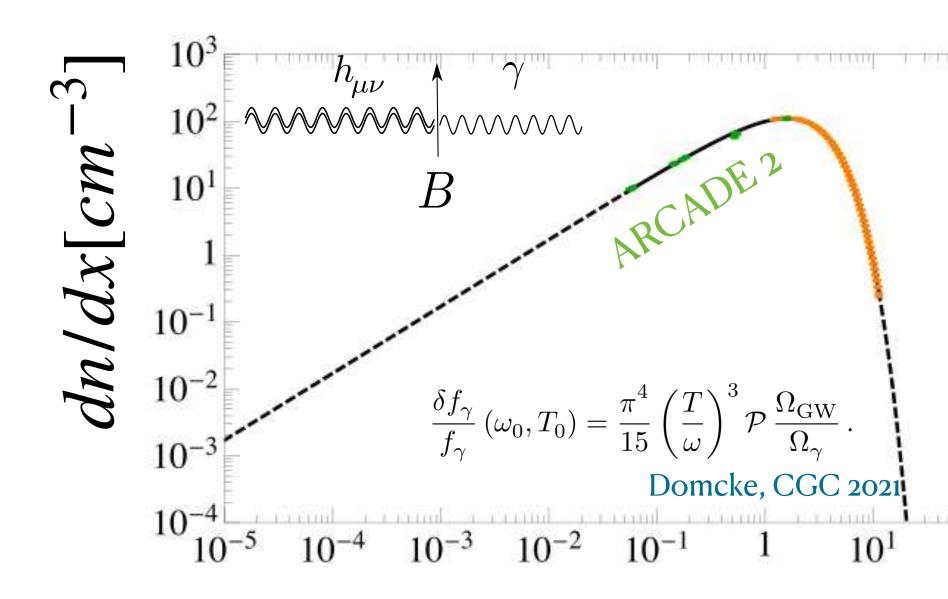
<sup>1</sup>Laboratoire Univers et Particules de Montpellier, UMR5299-CNRS, Université de Montpellier, 34095 Montpellier, France <sup>2</sup>Institute of Physics, Mathematics and Information Technology, Immanuel Kant Baltic Federal University, 236016 Kaliningrad, Russia <sup>3</sup>Faculty of Computational Mathematics and Cybernetics, Lomonosov Moscow State University, 119991 Moscow, Russia

(Received 8 May 2018; revised manuscript received 13 September 2018; published 10 July 2019)



Domcke, CGC 2021

# Rayleigh-Jeans tail



Largely unexplored with upcoming advances in radio astronomy probing it in the near future.

Puzzling signal by EDGES. (Experiment to Detect the Global Epoch of Reionization Signature)

### THE ASTROPHYSICAL JOURNAL

### ARCADE 2 MEASUREMENT OF THE ABSOLUTE SKY **BRIGHTNESS AT 3-90 GHz**

D. J. Fixsen<sup>1</sup>, A. Kogut<sup>2</sup>, S. Levin<sup>3</sup>, M. Limon<sup>4</sup>, P. Lubin<sup>5</sup>, P. Mirel<sup>6</sup>, M. Seiffert<sup>3</sup>, J. Singal<sup>7</sup>, E. Wollack<sup>2</sup>, T. Villela<sup>8</sup> + Show full author list Published 2011 May 17 • © 2011. The American Astronomical Society. All rights reserved. ophysical Journal, Volume 734, Number 1

### nature

An absorption profile centred at 78 megahertz in the sky-averaged spectrum

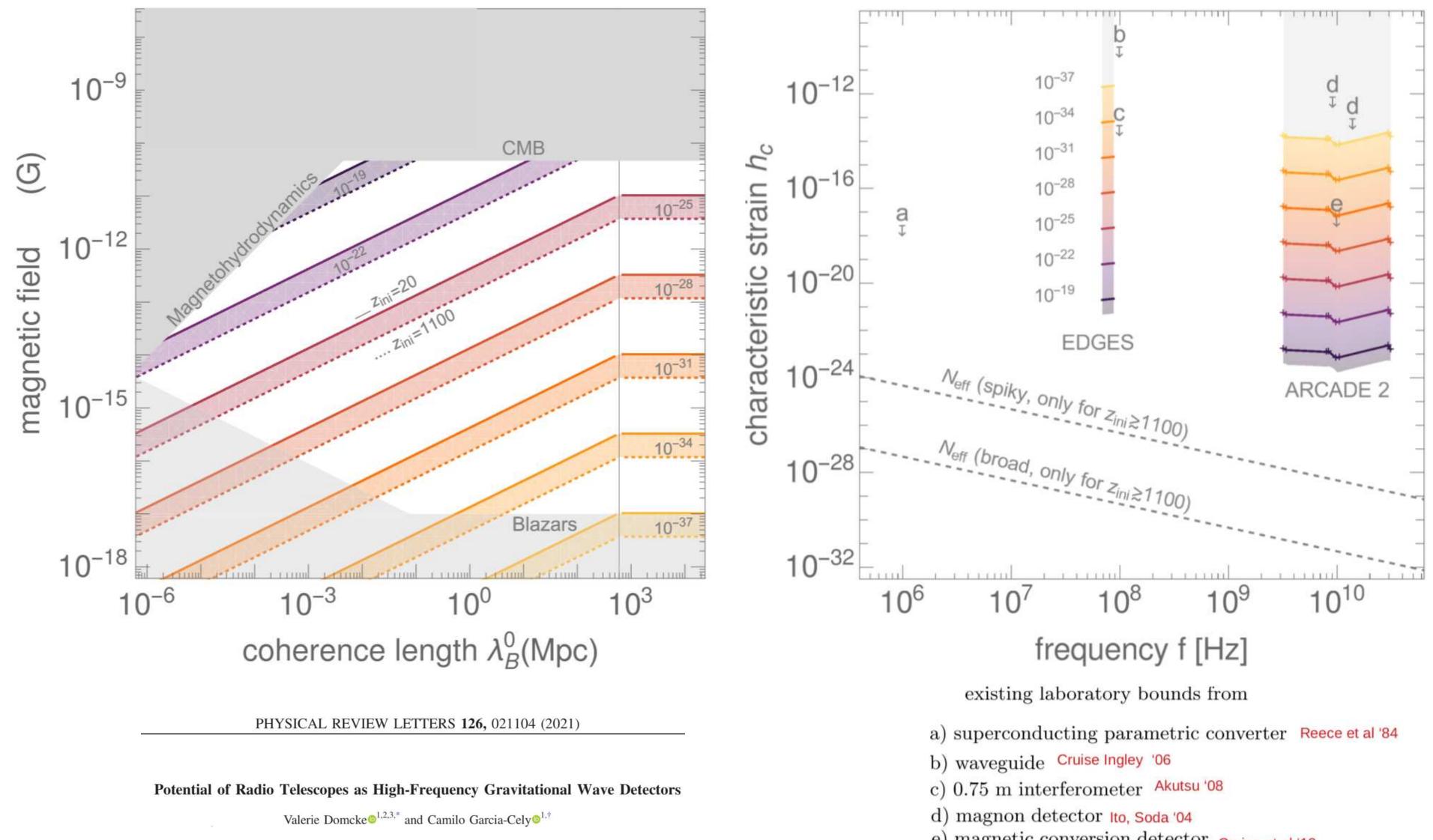
Judd D. Bowman ⊡, Alan E. E. Rogers, Raul A. Monsalve, Thomas J. Mozdzen & Nivedita Mahesh

*Nature* **555**, 67–70(2018) Cite this article

 $10^{2}$ 



### Upper bounds on stochastic gravitational waves



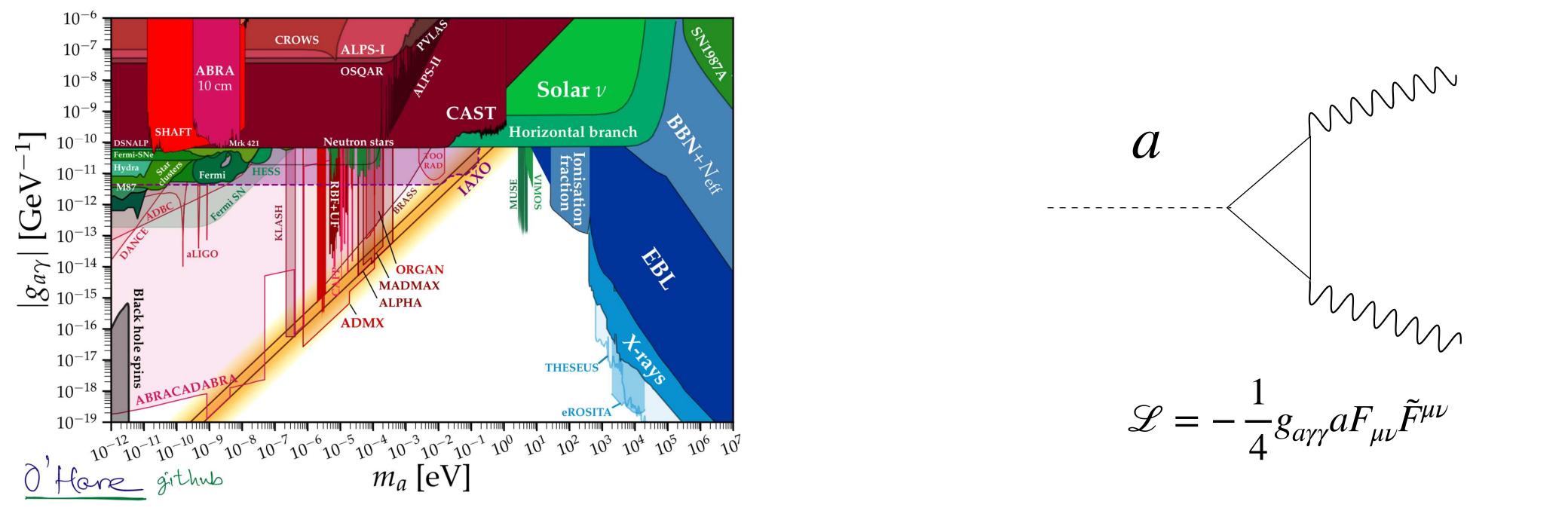
e) magnetic conversion detector Cruise et al '12

# Gravitational-Wave versus Axion electrodynamics

	electro
An example	Axion-Ph

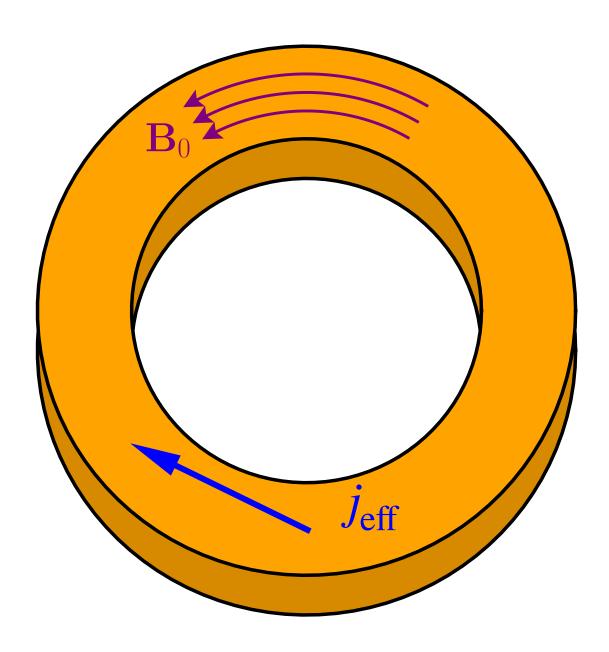
Axion odynamics	Gravitational wave electrodynamics
noton conversion	Gertsenshtein effect

# Axion electrodynamics



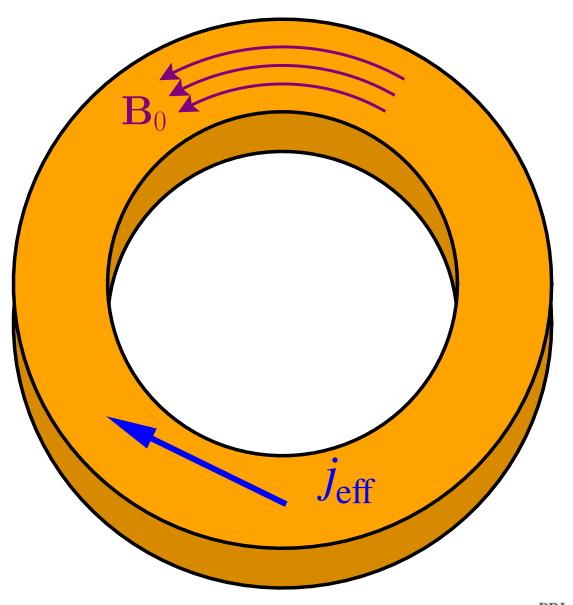
In the presence of an external electromagnetic field, axions act as a source term to Maxwell's equations,  $\partial_{\nu}F^{\mu\nu} = j_{\text{eff}}^{\mu} = \partial_{\nu}\left(g_{a\gamma\gamma}a\tilde{F}^{\nu\mu}\right)$ effectively inducing an electromagnetic current.

# Axion electrodynamics



In the presence of an external electromagnetic field, axions act as a source term to Maxwell's equations, effectively inducing an electromagnetic current.  $\nabla \times \mathbf{B} = \partial_t \mathbf{E} + g_{a\gamma\gamma} \partial_t a \mathbf{B}_0$ 

# Axion electrodynamics



PRL 117, 141801 (2016)

Yonatan Kahn,<sup>1,\*</sup> Benjamin R. Safdi,<sup>2,†</sup> and Jesse Thaler<sup>2,‡</sup> <sup>1</sup>Department of Physics, Princeton University, Princeton, New Jersey 08544, USA <sup>2</sup>Center for Theoretical Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA (Received 3 March 2016; published 30 September 2016)

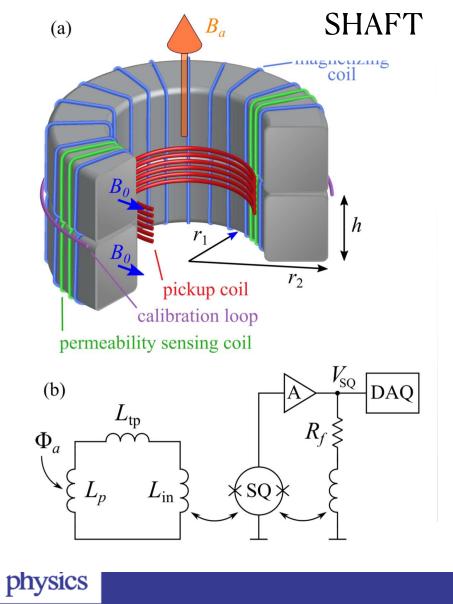
In the presence of an external electromagnetic field, axions act as a source term to Maxwell's equations, effectively inducing an electromagnetic current.

### haloscopes based on lumped-element detectors



PHYSICAL REVIEW LETTERS

### Broadband and Resonant Approaches to Axion Dark Matter Detection



### Search for axion-like dark matter with ferromagnets

Alexander V. Gramolin<sup>1</sup>, Deniz Aybas<sup>1,2</sup>, Dorian Johnson<sup>1</sup>, Janos Adam<sup>1</sup> and Alexander O. Sushkov (1,2,3 🖂

 $\nabla \times \mathbf{B} = \partial_t \mathbf{E} + g_{a\gamma\gamma} \partial_t a \mathbf{B}_{\mathbf{0}}$ 

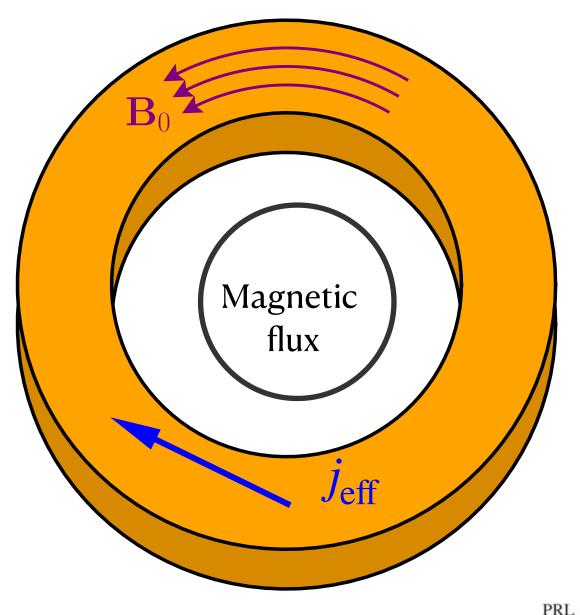
week ending

30 SEPTEMBER 2016

j<sub>eff</sub>







PRL 117, 141801 (2016)

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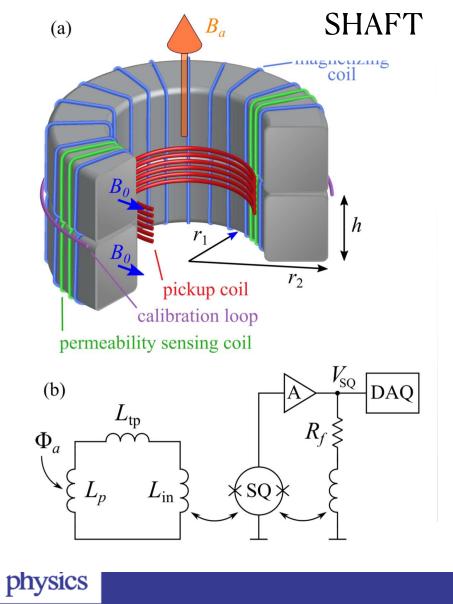
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week ending

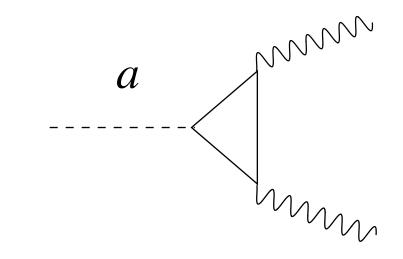
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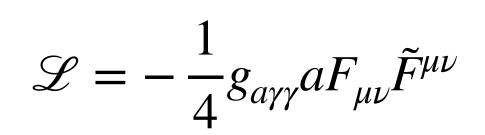
j<sub>eff</sub>



### **Effective currents**

### Axions





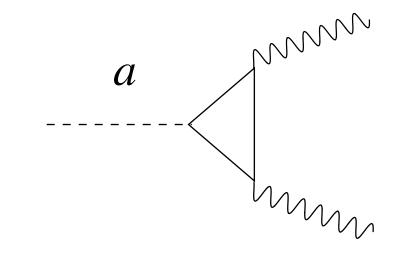
 $\nabla \cdot \mathbf{E} = -\nabla \cdot \mathbf{P}$  $\nabla \cdot \mathbf{B} = 0$  $\nabla \times \mathbf{E} = -\partial_t \mathbf{B}$  $\nabla \times \mathbf{B} = \partial_t \mathbf{E} + \nabla \times \mathbf{M} + \partial_t \mathbf{P}$ 

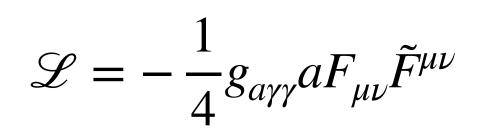
 $\mathbf{P} = g_{a\gamma\gamma} a \mathbf{B}, \quad \mathbf{M} = g_{a\gamma\gamma} a \mathbf{E}$ 

McAllister et al, 1803.07755 Tobar et al, 1809.01654 Ouellet et al, 1809.10709

### **Effective currents**

### Axions





 $\nabla \cdot \mathbf{E} = -\nabla \cdot \mathbf{P}$  $\nabla \cdot \mathbf{B} = \mathbf{0}$  $\nabla \times \mathbf{E} = -\partial_t \mathbf{B}$ 

 $\mathbf{P} = g_{a\gamma\gamma} a \mathbf{B}, \quad \mathbf{M} = g_{a\gamma\gamma} a \mathbf{E}$ 

McAllister et al, 1803.07755 Tobar et al, 1809.01654 Ouellet et al, 1809.10709

Gravitational waves

$$S \supset \int d^4x \sqrt{-g} \left( -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \right)$$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \qquad \left| h_{\mu\nu} \right| \ll 1$$

 $\nabla \times \mathbf{B} = \partial_t \mathbf{E} + \nabla \times \mathbf{M} + \partial_t \mathbf{P}$ 

$$P_i = -h_{ij}E_j \qquad M_i = -h_{ij}B_j$$

(in the TT gauge)

Domcke, CGC, Rodd, 2202.00695

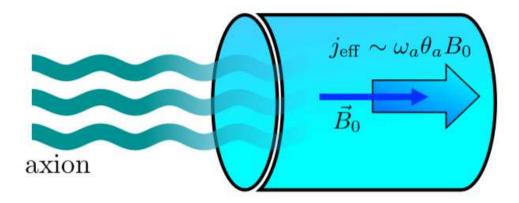


	Axion electrodynamics	Gravitational wave electrodynamics
An example	Axion-Photon conversion	Gertsenshtein effect
Effective current	$\mathbf{P} = g_{a\gamma\gamma} a \mathbf{B},  \mathbf{M} = g_{a\gamma\gamma} a \mathbf{E}$	$P_i = -h_{ij}E_j \qquad M_i = -h_{ij}B_j$
$j_{\text{eff}}^{\mu} = \left(-\nabla \cdot \mathbf{P}, \nabla \times \mathbf{M} + \partial_t \mathbf{P}\right)$	McAllister et al, 1803.07755 Tobar et al, 1809.01654 Ouellet et al, 1809.10709	( in the TT gauge) Domcke, CGC, Rodd, 2202.00695

Detecting High-Frequency Gravitational Waves with Microwave Cavities

Asher Berlin (New York U. and Fermilab), Diego Blas (Barcelona, Autonoma U. and Barcelona, IFAE), Raffaele Tito D'Agnolo (IPhT, Saclay), Sebastian A.R. Ellis (U. Geneva (main) and IPhT, Saclay), Roni Harnik (Fermilab) et al. (Dec 21, 2021)

e-Print: 2112.11465 [hep-ph]

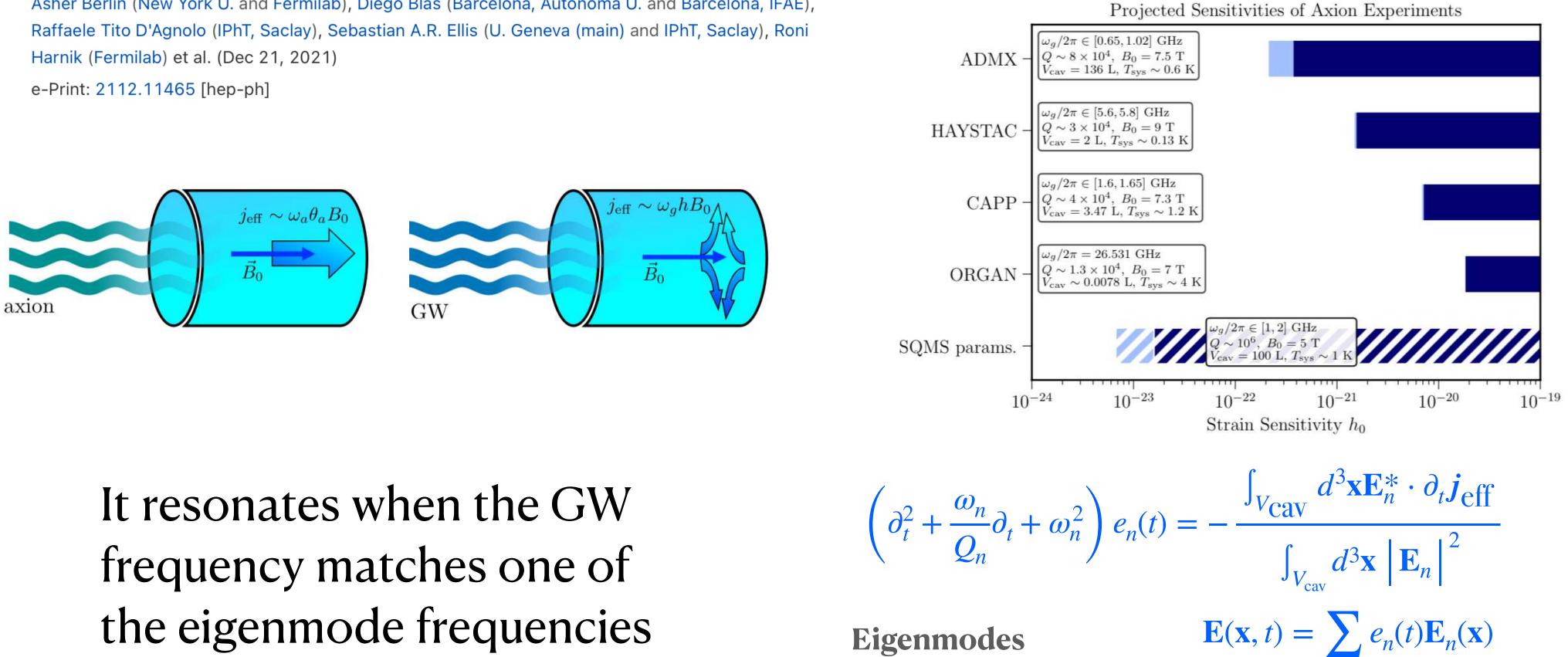


### It resonates when the GW frequency matches one of the eigenmode frequencies

$$\left(\partial_t^2 + \frac{\omega_n}{Q_n}\partial_t + \omega_n^2\right)e_n(t) = -\frac{\int_{V_{\text{CaV}}} d^3\mathbf{x}\mathbf{E}_n^* \cdot \partial_t \mathbf{j}_{\text{eff}}}{\int_{V_{\text{cav}}} d^3\mathbf{x} \left|\mathbf{E}_n\right|^2}$$
  
**Eigenmodes** 
$$\mathbf{E}(\mathbf{x}, t) = \sum_n e_n(t)\mathbf{E}_n(\mathbf{x})$$

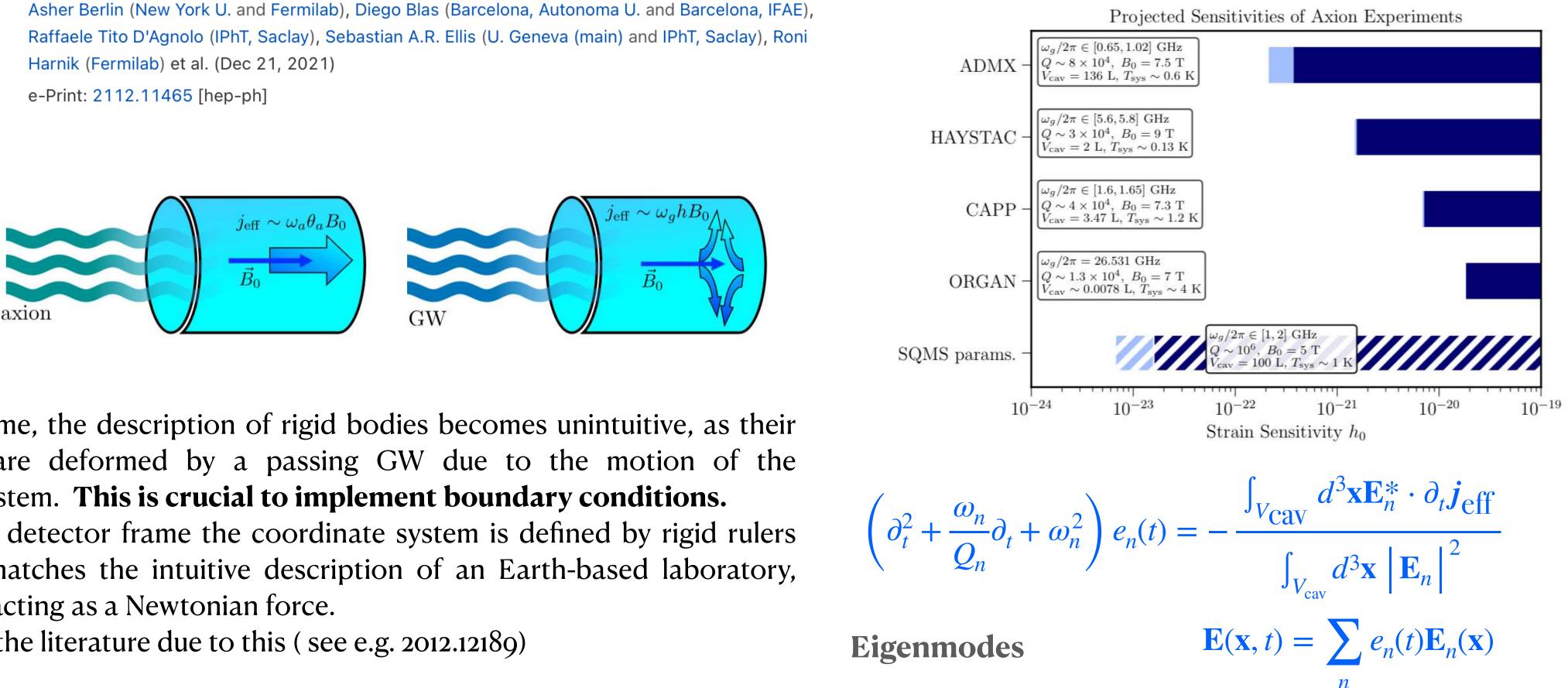
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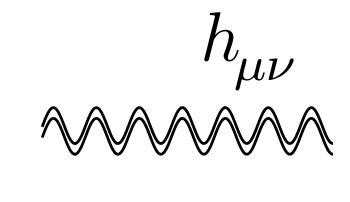
### Subtleties due to gauge fixing (TT vs detector frame gauge)

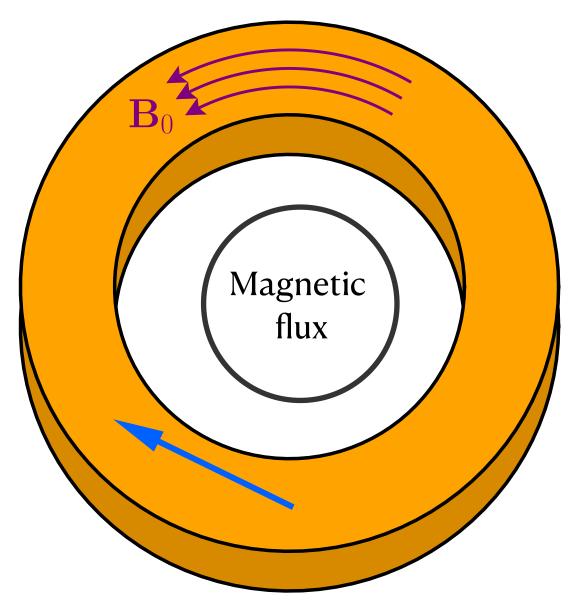
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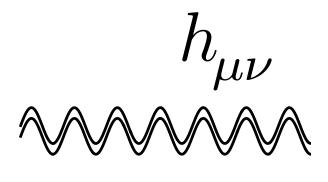
- In the TT frame, the description of rigid bodies becomes unintuitive, as their coordinates are deformed by a passing GW due to the motion of the coordinate system. This is crucial to implement boundary conditions.
- In the proper detector frame the coordinate system is defined by rigid rulers and closely matches the intuitive description of an Earth-based laboratory, with the GW acting as a Newtonian force.
- Confusion in the literature due to this (see e.g. 2012.12189)

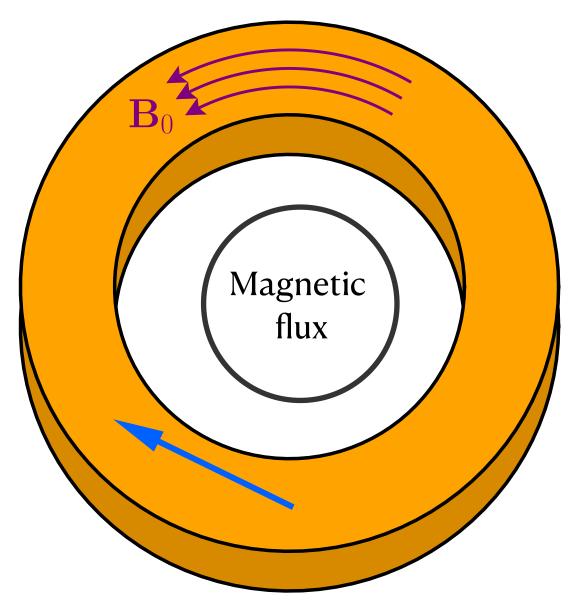
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$J_{\text{eff}} - (-\mathbf{v} \cdot \mathbf{I}, \mathbf{v} \times \mathbf{I} \mathbf{v} + \mathbf{O}_t \mathbf{I})$	McAllister et al, 1803.07755 Tobar et al, 1809.01654 Ouellet et al, 1809.10709	Domcke, CGC, Rodd, 2202.00695

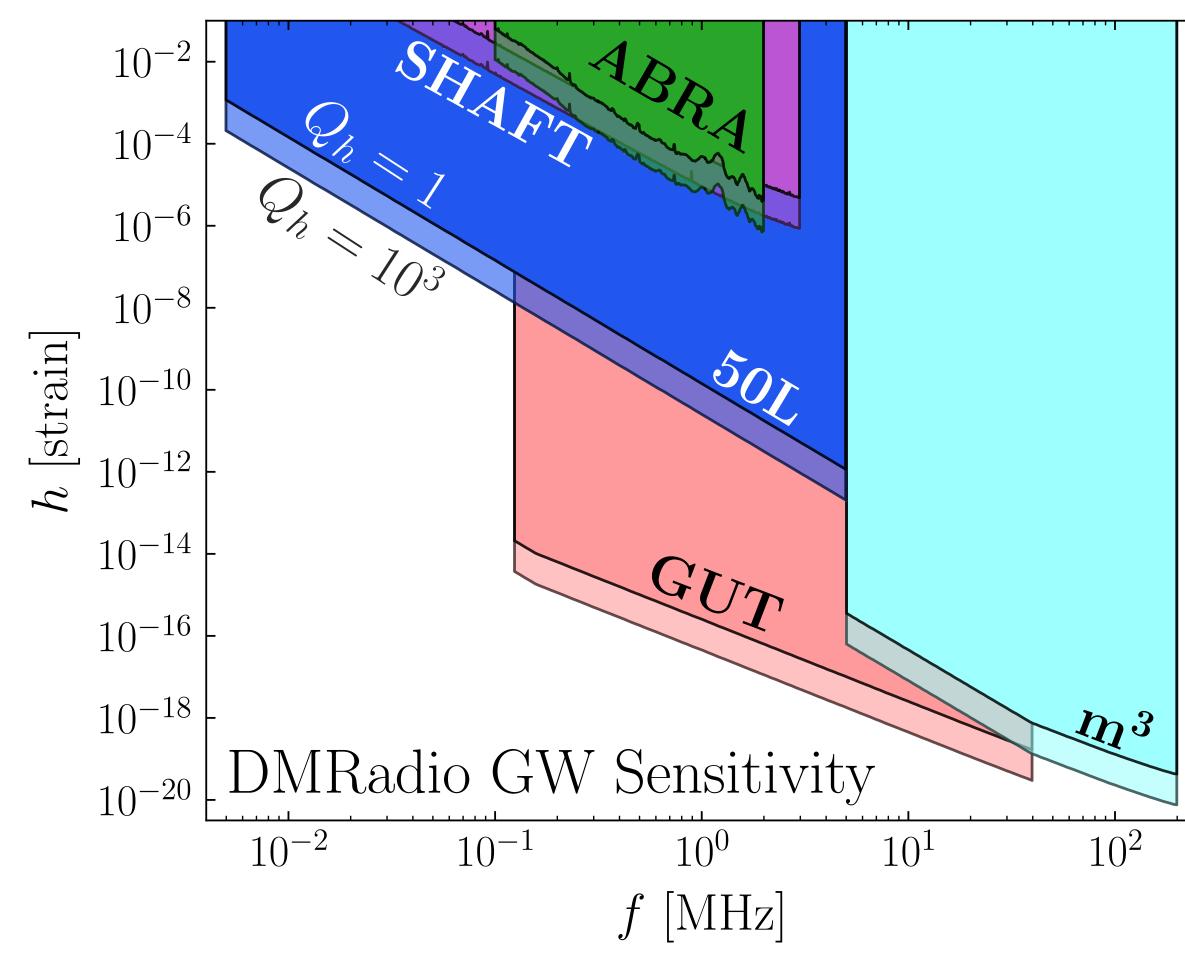




Domcke, CGC, Rodd, 2202.00695

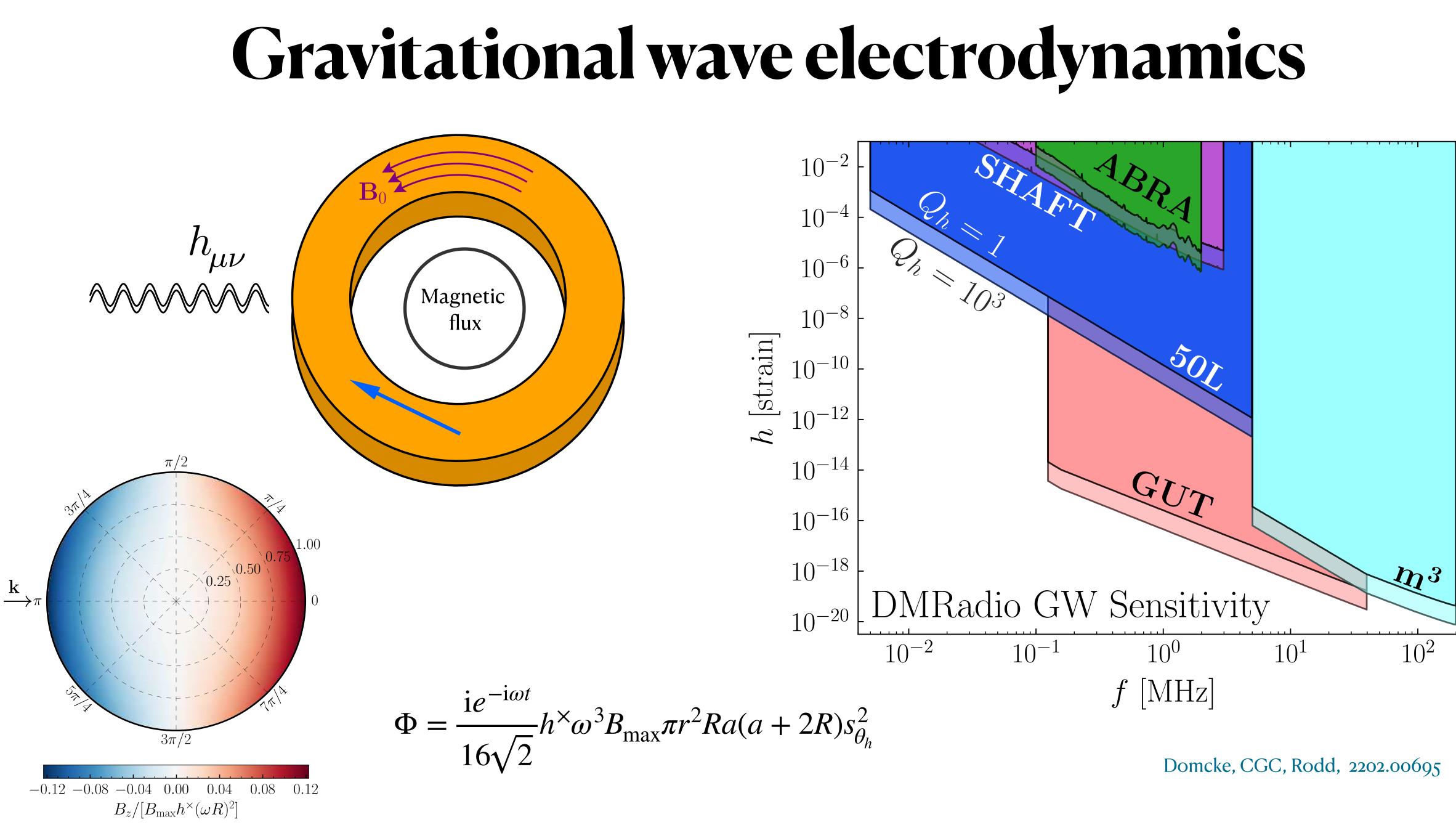




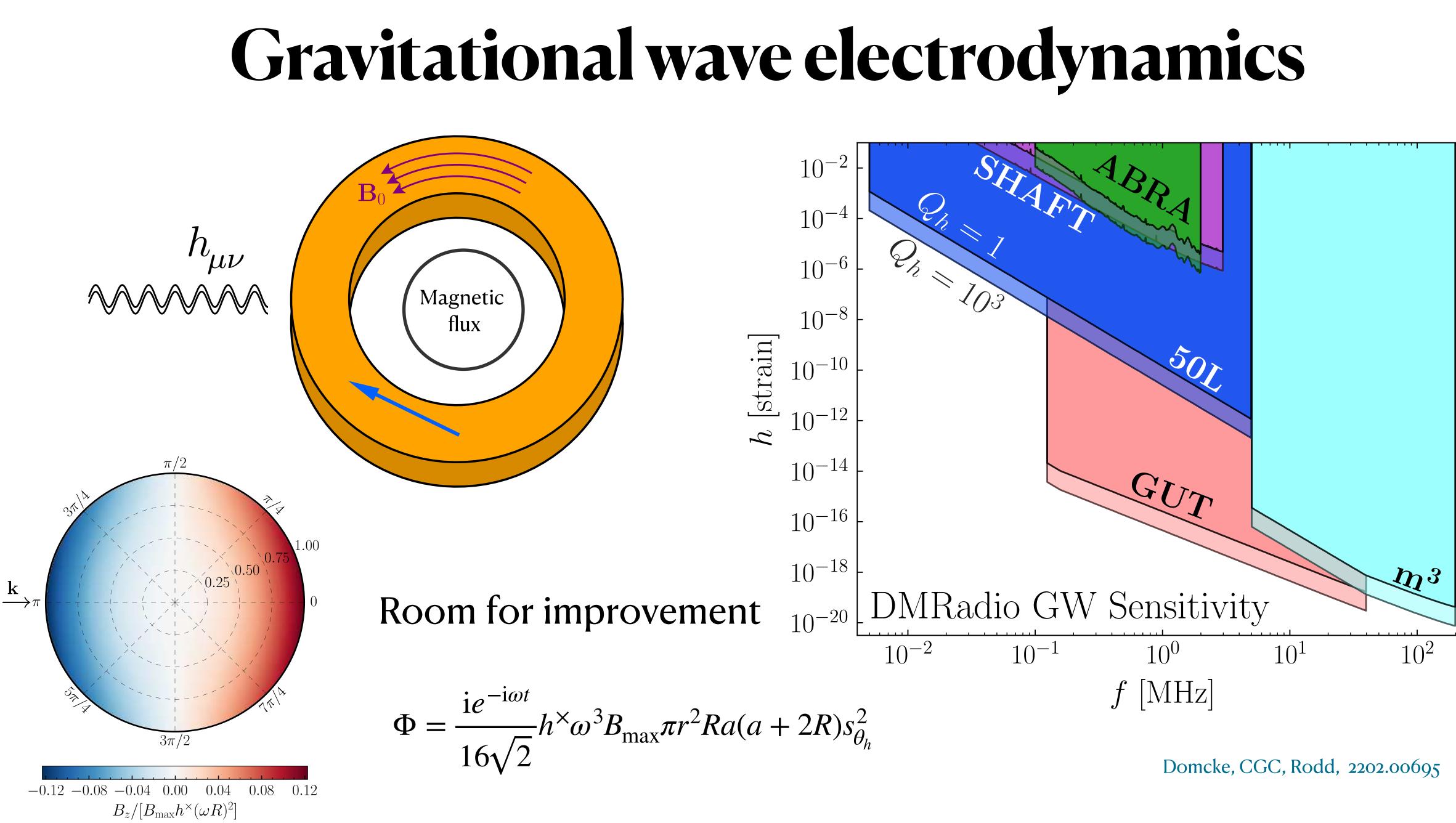


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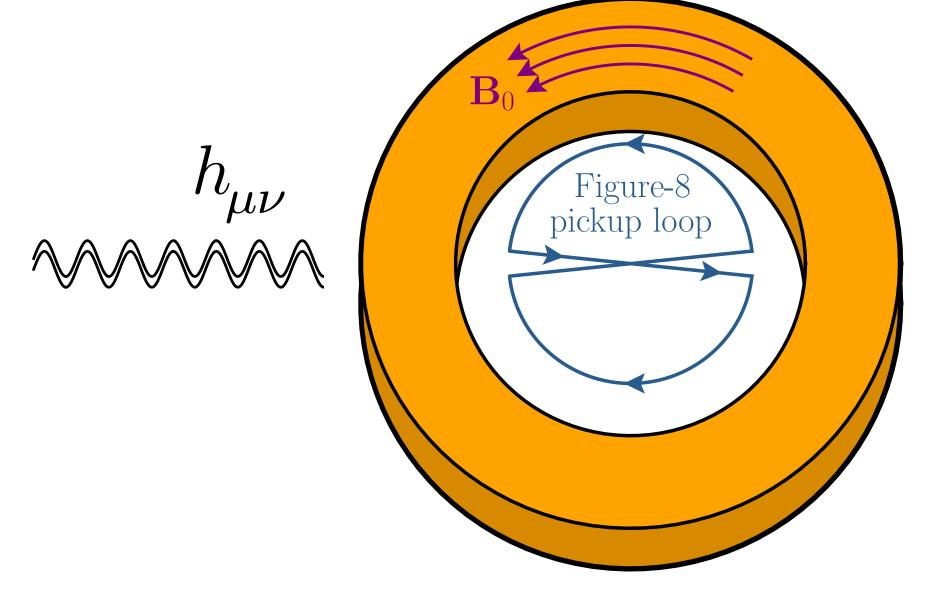




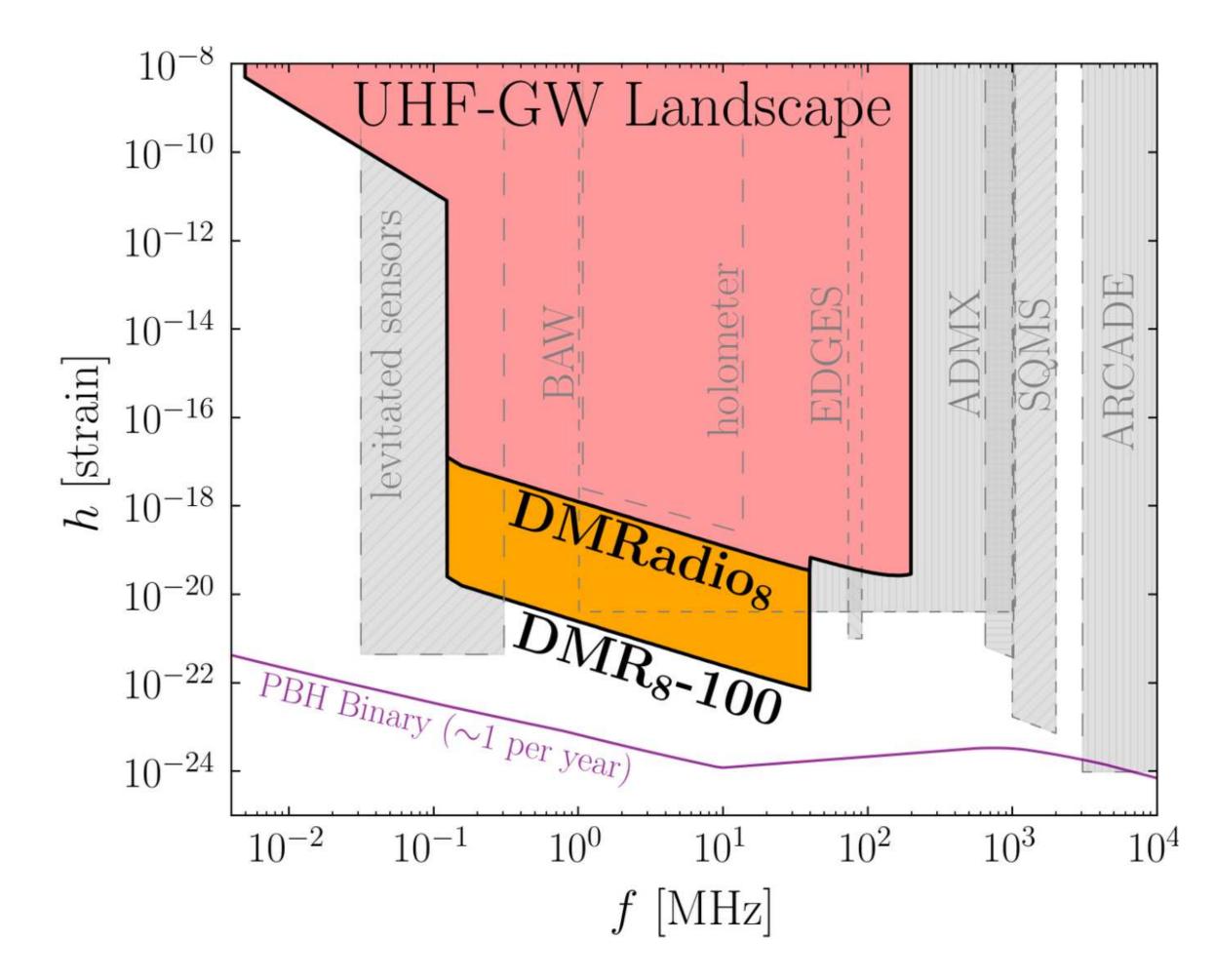






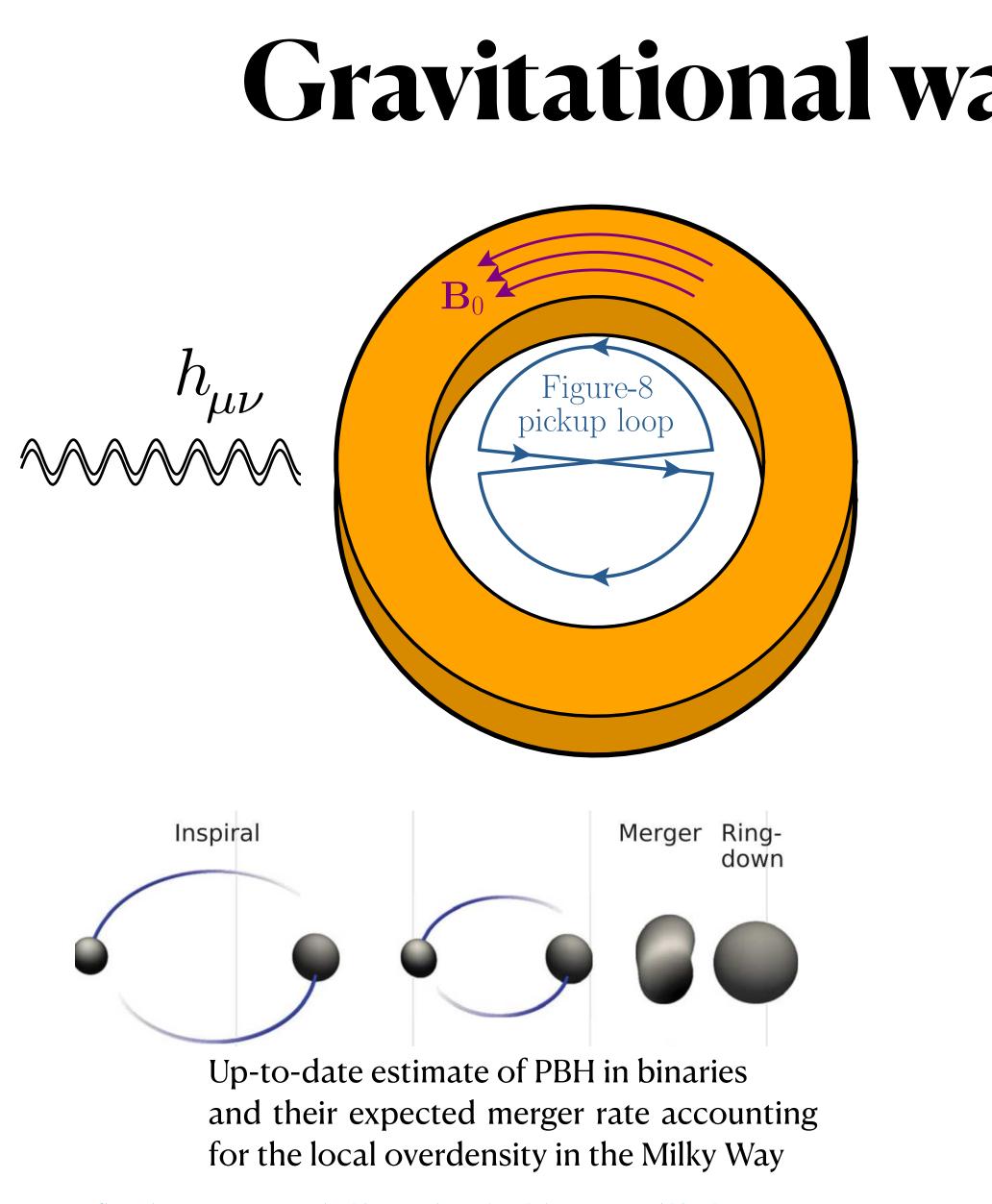


$$\Phi_8 = \frac{e^{-i\omega t}}{3\sqrt{2}} \omega^2 B_{\max} r^3 R \ln(1 + a/R) s_{\theta_h}$$
$$\times \left(h^* s_{\phi_h} - h^+ c_{\theta_h} c_{\phi_h}\right).$$



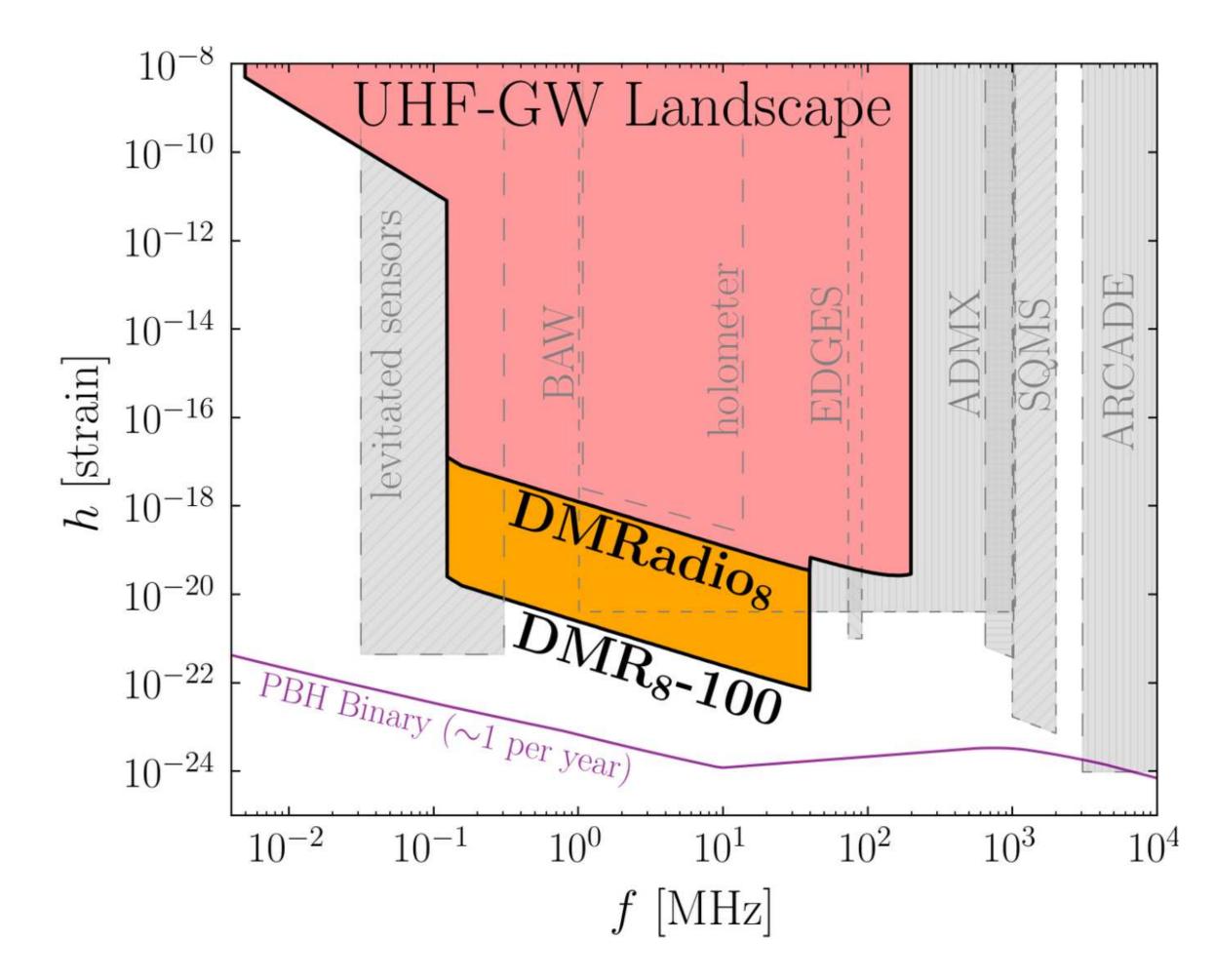
Domcke, CGC, Rodd, 2202.00695





See also 2205.02153 by Franciolini, A. Maharana, and F. Muia,

### Gravitational wave electrodynamics



Domcke, CGC, Rodd, 2202.00695



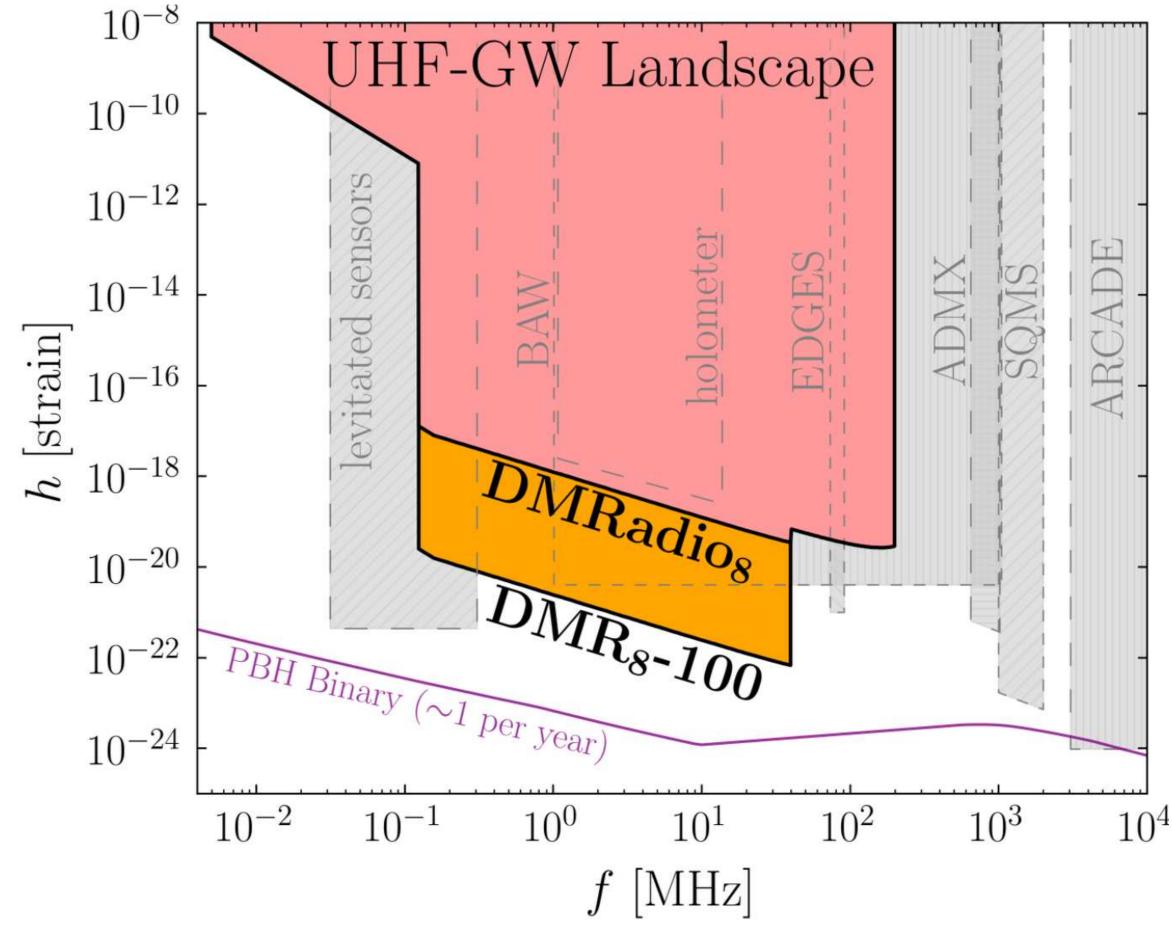
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Benchmark	QCD axion $g_{a\gamma\gamma}a \sim \frac{\alpha\sqrt{\rho_{\rm DM}}}{2\pi m_a f_a} \sim \frac{\alpha\sqrt{\rho_{\rm DM}}}{2\pi m_\pi f_\pi} \sim 10^{-22}$	$h \sim 10^{-22}$

# Conclusions

The Gertsenshtein effect during the dark ages provides a powerful way to probe gravitational waves in the MHz-GHz range from distortions of the Rayleigh-Jeans CMB tail.

I provide a formulation of GW electrodynamics which demonstrates that low-mass axion haloscopes are also UHF-GW telescopes.

A number of distinct experimental proposals have coalesced on a strain sensitivity of  $10^{-22}$  for MHz GWs, a level that is still orders of magnitude away from any signal of the early Universe. Whether we can hope to probe such strain sensitivities remains to be determined.





### Cosmic magnetic fields

$$\langle \mathbf{B}_{i}(\mathbf{x})\mathbf{B}_{j}(\mathbf{x}')\rangle = \frac{1}{(2\pi)^{3}a(t)^{4}} \int d^{3}k e^{i\mathbf{k}\cdot(\mathbf{x}')}$$

Durrer, Neronov, 2013

$$\begin{split} \langle B^2 \rangle &= \frac{1}{\pi^2 a(t)^4} \int_0^\infty dk k^2 P_B(k) = \int_{-\infty}^\infty d\log \lambda B_\lambda^2 \\ &\text{where} \quad B_\lambda^2 \equiv \frac{8\pi}{\lambda^3 a(t)^4} P_B\left(\frac{2\pi}{\lambda}\right) \,, \end{split}$$
$$\lambda_B &= \int_0^\infty d\lambda \frac{B_\lambda^2}{\langle B^2 \rangle} \end{split}$$

 $\mathbf{x}'-\mathbf{x}$   $\left(\left(\delta_{ij}-\hat{k}_{i}\hat{k}_{j}\right)P_{B}(k)-i\epsilon_{ijk}\hat{k}_{k}P_{aB}(k)\right),$ 

The adiabatic evolution of the magnetic field due to cosmic expansion is determined by the scale factor.

average magnetic field

the coherence length

## Oscillations after the formation of the CMB

$$\left(\Box + \omega_{\rm pl}^2\right) A_{\lambda} = -B\partial_{\ell}h_{\lambda}$$
$$\Box h_{\lambda} = 16\pi GB \partial_{\ell}A_{\lambda}$$

$$\left< \Gamma_{g \leftrightarrow \gamma} \right> = \frac{2\pi G B^2 \ell_{\rm OSC}^2}{\Delta \ell}$$

$$\mathscr{P} \equiv \int_{l.o.s.} \langle \Gamma_{g\leftrightarrow\gamma} \rangle dt = \int_{0}^{z \text{ini}} \frac{\langle \Gamma_{g\leftrightarrow\gamma} \rangle}{(1+z) H}$$

$$\omega_{\rm pl} = \sqrt{e^2 n_e/m_e}$$

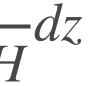
The plasma frequency acts as an effective mass term

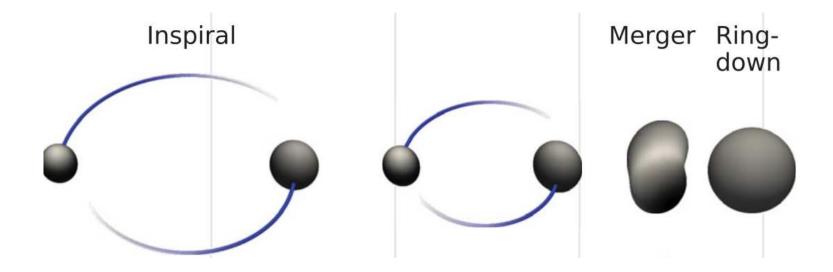
$$\ell_{\rm osc} \simeq 4\omega/\omega_{\rm pl}^2$$

Although cosmic magnetic fields are not expected to be perfectly homogeneous, coherent oscillations take place in highly homogeneous patches.

$$\ell_{\rm OSC} = 4\omega/(1+z)^2 X_e(z)\omega_{\rm pl,0}^2 \ll 1 \,{\rm pc}$$

Domcke, CGC 2021



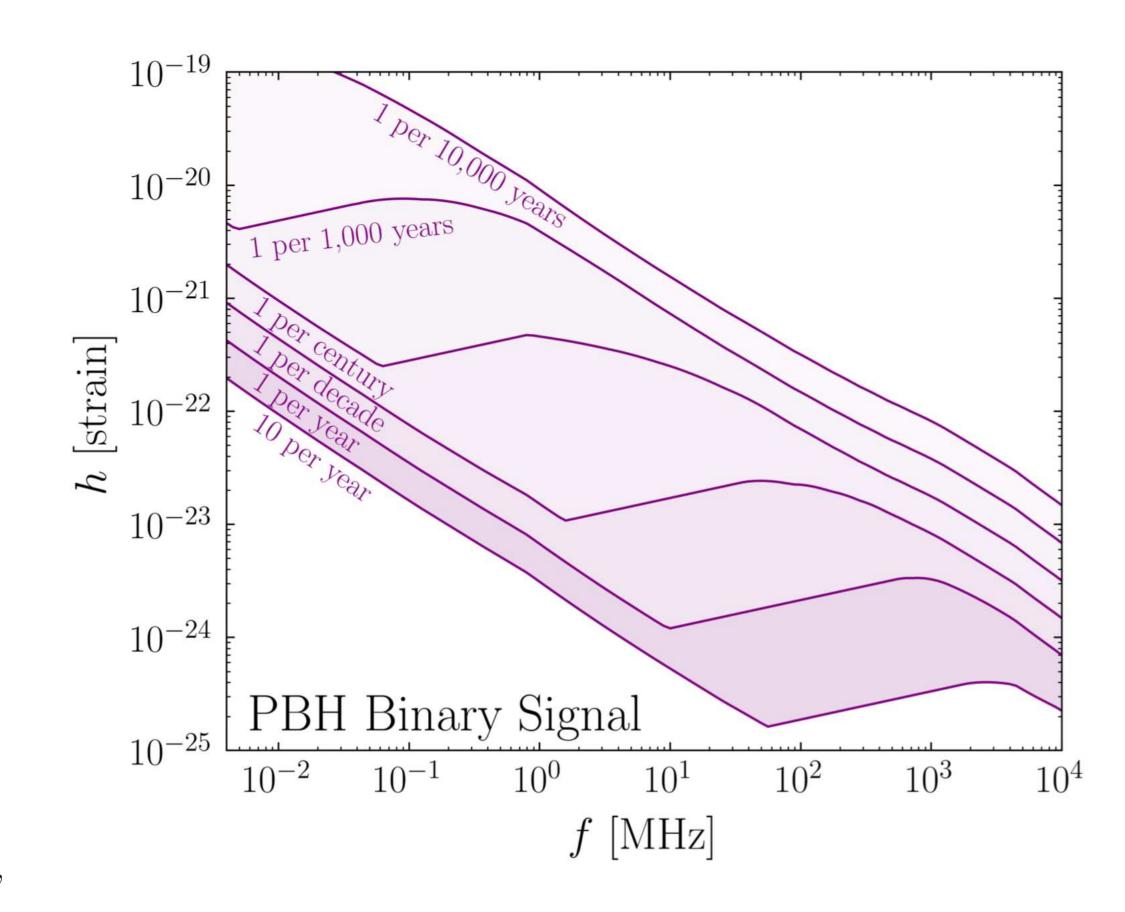


Up-to-date estimate of PBH in binaries and their expected merger rate accounting for the local overdensity in the Milky Way

### Merger rates

$$\begin{split} R_0(m_{\rm PBH}, f_{\rm PBH}) &\simeq 6.6 \times 10^{-8} \, {\rm kpc}^{-3} \, {\rm yr}^{-1} \, f_{\rm PBH}^{53/37} \left(\frac{m_{\rm PBH}}{10^{-5} M_\odot}\right)^{-32/37} S_{\rm early}(f_{\rm P}) \\ S_{\rm early}(f_{\rm PBH}) &= \min \left\{ 1, \left(\frac{f_{\rm PBH}}{0.01}\right)^{1/2} \right\}, \\ S_{\rm late}(f_{\rm PBH}) &= \min \left\{ 1, 9.6 \times 10^{-3} f_{\rm PBH}^{-0.65} e^{0.03 \ln^2 f_{\rm PBH}} \right\}, \end{split}$$

See also 2205.02153 by Franciolini, A. Maharana, and F. Muia,



 $_{\scriptscriptstyle \mathrm{PBH}}) S_{\mathrm{late}}(f_{\scriptscriptstyle \mathrm{PBH}}),$