Perspectives on the Dark Sector

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Three Mysteries

- Nature of Dark Energy
- Nature of Dark Matter
- Origin of Cosmic Structure

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Three Simple Explanations

- **Nature of Dark Energy**
- Dark energy is a Cosmological Constant
- **Nature of Dark Matter**
- Dark matter is a WIMP
- **Origin of Cosmic Structure**
- Inflationary Scenario is the **current paradigm** of **early Universe cosmology.**

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- Ruled out based on the Trans-Planckian Censorship Conjecture et al.
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- WIMP explanation is constrained experimentally and **many alternatives** are being actively discussed.
- **Origin of Cosmic Structure**
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Back to the Drawing Board

- Dark Energy: “The only thing which we know about Dark Energy is that we do not know anything.”
- Dark Matter: A lot of alternatives to WIMPS have been proposed and make interesting predictions for astrophysics and cosmology.
- Origin of Structure: Good alternatives to inflation exist.

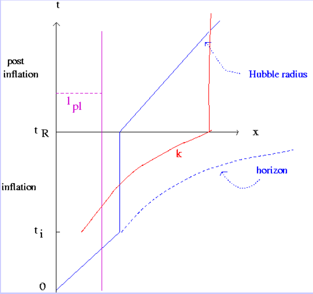
Outline

Plan

1. Trans-Planckian Censorship
2. Scenarios for a Successful Early Universe Cosmology
3. Unified Dark Sector Model
4. Emergent Cosmology from Matrix Theory
5. Conclusions

Trans-Planckian Problem

J. Martin and R.B., *Phys. Rev. D63, 123501 (2002)*



- **Success of inflation**: At early times scales are inside the Hubble radius → causal generation mechanism is possible.
- **Problem:** If time period of inflation is more than $70H^{-1}$, then $\lambda_p(t) < l_{pl}$ at the beginning of inflation.
- → breakdown of effective field theory; new physics MUST be taken into account when computing observables from inflation.

Trans-Planckian Censorship Conjecture (TCC)

A. Bedroya and C. Vafa., arXiv:1909.11063

No trans-Planckian modes exit the Hubble horizon.

$$ds^2 = dt^2 - a(t)^2 d\mathbf{x}^2$$

$$H(t) \equiv \frac{\dot{a}}{a}(t)$$

$$\boxed{\frac{a(t_R)}{a(t_i)} l_{pl} < H(t_R)^{-1}}$$

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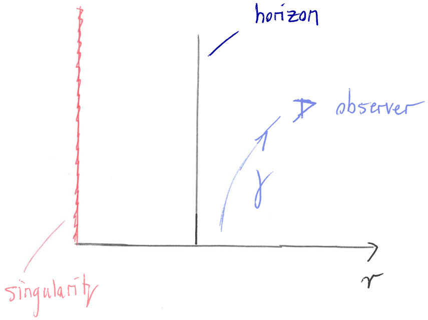
Justification

R.B. arXiv:1911.06056

Analogy with Penrose's Cosmic Censorship Hypothesis:



Black hole $Q < M$

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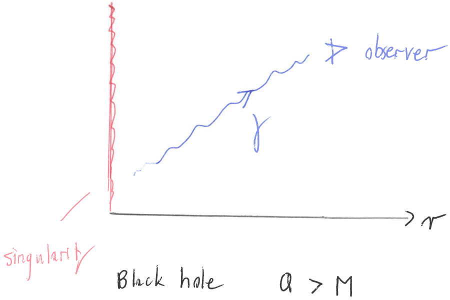

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- Effective field theory of General Relatiivty allows for solutions with timelike singularities: super-extremal black holes.
- → Cauchy problem not well defined for observer external to black holes.
- Evolution non-unitary for external observer.
- Conjecture: ultraviolet physics → **external observer** shielded from the singularity and non-unitarity by horizon.

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Cosmological Version of the Censorship Conjecture

Translation

- Position space → momentum space.
- Singularity → trans-Planckian modes.
- Black Hole horizon → Hubble horizon.

Observer measuring super-Hubble horizon modes must be shielded from trans-Planckian modes.

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Why Hubble Horizon?

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- Recall: Fluctuations only oscillate on sub-Hubble scales.
- Recall: Fluctuations freeze out, become **squeezed states** and **classicalize** on super-Hubble scales.
- Demand: **classical region be insensitive to trans-Planckian region.**
- → no trans-Planckian modes ever exit Hubble horizon.

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Unitarity Problem

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- Recall: non-unitarity of **effective field theory** in an expanding universe (N. Weiss, Phys. Rev. D32, 3228 (1985); J. Cotler and A. Strominger, arXiv:2201.11658).
- $\mathcal{H}$ is the product Hilbert space of a harmonic oscillator Hilbert space for all **comoving** wave numbers k
- UV cutoff: time dependent $k_{max} : k_{max}(t)a(t)^{-1} = m_{pl}$
- Continuous mode creation → non-unitarity.
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Effective Field Theory (EFT) and the CC Problem

- EFT: expand **fields** in comoving Fourier space.
- Quantize each Fourier mode like a harmonic oscillator $\rightarrow$ ground state energy.
- Add up ground state energies $\rightarrow$ CC problem.
- The usual quantum view of the CC problem is an artefact of an EFT analysis!

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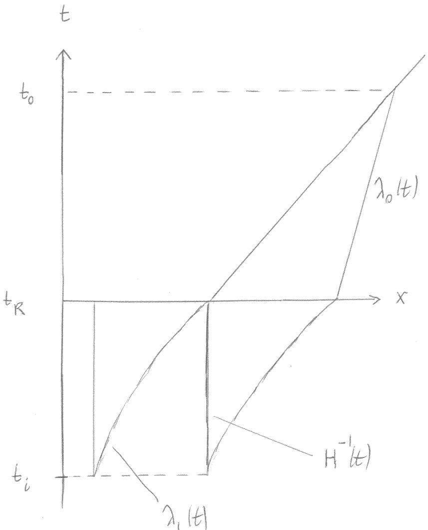
Application of the Second Law of Thermodynamics

S. Brahma, O. Alaryani and RB, arXiv:2005.09688

- Consider entanglement entropy density $s_E(t)$ between sub- and super-Hubble modes.
- Consider an **phase of inflationary expansion.**
- $s_E(t)$ increases in time since the phase space of super-Hubble modes grows.
- **Demand**: $s_E(t)$ remain smaller than the post-inflationary thermal entropy.
- → Duration of inflation is bounded from above, consistent with the TCC.

Application to EFT Description of Inflation

A. Bedroya, R.B., M. Loverde and C. Vafa., arXiv:1909.11106



Application to EFT Descriptions of Inflation

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TCC implies:

$$\frac{a(t_R)}{a(t_*)} l_{pl} < H(t_R)^{-1}$$

Demanding that inflation yields a causal mechanism for generating CMB anisotropies implies:

$$H_0^{-1} \frac{a(t_0)}{a(t_R)} \frac{a(t_R)}{a(t_*)} < H^{-1}(t_*)$$

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Implications

Upper bound on the **energy scale of inflation**:

$$V^{1/4} < 3 \times 10^9 \mathrm{GeV}$$

→ upper bound on the **primordial tensor to scalar ratio** r:

$$r < 10^{-30}$$

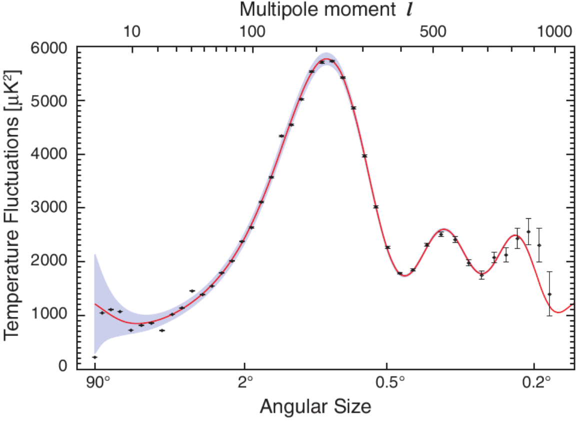
Note: Secondary tensors will be larger than the primary ones.

Implications for Dark Energy

Dark Energy cannot be a bare cosmological constant.

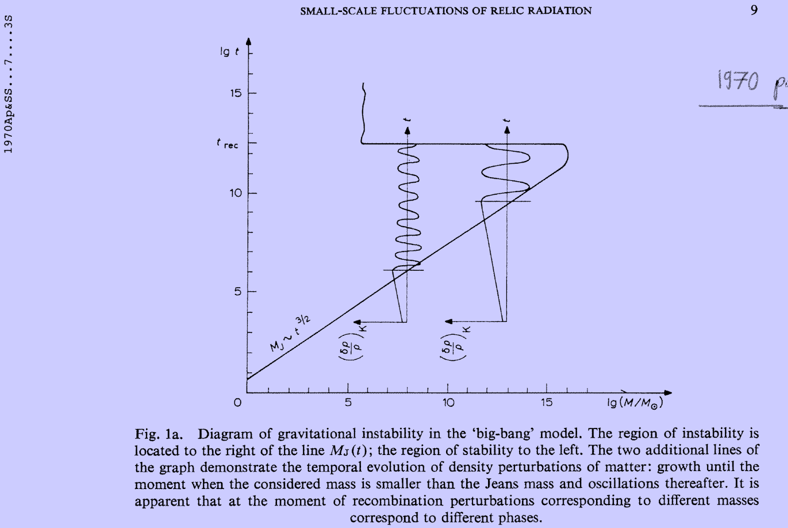
Plan

Angular Power Spectrum of CMB Anisotropies



Credit: NASA/WMAP Science Team

Early Work



1970 p

Fig. 1a. Diagram of gravitational instability in the 'big-bang' model. The region of instability is located to the right of the line $M_J(t)$; the region of stability to the left. The two additional lines of the graph demonstrate the temporal evolution of density perturbations of matter: growth until the moment when the considered mass is smaller than the Jeans mass and oscillations thereafter. It is apparent that at the moment of recombination perturbations corresponding to different masses correspond to different phases.

Key Realization

R. Sunyaev and Y. Zel'dovich, Astrophys. and Space Science **7**, 3 (1970); P. Peebles and J. Yu, Ap. J. **162**, 815 (1970).

- Given a scale-invariant power spectrum of adiabatic fluctuations on "super-horizon" scales before t_{eq}, i.e. standing waves.
- → "correct" power spectrum of galaxies.
- → acoustic oscillations in CMB angular power spectrum.

Early Work

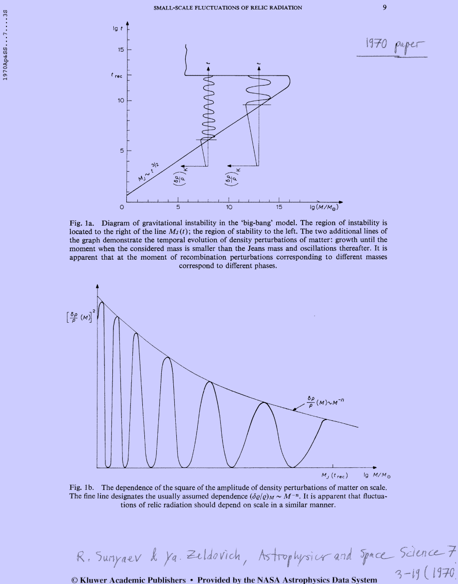


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Fig. 1b. The dependence of the square of the amplitude of density perturbations of matter on scale. The fine line designates the usually assumed dependence $(\delta\varrho/\varrho)_M \sim M^{-n}$. It is apparent that fluctuations of relic radiation should depend on scale in a similar manner.

R. Sunyaev & Ya. Zeldovich, Astrophysics and Space Science 7 3-19 (1970

Predictions from 1970

R. Sunyaev and Y. Zel'dovich, Astrophys. and Space Science **7**, 3 (1970); P. Peebles and J. Yu, Ap. J. **162**, 815 (1970).

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- → "correct" power spectrum of galaxies.
- → acoustic oscillations in CMB angular power spectrum.
- → baryon acoustic oscillations in matter power spectrum.

Criteria for a Successful Early Universe Scenario

- Horizon $\gg$ Hubble radius in order for the scenario to solve the “horizon problem” of Standard Big Bang Cosmology.
- Scales of cosmological interest today originate inside the Hubble radius at early times in order for a causal generation mechanism of fluctuations to be possible.
- Mechanism for producing a scale-invariant spectrum of curvature fluctuations on super-Hubble scales.

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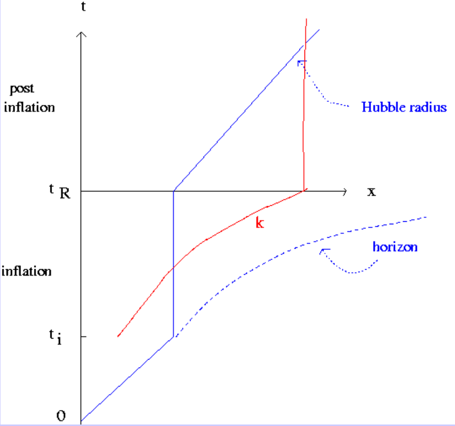
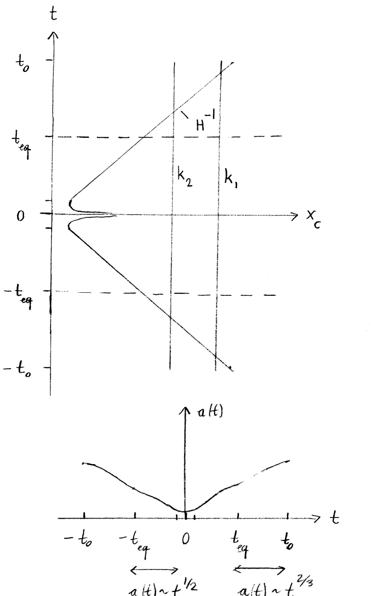
Inflation as a Solution



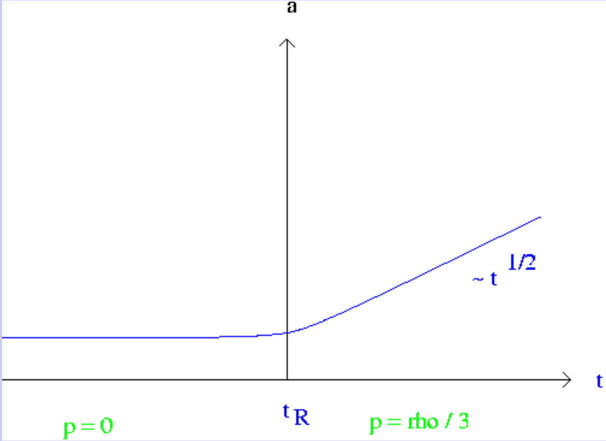
Bouncing Cosmology as a Solution

F. Finelli and R.B., *Phys. Rev. D65, 103522 (2002)*, D. Wands, *Phys. Rev. D60 (1999)*



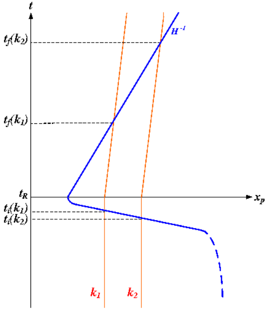
Emergent Universe

R.B. and C. Vafa, *Nucl. Phys. B316:391 (1989)*



Emergent Universe as a Solution

A. Nayeri, R.B. and C. Vafa, *Phys. Rev. Lett. 97:021302 (2006)*



Trans-Planckian Censorship and Cosmological Scenarios

- **Bouncing cosmologies** are consistent with the TCC provided that the energy scale at the bounce is lower than the Planck scale.
- **Emergent cosmologies** are consistent with the TCC provided that the energy scale of the emergence phase is lower than the Planck scale.
- **Inflationary cosmologies** are inconsistent with the TCC unless the energy scale of inflation is fine tuned.

All early universe scenarios require going beyond EFT.

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Motivation

- Quintessence is a much studied model for **dark energy**.
- **Quintessence realization**: Scalar field with a potential $V(\phi) = V_0 e^{-\phi/f}$ with $f > m_{pl}$.
- It would be nice to obtain several results for cosmology using one input.
- $\rightarrow$ search for unified dark sector model which also provides an explanation for the observed baryon to entropy ratio.

Unified Dark Sector Mode: Toy Modell

R.B. and J. Froehlich, **JCAP 04**, 030 (2021)

Consider a **complex scalar field** $z = \sigma + i\theta$ with potential

$$V(\sigma, \theta) \simeq \left[\Lambda + \frac{1}{2}\mu^4 \sin^2(\theta/f)\right] e^{-2\sigma/f}\,.$$

Note: canonical kinetic terms for σ and θ.

σ: dark energy field, slowly rolling

θ: dark matter field, oscillating about $\theta = 0$.

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Dynamics

R.B. and J. Froehlich, **JCAP 04**, 030 (2021)

- **Phase 1**: radiation dominates, θ frozen at $\theta \sim f$, σ slowly rolling.
- Note: **baryogenesis** phase.
- **Phase 2**: θ liberated, begins damped oscillations about $\theta = 0$, σ slowly rolling
- Note: dark matter comes to dominate: **dark matter phase.**
- **Phase 3**: energy density in σ becomes dominant $\rightarrow \sigma$ is rollilng.
- Note: **dark energy phase**

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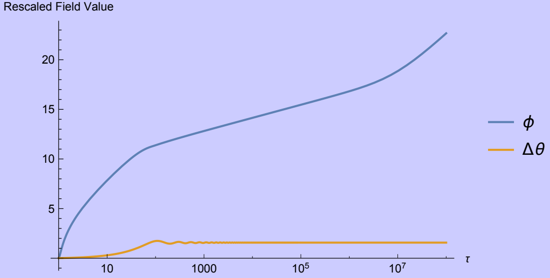
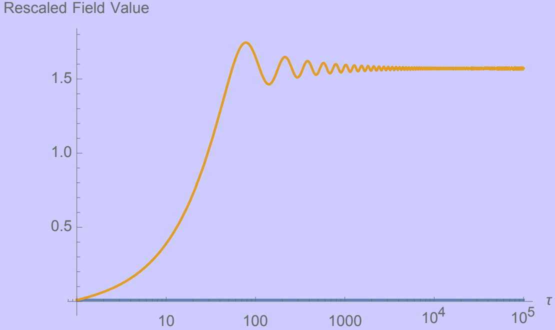
Field Evolution




Rescaled Field Value
1.5
1.0
0.5
10
100
1000
10^4
10^5
τ

Parameter Values

R.B. and J. Froehlich, **JCAP 04**, 030 (2021)

- Onset of Dark Energy domination at the present time t_0:

$$\Lambda e^{-2\sigma(t_0)/f} \sim T_0^4 z_{eq}$$

- Reproduce observed density of Dark Matter

$$\mu^4 \frac{\mathcal{A}^2(T_0)}{f^2} e^{-2\varphi(t_0)/f} \sim T_0^4 z_{eq}$$

- where $\mathcal{A}$ is the amplitude of oscillation of θ.
- Making use of $\mathcal{A} \sim T^{3/2}$ we obtain

$$\mu^4 \left(\frac{T_0}{T_c}\right)^3 \sim T_0^4 z_{eq} \, .$$

- where T_c is the temperature at the beginning of Phase 2.

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Predictions

Relationship between μ and the dark matter mass m_a and the phase transition temperature T_c:

$$\begin{aligned} \mu &\sim m_a^{1/2} 10^5 \mathrm{GeV} \\ T_c &\sim m_a 10^{14} \mathrm{GeV} \, . \end{aligned}$$

Towards a Unified Dark Sector Model from String Theory

H. Bernardo, R.B. and J. Froehlich, arXiv:2201.04668

- Starting point: **Heterotic superstring theory.**
- Model-independent axio-dilaton modulus field $S = e^{-\Phi} + ia$
- Volume modulus field $T = e^{\Psi} + ib$
- $a \rightarrow$ **dark matter field**
- $\Psi \rightarrow$ **dark energy field**

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Challenges

- How can we obtain low dark energy scale from string scale physics?
- How can we obtain a sufficiently flat potential?

Towards a Unified Dark Sector Model from String Theory

H. Bernardo, R.B. and J. Froehlich, arXiv:2201.04668

- Model-independent axio-dilaton modulus field $S = e^{-\Phi} + ia$
- a will be our dark matter field.
- Volume modulus field $T = e^{\Psi} + ib$
- Ψ will be related to our dark energy field.
- Assumption $b = 0$
- Gaugino condensation → dilaton fixed.
- Gaugino condensation → non-vanishing superpotential W

$$W = W_0 - Ae^{-a_0 S}$$

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- Standard Kaehler potential

$$\kappa^2 K = -\ln(S+\bar{S}) - 3\ln(T+\bar{T})$$

- Standard formula for the induced scalar potential:

$$V(S,T) = e^{\kappa^2 K}\left(K^{I\bar{J}} D_I W \overline{D_J W} - 3\kappa^2 |W|^2\right)$$

- Dimensional reduction and canonical field normalization.

$$\Psi = \sqrt{\frac{2}{3}}\frac{\sigma}{m_{pl}}$$

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Result

H. Bernardo, R.B. and J. Froehlich, arXiv:2201.04668

Resulting potential:

$$V(\phi,\sigma,a) = \frac{e^{-\sqrt{6}(\sigma/m_{pl})}}{8} e^{-\sqrt{2}(\phi/m_{pl})} \kappa^2 \Big[A^2 e^{-2a_0 e^{-\sqrt{2}(\phi/m_{pl})}} \Big(a_0 + \frac{1}{2} e^{\sqrt{2}(\phi/m_{pl})} \Big)^2 + \frac{W_0^2}{4} e^{2\sqrt{2}(\phi/m_{pl})} - A W_0 \Big(a_0 + \frac{1}{2} e^{\sqrt{2}(\phi/m_{pl})} \Big) e^{\sqrt{2}(\phi/m_{pl})} e^{-a_0 e^{-\sqrt{2}(\phi/m_{pl})}} \cos(a_0 a) \Big] .$$

Key features:

- Common factor of $e^{-\sqrt{6}\sigma/m_{pl}}$
- Axion potential has the right form .
- No term of the form $\Lambda e^{-b\sigma}$.

Comparison

$$V(\sigma,\theta) \simeq \left[\Lambda + \frac{1}{2}\mu^4 \sin^2(\theta/f)\right] e^{-2\sigma/f} \, .$$

Towards a Unified Dark Sector Model from String Theory

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Integrate out fluctuations of the axio-dilaton field.

$$V_{1l} \approx \frac{\Lambda_c^2 \gamma^2}{16\pi^2} e^{-\sqrt{6}\psi}$$

$$\begin{aligned} m_{\tilde{g}}^2 &= \kappa^4 \left\langle e^{\kappa^2 K} |W|^2 \right\rangle = \kappa^4 \frac{\left(2Aa_0 S_0 e^{-a_0 S_0}\right)^2}{2S_0(T+\bar{T})^3} \\ &= \kappa^4 \frac{A^2 a_0 (2a_0 S_0) e^{-2a_0 S_0}}{8} e^{-\sqrt{6}\psi} =: \gamma^2 e^{-\sqrt{6}\psi}, \end{aligned}$$

Towards a Unified Dark Sector Model from String Theory

H. Bernardo, R.B. and J. Froehlich, arXiv:2201.04668

Conclusions:

- A unified dark sector action arises naturally from heterotic superstring theory.
- Mild tuning of S_0 yields the required dark energy scale.
- To obtain a **sufficiently flat potential** need to keep 5 of the 6 internal dimensions fixed.
- Axion mass: $m_\theta \sim 10^{-24} e^{\sqrt{1/6}\psi} \mathrm{eV}$
- → ultralight DM.

Plan

Matrix Theory Cosmology

S. Brahma, R.B. and S. Laliberte, arXiv:2108.1152

Starting point: BFSS matrix model at high temperatures.

- BFSS model is a quantum mechanical model of 10 $N \times N$ Hermitean matrices.
- Note: no space!
- Note: no singularities!
- Note: BFSS matrix model is a proposed non-perturbative definition of M-theory: 10 dimensional superstring theory emerges in the $N \to \infty$ limit.

BFSS Model (bosonic sector)

T. Banks, W. Fischler, S. Shenker and L. Susskind, Phys. Rev. D **55**, 5112 (1997)

$$L = \frac{1}{2g^2}\left[\mathrm{Tr}\left(\frac{1}{2}(D_t X_i)^2 - \frac{1}{4}[X_i, X_j]^2\right)\right]$$

- $X_i, i = 1, ...9$ are $N \times N$ Hermitean matrices.
- D_t: gauge covariant derivative (contains a matrix A_0)

't Hooft limit: $N \to \infty$ with $\lambda \equiv g^2 N = g_s l_s^{-3} N$ fixed.

Thermal Initial State

N. Kawahara, J. Nishimura and S. Takeuchi, JHEP **12**, 103 (2007)

- Consider a high temperature state.
- At high temperatures, the bosonic sector of the (Euclidean) BFSS model is equivalent to the bosonic sector of the (Euclidean) IKKT matrix model.
- Matsubara expansion:

$$X_i(t) = \sum_n X_i^n e^{2\pi i T t}$$

$$A_i \equiv T^{-1/4} X_i^0$$

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IKKT Matrix Model

N. Ishibashi, H. Kawai, Y. Kitazawa and A. Tsuchiya, Nucl. Phys. B **498**, 467 (1997).

Proposed as a non-perturbative definition of the IIB Superstring theory.

Action:

$$S = -\frac{1}{g^2}\mathrm{Tr}\Big(\frac{1}{4}[A^a, A^b][A_a, A_b] + \frac{i}{2}\bar{\psi}_\alpha(\mathcal{C}\Gamma^a)_{\alpha\beta}[A_a, \psi_\beta]\Big) ,$$

Partition function:

$$Z = \int dA d\psi e^{iS}$$

Matrix Theory Cosmology

Y. Ito, J. Nishimura and A. Tsuchiya, arXiv:1506.04795

- Eigenvalues of A_0 become emergent time.
- Work in the basis in which A_0 is diagonal.
- Numerical studies: $\frac{1}{N}\langle \mathrm{Tr} A_0^2 \rangle \sim \kappa N$
- $\rightarrow t_{max} \sim \sqrt{N}$
- $\rightarrow \Delta t \sim \frac{1}{\sqrt{N}}$
- $\rightarrow$ infinite continuous time.

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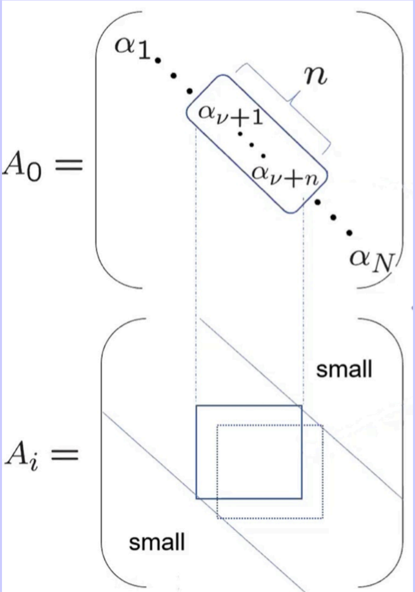
- $\sum_i \langle |A_i|^2_{ab} \rangle$ decays when $|a - b| > n_c$
- $\sum_i \langle |A_i|^2_{ab} \rangle \sim$ constant when $|a - b| < n_c$
- $n_c \sim \sqrt{N}$

Matrix Theory Cosmology

S. Kim, J. Nishimura and A. Tsuchiya, arXiv:1108.1540

- Eigenvalues of A_0 become emergent time, continuous in $N \to \infty$ limit.
- Work in the basis in which A_0 is diagonal: A_i matrices elements decay when going away from the diagonal.
- Pick $n \times n$ blocks $\tilde{A}_i(t)$ about the diagonal ($n < n_c$)


$A_0 =$
α_1
n
$\alpha_{\nu+1}$
$\alpha_{\nu+n}$
α_N
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small
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Matrix Theory Cosmology

J. Nishimura, PoS CORFU **2019**, 178 (2020) [arXiv:2006.00768 [hep-lat]].

- Eigenvalues of A_0 become emergent time, continuous in $N \to \infty$ limit.
- Work in the basis in which A_0 is diagonal.
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- Extent of space in direction i

$$x_i(t)^2 \equiv \left\langle \frac{1}{n}\mathrm{Tr}(\bar{A}_i)(t))^2 \right\rangle ,$$

- In a thermal state there is spontaneous symmetry breaking: $SO(9) \to SO(6) \times SO(3)$: three dimensions of space become larger, the others are confined. [J. Nishimura and G. Vernizzi, JHEP **0004**, 015 (2000);]S.-W. Kim, J. Nishimura and A. Tsuchiya, Phys. Rev. Lett. **109**, 011601 (2012)]

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S. Brahma, R.B. and S. Laliberte, in preparation

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- Work in the basis in which A_0 is diagonal: pick n (comoving spatial coordinate) and consider the block matrix $\tilde{A}_i(t)$.
- Physical distance between $n = 0$ and n (emergent space):

$$l^2_{phys,i}(n) \equiv \left\langle \mathrm{Tr}(\bar{A}_i)(t))^2 \right\rangle ,$$

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- Emergent metric (S. Brahma, R.B. and S. Laliberte, in preparation).

$$g_{ii}(n)^{1/2} = \frac{d}{dn} l_{phys,i}(n)$$

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Matrix Theory Cosmology: Results

S. Brahma, R.B. and S. Laliberte, in preparation

Emergent metric:

$$g_{ii}(n)^{1/2} = \frac{d}{dn} l_{phys,i}(n)$$

Result:

$$g_{ii}(n,t) = \mathcal{A}(t)\delta_{ii} \quad i = 1, 2, 3$$

$SO(3)$ symmetry $\rightarrow$

$$g_{ij}(n,t) = \mathcal{A}(t)\delta_{ij} \quad i = 1, 2, 3$$

$\rightarrow$ spatially flat.

Note: Local Lorentz invariance emerges in $N \rightarrow \infty$ limit.

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Late Time Dynamics

$$\mathcal{A}(t) \sim t^{1/2}$$

Note: no sign of a cosmological constant.

Matrix Theory Cosmology

S. Brahma, R.B. and S. Laliberte, arXiv:2108.1152

- We assume that the spontaneous symmetry breaking observed in the IKKT model also holds in the BFSS model.
- Method: generalize the Gaussian approximation method used to demonstrate the existence of the phase transition in the IKKT model to the BFSS theory (S. Brahma et al, in preparation).
- **Thermal correlation functions** in the three large spatial dimensions calculated in the high temperature state of the BFSS model (following the formalism developed in String Gas Cosmology).
- → curvature fluctuations and gravitational waves.

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Matrix Theory Cosmology: Thermal Fluctuations

S. Brahma, R.B. and S. Laliberte, arXiv:2108.1152

Method:

- Consider BFSS finite temperature partition function
- Take partial derivatives with respect to T and R_i
- Obtain energy density and pressure fluctuations.

Matrix Theory Cosmology: Results

S. Brahma, R.B. and S. Laliberte, arXiv:2108.1152

Obtain the same results as in **String Gas Cosmology**.

- Scale-invariant spectrum of curvature fluctuations
- With a Poisson contribution for UV scales.
- Scale-invariant spectrum of gravitational waves.

→ BFSS matrix model yields emergent infinite space, emergent infinite time, emergent spatially flat metric and an emergent early universe phase with thermal fluctuations leading to scale-invariant curvature fluctuations and gravitational waves.

Note: Horizon problem automatically solved.

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Open Problems

- Understand **phase transition** to the expanding phase of Big Bang Cosmology.
- Spectral indices?
- What about Dark Energy?

Plan

Conclusions

- In light of the TCC and other conceptual problems Dark Energy cannot be a cosmological constant.
- In light of the TCC we need to go beyond point particle EFT in order to describe the very early universe.
- Starting from heterotic superstring theory a unified dark sector model can be obtained without too much fine tuning.

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Conclusions

- BFSS matrix model is a proposal for a non-perturbative definition of superstring theory. Consider a high temperature state of the BFSS model.
- → emergent time, space and metric. Emergent space is spatially flat and infinite.
- **Thermal fluctuations** of the BFSS model → scale-invariant spectra of cosmological perturbations and gravitational waves.
- Transition from an emergent phase to the radiation phase of expansion. No cosmological constant.

Obtaining an Emergent Cosmology: String Gas Cosmology

R.B. and C. Vafa, *Nucl. Phys. B316:391 (1989)*

Idea: make use of the new symmetries and new degrees of freedom which string theory provides to construct a new theory of the very early universe.
Assumption: Matter is a gas of fundamental strings
Assumption: Space is compact, e.g. a torus.
Key points:

- New degrees of freedom: string oscillatory modes
- Leads to a maximal temperature for a gas of strings, the Hagedorn temperature
- New degrees of freedom: string winding modes
- Leads to a new symmetry: physics at large R is equivalent to physics at small R

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T-Duality

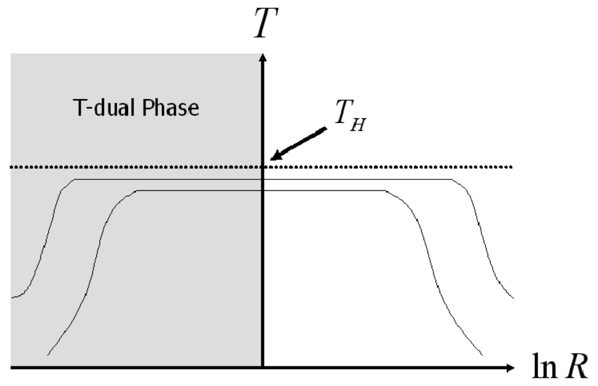
T-Duality

- Momentum modes: $E_n = n/R$
- Winding modes: $E_m = mR$
- Duality: $R \rightarrow 1/R \quad (n, m) \rightarrow (m, n)$
- Mass spectrum of string states unchanged
- Symmetry of vertex operators
- Symmetry at non-perturbative level → existence of D-branes

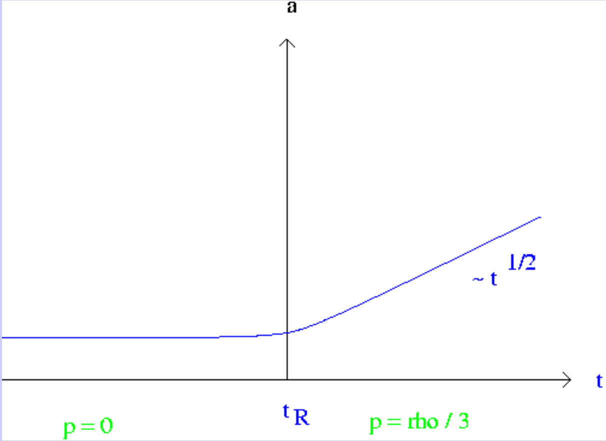
Adiabatic Considerations

R.B. and C. Vafa, *Nucl. Phys. B316:391 (1989)*

Temperature-size relation in string gas cosmology



Background for string gas cosmology



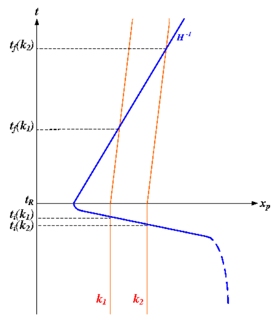
Challenge for Emergent Cosmologies

Dynamics of the emergent phase?

Require an analysis beyond EFT.

Structure formation in string gas cosmology

A. Nayeri, R.B. and C. Vafa, *Phys. Rev. Lett. 97:021302 (2006)*



N.B. Perturbations originate as thermal string gas fluctuations.

Method

- Calculate matter correlation functions in the Hagedorn phase (neglecting the metric fluctuations)
- For fixed k, convert the matter fluctuations to metric fluctuations at Hubble radius crossing $t = t_i(k)$
- Evolve the metric fluctuations for $t > t_i(k)$ using the usual theory of cosmological perturbations

Note: the matter correlation functions are given by partial derivatives of the **finite temperature string gas partition function** with respect to T (density fluctuations) or R (pressure perturbations).

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Extracting the Metric Fluctuations

Ansatz for the metric including cosmological perturbations and gravitational waves:

$$ds^2 \, = \, a^2(\eta)\big((1+2\Phi)d\eta^2 - [(1-2\Phi)\delta_{ij} + h_{ij}]dx^i dx^j\big) \, .$$

Inserting into the perturbed Einstein equations yields

$$\langle|\Phi(k)|^2\rangle \, = \, 16\pi^2 G^2 k^{-4} \langle\delta T^0{}_0(k) \delta T^0{}_0(k)\rangle \, ,$$

$$\langle|h(k)|^2\rangle \, = \, 16\pi^2 G^2 k^{-4} \langle\delta T^i{}_j(k) \delta T^i{}_j(k)\rangle \, .$$

Power Spectrum of Cosmological Perturbations

Key ingredient: For thermal fluctuations:

$$\langle \delta\rho^2 \rangle = \frac{T^2}{R^6} C_V .$$

Key ingredient: For string thermodynamics in a compact space

$$C_V \approx 2 \frac{R^2/\ell_s^3}{T(1 - T/T_H)} .$$

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Power spectrum of cosmological fluctuations

$$\begin{aligned} P_\Phi(k) &= 8G^2 k^{-1} < |\delta\rho(k)|^2 > \\ &= 8G^2 k^2 < (\delta M)^2 >_R \\ &= 8G^2 k^{-4} < (\delta\rho)^2 >_R \\ &= 8G^2 \frac{T}{\ell_s^3} \frac{1}{1 - T/T_H} \end{aligned}$$

Key features:

- scale-invariant like for inflation
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Spectrum of Gravitational Waves

R.B., A. Nayeri, S. Patil and C. Vafa, *Phys. Rev. Lett. (2007)*

$$\begin{aligned} P_h(k) &= 16\pi^2 G^2 k^{-1} < |T_{ij}(k)|^2 > \\ &= 16\pi^2 G^2 k^{-4} < |T_{ij}(R)|^2 > \\ &\sim 16\pi^2 G^2 \frac{T}{\ell_s^3}(1 - T/T_H) \end{aligned}$$

Key ingredient for string thermodynamics

$$< |T_{ij}(R)|^2 > \sim \frac{T}{l_s^3 R^4}(1 - T/T_H)$$

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Where do we stand?

- String Gas Cosmology appears to be an alternative to inflation for generating the structures we see in cosmological observations.
- String Gas Cosmology: nonsingular, solves the horizon problem.
- Achilles heel: how do we describe the emergent phase mathematically?

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Relationship between IKKT Model and Type IIB String Theory

Consider action of the Type IIB string theory in Schild gauge

$$S_{\rm Schild} = \int d^2\sigma\alpha\left[\sqrt{g}\left(\frac{1}{4}\{X^\mu, X^\nu\} - \frac{i}{2}\bar{\psi}\Gamma^\mu\{X^\mu, \psi\}\right) + \beta\sqrt{g}\right].$$

$$\text{Partition function}: Z = \int \mathcal{D}\sqrt{g}\mathcal{D}X\mathcal{D}\psi e^{-S_{\rm Schild}}.$$

$$\begin{aligned} \text{Correspondence}: \quad \{,\} &\rightarrow -i[,] \\ \int d^2\sigma\sqrt{g} &\rightarrow \mathrm{Tr} \end{aligned}$$

Obtain grand canonical partition function of IKKT model.

Some Details

Starting point: finite temperature partition function:

$$Z(\beta) = \int \mathcal{D}A\mathcal{D}X_i e^{-S(\beta)}$$

Internal energy

$$E = -\frac{d}{d\beta}\ln Z(\beta)$$

$$E = -\frac{3}{4}\lambda^{-1}\frac{N}{\beta}\int_0^\beta dt \mathrm{Tr}[X_i,X_j]^2$$

Matsubara expansion:

$$X_i = \sum_n X_i^n e^{i(2\pi n\ \beta)t}$$

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Matsubara expansion of the action:

$$S_{BFSS} = S_0 + S_{kin} + S_{int}$$

At high temperature: S_{kin} and S_{int} suppressed compared to S_0.

To next to leading order:

$$E \simeq \lambda^{-1}\frac{3N^2}{4}\chi_2 T - \lambda^{-1}\frac{3N^2}{4}\mathcal{O}(1)\chi_2\chi_1 T^{-1/2}$$

where $\chi_1 \simeq R^2\lambda^{4/3}T^{-1/2}$.

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Some Details

- Derivative w.r.t. $T \rightarrow$ density fluctuations: both terms contribute.
- Derivative w.r.t. $R \rightarrow$ pressure fluctuations: only second term contributes.

Power spectrum $P(k)$ of density fluctuations: $(k = R^{-1})$

- First term dominates in the UV: Poisson spectrum.
- Second term dominated in the IR: Scale-invariant spectrum.

$$P(k) = 16\pi^2 G^2 \lambda^{4/3} N^2 \mathcal{O}(1) \sim (l_s m_{pl})^{-4}$$

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Obtaining Inflation

- EFT framework: General Relativity + field theory matter.
- → require matter with $p < -\frac{1}{3}\rho$.
- Consider scalar field φ as matter: potential energy term has an equation of state $p = -\rho$.
- But one needs to ensure that potential energy dominates over other forms of energy!
- Require a slowly rollling scalar field:

$$\frac{V'}{V} \ll \frac{1}{m_{pl}} .$$

- Require rolling over large distances

$$\Delta\varphi > m_{pl} .$$

- → EFT breaks down.

Challenge for Inflation

- TCC → require non-perturbative analysis of inflation.
- Constructions exist:
- G. Dvali et al: inflationary phase as a **condensate of gravitons** about Minkowski space-time (arXiv:1701.08776 [hep-th]).
- H. Bernardo, S. Brahma, K. Dasgupta et al: inflationary phase as a **coherent state in string theory** (arXiv:2007.00786 [hep-th]; arXiv:2007.11611 [hep-th]; arXiv:2009.04504 [hep-th]).

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Obtaining an Ekpyrotic Bounce

J. Khoury, B. Ovrut, P. Steinhardt and N. Turok, Phys. Rev. D, 2001

- Space-time described by Einstein-Hilbert action.
- Idea: Slow contraction given by matter with equation of state $w \gg 1$.
- Obtained by assuming that matter is dominated by a scalar field φ with a negative exponential potential.
- Anisotropies diluted, creates spatial flatness
- Global attractor in initial condition space (A. Ijjas et al, arXiv:2103.00584)
- Note: **Negative exponential potentials are ubiquitous in string theory.**

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Challenge for Bouncing Cosmologies

Challenges for Ekpyrotic Cosmology:

- How do we get the bounce?
- How do we obtain a scale-invariant spectrum of curvature fluctuations?
- Can we obtain a spectrum of gravitational waves relevant to current observations?

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S-Brane and Ekpyrosis

RB and Z. Wang, arXiv:2001.00638, arXiv:2004.06437

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Adding an S-Brane to the EFT action can solve all three problems, and leads to two consistency relations for cosmological observables.

Action

Idea: At the string scale a new tower of string states becomes comparable in mass to the usual low energy degrees of freedom;
→ they must be included in the low energy effective action.

Included as an S-Brane.

$$S = \int d^4x\sqrt{-g}\left[R + \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - V(\varphi)\right] - \int d^4x\kappa\delta(\tau-\tau_B)\sqrt{\gamma}\,,$$

$$\kappa \equiv N\eta_s^3\,,$$

Note: The S-brane has $\rho = 0$ and $p < 0$ → can mediate the transition between contraction and expansion.

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