Dark photon (and axion) stars

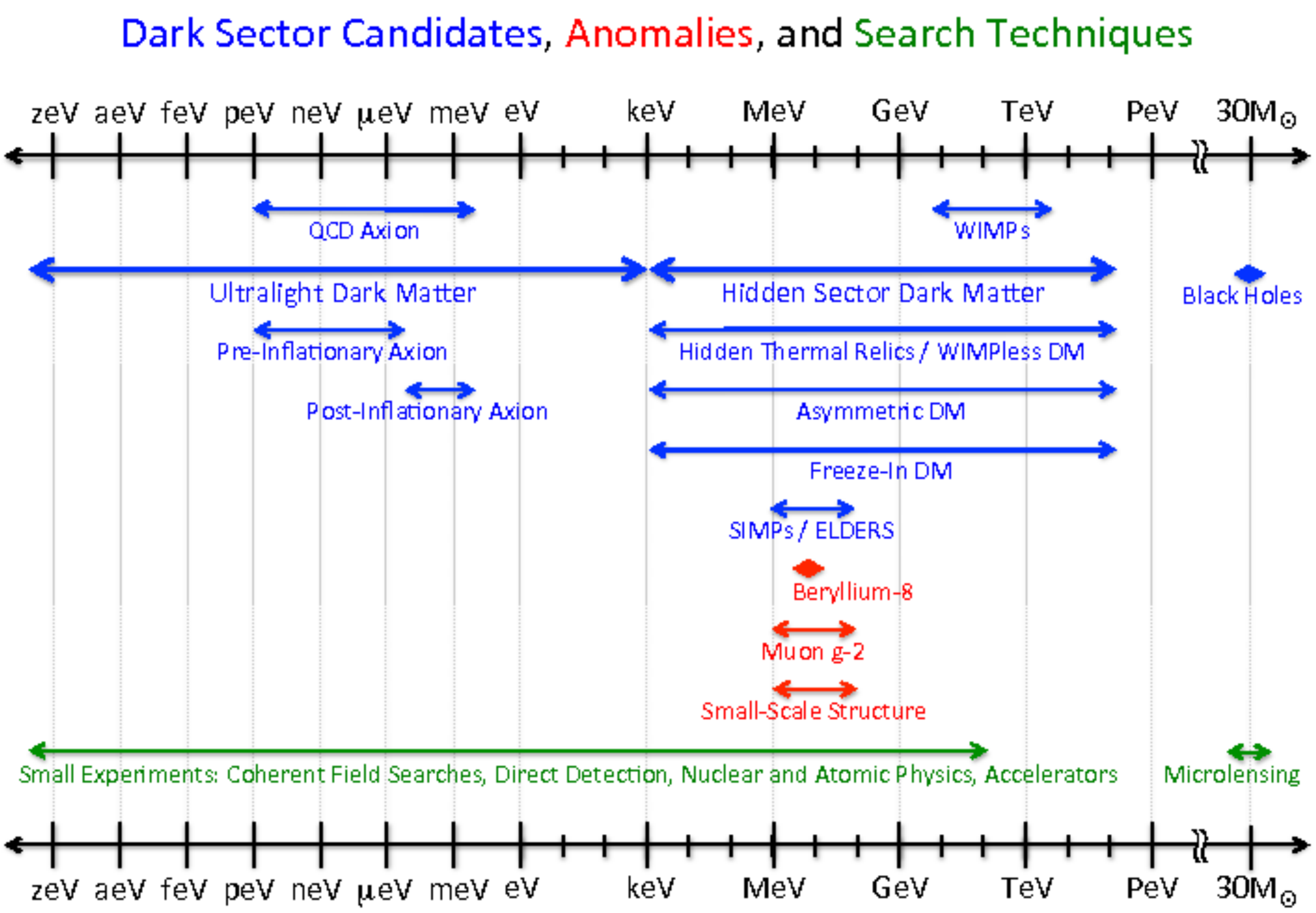
Ed Hardy

UNIVERSITY OF LIVERPOOL

With M. Gorghetto, J. March-Russell, N. Song, S. West

[2203.10100 + ongoing]

Dark matter candidates



Experimental Searches

Axion, a

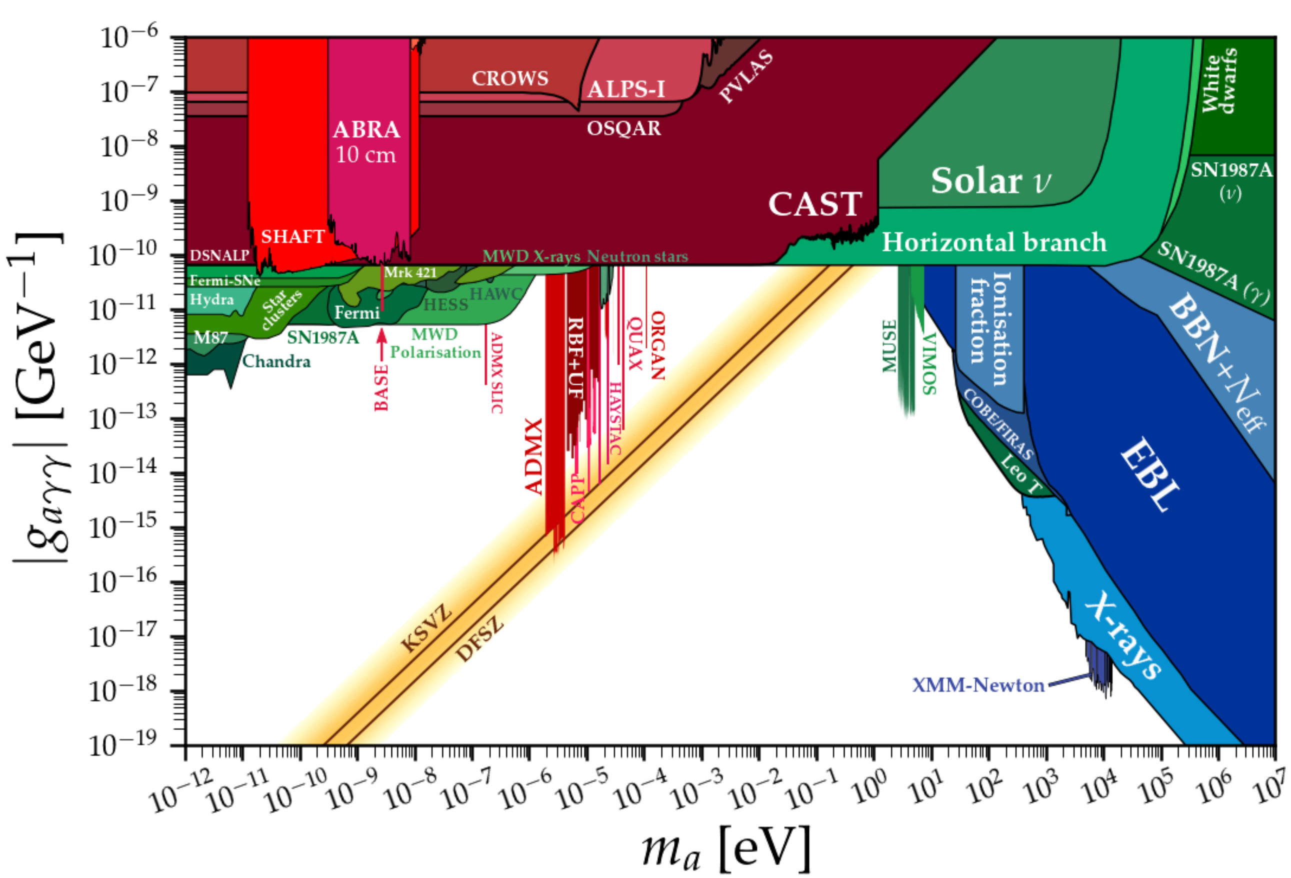
- Spin 0 pseudo-scalar
- Shift symmetry $a \rightarrow a + c$

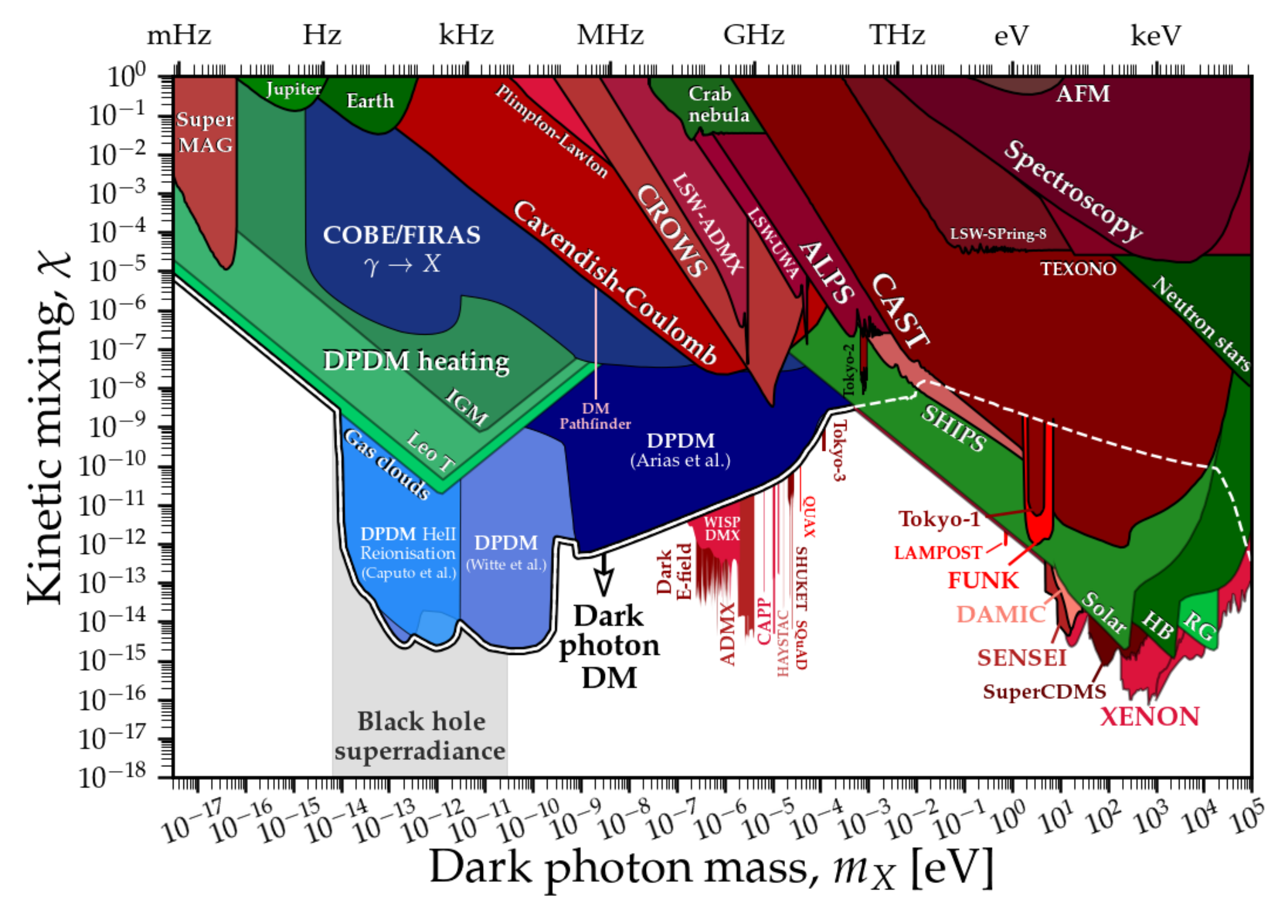
$$\mathscr{L} \supset -\frac{1}{4} g_{a\gamma\gamma} a \, (F_{\mu\nu})^{\rm SM} (\tilde{F}^{\mu\nu})^{\rm SM}$$

Dark photon, A

- Spin 1 vector $\quad S = \int dt d^3x \sqrt{-g} \left[-\frac{1}{4} g^{\mu\rho} g^{\nu\sigma} F_{\mu\nu} F_{\rho\sigma} + \frac{1}{2} m^2 g^{\mu\nu} A_\mu A_\nu \right]$
- New U(1) gauge symmetry

$$\mathscr{L} \supset -\frac{\chi}{2} (F_{\mu\nu})^{\rm SM} (F^{\mu\nu})^{\rm dark\ photon} + J_\mu (A^\mu)^{\rm SM}$$





https://cajohare.github.io/AxionLimits

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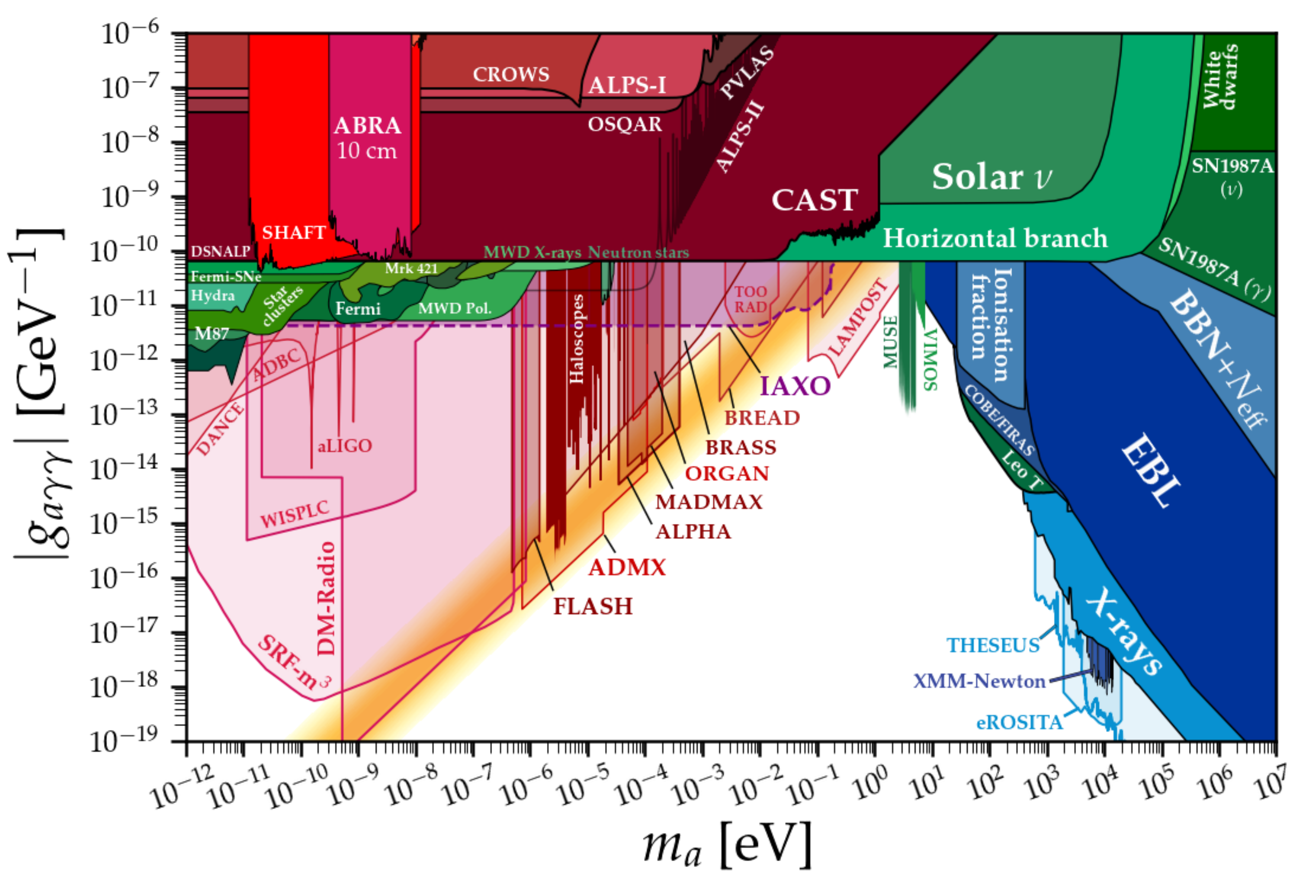

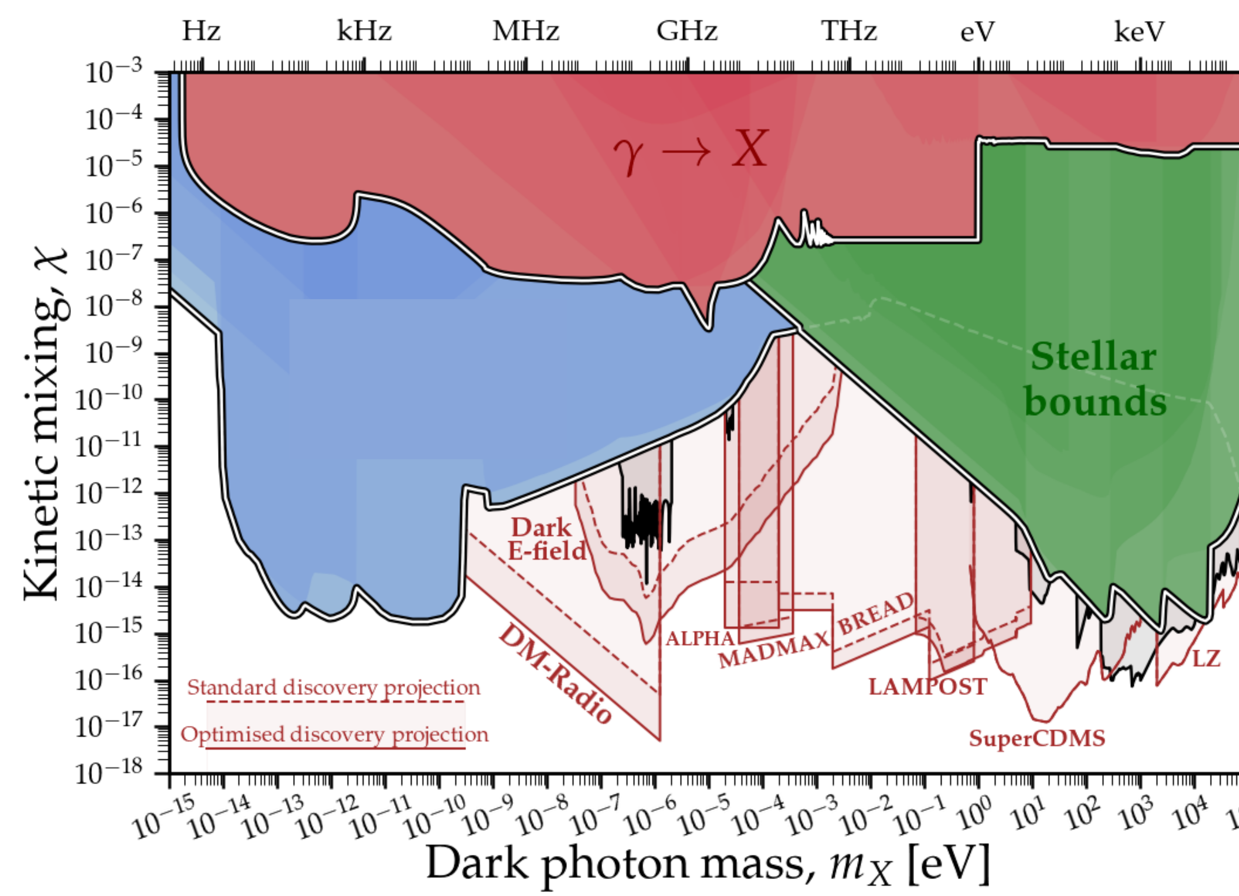

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Basic definitions

Dark matter over-density field

$$\delta(x) \equiv \frac{\rho(x) - \bar{\rho}}{\bar{\rho}}$$

Dark matter density

Density power spectrum

$$\langle \tilde{\delta}^*(\vec{k})\tilde{\delta}(\vec{k}')\rangle \equiv \frac{2\pi^2}{k^3}\delta^3(\vec{k}-\vec{k}')\mathcal{P}_\delta(|\vec{k}|)$$

Statistical homogeneity + isotropy

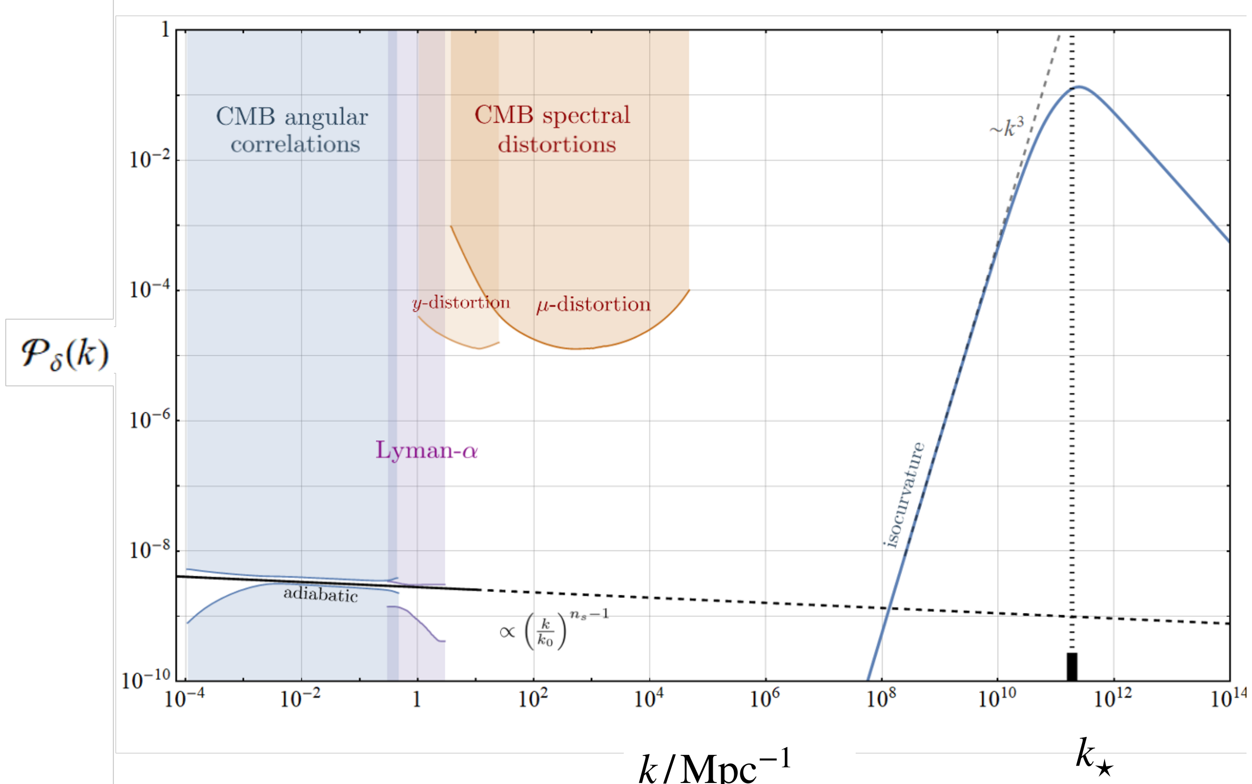
Production mechanisms

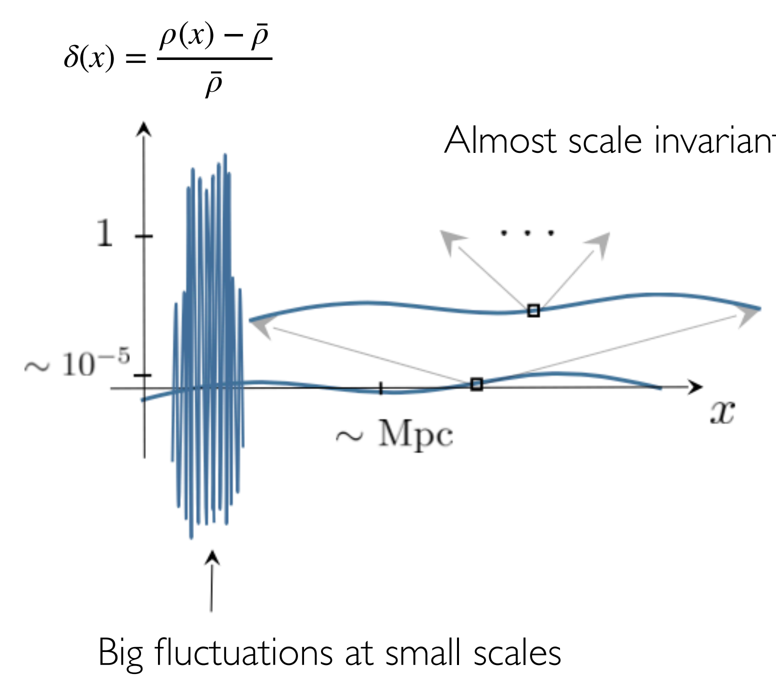
Dark photon

- Inflationary fluctuations
- From local strings
- Parametric resonance

Axion

- Post-inflationary ALP
- Axion coupled to dark photon





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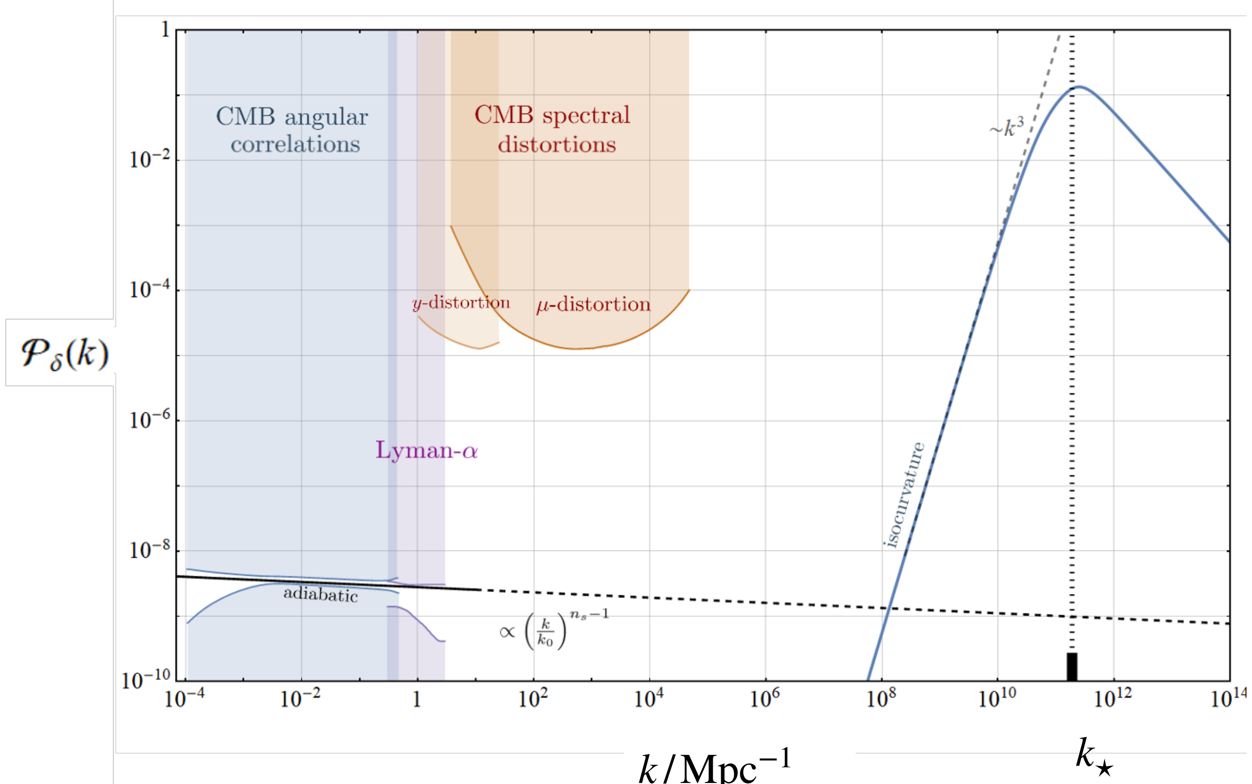

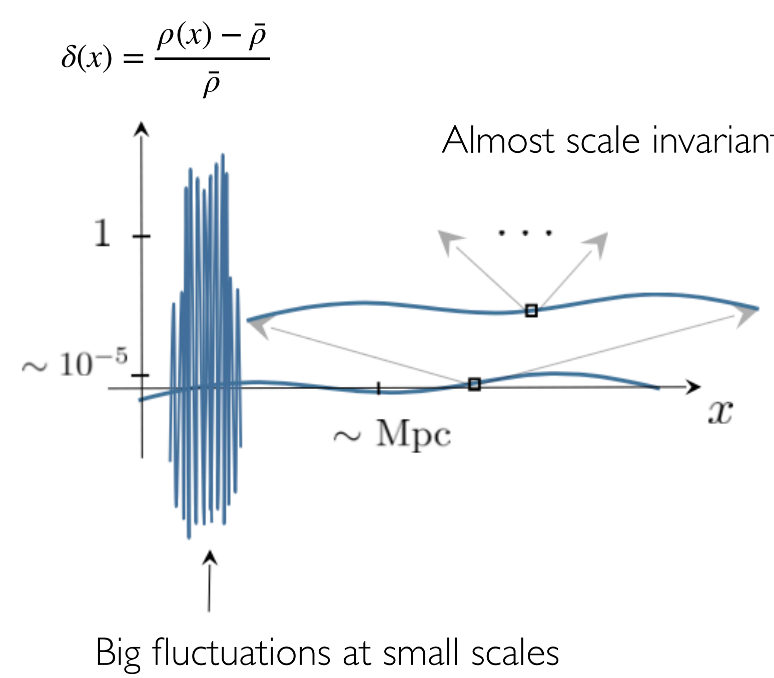

Dark photon dark matter from inflation

$\rightarrow$ **massive vector field** during inflation

$$S = \int dt d^3x \sqrt{-g} \left[-\frac{1}{4} g^{\mu\rho} g^{\nu\sigma} F_{\mu\nu} F_{\rho\sigma} + \frac{1}{2} m^2 g^{\mu\nu} A_\mu A_\nu \right]$$

$m \ll H_I$

$$ds^2 = -dt^2 + a^2(t) d\vec{x}^2$$

$$\rho(a_0) \simeq \frac{H_I^4}{(2\pi)^2} \left(\frac{k_\star / H_I}{a_\star} \right)^2 \left(\frac{a_\star}{a_0} \right)^3$$

$$= \rho_{\rm DM} \sqrt{\frac{m}{6 \cdot 10^{-6}\,\mathbf{eV}}} \left(\frac{H_I}{10^{14}\,\mathbf{GeV}} \right)^2$$

$H_I \lesssim 10^{13}\,\mathrm{GeV}$ (tensor perturbations)

$\rightarrow$ can comprise the DM for: $m \gtrsim 10^{-5}$ eV

[Graham, Mardon, and Rajendran]

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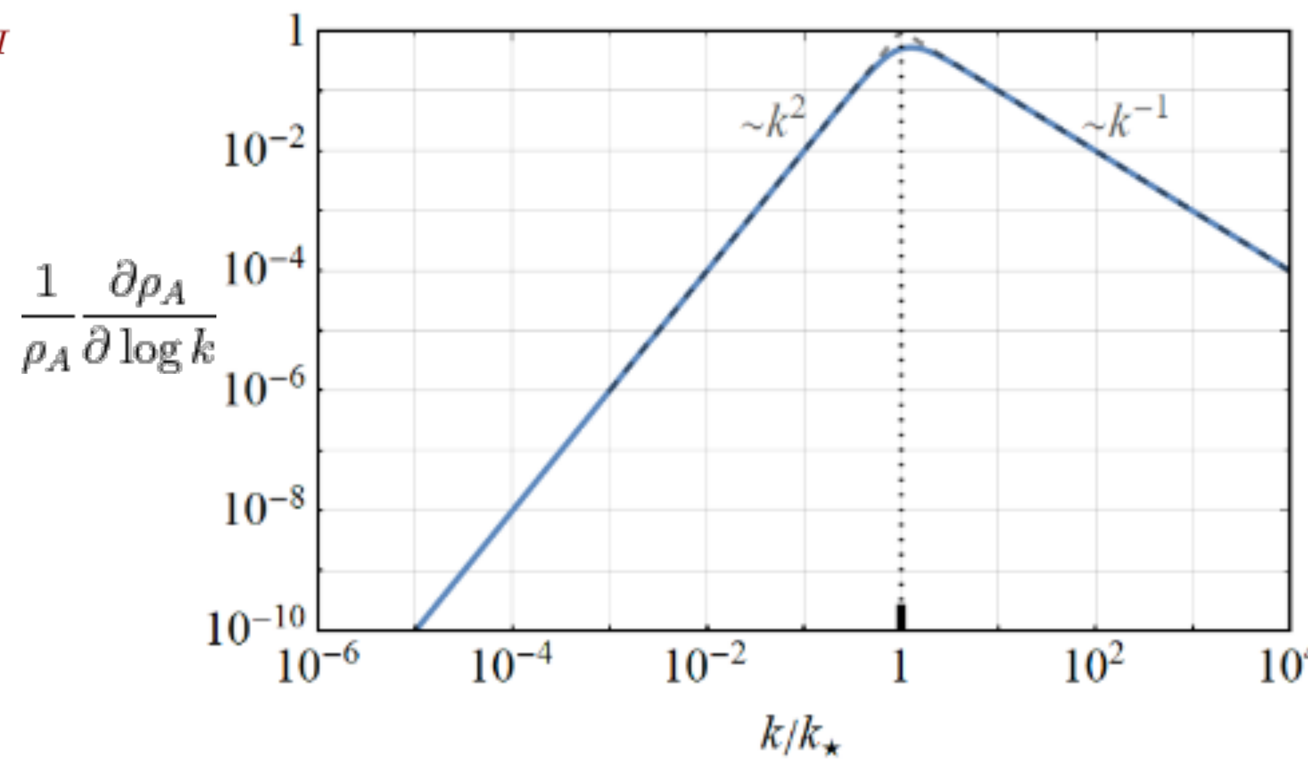


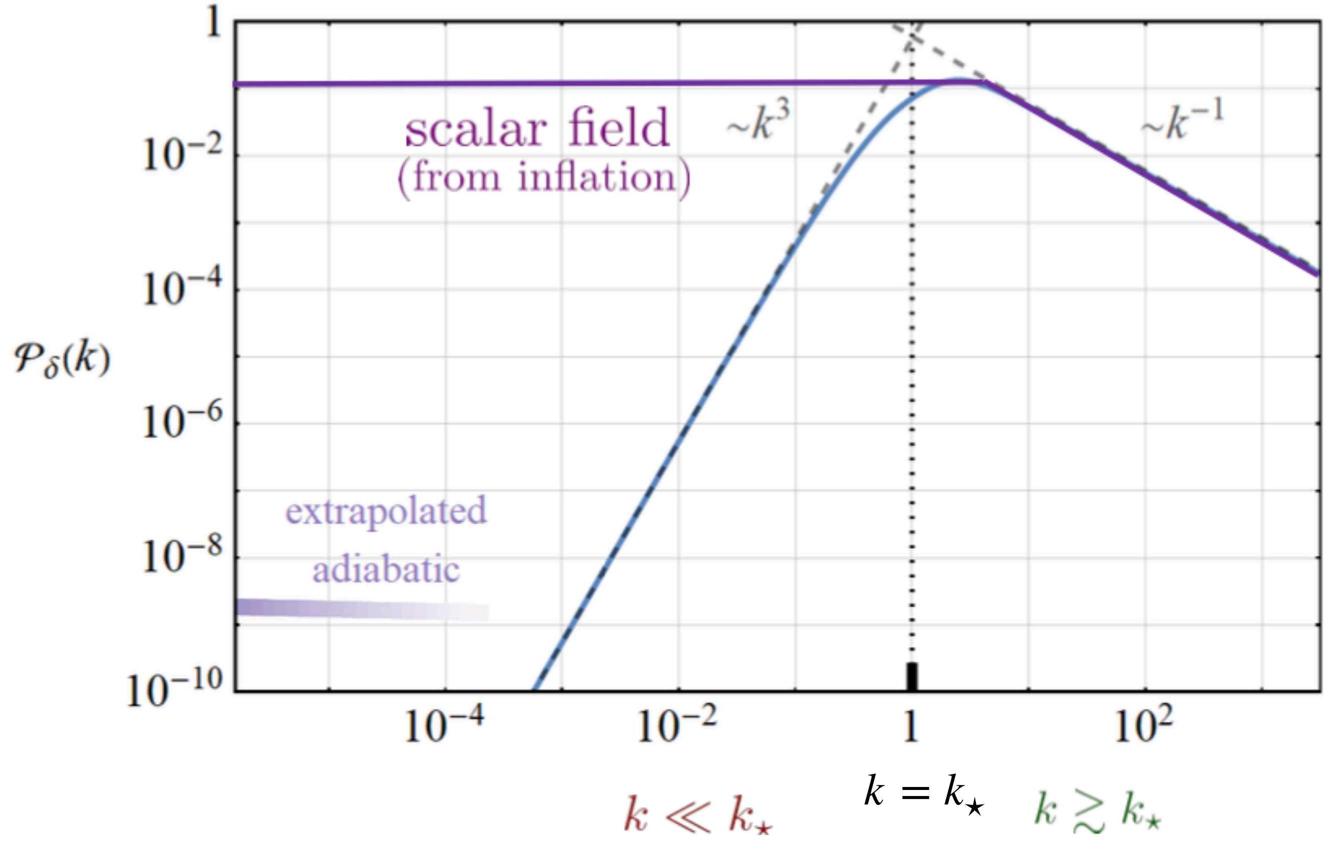
$k_\star/a_\star = H$ when $H = m$

[Graham, Mardon, and Rajendran]

Initial density power spectrum

$$\langle \rho \rangle = \int d\log k \, \frac{1}{2a^2}\left[\frac{a^2m^2}{k^2+a^2m^2}\mathcal{P}_{\partial_t A_L} + m^2\mathcal{P}_{A_L}\right]$$

$$\mathcal{P}_\delta(t,k) = \frac{k^2}{8\langle A_L^2\rangle^2}\int_0^\infty dq \int_{|q-k|}^{q+k} dp \frac{(k^2-q^2-p^2)^2}{q^4p^4}\mathcal{P}_{A_L}(t,p)\mathcal{P}_{A_L}(t,q)$$



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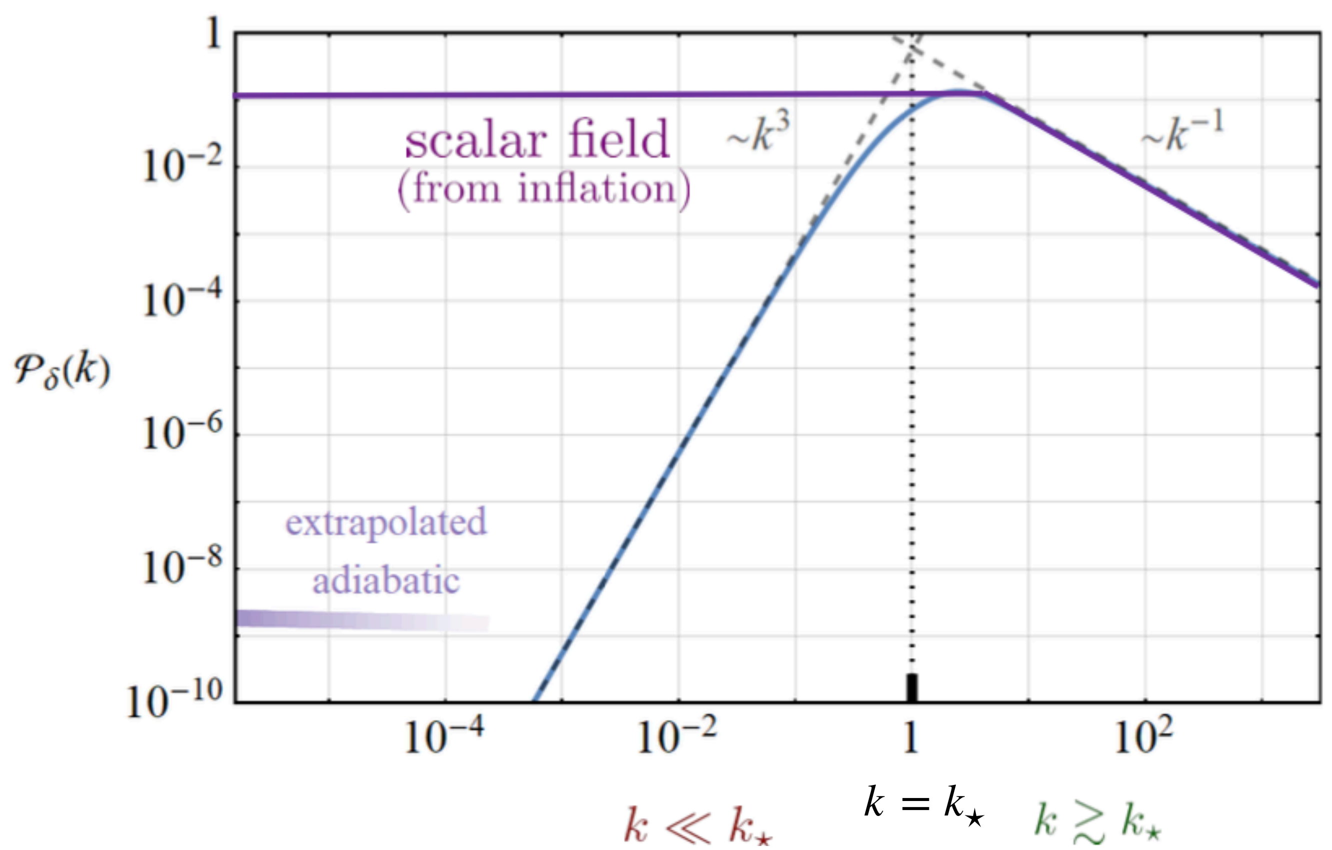
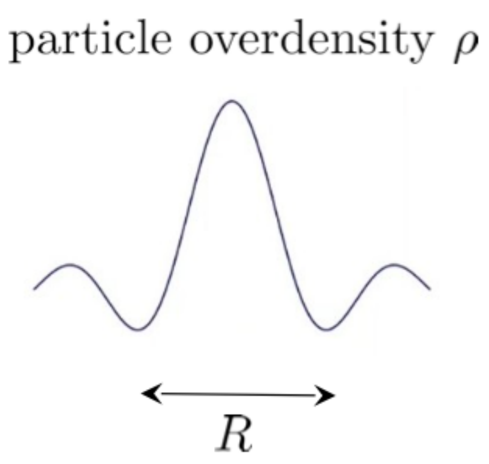


$$a_0\lambda_\star = \frac{2\pi a_0}{m a_\star} \simeq 10^{11}\text{km}\left(\frac{10^{-5}\text{eV}}{m}\right)^{1/2}$$

Gravitational collapse and wave effects

An order one over-density $\delta = \delta\rho/\bar{\rho}$ becomes nonlinear around matter radiation equality, at $a = a_{\rm eq}/\delta$



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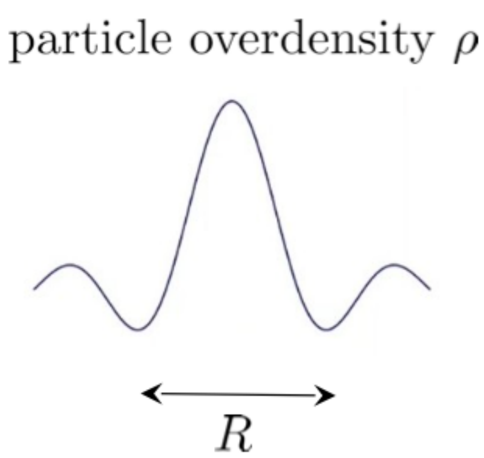

de Broglie wavelength of a particle in the resulting clump

$$\lambda_{dB} = \frac{1}{mv} = \frac{1}{m(GM/R)^{1/2}} = \frac{1}{R(4\pi G\rho m^2)^{1/2}}$$

$$R_{crit} \simeq \lambda_J \simeq (16\pi G\rho m^2)^{-1/4}$$

$$\frac{k_J}{a} = (16\pi G\rho m^2)^{1/4}$$

A coincidence

$$\left.\frac{k_J(\bar{\rho})}{k_\star}\right|_{a=a_{\rm eq}} = \frac{(16\pi G\,\bar{\rho}(a_{\rm eq})\,m^2)^{1/4}}{m(a_\star/a_{\rm eq})}$$

A coincidence

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$$= \frac{T_{\rm eq}}{T_\star} \simeq \left(\frac{G}{m^2}\,\bar\rho_{\rm M}(a_{\rm eq})\right)^{1/4}$$

$$H_\star^2 = m^2 \simeq G T_\star^4$$

$$2\bar\rho_{\rm M}(a_{\rm eq}) \simeq T_{\rm eq}^4$$

A coincidence

$$\left.\frac{k_J(\bar\rho)}{k_\star}\right|_{a=a_{\rm eq}} = \frac{(16\pi G\,\bar\rho(a_{\rm eq})\,m^2)^{1/4}}{m(a_\star/a_{\rm eq})} \simeq \left(\frac{\bar\rho(a_{\rm eq})}{\bar\rho_{\rm M}(a_{\rm eq})})\right)^{\frac{1}{4}} \simeq \left(\frac{\Omega_A}{\Omega_{\rm M}}\right)^{1/4} \quad (\ \approx 1.9\ )$$

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$$H_\star^2 = m^2 \simeq GT_\star^4$$

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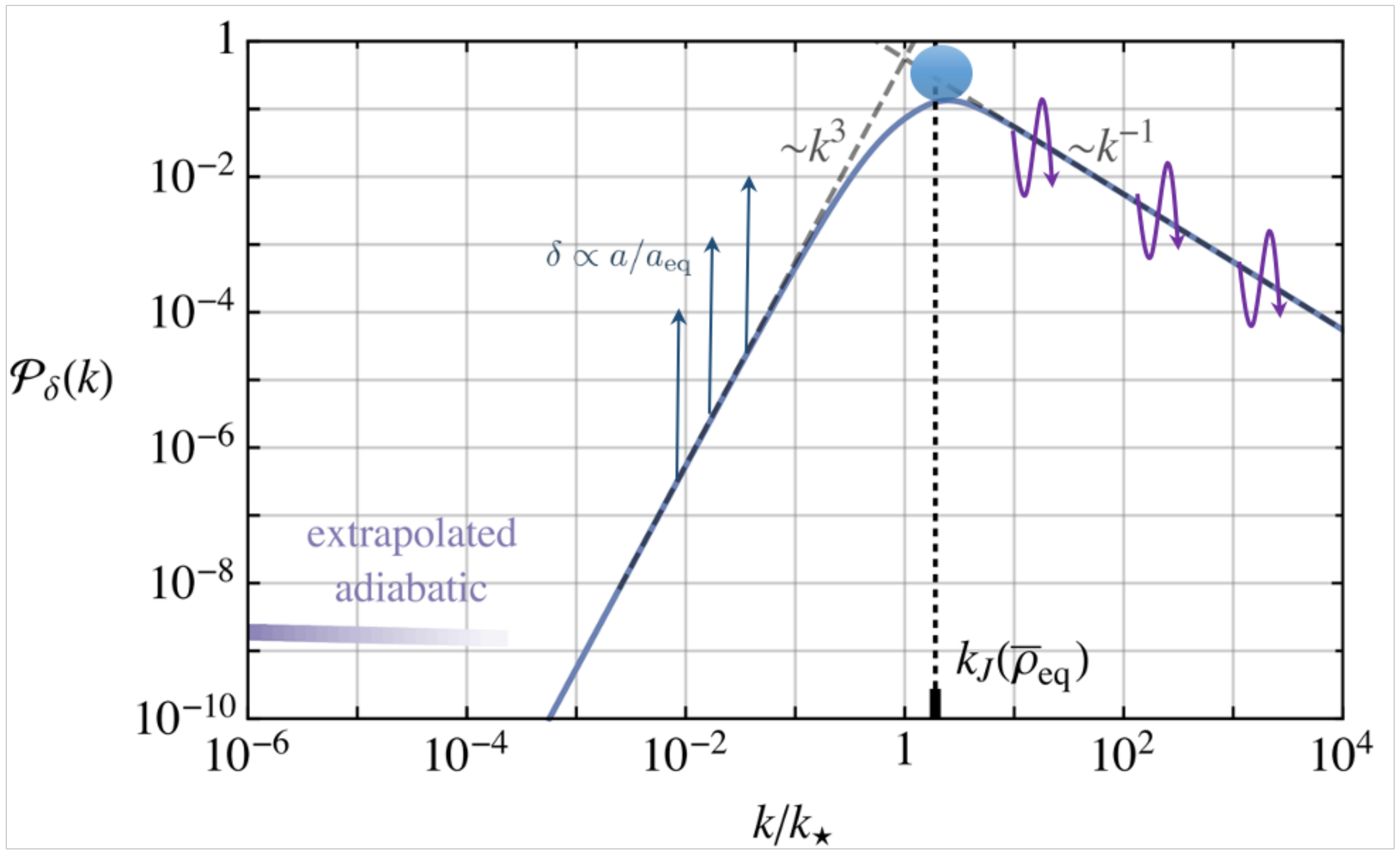
Quantum pressure cannot be neglected and affects the evolution of the order one overdensities at $a = a_{\rm eq}$

Evolution of different modes



at $a = a_{\rm eq}$

$k \ll k_J$

quantum pressure negligible

- perturbative ($\delta < 1$)
- grow, initially linearly $\delta \propto a/a_{\rm eq}$
- collapse (when nonperturbative, $\delta \simeq 1$)

$k \simeq k_J$

at the edge of being affected by quantum pressure at MRE

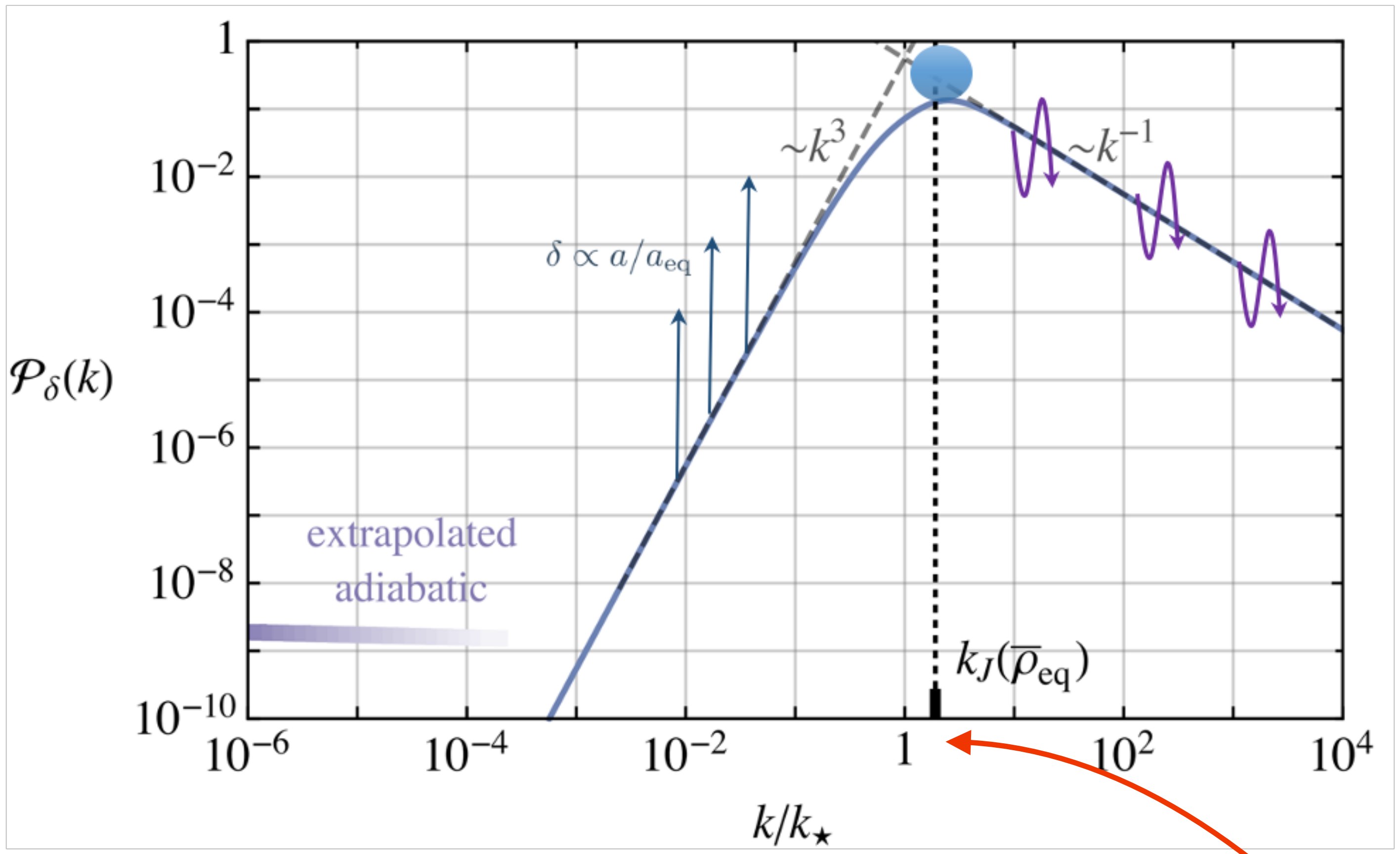
- already nonperturbative ($\delta = O(1)$)
- collapse at around MRE

$k \gg k_J$

quantum pressure dominant

- prevented from collapsing and oscillate (whether perturbative or not)

Evolution of different modes



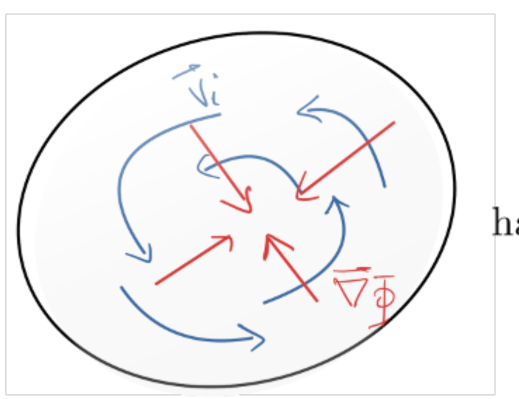
$$M_J(a_{\rm eq}) = 1.6 \cdot 10^{-15} M_\odot \left(\frac{10^{-5}\ {\rm eV}}{m}\right)^{\frac{3}{2}}$$

Halos vs Solitons

Halos

$\Phi_Q = 0$

$\rightarrow$ gravitational potential balanced by the velocity terms



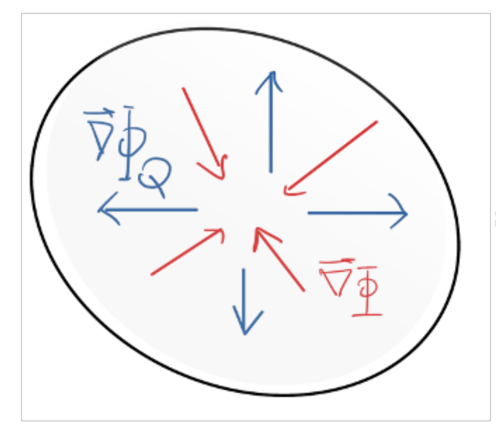
halo

angular momentum 'supports' the gravitational potential

Solitons

$\Phi_Q = -\Phi \quad \longleftrightarrow \quad \vec{v}_i = 0$

$\rightarrow$ gravitational potential balanced by the quantum pressure



soliton

quantum pressure 'supports' the gravitational potential

Simulations

Non-relativistic limit $A_i \equiv \frac{1}{\sqrt{2m^2a^3}}(\psi_i e^{-imt} + \text{c.c.})$

$$
\begin{cases}
\text{Schroedinger:} & \left(i\partial_t + \frac{\nabla^2}{2m} - m\Phi\right)\psi_i = 0 \, , \\
\text{Poisson:} & \nabla^2\Phi = \frac{4\pi G}{a}\sum_i \left(|\psi_i|^2 - \langle|\psi_i|^2\rangle\right)
\end{cases}
$$

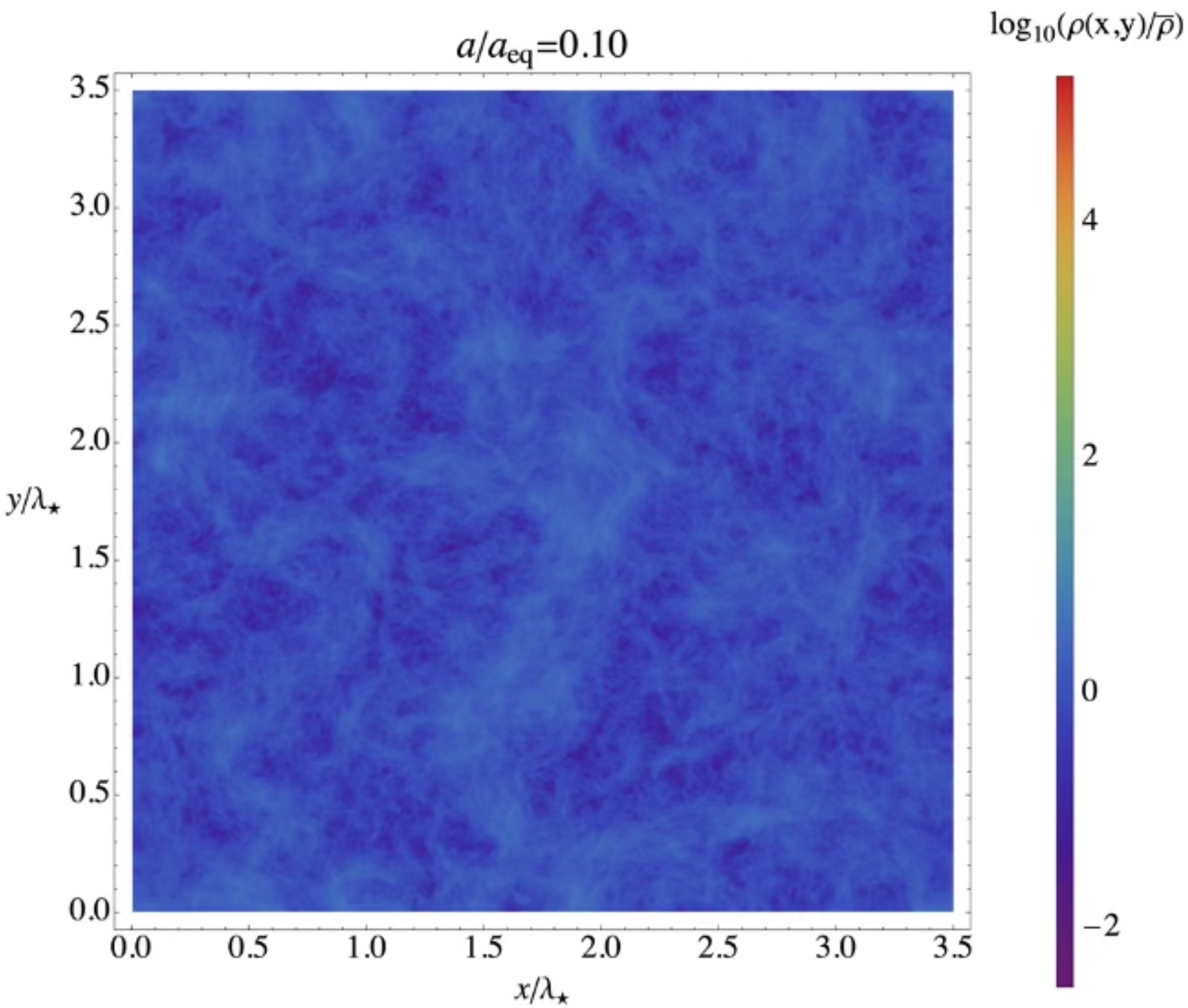
Solve the Schrodinger Poisson equations on a discrete lattice

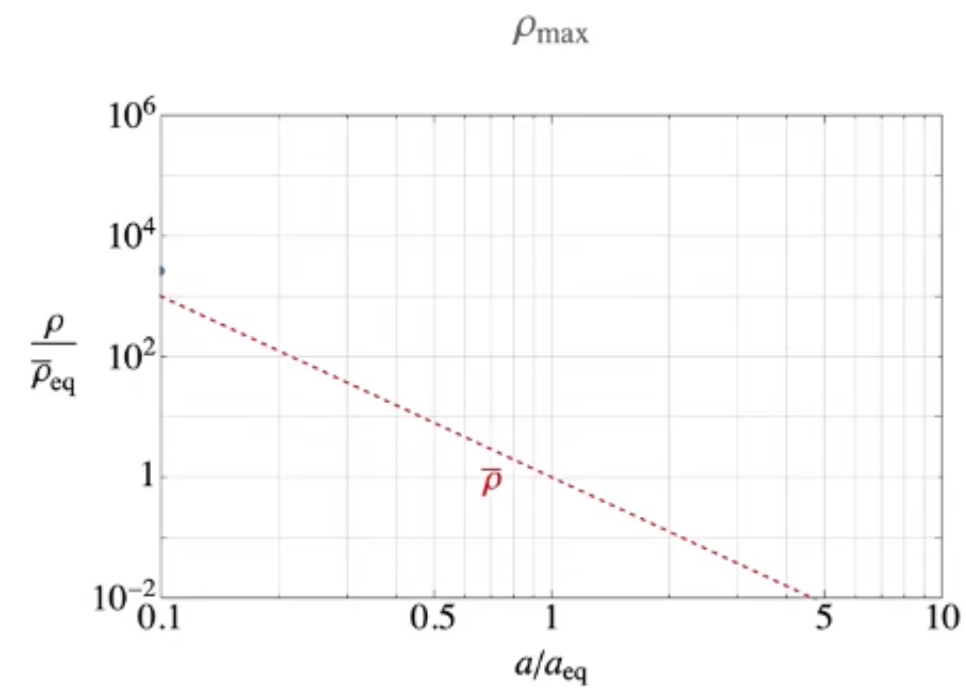
Start with a realisation of the initial conditions at $a \ll a_{\rm eq}$ and evolve through matter-radiation equality

Fluctuations collapse into bound dark matter clumps and can easily be identified

Comoving distance between lattice points is constant; end the simulation once the soliton cores are no longer resolved

Simulations





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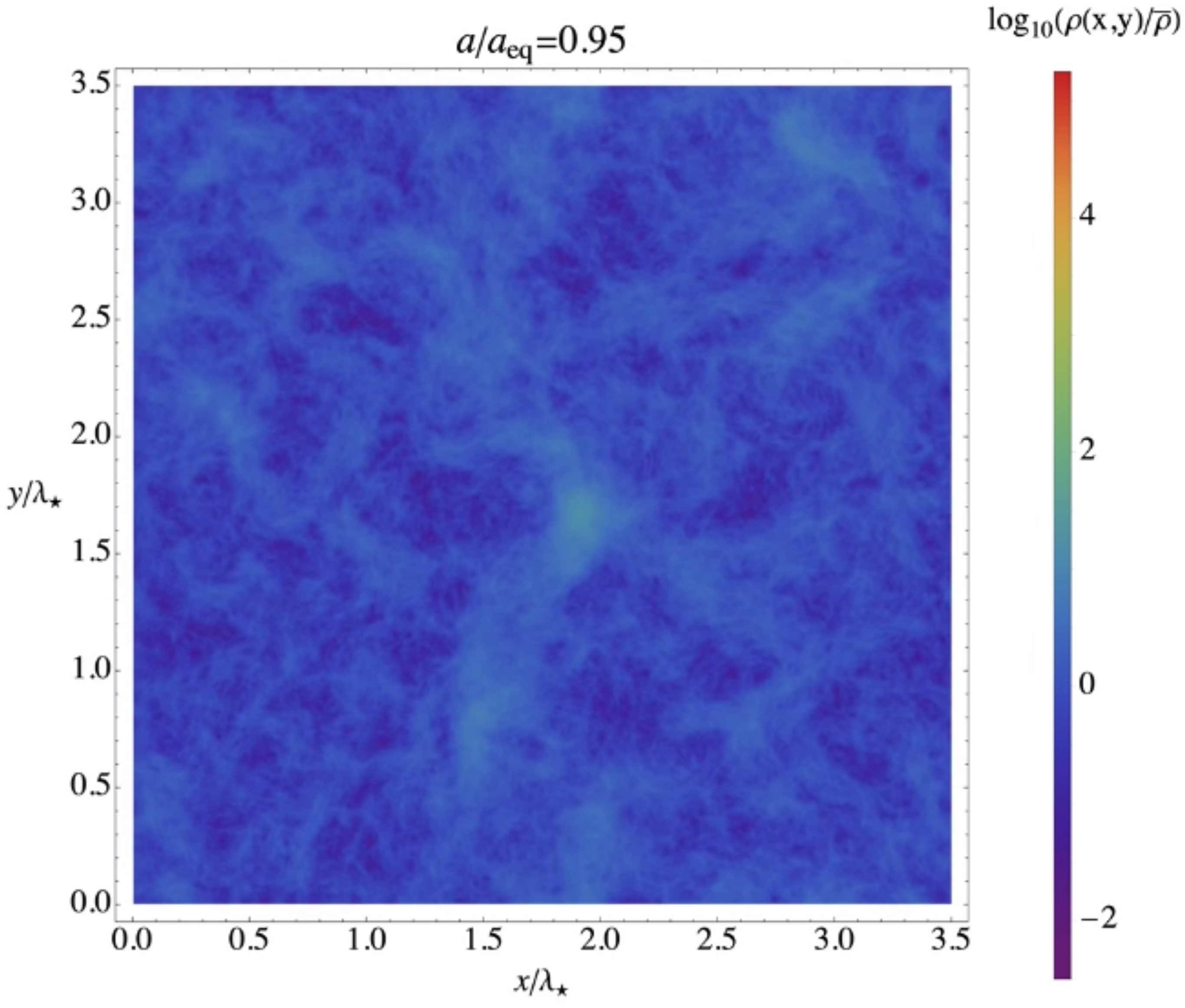

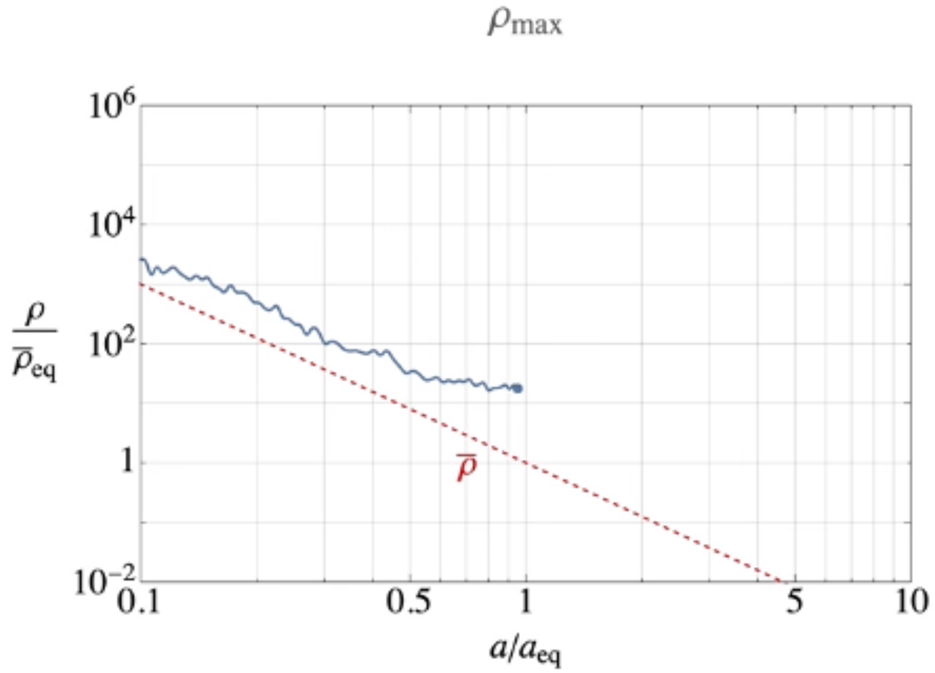

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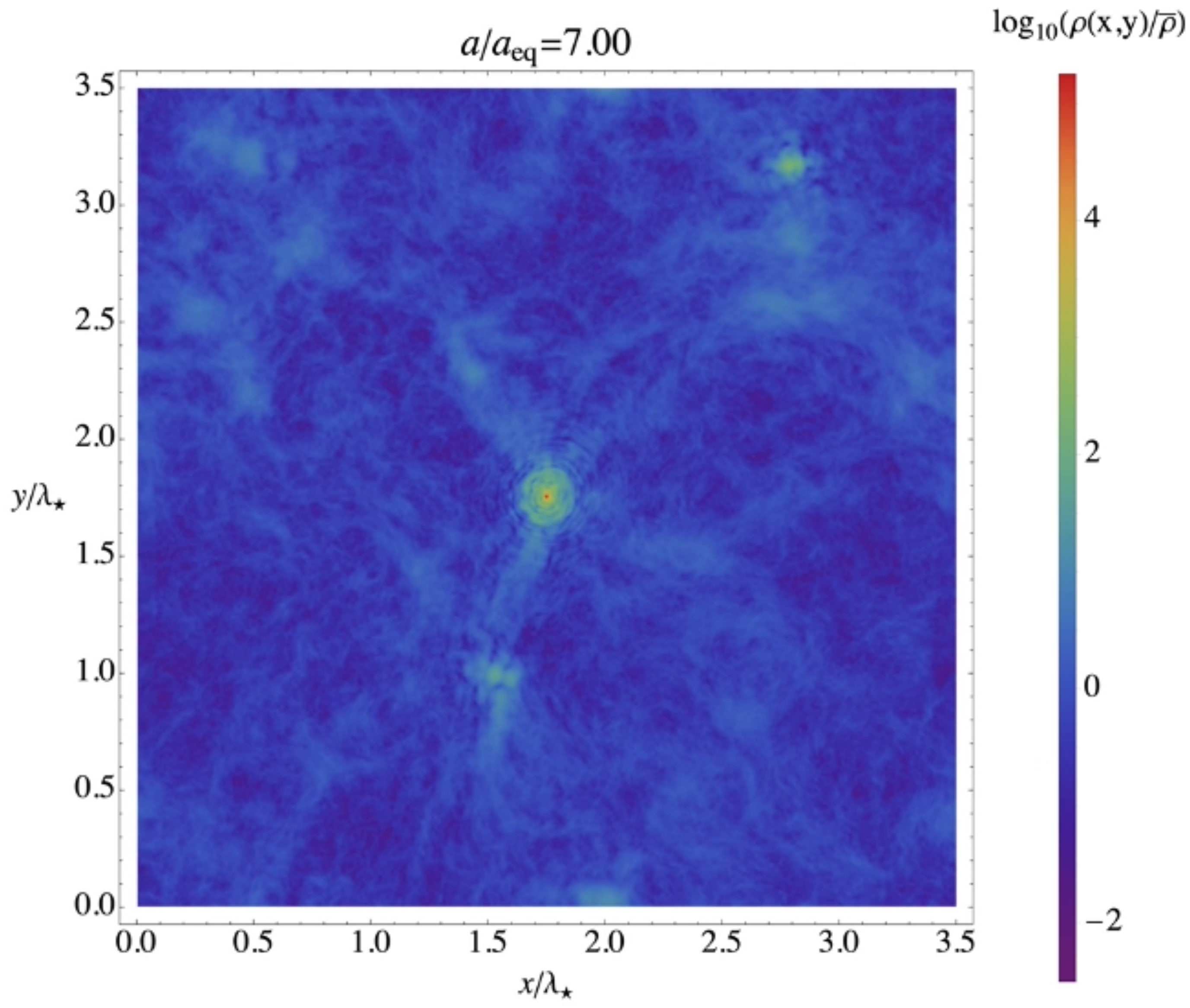
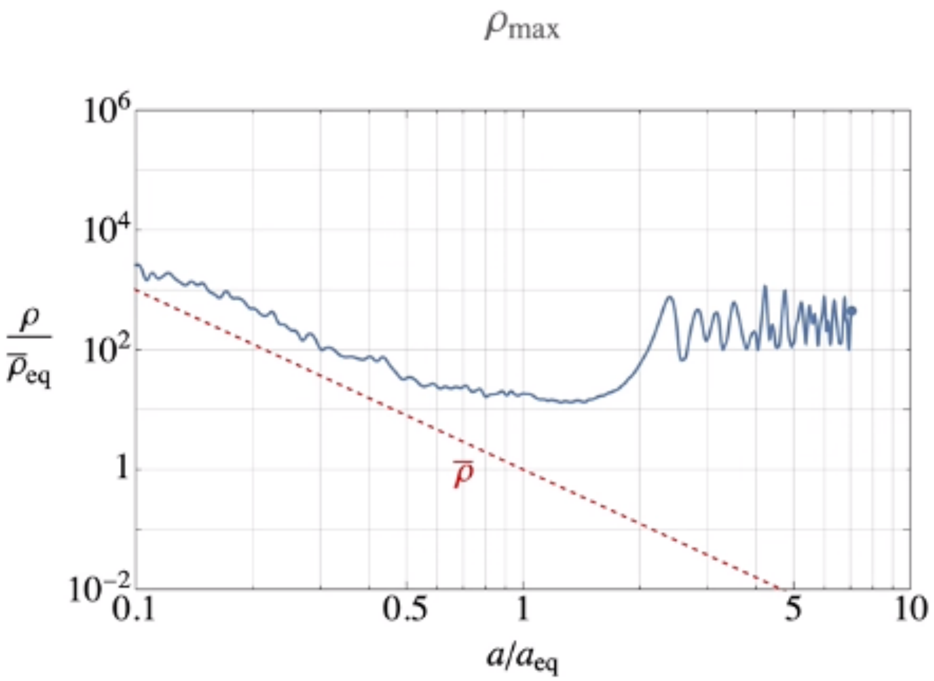
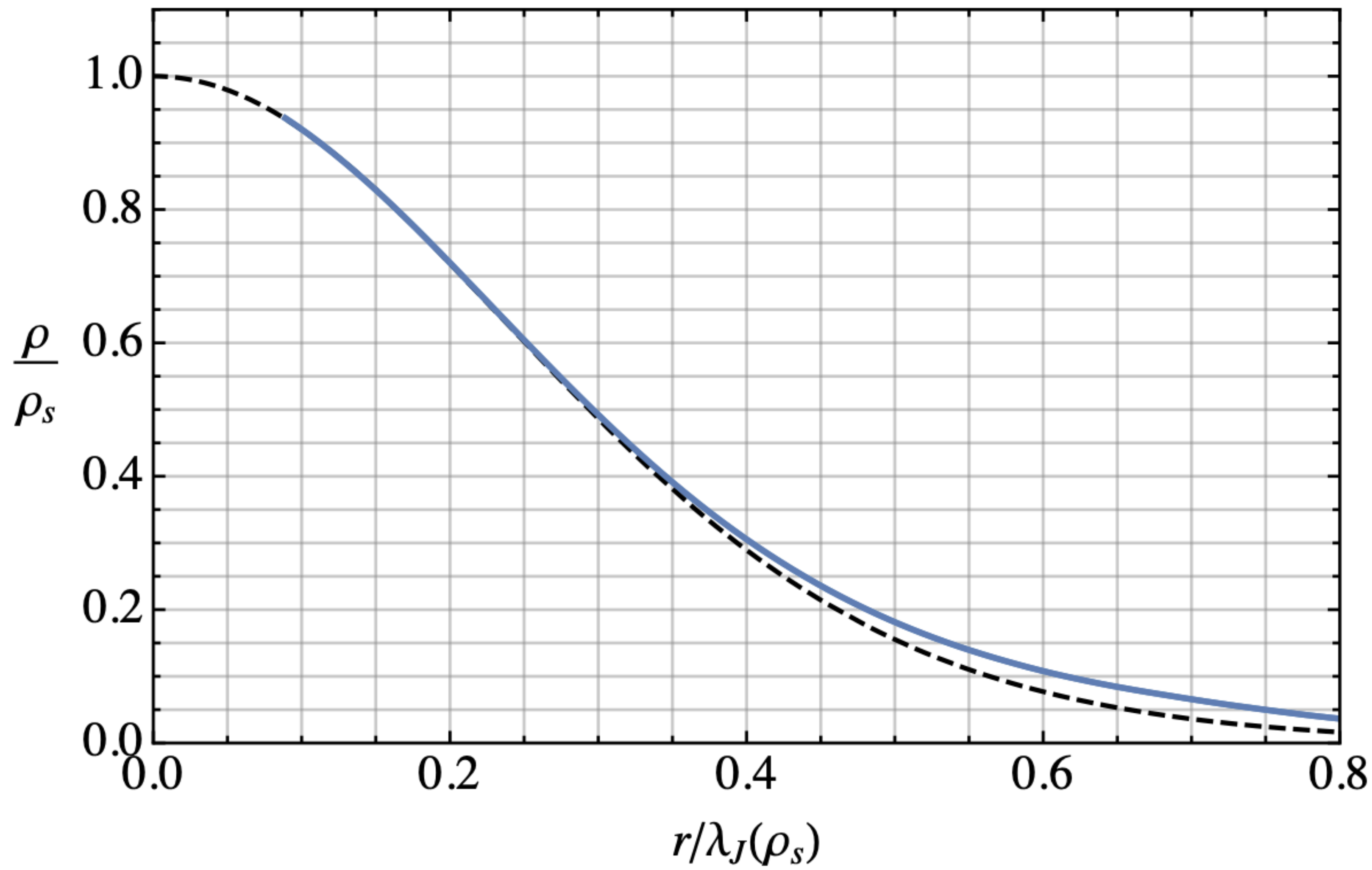




Identified objects

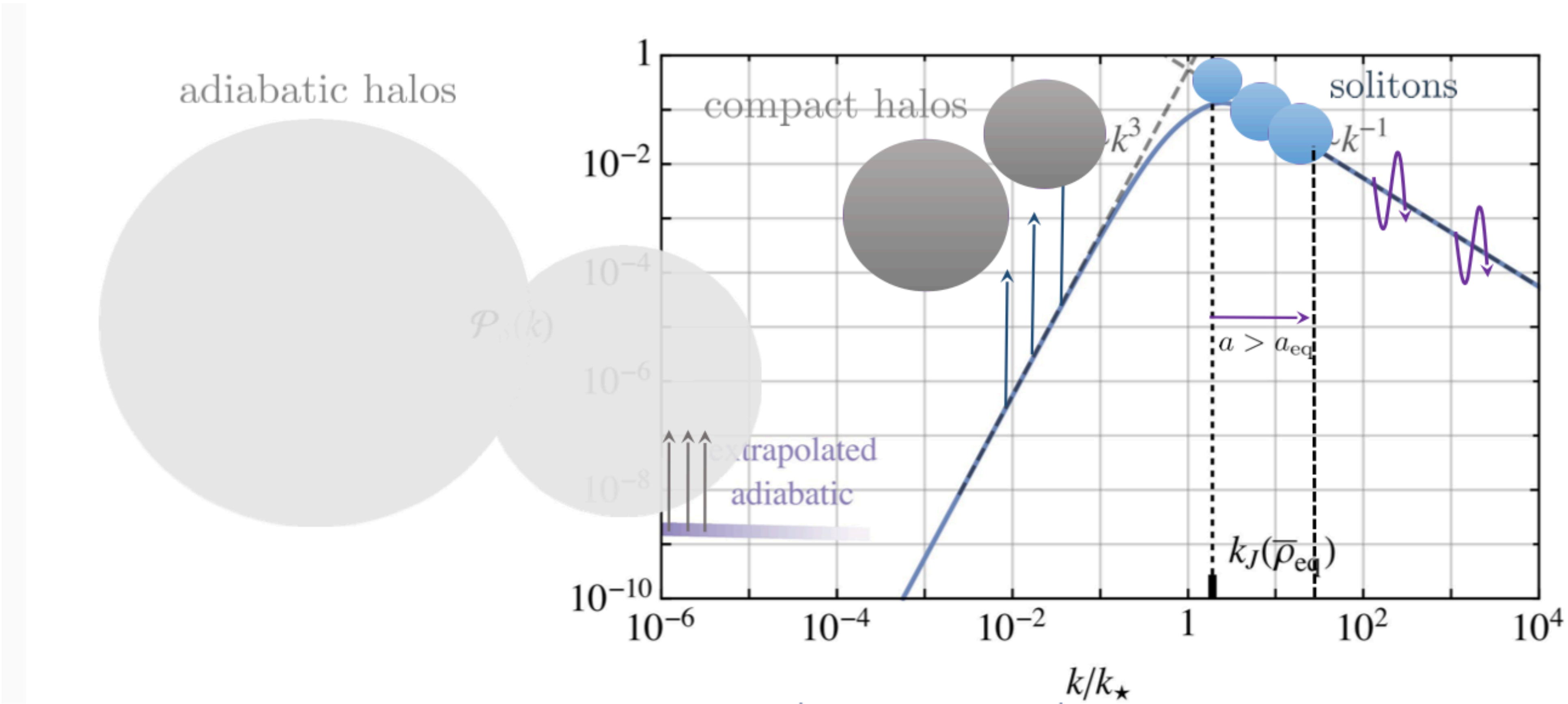
Spherical average over ~100 objects



The solitons contain about 10% of the dark matter

Typical central density $\rho_s \simeq (10 \div 10^3)\bar{\rho}_{\rm eq} \simeq (1 \div 10^3){\rm eV}^4$

Halos vs Solitons



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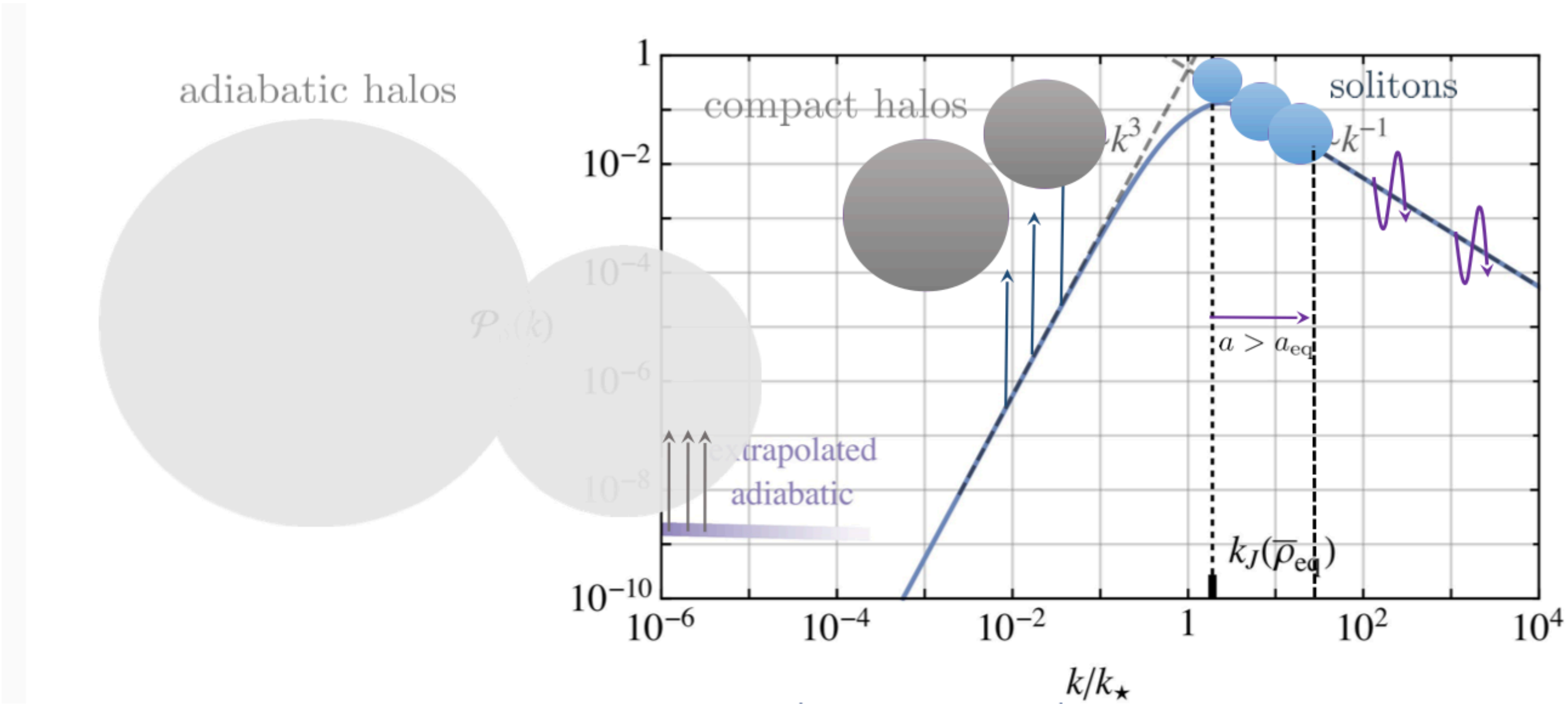
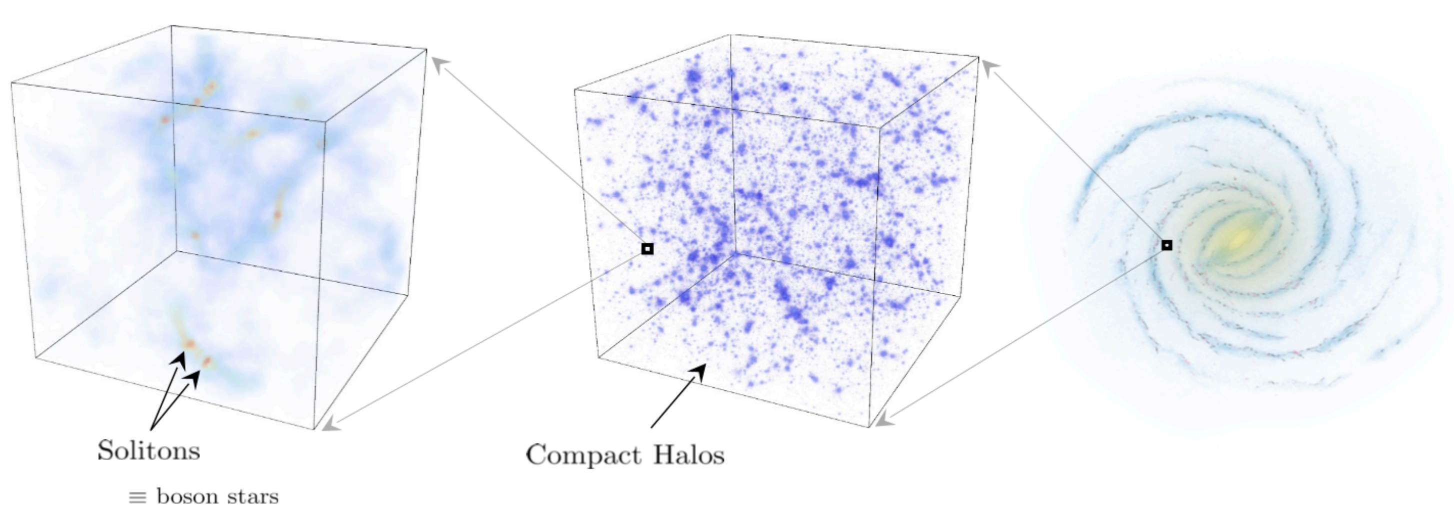
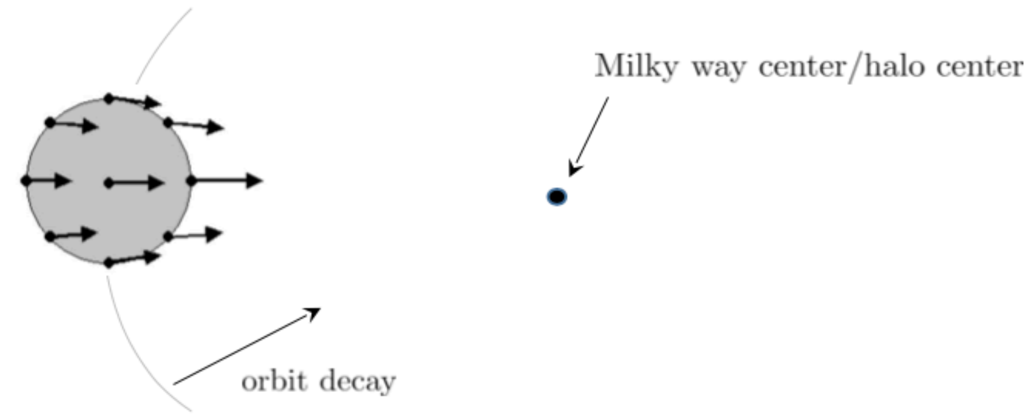


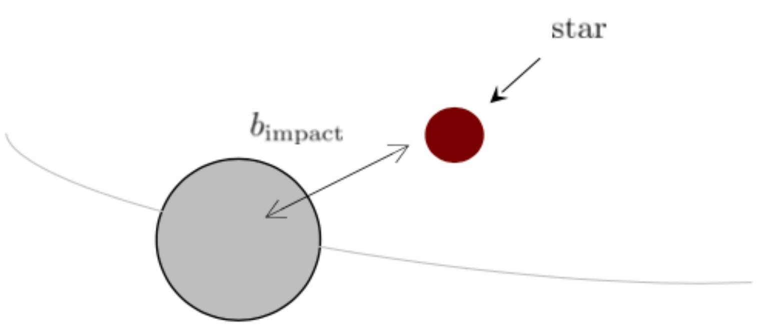


Survival of the substructure

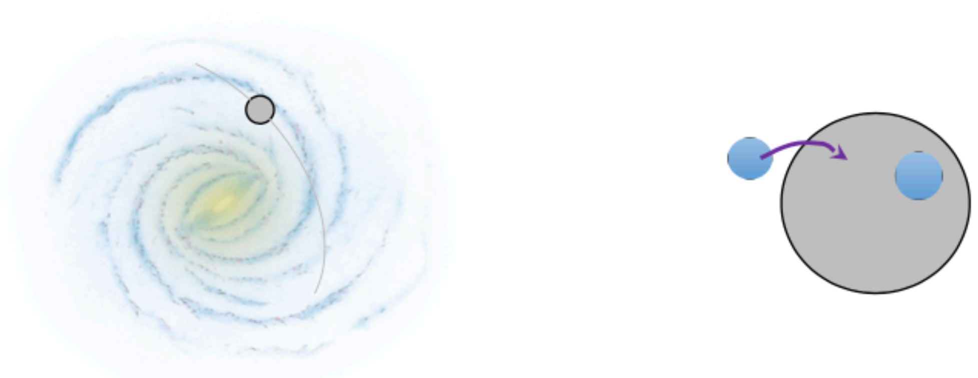
- tidal forces/disruption by a central potential
- dynamical friction $\implies$ orbit decay



- collisions with stars



- tidal shocks by the galactic disk
- tidal shocks during formation of halos/merging?



Encounter rate with the Earth

Dark photon star number density

$$n = f_s \bar{\rho}(t_0)/M \quad \simeq 10^{20} \mathrm{pc}^{-3} \left(\frac{m}{\mathrm{eV}}\right)^{3/2}$$

Encounter rate with the Earth

$$\Gamma \simeq n \pi R^2 v_{\mathrm{rel}}$$

$$\simeq \frac{0.1}{\mathrm{yr}} \left(\frac{m}{\mathrm{eV}}\right)^{1/2}$$

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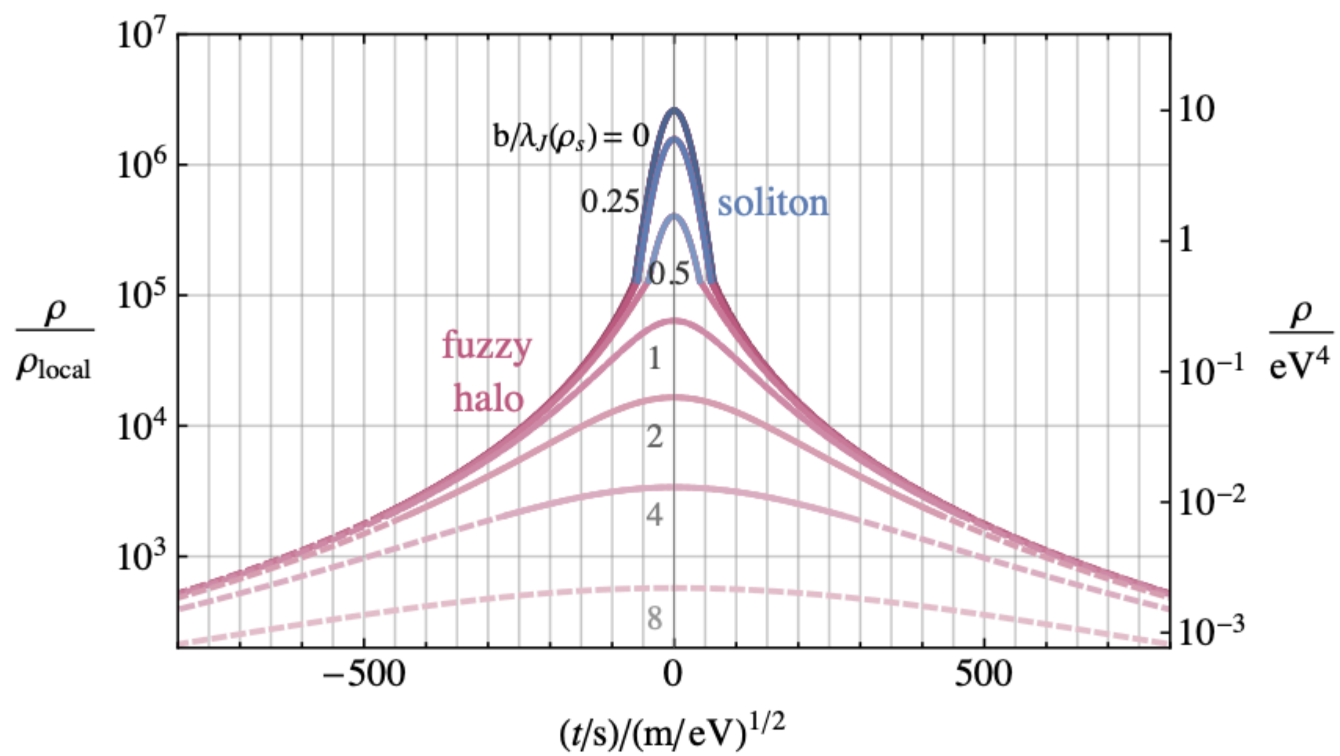
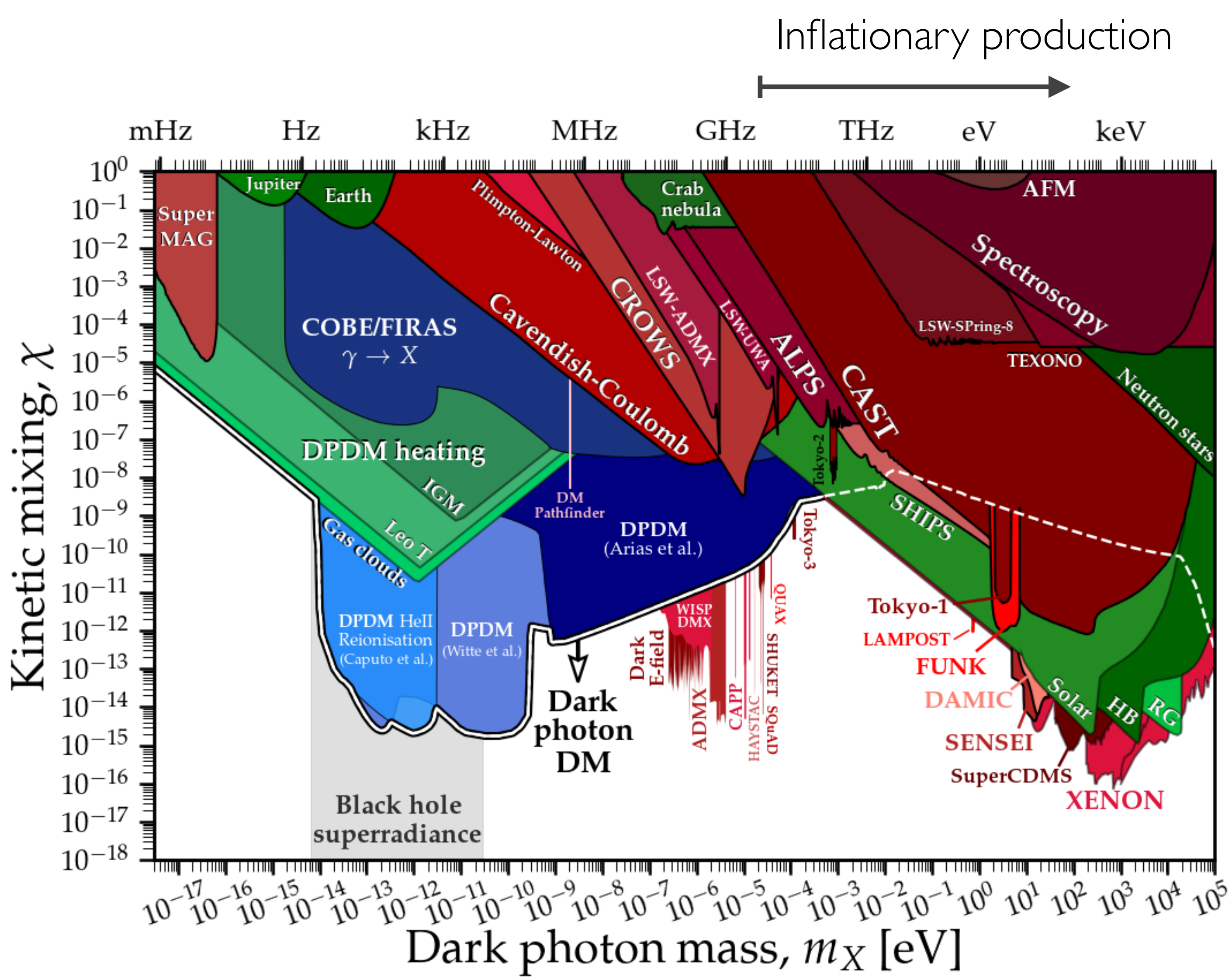


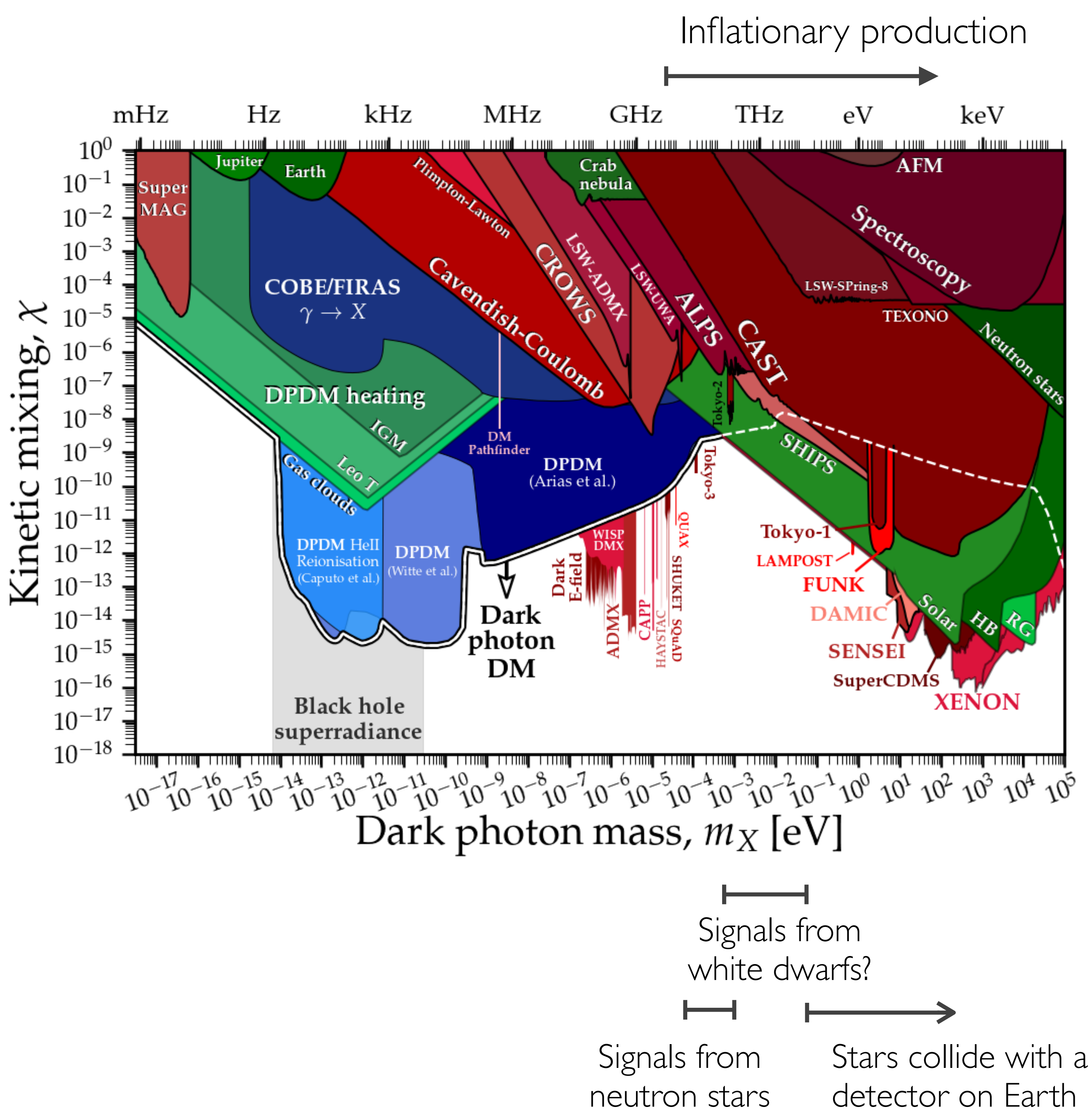
$$t_{\mathrm{collision}} \simeq 10^2 \ \mathrm{s} \left(\frac{0.1 M_J^{\mathrm{eq}}}{M}\right) \left(\frac{\mathrm{eV}}{m}\right)^{1/2}$$

Signals



Inflationary production
mHz
Hz
kHz
MHz
GHz
THz
eV
keV
Kinetic mixing, χ
Dark photon mass, m_X [eV]
Super MAG
Jupiter
Earth
Plimpton-Lawton
CROWS
LSW-ADMX
LSW-UWA
ALPS
CAST
Crab nebula
AFM
Spectroscopy
LSW-SPring-8
TEXONO
Neutron stars
COBE/FIRAS
$\gamma \to X$
Cavendish-Coulomb
DPDM heating
IGM
Leo T
Gas clouds
DM Pathfinder
DPDM (Arias et al.)
Tokyo-2
Tokyo-3
SHIPS
DPDM HeII Reionisation (Caputo et al.)
DPDM (Witte et al.)
Dark photon DM
Dark E-field
WISP DMX
ADMX
CAPP
HAYSTAC
QUAX
SHUKET
SQuAD
Tokyo-1
LAMPOST
FUNK
DAMIC
SENSEI
SuperCDMS
XENON
Solar
HB
RG
Black hole superradiance

Signals



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TEXONO
Neutron stars
Signals from white dwarfs?
Signals from neutron stars
Stars collide with a detector on Earth

Other production mechanisms

Dark photon

- Inflationary fluctuations
- From local strings
- Parametric resonance

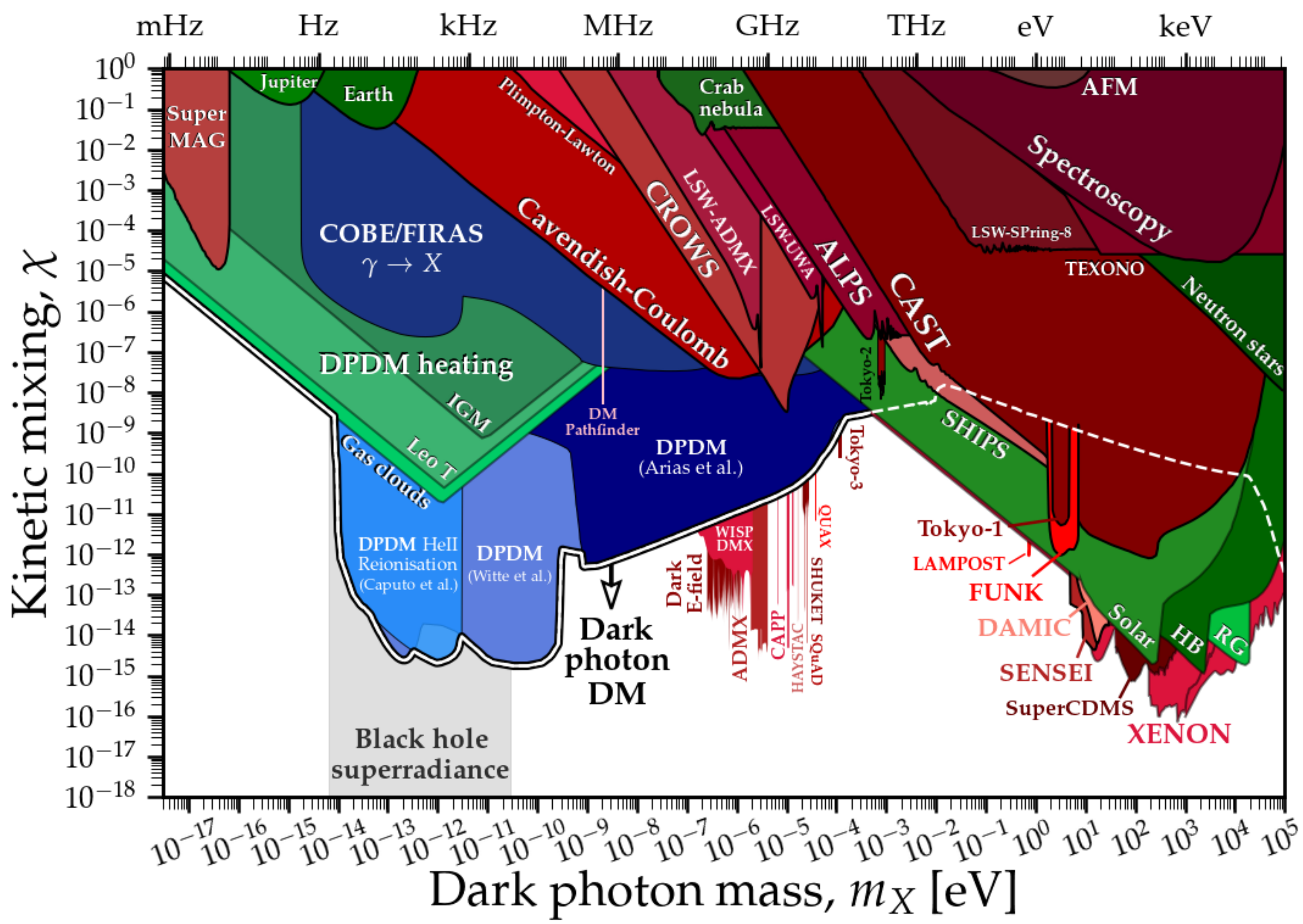
Axion

- Post-inflationary ALP
- Axion coupled to dark photon

The time when $H = m$ plays a key role in all of these

Other production mechanisms

Inflationary production



Production from a string network

Production from an axion

$$M_J(a_{\rm eq}) = 5.2 \cdot 10^{-23} M_\odot \left(\frac{\rm eV}{m}\right)^{3/2}$$

Conclusions

- New light particles are good dark matter candidates and can have interesting substructure
- A dark vector produced from inflationary fluctuations is a minimal, easily calculable theory
- A coincidence means that solitonic `dark photon stars' automatically form at matter radiation equality
- These contain about 5 to 10% of the dark matter abundance and are likely to survive to the present day

Future:

- Observational signals
- Similar dynamics for other dark photon production mechanisms, also for axions/ axion-like-particles

Thanks

More rigorously

- more rigorously from the equations of motion of the vector field: $D_\mu F^{\mu\nu} = m^2 A^\nu$

$$A_i \equiv \frac{1}{\sqrt{2m^2a^3}}(\psi_i e^{-imt} + \text{c.c.})$$

non-relativistic limit: $\dot{\psi}_i \ll m\psi_i$ and $\ddot{\psi}_i \ll m^2\psi_i$

Schroedinger: $\left(i\partial_t + \frac{\nabla^2}{2m} - m\Phi\right)\psi_i = 0\ ,$ $\quad \leftarrow D_\mu F^{\mu\nu} = m^2 A^\nu$

Poisson: $\nabla^2\Phi = \frac{4\pi G}{a}\sum_i \left(|\psi_i|^2 - \langle|\psi_i|^2\rangle\right)$ $\quad \leftarrow G^{00} = 8\pi G T^{00}$ Einstein eq.

Nonlinear, A_L and A_T are now coupled together

More rigorously

$$\left(i\partial_t + \frac{\nabla^2}{2m} - m\Phi\right)\psi_i = 0\ ,$$
$$\nabla^2\Phi = \frac{4\pi G}{a}\sum_i \left(|\psi_i|^2 - \langle|\psi_i|^2\rangle\right)$$

Magdelung transformation:

$$\psi_i = \sqrt{\rho_i}e^{i\theta_i}$$
$$\vec{v}_i = \frac{1}{m}\nabla\theta_i$$

$\longleftrightarrow$

Continuity: $\partial_t\rho_i + 3H\rho_i + a^{-1}\nabla\cdot(\rho_i\vec{v}_i) = 0$

Euler: $\partial_t\vec{v}_i + H\vec{v}_i + a^{-1}(\vec{v}_i\cdot\nabla)\vec{v}_i = -\,a^{-1}(\nabla\Phi + \nabla\Phi_{Qi})$

$$\nabla^2\Phi = 4\pi G a^2(\rho - \bar{\rho})$$

(3-component) perfect fluid
with 'quantum pressure' term:

$$\Phi_{Qi} \equiv -\frac{\hbar^2}{2a^2m^2}\frac{\nabla^2\sqrt{\rho_i}}{\sqrt{\rho_i}}$$

More rigorously

$$\left(i\partial_t + \frac{\nabla^2}{2m} - m\Phi\right)\psi_i = 0\ ,$$
$$\nabla^2\Phi = \frac{4\pi G}{a}\sum_i\left(|\psi_i|^2 - \langle|\psi_i|^2\rangle\right)$$

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(3-component) perfect fluid with 'quantum pressure' term:

$$\Phi_{Qi} \equiv -\frac{\hbar^2}{2a^2m^2}\frac{\nabla^2\sqrt{\rho_i}}{\sqrt{\rho_i}}$$

- quantum pressure negligible if:

$$\nabla\Phi \gg \nabla\Phi_Q$$

$\nabla\cdot$

$$4\pi G a^2\rho \gg \tfrac{1}{2a^2m^2}\nabla^2\tfrac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}} \simeq \tfrac{1}{2a^2m^2}\tfrac{k^4}{2}$$

$\downarrow$

$$\left(\ k_J^{\rm phys} =\ \right)\qquad \frac{k_J}{a} = (16\pi G\rho m^2)^{1/4}$$

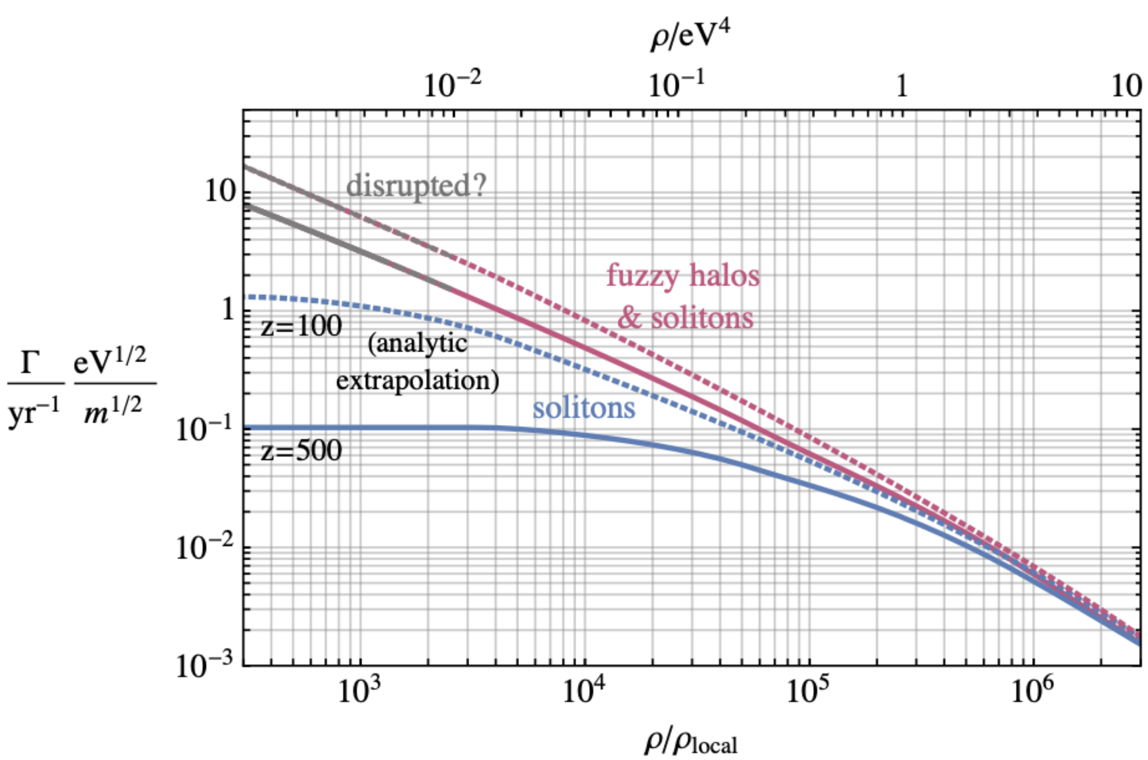
$k \ll k_J$ overdensities are dominated by Φ $\rightarrow$ grow and collapse

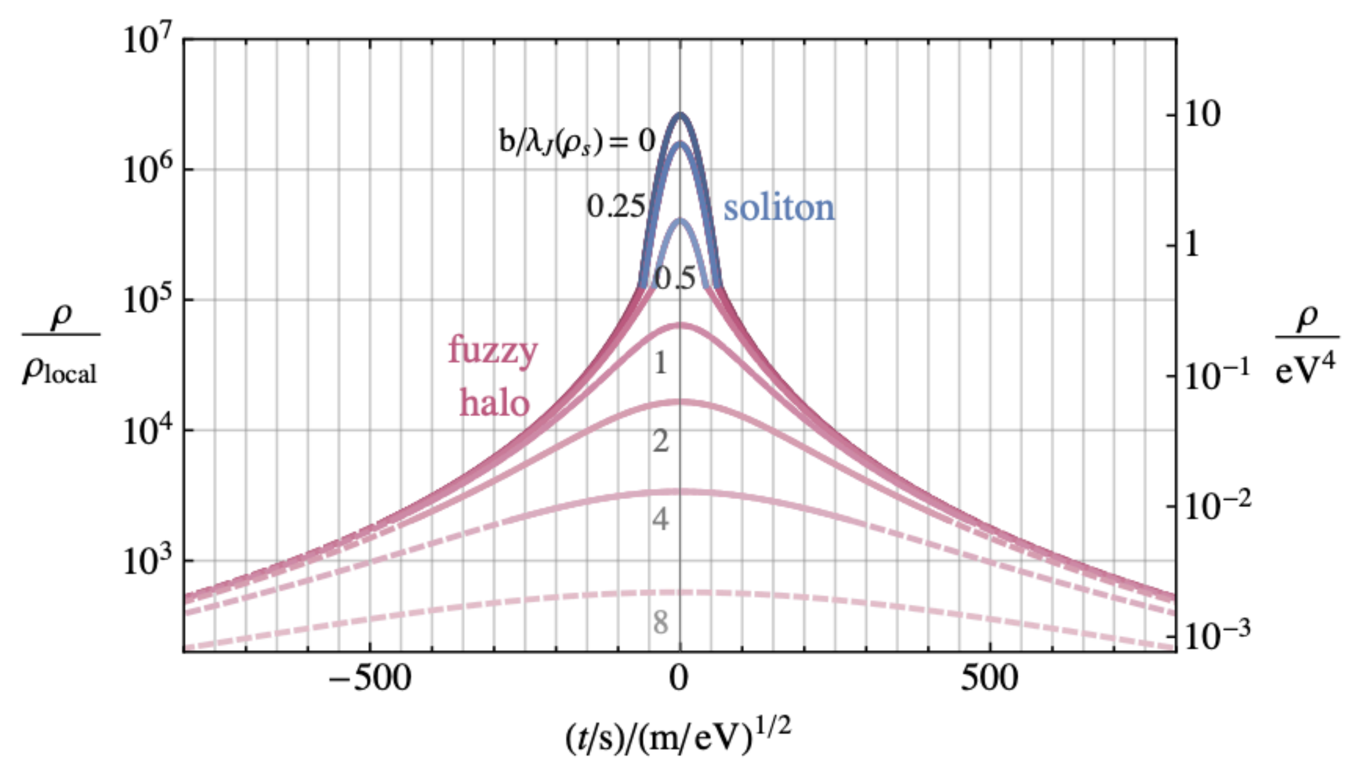
$k \gg k_J$ overdensities are dominated by Φ_Q $\rightarrow$ prevented from collapsing and oscillate

Encounter rate with the Earth

Dark photon star number density $$n = f_s \bar{\rho}(t_0)/M \simeq 10^{20}\mathrm{pc}^{-3}\left(\frac{f_s}{0.05}\right)\left(\frac{\rho_{\rm local}}{0.5\,\mathrm{GeV/cm^3}}\right)\left(\frac{0.1M_J^{\rm eq}}{M}\right)\left(\frac{m}{\mathrm{eV}}\right)^{3/2}$$

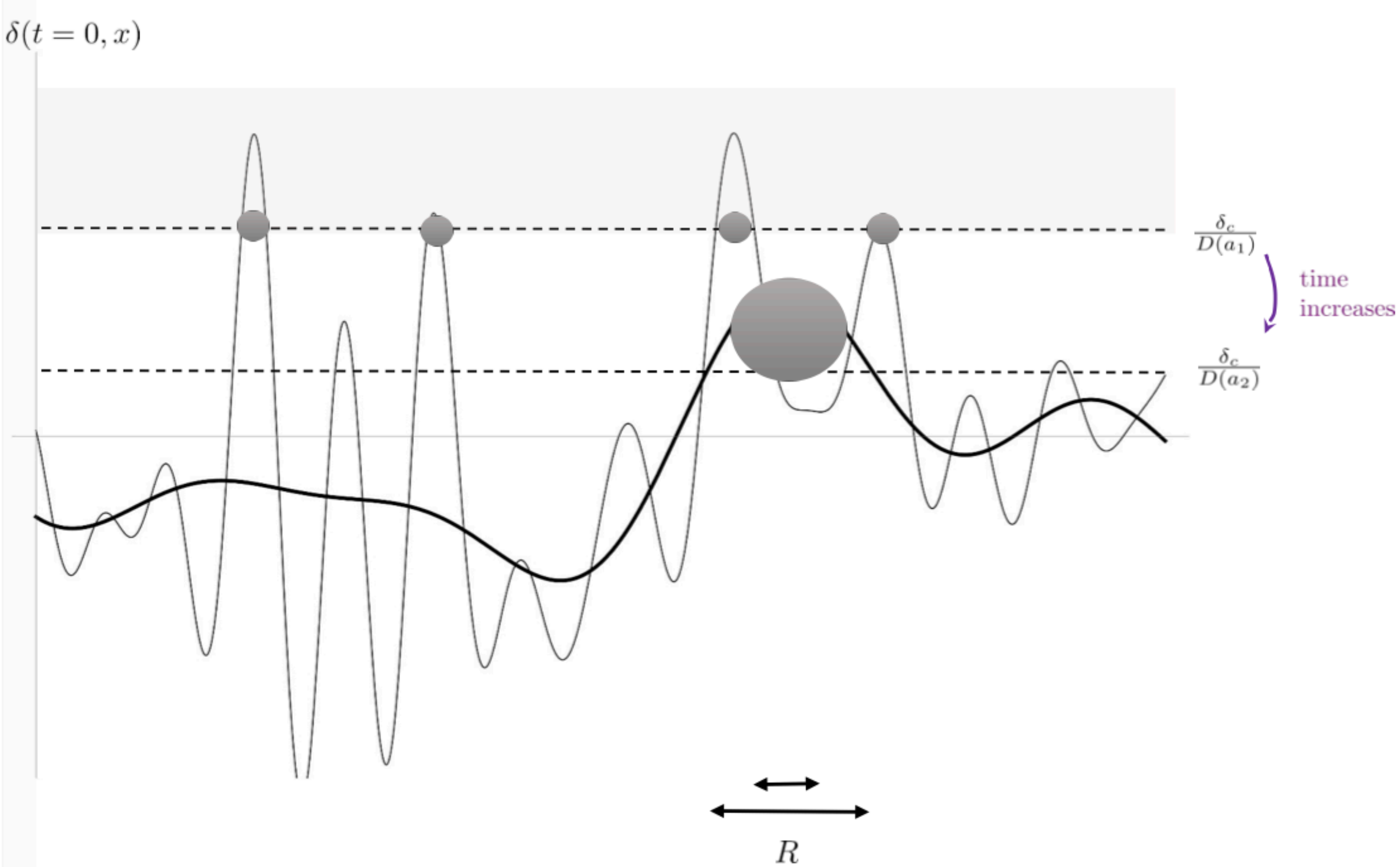
Encounter rate with the Earth $$\Gamma \simeq n\pi R^2 v_{\rm rel}$$
$$\simeq \frac{0.1}{\mathrm{yr}}\left(\frac{m}{\mathrm{eV}}\right)^{1/2}\left(\frac{0.1M_J^{\rm eq}}{M}\right)^3\left(\frac{v_{\rm rel}}{10^{-3}}\right)\left(\frac{f_s}{0.05}\right)\left(\frac{\rho_{\rm local}}{0.5\,\mathrm{GeV/cm^3}}\right)$$





$$t_{\rm collision} \simeq 10^2\ \mathrm{s}\left(\frac{0.1M_J^{\rm eq}}{M}\right)\left(\frac{\mathrm{eV}}{m}\right)^{1/2}$$

Press Schechter



- while perturbative, overdensities grow linearly:

$$\delta \propto 1 + \frac{3}{2}\frac{a}{a_{\rm eq}} \equiv D(a)$$

- when $\delta(x) \simeq \delta_c = O(1)$ the overdensity collapses into a halo

- the mass of this halo is determined by 'smoothing' the field over distances $R(M) = (\frac{3M}{4\pi\rho})^{\frac{1}{3}}$

$\rightarrow$ the regions of the smoothed field in which $D(a)\delta(t=0,x) > \delta_c$ (i.e. $\delta(t=0,x) > \delta_c/D(a)$) are expected to have collapsed into a halo of mass $> M$

$\rightarrow$ heavier and heavier halos as time increases

Vector solitons

$$\psi_i = \frac{m\alpha^2}{\sqrt{4\pi G}} e^{-i\alpha^2 \gamma m t} \chi_1(\alpha m r)\, u_i$$

- quantum pressure is relevant for this solution:

$\rightarrow$ density at the center $\equiv$ $\rho_s = \frac{\alpha^4 m^2}{4\pi G} \simeq \frac{1}{G m^2 R^4} \simeq \frac{G^3 m^6 M^4}{64\pi}$

$$\lambda_J(\rho_s) = 2\pi/(16\pi G \rho_s m^2)^{1/4} \simeq 2.3R$$

Jeans scale comparable to the soliton radius

$\rightarrow$ fully 'supported' by quantum pressure

$$\psi_i = \sqrt{\rho_i} e^{i\theta_i}$$

$$\vec{v}_i = \tfrac{1}{m}\nabla\theta_i = 0$$

Euler $\quad 0 \leftarrow \partial_t \vec{v}_i + H\vec{v}_i + a^{-1}(\vec{v}_i \cdot \nabla)\vec{v}_i = -\, a^{-1}(\nabla\Phi + \nabla\Phi_{Qi}) = 0 \quad \longleftrightarrow \quad \Phi_Q = -\Phi$ (alternative definition of solitons)

Vector solitons

$\rightarrow$ density at the center $\equiv \quad \rho_s = \frac{\alpha^4 m^2}{4\pi G} \simeq \frac{1}{Gm^2 R^4} \simeq \frac{G^3 m^6 M^4}{64\pi}$

- cosmological solitons produced with mass $M(a) = c_M M_J(a)$

$$M_J(a) = \frac{4\pi}{3}\bar{\rho}(a)\left[\frac{2\pi}{(16\pi G m^2 \bar{\rho}(a))^{\frac{1}{4}}}\right]^3 = O(1)\cdot\left[\frac{\bar{\rho}(a)}{G^3 m^6}\right]^{\frac{1}{4}}$$

$$\rho_s(a) = 4.5\cdot 10^4 c_M^4 \bar{\rho}(a) \propto a^{-3}$$

$\rightarrow$ the density of the solitons produced is parametrically the average DM density – indepedent of m!

$\rightarrow$ the first are produced with density $\simeq \bar{\rho}(a_{\rm eq})$; as time increases, solitons produced with smaller $\rho_s \propto a^{-3}$, smaller M and larger R

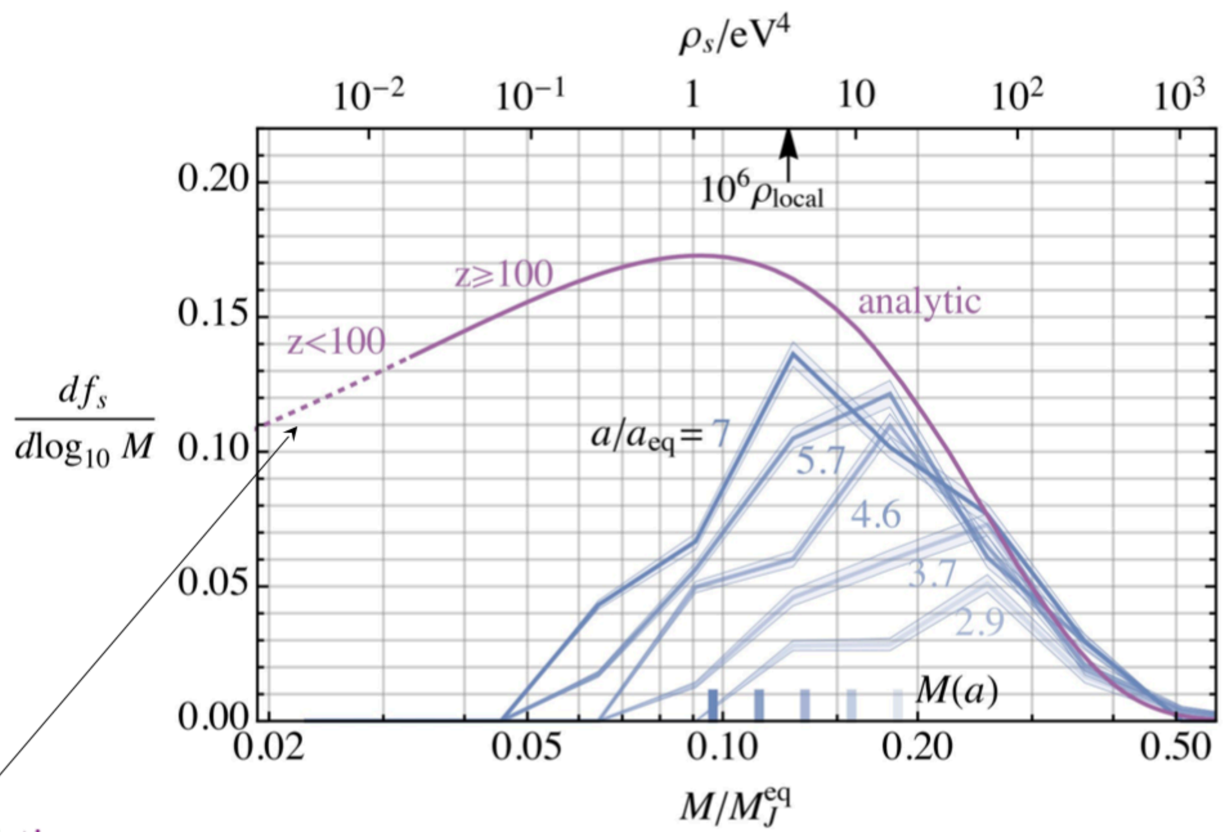
$\rightarrow M(a) = c_M M_J(a) = c_M M_J(a_{\rm eq})\left(\frac{a_{\rm eq}}{a}\right)^{\frac{3}{4}}$

$$M_J(a_{\rm eq}) \simeq 1.6\cdot 10^{-15} M_\odot \left(\frac{10^{-5}\ {\rm eV}}{m}\right)^{\frac{3}{2}}$$

$\rightarrow R(a) \simeq a\lambda_J(\bar{\rho}(a)) = \lambda_J(\bar{\rho}(a_{\rm eq}))(\frac{a}{a_{\rm eq}})^{\frac{3}{4}}$

$$\lambda_J(\bar{\rho}(a_{\rm eq})) \simeq 10^6\ {\rm km}\left(\frac{10^{-5}\ {\rm eV}}{m}\right)^{\frac{1}{2}}$$

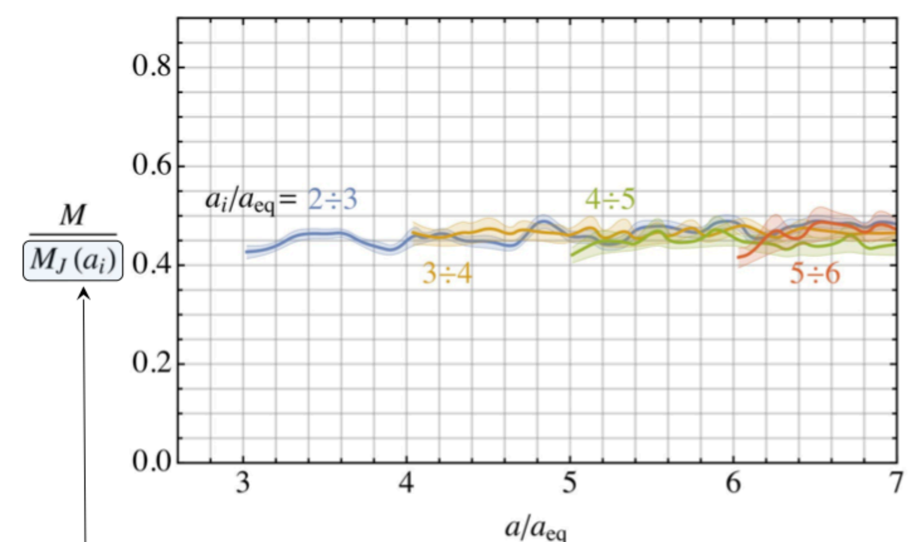
Mass distribution



analytic prediction:

$$M_J(a_{\rm eq}) \simeq 1.6\cdot 10^{-15} M_\odot \left(\frac{10^{-5}\ \mathrm{eV}}{m}\right)^{\frac{3}{2}}$$

$$\lambda_J(\bar\rho(a_{\rm eq})) \simeq 10^6\,\mathrm{km}\left(\frac{10^{-5}\ \mathrm{eV}}{m}\right)^{\frac{1}{2}}$$



analytic expectation:

$M_J(a) = M_J(a_{\rm eq})\left(\frac{a_{\rm eq}}{a}\right)^{\frac{3}{4}}$

$\rightarrow M(a) = c_M M_J(a)$, with $c_M \simeq 0.45$

Compact halos

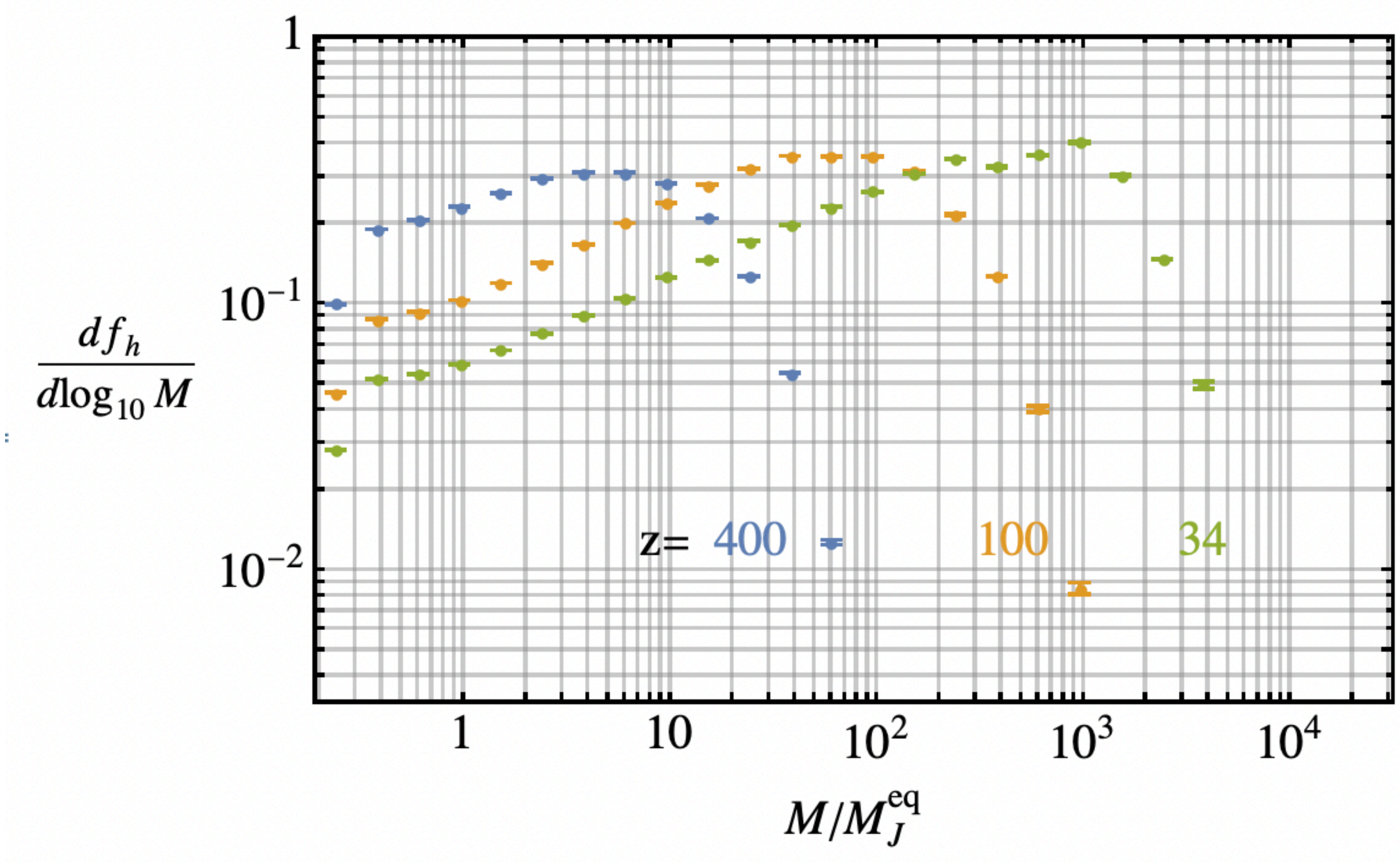
- single component perfect fluid (without quantum pressure) with density ρ and velocity $v \simeq 0$
- perturbative initial conditions with power spectrum $\mathcal{P}_\delta(k) \approx (k/k_\star)^3$ [the field is Gaussian at $L \gg \lambda_\star$ and determined only by $\mathcal{P}_\delta$]

Press–Schechter method [analytic]

N-body simulations [numerical]

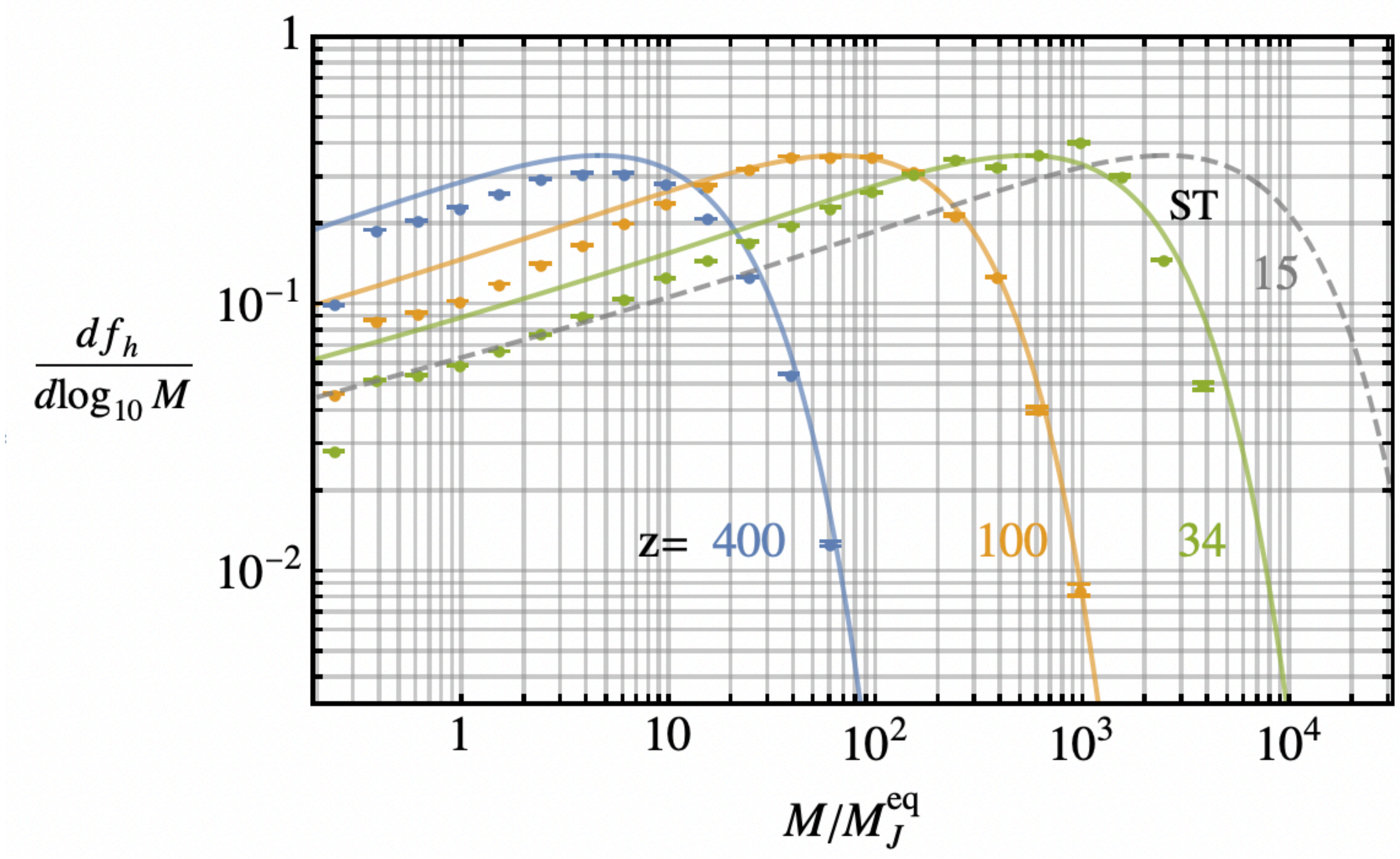
Compact halos

$$M_J(a_{\rm eq}) = 1.6 \cdot 10^{-15} M_\odot \left(\frac{10^{-5}\ \rm eV}{m}\right)^{\frac{3}{2}}$$



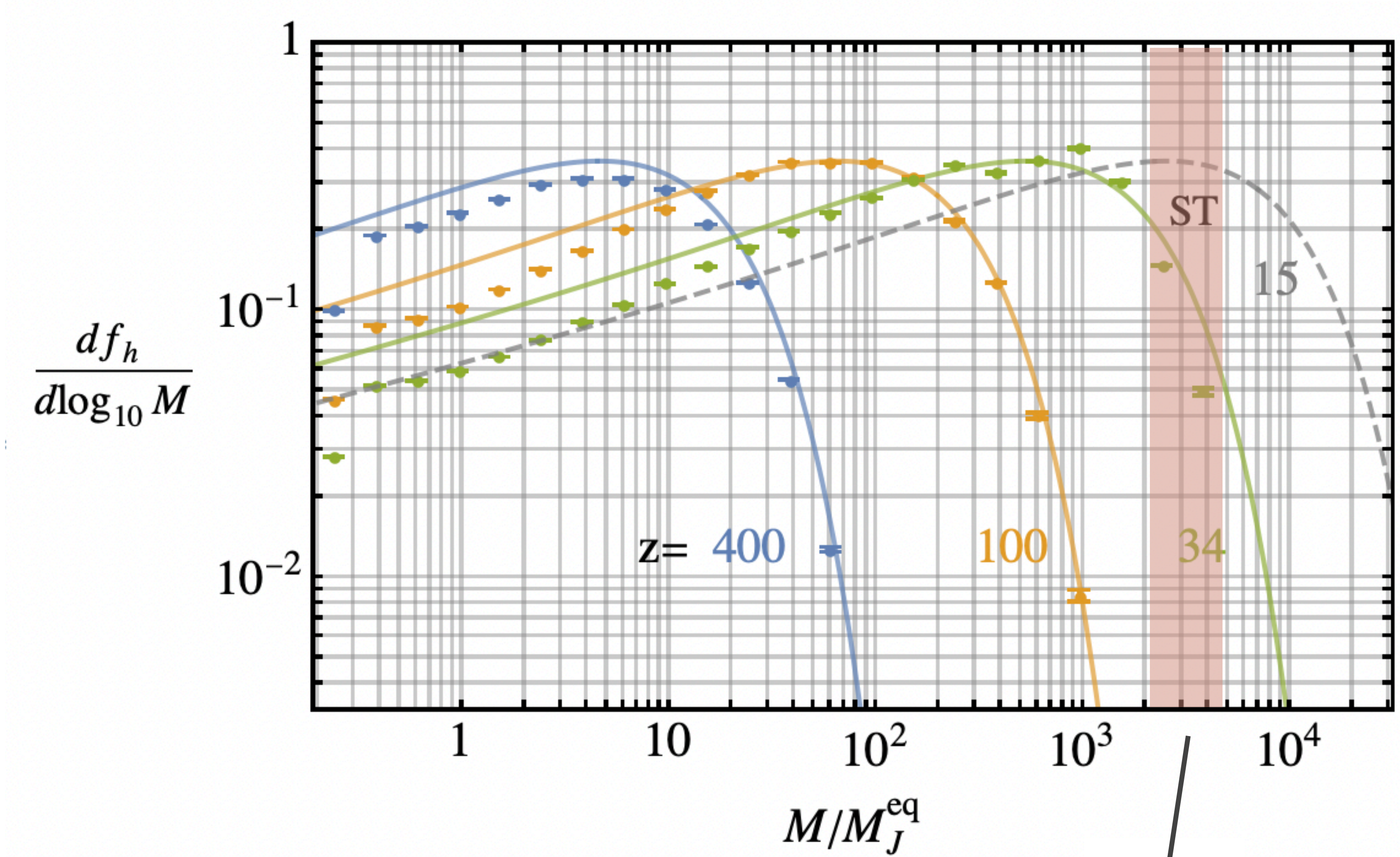
Compact halos

$$M_J(a_{\rm eq}) = 1.6 \cdot 10^{-15} M_\odot \left(\frac{10^{-5}\ \text{eV}}{m}\right)^{\frac{3}{2}}$$



Compact halos

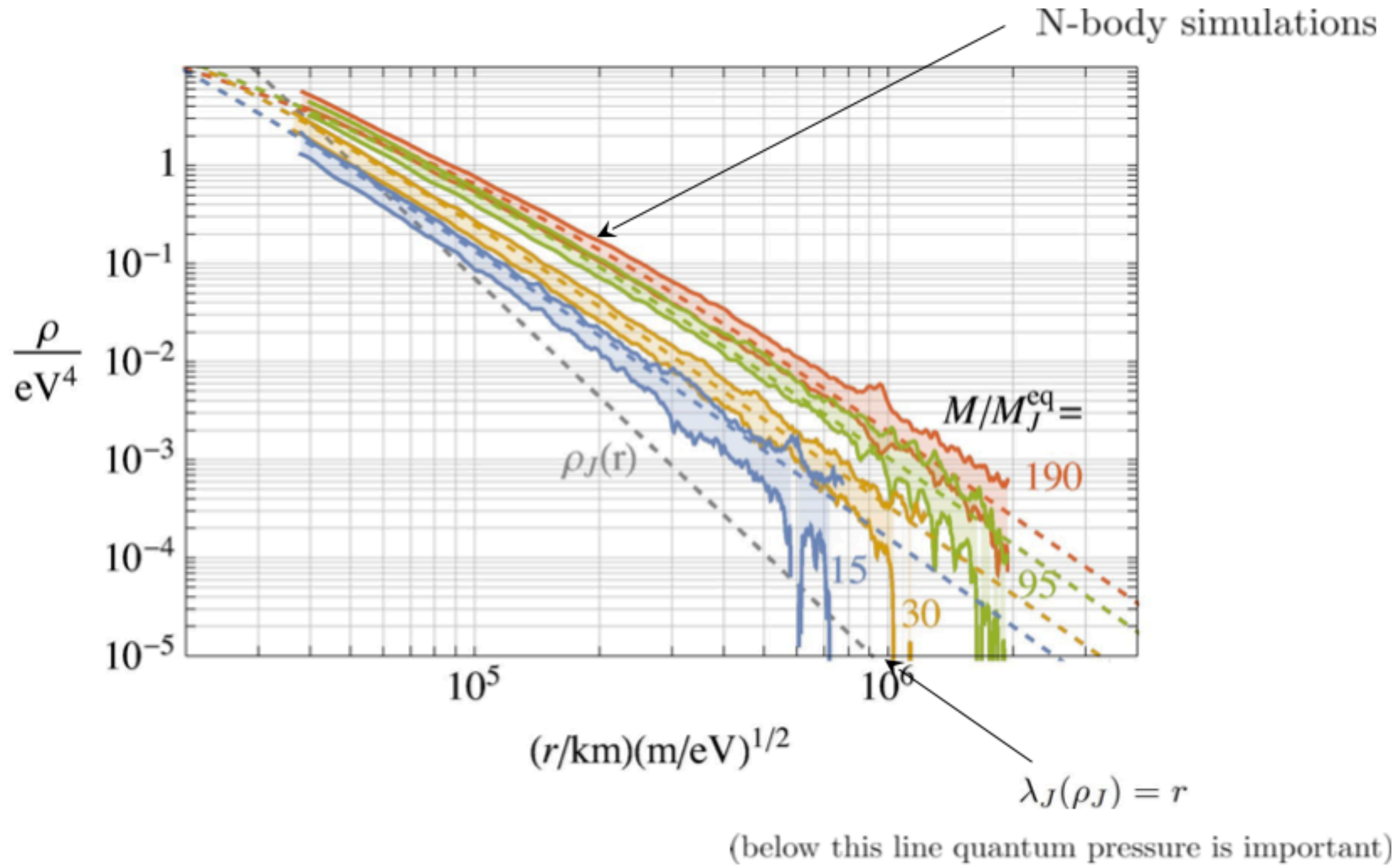
$$M_J(a_{\rm eq}) = 1.6 \cdot 10^{-15} M_\odot \left(\frac{10^{-5}\ \text{eV}}{m}\right)^{\frac{3}{2}}$$



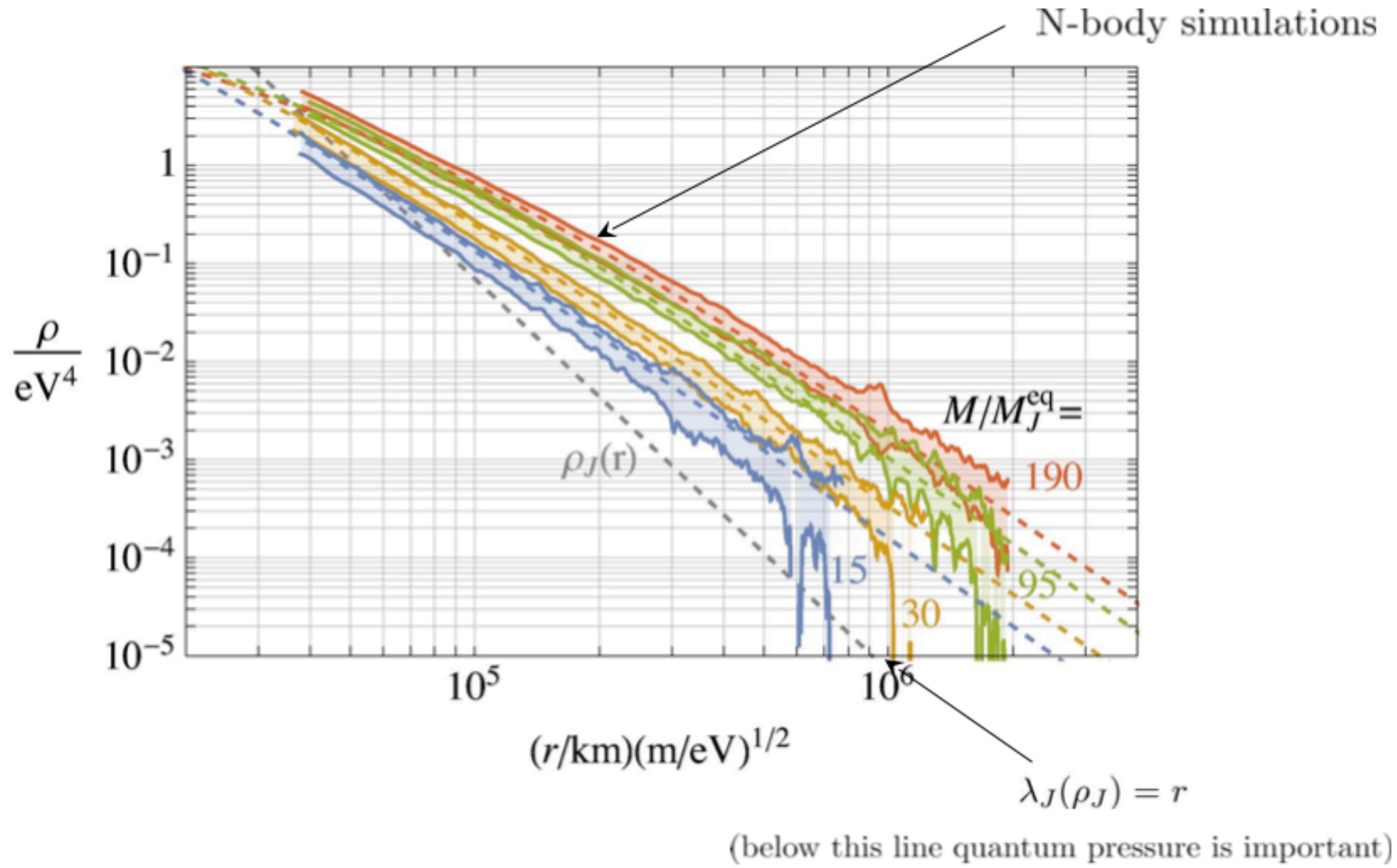
peaked at:

$$M_{\rm h}(z) \simeq 65 M_J^{\rm eq} \left[\frac{1.7}{\delta_c}\right]^2 \left[\frac{100}{z+1}\right]^2 \xrightarrow{z \to 10 \div 15} M_{\rm h} = (3 \div 5) \cdot 10^3 M_J^{\rm eq} \simeq (5 \div 9) \cdot 10^{-12} M_\odot \left(\frac{10^{-5}\text{eV}}{m}\right)^{3/2}$$

Halo Profiles



Halo Profiles



$$\rho_0 \simeq 0.7 \left[\frac{10^3 M_J^{eq}}{M}\right]^{3/2} \bar{\rho}^{eq} \simeq 0.3\,\text{eV}^4 \left[\frac{10^3 M_J^{eq}}{M}\right]^{3/2}$$

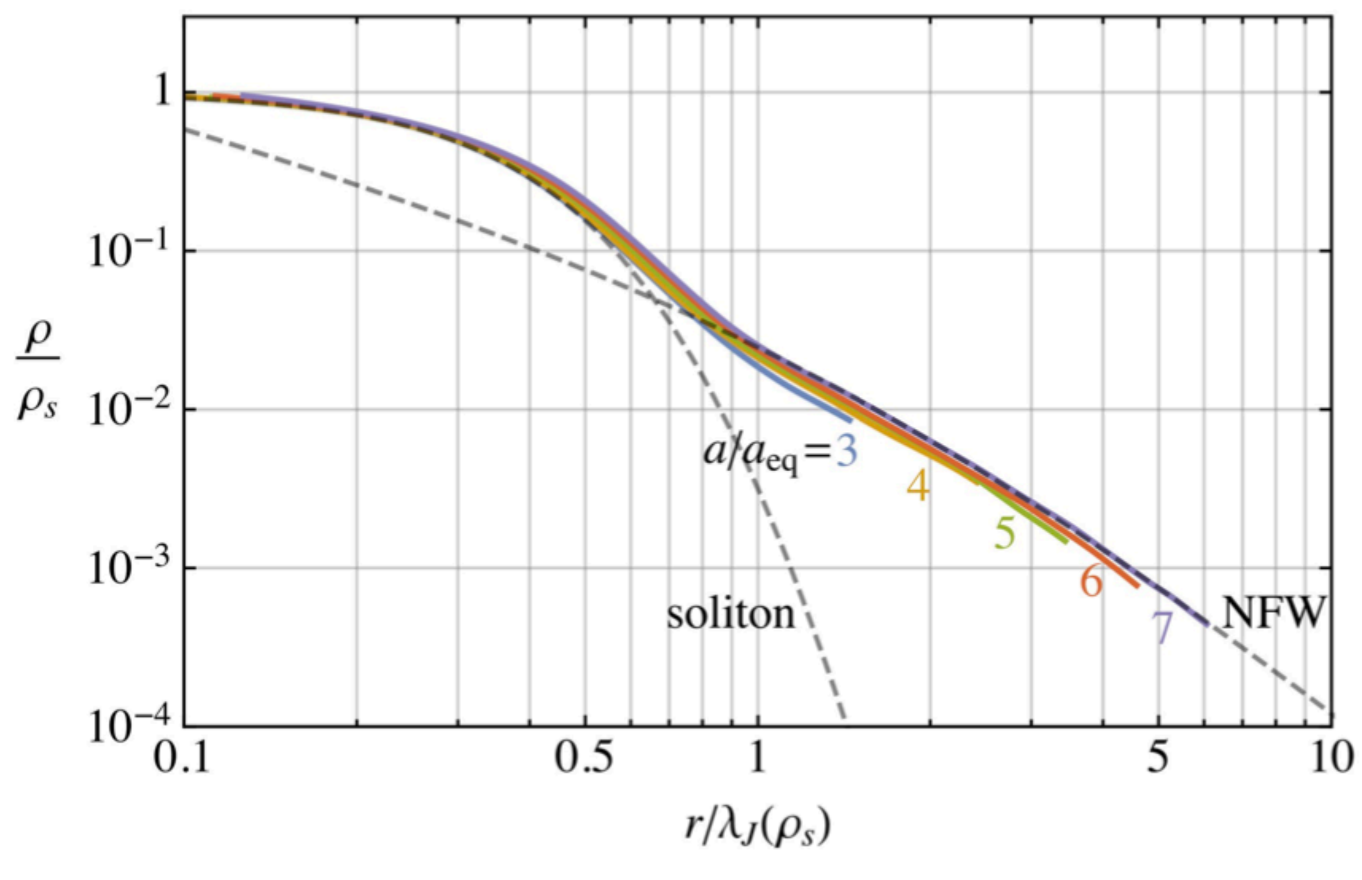
$[\rho_{loc} \simeq 2 \cdot 10^{-6} \text{eV}^4]$

NFW: $$\rho(r) = \frac{\rho_0}{r/r_0\,(1+r/r_0)^2}$$

$$r_0 \simeq 5.4 \left[\frac{M}{10^3 M_J^{eq}}\right]^{5/6} \lambda_J^{eq} \simeq 2 \cdot 10^8\,\text{km} \left[\frac{M}{10^3 M_J^{eq}}\right]^{5/6} \left[\frac{10^{-5}\ \text{eV}}{m}\right]^{1/2}$$

– lower mass halos have a smaller density [density determined parametrically by the background DM density at formation]

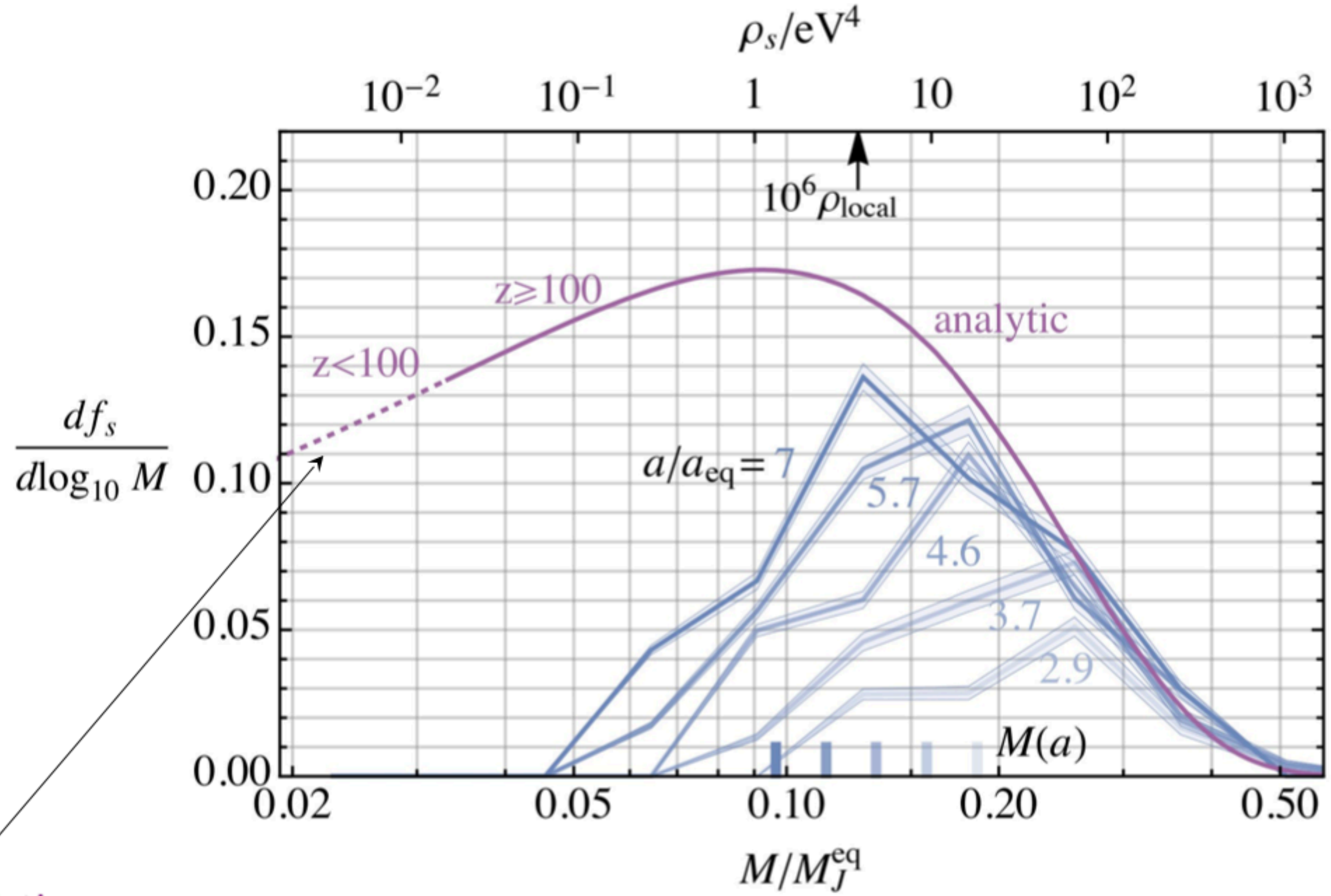
Fuzzy halos



$$\lambda_J(a_{eq}) \simeq 10^6\,\mathrm{km}\left(\frac{10^{-5}\ \mathrm{eV}}{m}\right)^{\frac{1}{2}}$$

- soliton is an exact solution only in vacuum
- 'fuzzy' halo around the solitons that follows an NFW profile
- extends out of a few soliton radii
- maximum density two orders of magnitude less than the soliton's

Mass distribution



analytic prediction:

$$M_J(a_{\mathrm{eq}}) \simeq 1.6 \cdot 10^{-15} M_\odot \left(\frac{10^{-5}\ \mathrm{eV}}{m}\right)^{\frac{3}{2}}$$

$$\lambda_J(\bar{\rho}(a_{\mathrm{eq}})) \simeq 10^6\,\mathrm{km} \left(\frac{10^{-5}\ \mathrm{eV}}{m}\right)^{\frac{1}{2}}$$

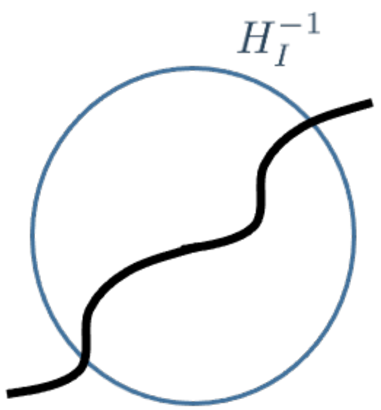
Vector dark matter from inflation

→ **massive vector field** during inflation

$$S = \int dt d^3x \sqrt{-g} \left[-\frac{1}{4} g^{\mu\rho} g^{\nu\sigma} F_{\mu\nu} F_{\rho\sigma} + \frac{1}{2} m^2 g^{\mu\nu} A_\mu A_\nu \right]$$

$m \ll H_I$

$$ds^2 = -dt^2 + a^2(t) d\vec{x}^2$$



consider a generic mode: $k_{\text{phys}} \equiv \frac{k}{a} \quad > H_I \gg m$

subhorizon during inflation $\quad \Rightarrow$ relativistic

$\tilde{A} = \{A_T, A_L\}$

Transverse $\equiv \quad \vec{k} \cdot \vec{A}_T = 0$

Longitudinal $\equiv \quad \vec{k} \cdot \vec{A} = k A_L$

Vector dark matter from inflation

$S = S_T + S_L$ (tranverse and longitudinal decoupled)

Transverse

$$S_T = \int \frac{a^3 d^3k\, dt}{(2\pi)^3} \frac{1}{2a^2} \left[|\partial_t \vec{A}_T|^2 - \left(\frac{k^2}{a^2} + m^2 \right) |\vec{A}_T|^2 \right] \xrightarrow{d\eta = \frac{dt}{a}} (2\pi)^{-3} \int d^3k\, d\eta \frac{1}{2} \left(|\partial_\eta \vec{A}_T|^2 - (k^2 + a^2 m^2) |\vec{A}_T|^2 \right)$$

$a^2 m^2 \to 0$ (relativistic)

time-translation invariant $\longrightarrow$ vacuum does not change

transverse modes are not produced

Vector dark matter from inflation

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($a^2 m^2 \to 0$ (relativistic))

time-translation invariant $\longrightarrow$ vacuum does not change
transverse modes are not produced

Longitudinal

$$S_L = \int \frac{a^3 d^3k\, dt}{(2\pi)^3} \frac{1}{2a^2} \left[\frac{a^2 m^2}{k^2 + a^2 m^2} |\partial_t A_L|^2 - m^2 |A_L|^2 \right] \xrightarrow{\frac{k}{a} \gg m \quad \varphi \equiv \frac{A_L}{k}} \int a^3 d^3x dt \frac{1}{2} [(\partial_t \varphi)^2 - |\nabla \varphi|^2 / a^2]$$

(relativistic) scalar field $\longrightarrow$ produced with scale-invariant energy density spectrum at horizon exit $a_e = k/H_I$

Vector dark matter from inflation

$S = S_T + S_L$ (tranverse and longitudinal decoupled)

Transverse

$$S_T = \int \frac{a^3 d^3k\, dt}{(2\pi)^3} \frac{1}{2a^2} \left[|\partial_t \vec{A}_T|^2 - \left(\frac{k^2}{a^2} + m^2 \right) |\vec{A}_T|^2 \right] \xrightarrow{d\eta = \frac{dt}{a}} (2\pi)^{-3} \int d^3k d\eta \frac{1}{2} \left(|\partial_\eta \vec{A}_T|^2 - (k^2 + a^2 m^2) |\vec{A}_T|^2 \right)$$

($a^2 m^2 \to 0$, relativistic)

time-translation invariant → vacuum does not change; transverse modes are not produced

Longitudinal

$$S_L = \int \frac{a^3 d^3k\, dt}{(2\pi)^3} \frac{1}{2a^2} \left[\frac{a^2 m^2}{k^2 + a^2 m^2} |\partial_t A_L|^2 - m^2 |A_L|^2 \right] \xrightarrow{\frac{k}{a} \gg m \quad \varphi \equiv \frac{A_L}{k}} \int a^3 d^3x dt \frac{1}{2} [(\partial_t \varphi)^2 - |\nabla \varphi|^2 / a^2]$$

(relativistic) scalar field → produced with scale-invariant energy density spectrum at horizon exit $a_e = k/H_I$

Energy density:

$$\rho_{A_L} \equiv \int dk \frac{\partial \rho_{A_L}}{\partial k}$$

$$\left. \frac{\partial \rho_{A_L}}{\partial \log k} \right|_{a=a_e} \approx \frac{H_I^4}{(2\pi)^2} \qquad \left. \vec{A}_T \right|_{a=a_e} \approx 0$$

Vector dark matter from inflation

$$\left[\partial_t^2 + \frac{3k^2 + a^2m^2}{k^2 + a^2m^2} H\partial_t + \frac{k^2}{a^2} + m^2\right] A_L = 0$$

$$\rho = \frac{1}{2a^2}\left(\partial_t A_L \frac{a^2m^2}{a^2m^2 - \nabla^2}\partial_t A_L + m^2 A_L^2\right)$$

Vector dark matter from inflation

$$\left[\partial_t^2 + \frac{3k^2 + a^2m^2}{k^2 + a^2m^2} H \partial_t + \frac{k^2}{a^2} + m^2\right] A_L = 0$$

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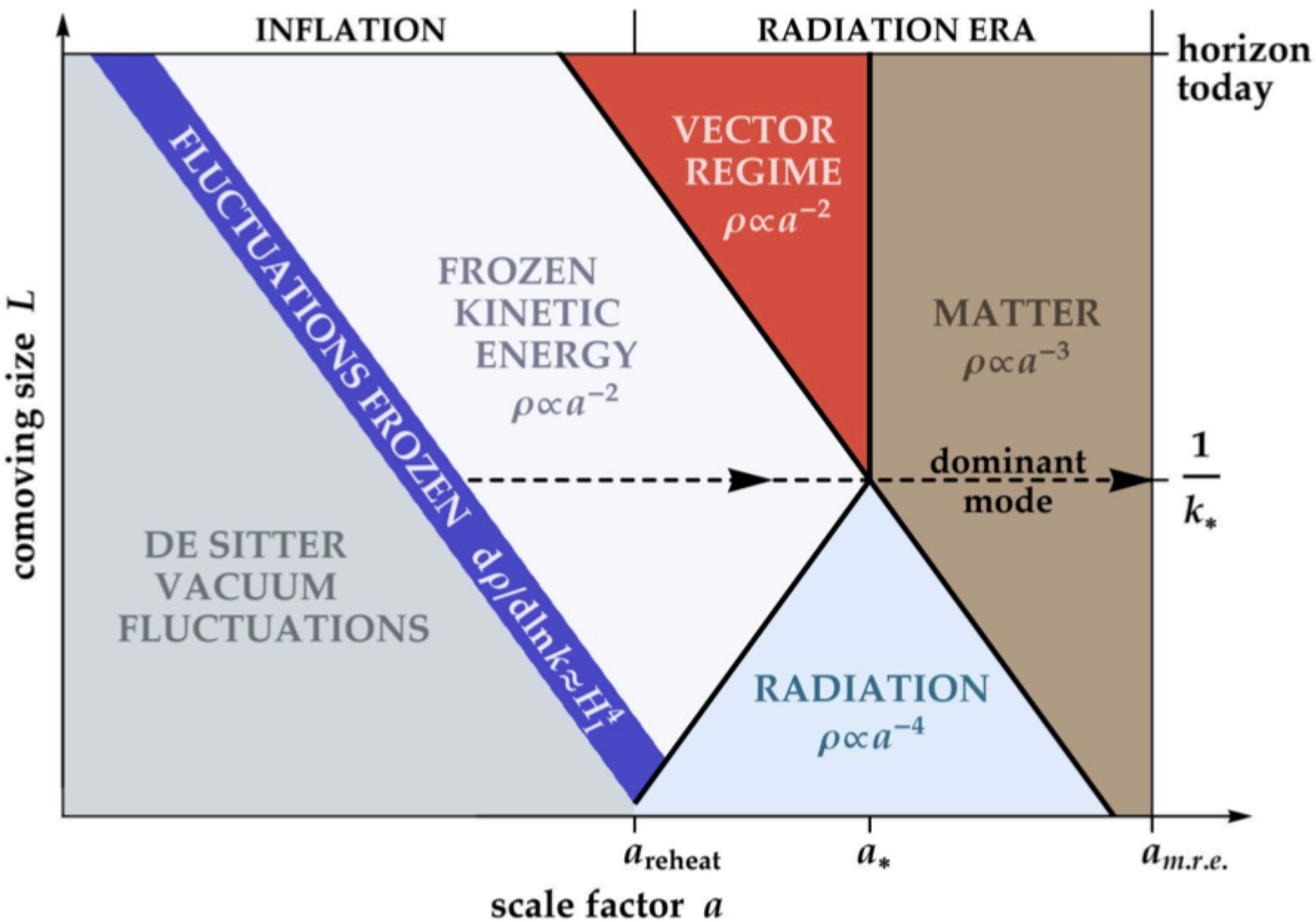

[Graham, Mardon, and Rajendran]

Vector dark matter from inflation

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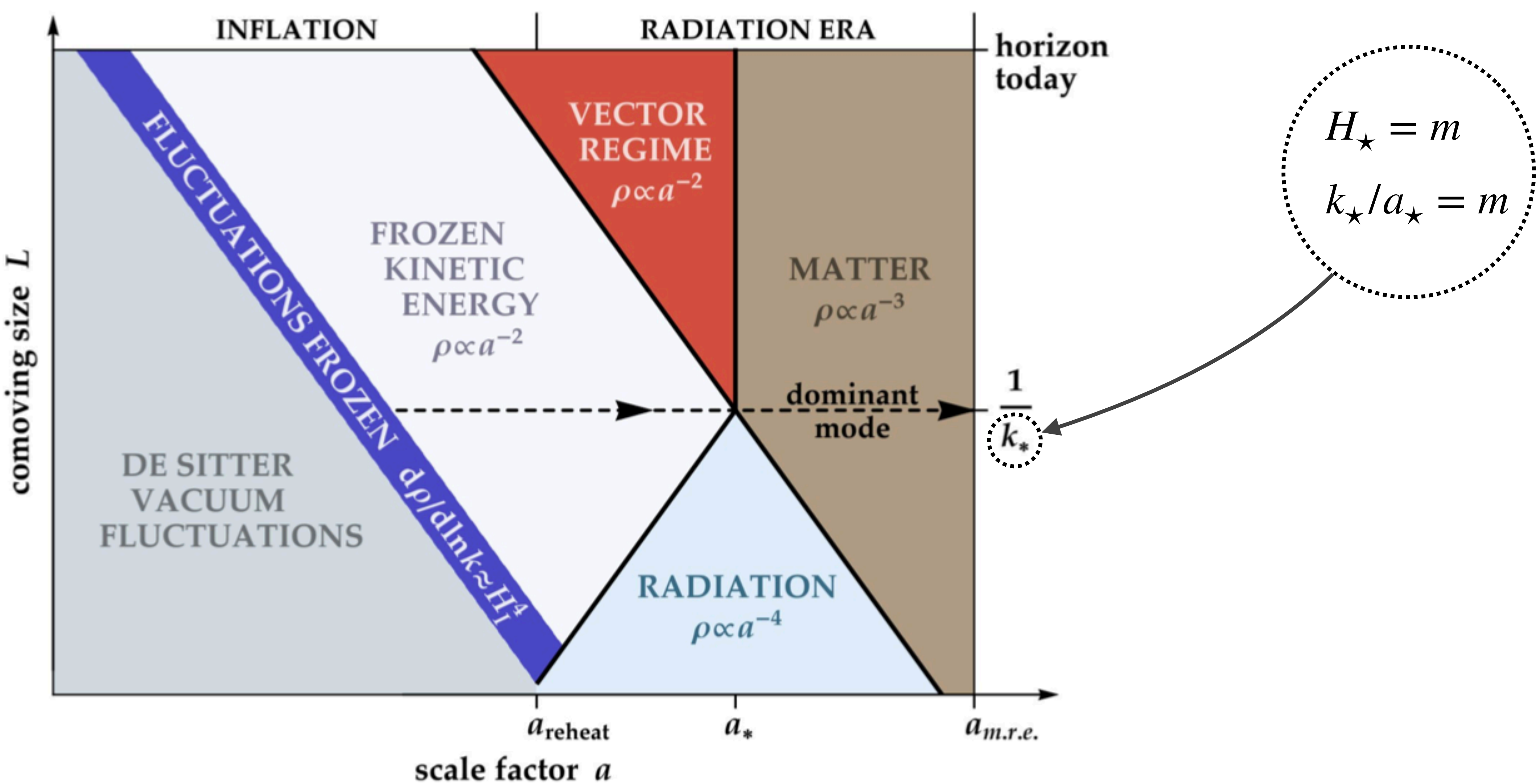

[Graham, Mardon, and Rajendran]

Vector dark matter from inflation

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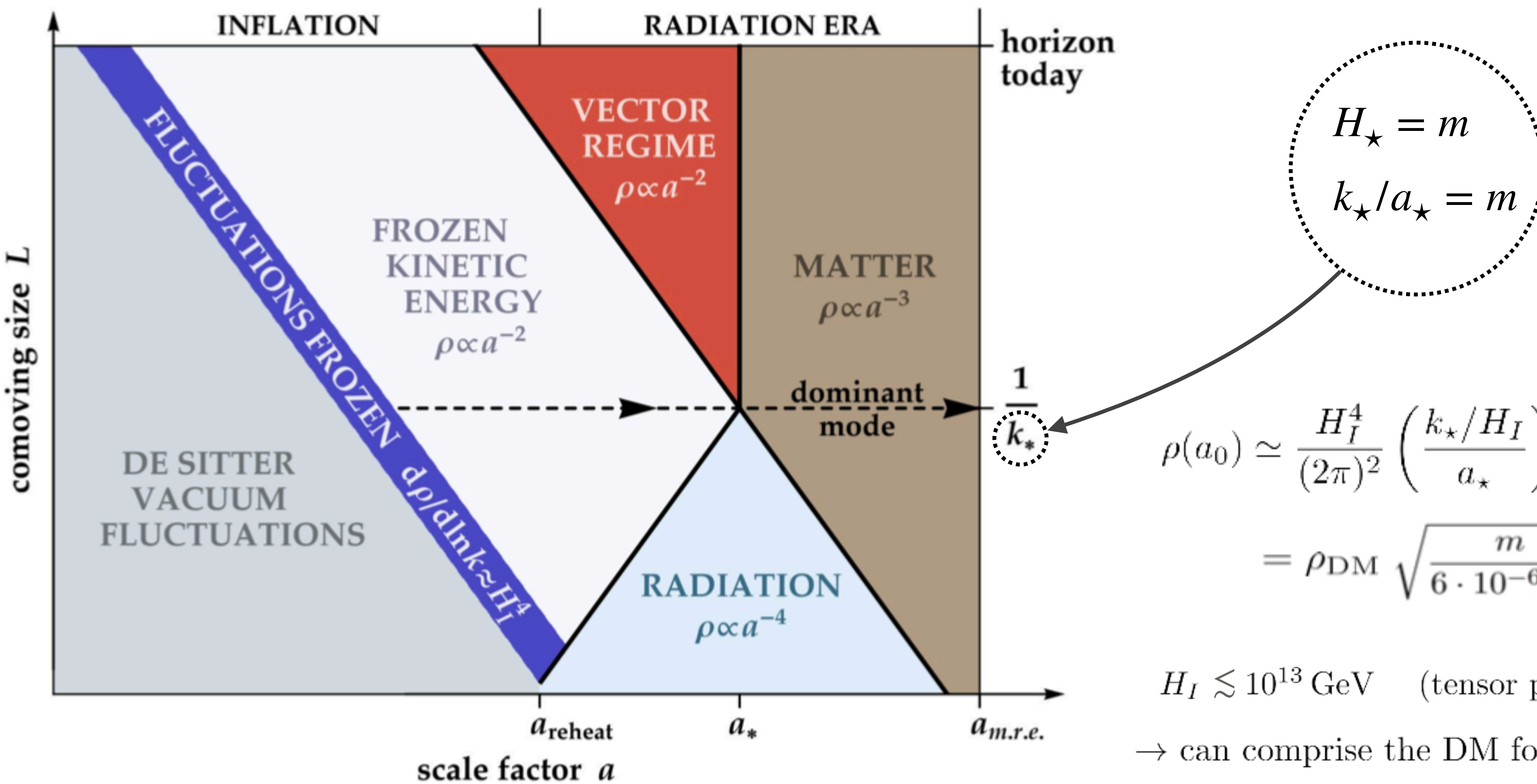


$$\rho(a_0) \simeq \frac{H_I^4}{(2\pi)^2}\left(\frac{k_\star/H_I}{a_\star}\right)^2\left(\frac{a_\star}{a_0}\right)^3$$

$$= \rho_{\rm DM}\sqrt{\frac{m}{6\cdot 10^{-6}\,{\rm eV}}}\left(\frac{H_I}{10^{14}\,{\rm GeV}}\right)^2$$

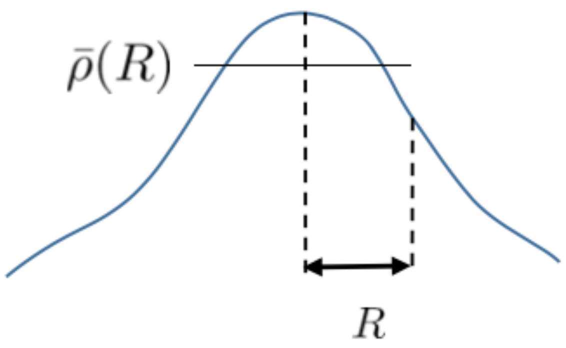
$H_I \lesssim 10^{13}$ GeV (tensor perturbations)

$\rightarrow$ can comprise the DM for: $m \gtrsim 10^{-5}$ eV

[Graham, Mardon, and Rajendran]

Survival of the substructure

Disruption probability related to the density of the objects



$$\bar{\rho}(R) = \frac{1}{4\pi R^3/3}\int_0^R d^3r\,\rho(r)$$

$\rightarrow$ if the mean density of an object $> 0.05\text{eV}^4 \simeq 10^5 \rho_{\rm loc} = O(10)\bar{\rho}_{\rm gal}$, the parts with $r < R$ survive

- **solitons**: $\rho_s = (0.1 \div 100)\text{eV}^4$, and $\bar{\rho}(R_{\rm edge}) \simeq 0.2\rho_s$

  $\rightarrow$ most of the solitons and the fuzzy halo around them survive undirsupted

- **compact halos**: those with mass $(10^2 \div 10^4)M_J^{\rm eq}$ have average density $(10^{-6} \div 10^{-3}\text{eV}^4)$

  $\rightarrow$ likely to be dirsupted except at their core

Halos vs Solitons

Non-relativistic limit $A_i \equiv \frac{1}{\sqrt{2m^2a^3}}(\psi_i e^{-imt} + \text{c.c.})$

$\Phi_Q \equiv -\frac{\hbar^2}{2a^2m^2}\frac{\nabla^2\sqrt{\rho_a}}{\sqrt{\rho_a}}$

$$\left(i\partial_t + \frac{\nabla^2}{2m} - m\Phi\right)\psi_i = 0\ ,$$
$$\nabla^2\Phi = \frac{4\pi G}{a}\sum_i \left(|\psi_i|^2 - \langle|\psi_i|^2\rangle\right)$$

$\psi_i = \sqrt{\rho_i}e^{i\theta_i}$

$\vec{v}_i = \frac{1}{m}\nabla\theta_i$

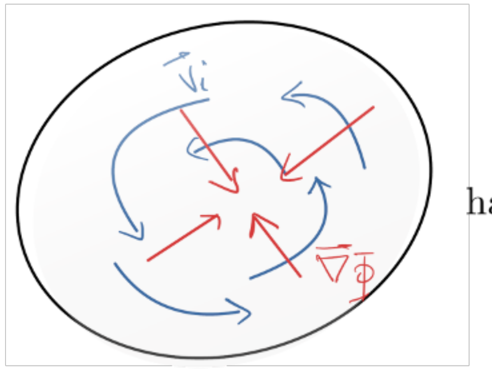
$\longleftrightarrow$

$$\begin{cases} \partial_t\rho_i + 3H\rho_i + a^{-1}\nabla\cdot(\rho_i\vec{v}_i) = 0 \\ \partial_t\vec{v}_i + H\vec{v}_i + a^{-1}(\vec{v}_i\cdot\nabla)\vec{v}_i = -a^{-1}(\nabla\Phi + \nabla\Phi_{Qi}) \\ \nabla^2\Phi = 4\pi G a^2(\rho - \bar{\rho})\ , \end{cases}$$

Halos

$\Phi_Q = 0$

$\rightarrow$ gravitational potential balanced by the velocity terms



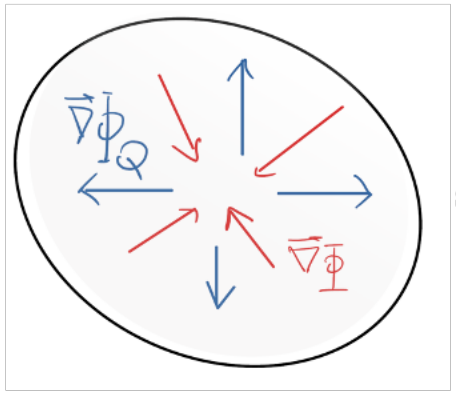
halo

angular momentum 'supports' the gravitational potential

Solitons

$\Phi_Q = -\Phi \quad \longleftrightarrow \quad \vec{v}_i = 0$

$\rightarrow$ gravitational potential balanced by the quantum pressure



soliton

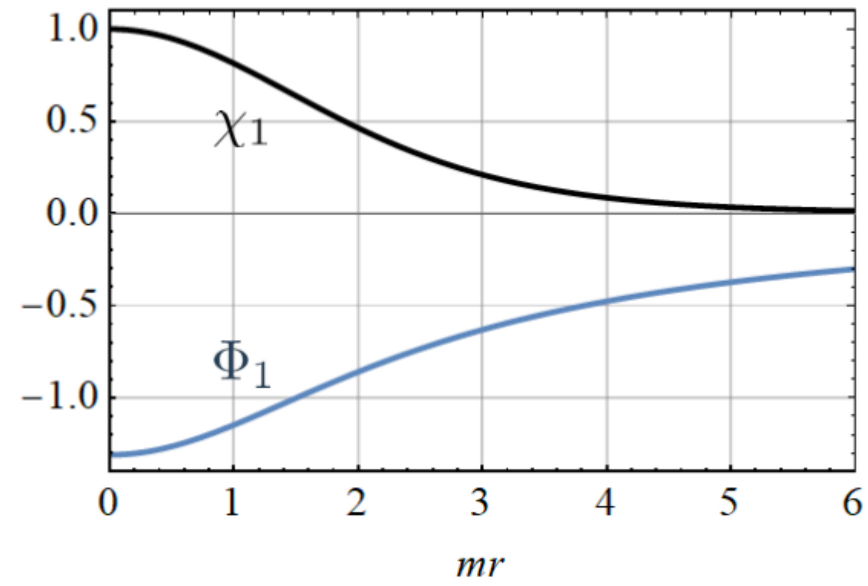
quantum pressure 'supports' the gravitational potential

Vector solitons

Basic ansatz:

$$\psi_i = \frac{m}{\sqrt{4\pi G}} e^{-i\gamma m t} \chi_1(mr) \boxed{u_i}, \qquad \Phi = \Phi_1(mr) \qquad \gamma \simeq -0.65$$

unit vector $u_i^* u_i = 1$



Energy localised at the centre

Other solutions obtained by rescaling

$\chi_1(x) \to \alpha^2 \chi_1(\alpha x)$, $\Phi_1(x) \to \alpha^2 \Phi_1(\alpha x)$ and $\gamma \to \alpha^2 \gamma$, for any $\alpha > 0$

$$M \simeq \frac{2\alpha}{Gm}, \qquad R \simeq \frac{1.9}{\alpha m}$$

$$\boxed{MR \simeq \frac{3.9}{Gm^2}}$$

⟶ Product MR depends only on the vector's mass