Compact objects in gravity theories

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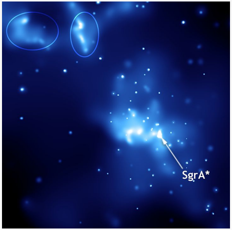
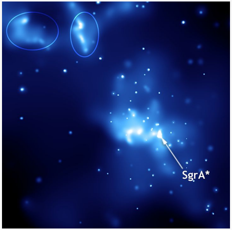



- GW signals from binaries at their ringdown phase : GW170817 neutron star merger, GW190814 and the large mass secondary at $2.59^{+0.08}_{-0.09}\ M_\odot$
- Array of radio telescopes, EHT : image of M87 black hole with its light ring
- GRAVITY VLT: observation of star trajectories orbiting SgrA central black hole : orbit characteristics give us tests of strong gravity which get better as precision increases.

Black holes and neutron stars, breakthrough in observational data



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- GRAVITY VLT: observation of star trajectories orbiting SgrA central black hole : orbit characteristics give us tests of strong gravity which get better as precision increases.
- What is the maximal mass of neutron stars? What is their equation of state? How rapid can their rotation be before instability?
- eg.: Is the compact secondary the heaviest neutron star or the lightest astrophysical black hole? How does this fit with GR?
- Can we find pulsars in the vicinity of SgrA and follow them around the central black hole?

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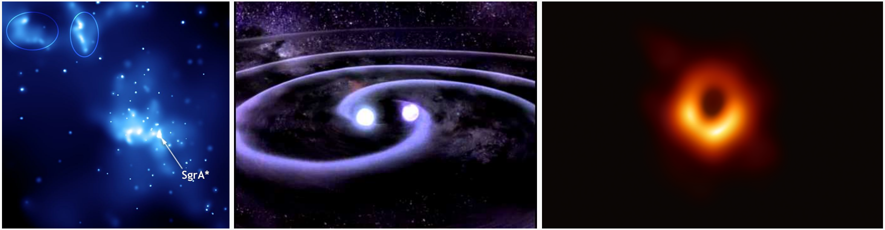

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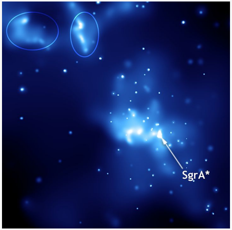
- GRAVITY VLT: observation of star trajectories orbiting SgrA central black hole : orbit characteristics give us tests of strong gravity which get better as precision increases.
- Can we find alternatives to GR black holes and stars as precise rulers of departure from GR? What about other compact objects like wormholes or regular black holes

Plan and tools of construction




- GR black holes and their characteristics
- Scalar tensor theories as a measurable departure from GR
- Use GR solutions to construct stealth solutions
- kinetic term $X = -\frac{1}{2}\partial_\mu\phi\partial_\nu\phi g^{\mu\nu}$ and geodesics
- disformal transformations [Zumalacarregui, Garcia Bellido]

GR black holes

In GR black holes are "unique" and characterised by a finite number of charges, essentially, J, M

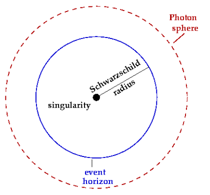
They are relatively simple solutions-they have no hair, quadrapole is $Q^2 = -J^2/M$

- During collapse, black holes lose their hair and relax to some stationary state of large symmetry. They are (mostly) vacuum solutions of Einstein's eqs, $G_{\mu\nu} = 0$
- Static and spherically symmetric Schwarzschild solution :

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2$$

with $f(r) = 1 - \frac{2M}{r}$, M mass black hole parameter

- The zero(s) of $f(r)$ are the horizon(s) of the black hole ($r_h = 2M$).
- An event horizon determines an absolute surface of no return. Its interior is the trapped region of the black hole hiding the curvature singularity at $r = 0$



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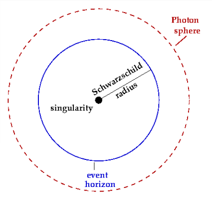
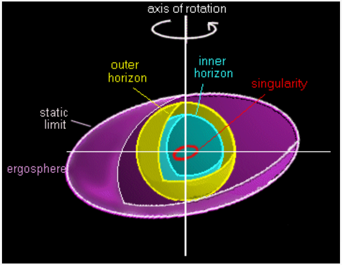


The rotating Kerr black hole

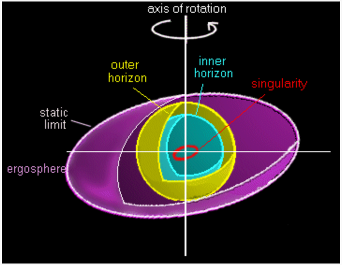


- Parameters mass M, angular momentum $J = aM$ create ergoregion and other goodies
- For Kerr, geodesics are integrable : In 4 dimensions we find 4 constants of motion describing test particles : L_z, E, m, Q.
  Geodesic equation is given as a first order diff eq using $S = -Et + L_z\varphi + S(r,\theta)$ [Carter],

$$\frac{\partial S}{\partial \lambda} = g^{\mu\nu}\frac{\partial S}{\partial x^\mu}\frac{\partial S}{\partial x^\nu} = -m^2$$

  Integrability for Kerr means that S is a completely known function parametrised by L_z, E, m, Q. Note that S shares the same definition as $X = -\frac{1}{2}\partial_\mu\phi\partial_\nu\phi g^{\mu\nu}$

Lets move on now to ST theories very briefly



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Simplest modified gravity theory with a single scalar degree of freedom

BD theory,..., Horndeski,..., beyond Horndeski,..., DHOST theories [Noui, Langlois, Crisostomi, Koyama et al]

- BD have only GR black hole solutions (no hair theorems)
- For hairy black holes we need to have higher derivative theories... Horndeski, Beyond and DHOST
- Nothing fundamental about ST theories, they are just sane and measurable departures from GR.
- They are limits of more complex fundamental theories (massive gravity, braneworld models, EFT from string theory, Lovelock theory etc.)
- Horndeski is parametrized by 4 functions of scalar and its kinetic energy, $G_i = G_i(\phi, X)$.

Scalar tensor theories : a robust measurable departure from GR

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Fix a theory and try to find a solution

Horndeski theory-Galileons

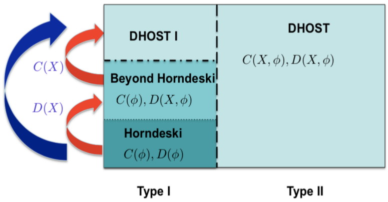
Typically we choose functions and study the system of eqs.
Example in Horndeski with G_2, G_4 linear functions of X; $G_3 = G_5 = 0$

$$S = \int d^4x\sqrt{-g}\left[R - 2\Lambda_b + X + \beta G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi\right],$$

- Kinetic term is $X = -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$.
- Conformal and Disformal transformations transport us in between theories
- General conformal and disformal map :

$$g_{\mu\nu} \longrightarrow \tilde{g}_{\mu\nu} = C(\phi, X)g_{\mu\nu} + D(\phi, X)\nabla_\mu\phi\nabla_\nu\phi$$

for given (regular) functions C and D.



[Langlois, 2018]

Horndeski and beyond

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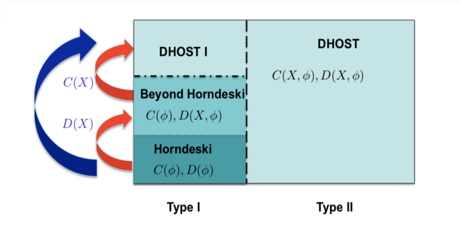

[Langlois, 2018]

Stealth solution of spherical symmetry

- Example Horndeski theory [Babichev, CC]

$$S = \int d^4x \sqrt{-g} \left[R - 2\Lambda_b - \eta X + \beta G^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \right],$$

- simple (stealth) solution reads

$$f = h = 1 - \frac{2\mu}{r} + \frac{\eta}{3\beta} r^2$$

,

$$\phi = qt \pm \int dr \, \frac{q}{h} \sqrt{1 - h}$$

with $q^2 = \frac{\zeta\eta + \Lambda_b \beta}{\beta\eta}$.

- Metric is a GR solution but scalar field is non trivial!
- Note that, $X = g^{\mu\nu}\phi_\mu\phi_\nu = -\frac{q^2}{h} + q^2 \frac{f(1-h)}{h^2} = -q^2$ is constant.
- Coordinate transformation shows that the disformed metric is a stealth black hole. $\mathcal{D}$(stealth)$\equiv$ stealth
- Can we construct rotating solutions?

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Going beyond spherical symmetry

We can find numerous solutions of spherical symmetry

Stealth solutions with X constant are generic in DHOST theories

- Can we construct stealth rotating solutions?
- Can we obtain a Kerr metric with $X = -q^2$ for some ST theory? We have a candidate metric, Kerr, but what can the scalar field be?
- The key is understanding what $X = -q^2$ signifies geometrically
  Kerr : Geodesics are constructed via S which is known [Carter],

$$\frac{\partial S}{\partial \lambda} = g^{\mu\nu} \frac{\partial S}{\partial x^\mu} \frac{\partial S}{\partial x^\nu} = -m^2$$

  Basically assume that the Kerr potential is the scalar field!
- Result : for a certain class of DHOST theories, (de Sitter) Kerr with $X = -q^2$ with a regular scalar field is an exact solution
- Stealth Kerr black hole in DHOST theory [Crisostomi, CC, Gregory, Stergioulas]
- What is $\mathcal{D}$(Kerr)?

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Stealth solutions with X constant are generic (boring) in DHOST theories

- $\mathcal{D}$(stealth)$\equiv$ stealth
- non trivial rotating BH solutions other than Kerr are not known analytically! [Herdeiro, Radu]
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Constructing non Kerr rotating solutions [Anson, Babichev, CC, Hassaine]

- Using disformal transformation we can construct analytically causal black holes other than Kerr.
- Disformed Kerr metrics,

$$g_{\mu\nu}^{Kerr} \longrightarrow \tilde{g}_{\mu\nu} = g_{\mu\nu}^{Kerr} + D(X)\nabla_\mu\phi\nabla_\nu\phi$$

for given D.

$$\mathrm{d}s^2 = -\left(1 - \frac{2\tilde{M}r}{\rho^2}\right)\mathrm{d}t^2 - \frac{4\sqrt{1+D}\tilde{M}ar\sin^2\theta}{\rho^2}\mathrm{d}t\mathrm{d}\varphi + \frac{\sin^2\theta}{\rho^2}\left[\left(r^2+a^2\right)^2 - a^2\Delta\sin^2\theta\right]\mathrm{d}\varphi^2$$

$$+ \frac{\rho^2\Delta - 2\tilde{M}(1+D)rD(a^2+r^2)}{\Delta^2}\mathrm{d}r^2 - 2D\frac{\sqrt{2\tilde{M}r(a^2+r^2)}}{\Delta}\mathrm{d}t\mathrm{d}r + \rho^2\mathrm{d}\theta^2 \ .$$

- For $D \neq 0$ and $a \neq 0$ $\mathcal{D}$(Kerr) in not an Einstein metric!
- Metric is causal, has an ergoregion, an event horizon
- For $D \neq 0$ we do not verify the GR no hair relation
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The $4d - EGB$ scalar tensor theory from higher dimensions

- Start with a higher dimensional metric theory in D dimensions, $S = \int d^D x (R^{(D)} + \alpha \mathcal{G}^{(D)})$
- Compactify on a $D-4$ manifold $S = \int d^D x (R^{(D)} + \alpha \mathcal{G}^{(D)}) \longrightarrow 4$ dim Horndeski theory
- If manifold is product of spheres we obtain black hole solutions [CC, Gouteraux, Kiritsis]
- Special singular limit $\tilde{\alpha} = \frac{\alpha}{D-4}$ while $D \longrightarrow 4$ [Glavan, Lin]
- Non trivial scalar tensor theory [Lu-Pang, Hennigar etal, Clifton, Fernandes etal]
- Particular Horndeski theory with all G's switched on (unique coupling constant α)

$$S = \frac{1}{2\kappa} \int \mathrm{d}^4 x \sqrt{-g} \left\{ R + \alpha \left[\phi \mathcal{G} + 4 G_{\mu\nu} \nabla^\mu \phi \nabla^\nu \phi - 4 (\nabla \phi)^2 \Box \phi + 2 (\nabla \phi)^4 \right] \right\} + S_{\mathrm{m}},$$

Admits the following solution...

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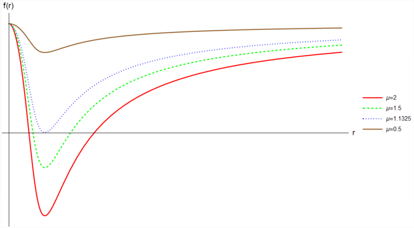
- For static and spherical symmetry, $\mathrm{d}s^2 = -f(r)\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{f(r)} + r^2\mathrm{d}\Omega^2$
- with $f(r) = 1 + \frac{r^2}{2\alpha}\left(1 - \sqrt{1 + \frac{8\alpha M}{r^3}}\right)$, $\phi(r) = \int \mathrm{d}r \frac{\sqrt{f} - 1}{r\sqrt{f}}$
- Far away solution is very much like GR, $f(r) \underset{r\to+\infty}{=} 1 - \frac{2M}{r} + \frac{4\alpha M^2}{r^4} + \mathcal{O}(r^{-5})$,



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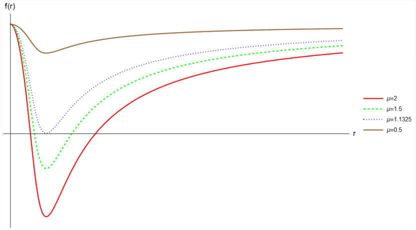
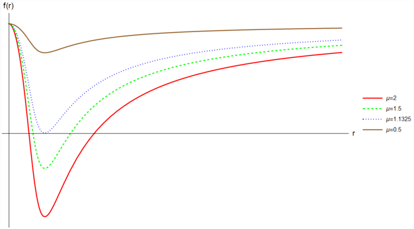
- Far away solution is very much like GR, $f(r) \underset{r\to+\infty}{=} 1 - \frac{2M}{r} + \frac{4\alpha M^2}{r^4} + \mathcal{O}(r^{-5})$,
- Differences : The solution is more regular than GR at $r \to 0$ but still not regular, $f \sim 1 - \sqrt{\frac{Mr}{2\alpha}}$
- Solution has event horizon $r_{\mathrm{h}} = M + \sqrt{M^2 - \alpha}$ if $\alpha > 0$ and $M > M_{\mathrm{min}} = \sqrt{\alpha}$.
- Higher order terms are smoothing out the geometry and allowing more compact objects!

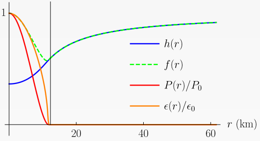


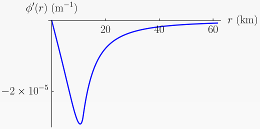
- For static and spherical symmetry, $\mathrm{d}s^2 = -f(r)\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{f(r)} + r^2\mathrm{d}\Omega^2$
- with $f(r) = 1 + \frac{r^2}{2\alpha}\left(1 - \sqrt{1 + \frac{8\alpha M}{r^3}}\right)$, $\quad \phi(r) = \int \mathrm{d}r \frac{\sqrt{f} - 1}{r\sqrt{f}}$
- Far away solution is very much like GR, $f(r) \underset{r\to+\infty}{=} 1 - \frac{2M}{r} + \frac{4\alpha M^2}{r^4} + \mathcal{O}(r^{-5})$,
- Constrants :$\alpha < 0$ excluded from probed atomic nuclei which are horizonless... For $R \sim 10^{-15}$ need $-\alpha < 10^{-30}$
- If $\alpha > 0$ since $M_{\min} = \sqrt{\alpha}$ then observed black holes from GW give us constraints on α
- For example if secondary of GW190814 is a black hole then $\alpha < 59 km^2$ etc.



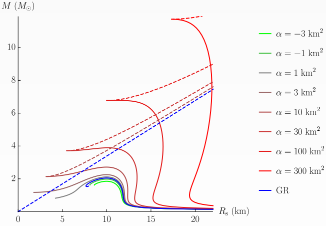
Neutron stars [CC, Lehebel, Smyrniotis, Stergioulas]

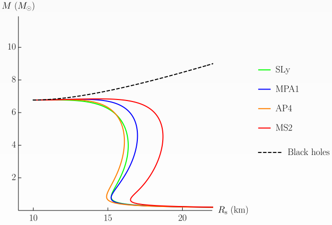
- Introducing, $T_{\mu\nu} = (\epsilon + P)u_\mu u_\nu + Pg_{\mu\nu}$ we find neutron star solutions





- For $\alpha > 0$ we have more compact neutron stars. GW19 is compatible with slowly rotating neutron star.





- Universal point of convergence for neutron stars and black holes for known EOS. No mass gap present in this theory unlike GR!

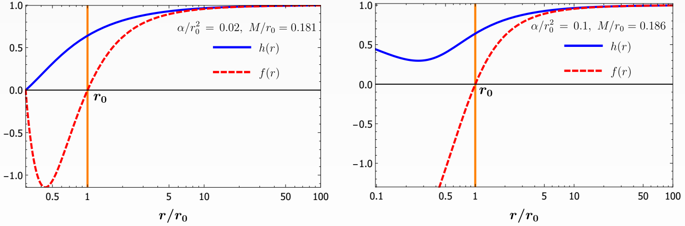
From a black hole to a wormhole : general characteristics [Morris&Thorne, Damour&Solodukhin]

- Start again with a static and spherically symmetric spacetime.
  $$\mathrm{d}s^2 = -h(r)\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{f(r)} + r^2\mathrm{d}\Omega^2$$
- In practical terms a wormhole has a throat (rather than an event horizon) where $f(r_0) = 0$ and $f \neq h$ so $h(r_0) \neq 0$.
- Wormhole : matter and light can tranverse in principle the throat both ways. If the throat were an event horizon then it d be a one way wormhole...
- The smaller $h(r_0)$ the bigger the redshift, the more black hole like is our wormhole

In GR wormholes are exotic objects. What about ST theories?

From a black hole to a wormhole [Bakopoulos, CC, Kanti]

- Start with the black hole :
$$\bar{h}(r) = \bar{f}(r) = 1 + \frac{r^2}{2\alpha}\left(1 - \sqrt{1 + \frac{8\alpha M}{r^3}}\right), \quad \bar{\phi}(r) = \int \mathrm{d}r \frac{\sqrt{\bar{h}} - 1}{r\sqrt{\bar{h}}}$$
- to construct the wormhole we undertake a disformal transformation, $g_{\mu\nu} = \bar{g}_{\mu\nu} - D(\bar{X})\, \nabla_\mu \bar{\phi}\, \nabla_\nu \bar{\phi}\,,$
- Disformal changes only the g_{rr} term : $h = \bar{h}\,, \qquad f = \frac{\bar{h}}{1+2D(\bar{X})\bar{X}} \equiv h\, W(\bar{X})^{-1}\,, \qquad \phi = \bar{\phi}\,,$
- Rough idea is to introduce a zero of f (but not h) ie., $f(r_0) = 0$ with $r_0 > r_h$
- In other words we need $W(\bar{X})^{-1} = 0|_{r=r_0}$ with $r_0 > r_h$.
- If we can glue together two patches of $[r_0, \infty]$ in a C^2, then we have a traversable wormhole



- Start with the black hole :
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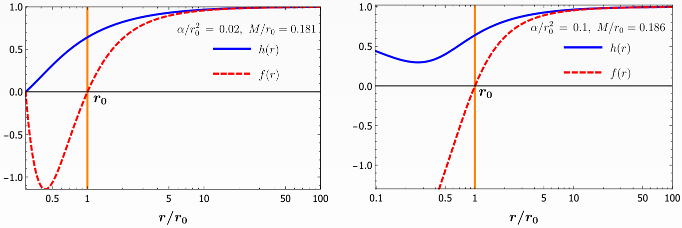

- Start with the black hole :

$$\bar{h}(r) = \bar{f}(r) = 1 + \frac{r^2}{2\alpha}\left(1 - \sqrt{1 + \frac{8\alpha M}{r^3}}\right), \quad \bar{\phi}(r) = \int \mathrm{d}r \frac{\sqrt{\bar{h}} - 1}{r\sqrt{\bar{h}}}$$

- to construct the wormhole we undertake a disformal transformation, $g_{\mu\nu} = \bar{g}_{\mu\nu} - D(\bar{X})\, \nabla_\mu \bar{\phi}\, \nabla_\nu \bar{\phi}$,
- Disformal changes only the g_{rr} term : $h = \bar{h}$, $\quad f = \frac{\bar{h}}{1+2D(\bar{X})\bar{X}} \equiv h\, W(\bar{X})^{-1}$, $\quad \phi = \bar{\phi}$,
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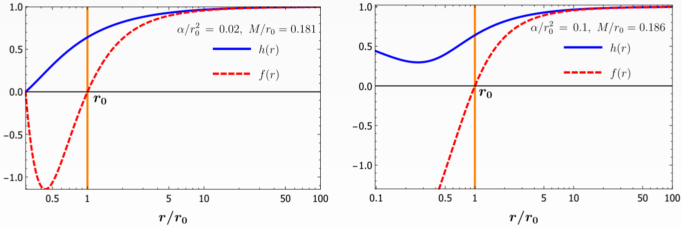
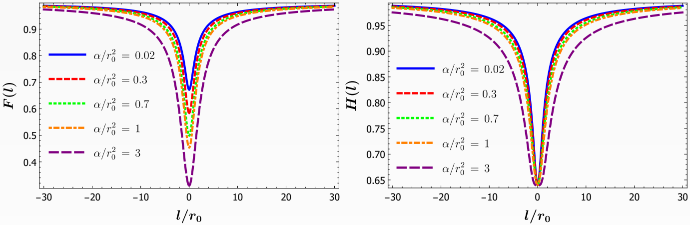


Wormhole solution [Bakopoulos, CC, Kanti]

- Choosing a generic shape function we can construct wormholes of variable mass and redshift
- For $M = 0$ we have flat spacetime, so throat is mass dependent
- Spacetime regular at $r = r0$. Consider, $r^2 = l^2 + r_0^2$
- $ds^2 = -H(l)\,dt^2 + \frac{dl^2}{F(l)} + (l^2 + r_0^2)\,d\Omega^2$, with $H(l) = h(r(l))$, and $F(l) = \frac{f(r(l))\,(l^2+r_0^2)}{l^2}$
- $H(l) = h_0 + h_2\,l^2 + O(l^4)\,, \qquad F(l) = f_0 + f_2\,l^2 + O(l^4)$



- The wormhole is therefore everywhere regular and needs no local or non local sources of matter. It is a vacuum solution just like black holes are in GR
- The throat is always a light ring or critical point! Light will accumulate at the throat of the wormhole

Conclusions

- There is an ongrowing multitude of hairy ST black holes and compact objects that we can construct
- We have stealth GR like solutions but also analytic rotating solutions which are non GR
- Starting from higher dimensional theories we can construct static black holes which are departing from GR
- We can construct neutron stars and wormholes with interesting and non trivial phenomenology
- nothing in general for stationary solutions apart from slow rotation
- are the solutions relevant? stable? resulting from gravitational collapse?