Precision predictions for the Migdal effect

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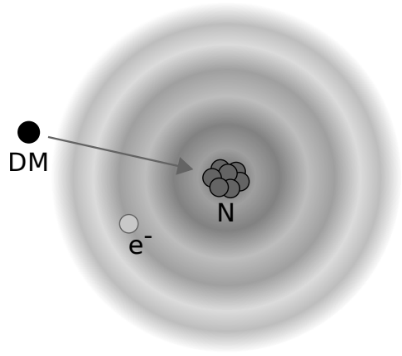
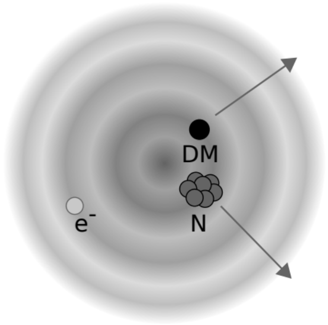
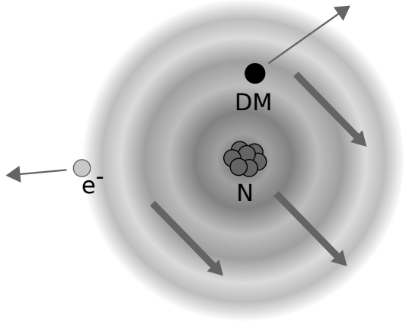
Migdal effect

Migdal 1939

- Ionisation/excitation due to displacement of nucleus after nuclear recoil

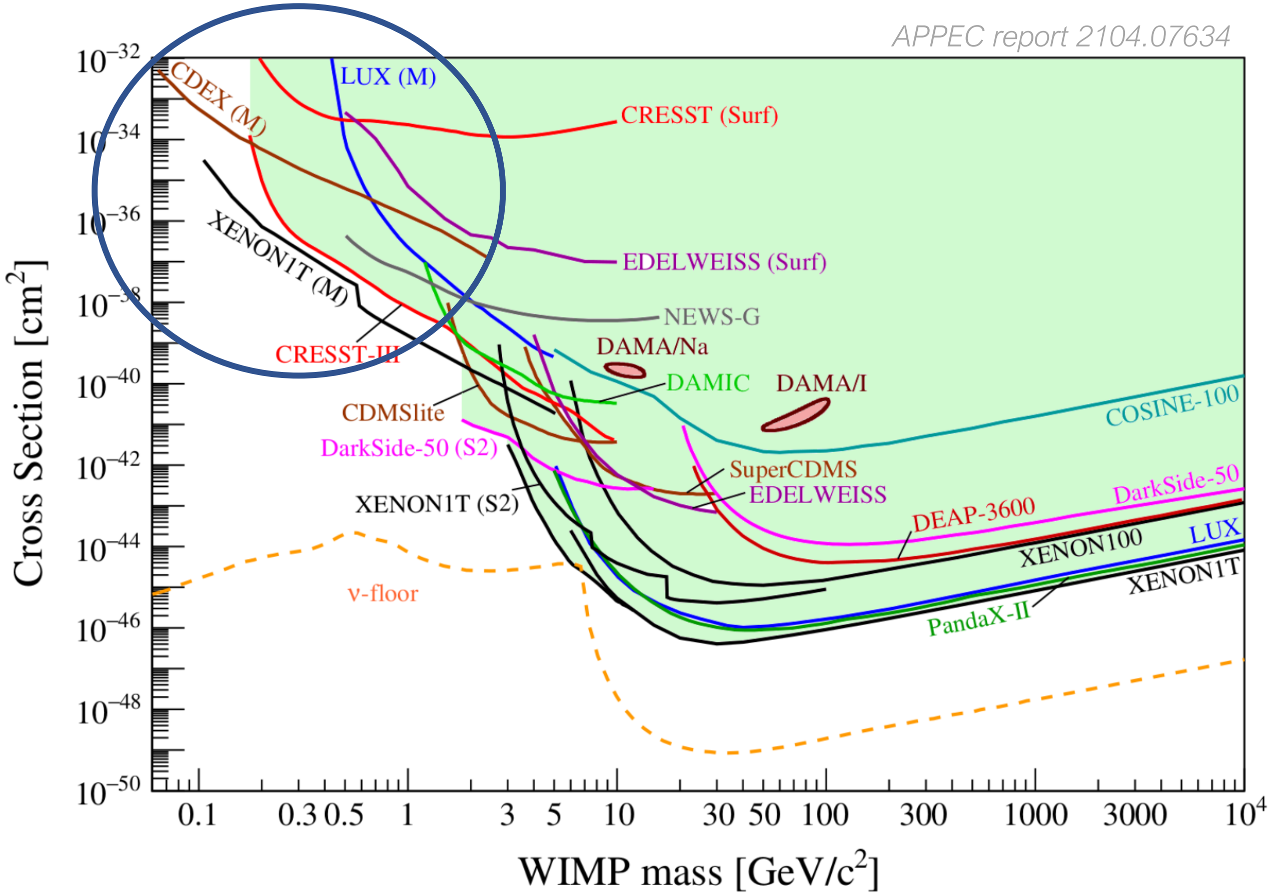






- Migdal effect observed in $\alpha, \beta^{\pm}$ decays
- Yet to be observed in neutron scattering

Migdal effect for sub-GeV DM



APPEC report 2104.07634

Elastic DM-nucleus scattering:

$$E_{NR} = \frac{q^2}{2m_N} \le \frac{2\mu^2 v_\chi^2}{m_N}$$

$$E_{NR}^{\max} \sim \underline{0.1\,\mathrm{keV}} \left(\frac{131}{A}\right)\left(\frac{m_\chi}{\mathrm{GeV}}\right)^2$$

Migdal (inelastic):

$$\omega = \boldsymbol{v}\cdot\boldsymbol{q} - \frac{q^2}{2m_\chi} \le \frac{1}{2}\mu v_\chi^2$$

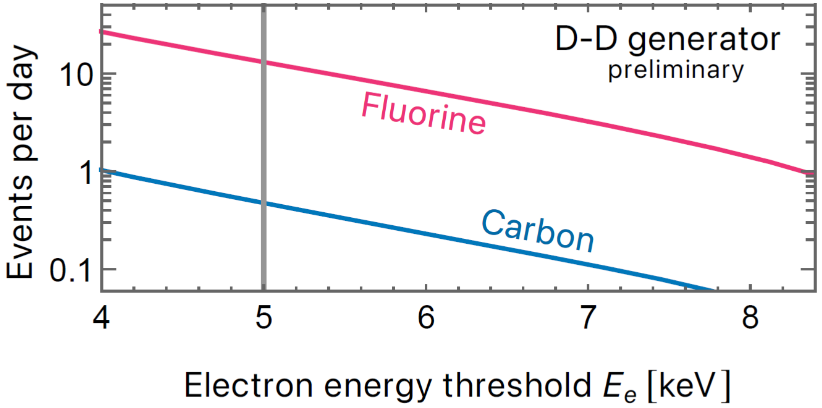
$$\omega_{\max} \sim \underline{3\,\mathrm{keV}}\left(\frac{m_\chi}{\mathrm{GeV}}\right)$$

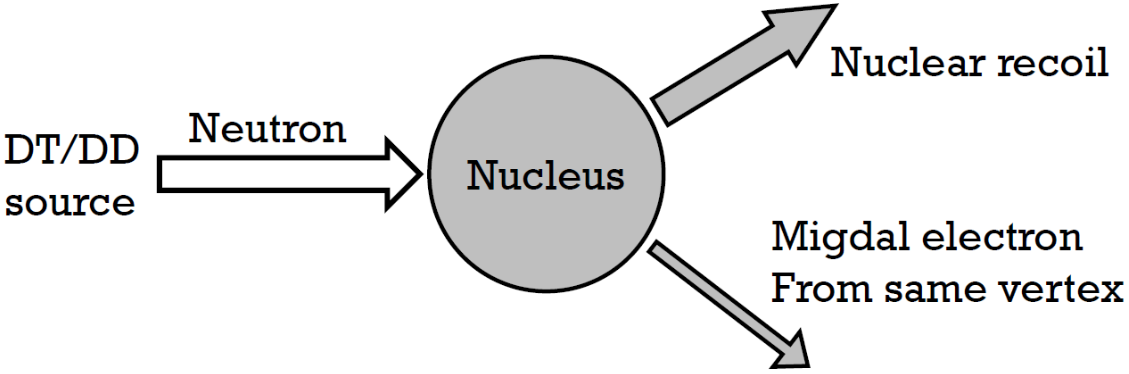
MIGDAL

Migdal In Galactic Dark mAtter expLoration

- Observe Migdal effect in neutron scattering using optical TPC

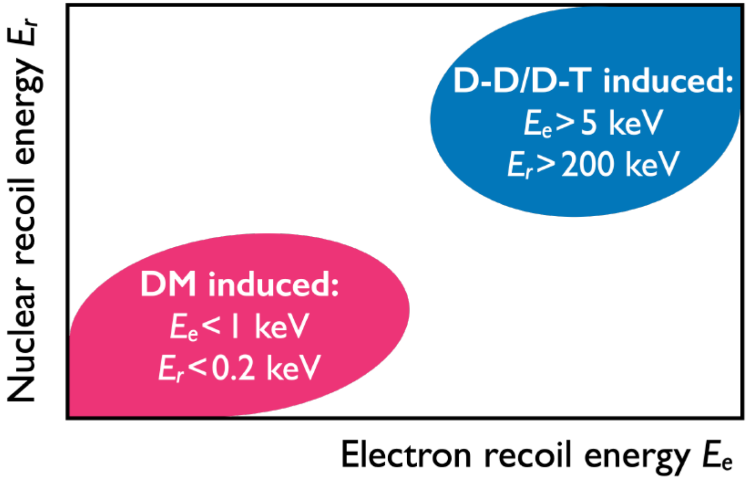
- Phase 1: $\mathrm{CF_4}$
  Phase 2: $\mathrm{CF_4}$ + noble gases





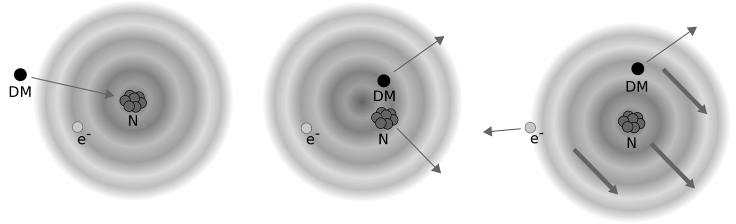
D-D: E_n = 2.45 MeV, flux 10^9 n/s

D-T: E_n = 14.1 MeV, flux 10^{10} n/s



Calculating the Migdal effect

Migdal 1939



- Impulse approximation

- In rest frame of nucleus, wavefunction of moving electron cloud obtained by Galilean boost:

$$\left|\Psi'_{ec}\right\rangle = U(\boldsymbol{v})\left|\Psi_{ec}\right\rangle = e^{-im_e \sum_i \boldsymbol{v}\cdot\boldsymbol{r}_i}\left|\Psi_{ec}\right\rangle$$

Transition probability:

$$P(i \to f) = \left|\left\langle \Psi_f \middle| \exp\left(im_e \sum_{i=1}^{N} \boldsymbol{v}\cdot\boldsymbol{r}_i\right) \middle| \Psi_i \right\rangle\right|^2$$

eigenstates of $v = 0$ Hamiltonian

Migdal effect also considered in

- Molecules (Yoni Kahn’s talk)
- Semiconductors (e.g. Knapen et. al. ’20; Liang et. al. ‘22)

Dipole approximation

$$\left\langle \Psi_f \middle| \exp\left(i m_e \sum_{i=1}^{N} \boldsymbol{v} \cdot \boldsymbol{r}_i \right) \middle| \Psi_i \right\rangle$$

Usual approach: dipole approximation

$$\exp\left(i m_e \boldsymbol{v} \cdot \sum_{i=1}^{N} \boldsymbol{r}_i \right) \to i m_e \boldsymbol{v} \cdot \sum_{i=1}^{N} \boldsymbol{r}_i$$

Expected to breakdown when

$$v \gtrsim (a_0 m_e)^{-1} \sim 0.007$$

- Reduces to single electron matrix element: $\langle \chi_j | i m_e \boldsymbol{v} \cdot \boldsymbol{r} | \psi_i \rangle$
- Migdal probability $\propto v^2$

Theory improvements

$$\left\langle \Psi_f \middle| \exp\left(i m_e \sum_{i=1}^{N} \boldsymbol{v} \cdot \boldsymbol{r}_i \right) \middle| \Psi_i \right\rangle$$

Beyond the dipole approximation

- Wavefunction is Slater determinant of single electron orbitals:

$$\Psi(\boldsymbol{r}_1, \boldsymbol{r}_2 \ldots \boldsymbol{r}_N) = \mathcal{A}\left(\psi_1(\boldsymbol{r}_1)\psi_2(\boldsymbol{r}_2)\ldots\psi_N(\boldsymbol{r}_N)\right)$$

- Full matrix element can be written in terms of single electron matrix elements:

$$A_{ji} = \langle \chi_j | \exp(i m_e \boldsymbol{v} \cdot \boldsymbol{r}) | \psi_i \rangle$$

$$P = \det\left(A^\dagger A\right)$$

(Talman & Frolov '06)

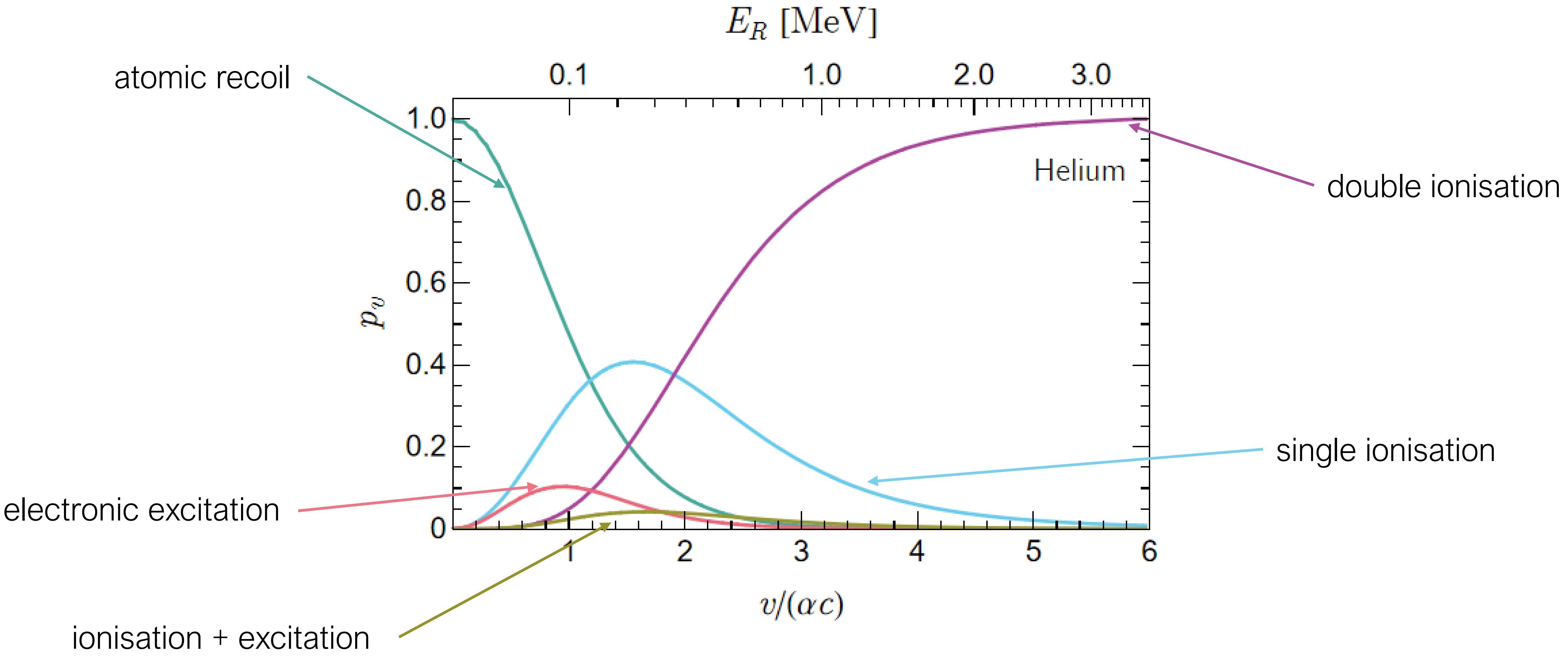
Dirac-Hatree-Fock method

- Relativistic effects important for large atoms
- Include full non-local exchange potential
  (c.f. local effective potential in previous calculations)

Use two complementary approaches:

- Gaussian basis set method (BERTHA)
- Finite difference self-consistent field (GRASP/RATIP)

Beyond dipole: multiple ionisation



atomic recoil

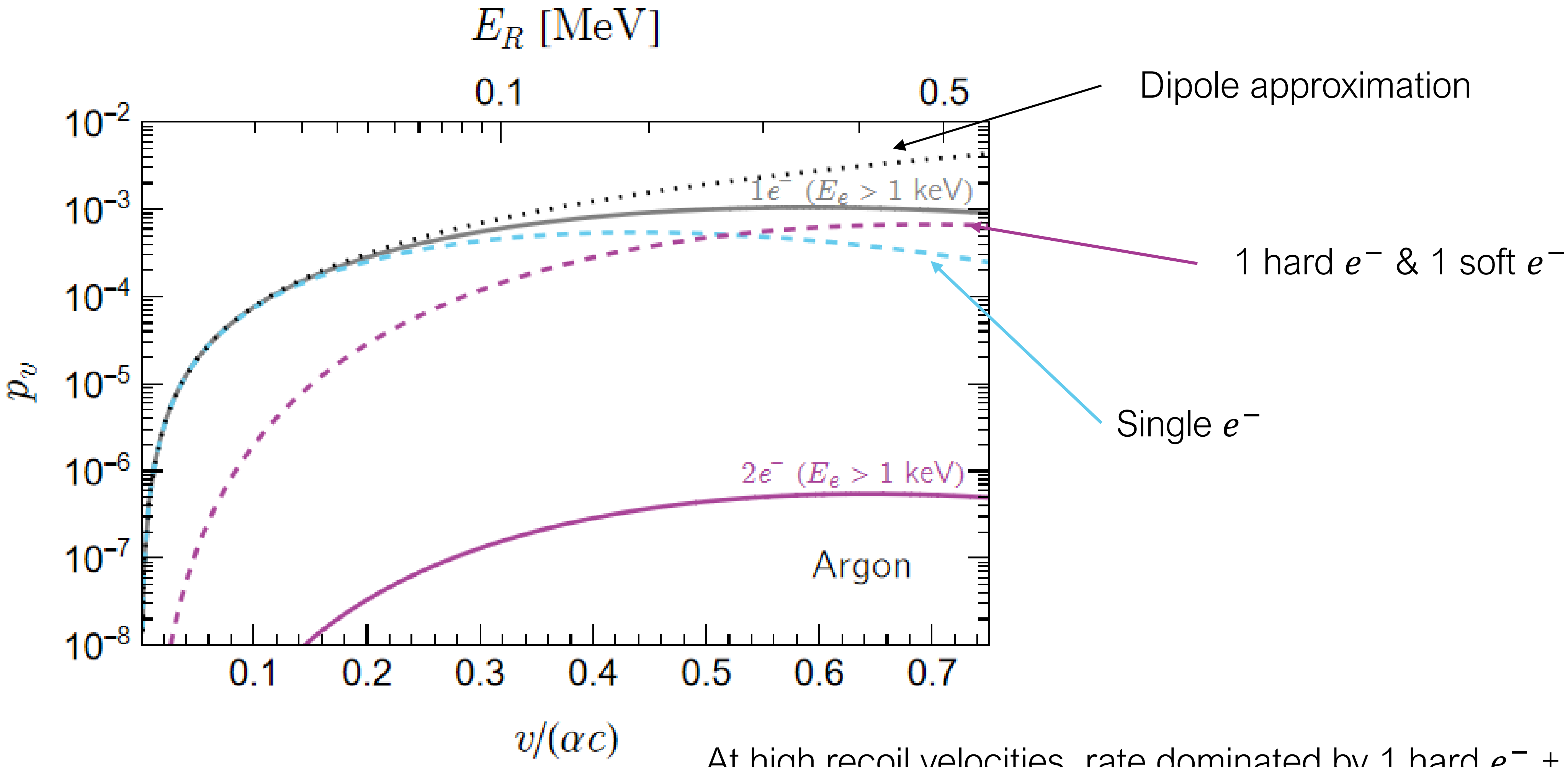
double ionisation

single ionisation

electronic excitation

ionisation + excitation

Single & double ionisation



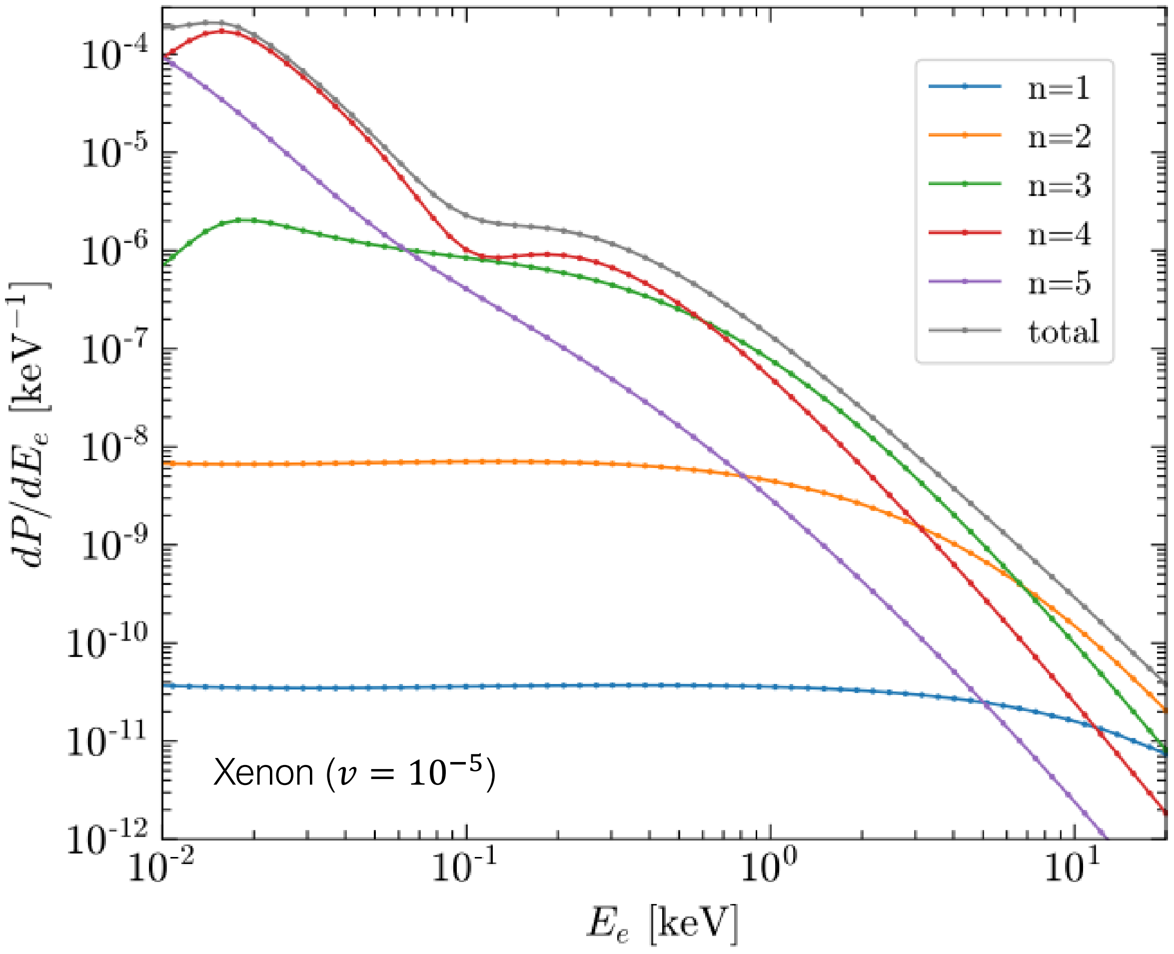
At high recoil velocities, rate dominated by 1 hard e^- + soft electrons

Differential probabilities

- Valence electrons dominate rate at very low e^- energies

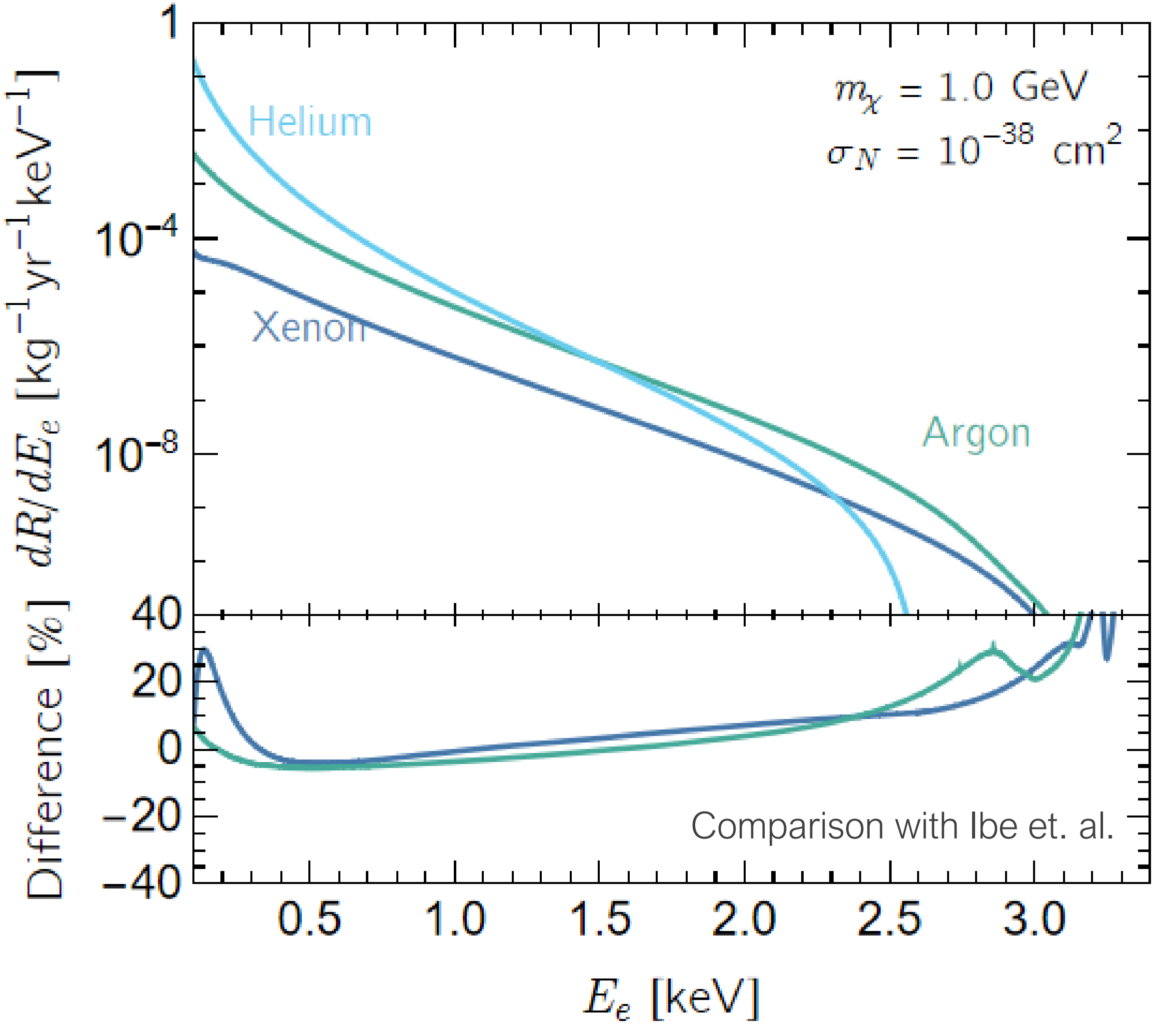
- Inner shells dominate at high energies

⟶ Additional x-ray / auger electrons from de-excitation



Dark matter Migdal rates

- Good agreement with previous calculation (Ibe et. al. '17)
- Differences due to atomic potential, particularly at low energies
- Dipole approximation OK for dark matter



$$\frac{dR}{dE_e} = \frac{\rho_\chi}{m_\chi}\frac{\sigma_N}{2\mu^2}\int dE_R\, g_\chi(v_{min})|F_N|^2 \left(\sum_{n\kappa}\frac{dp_v(n\kappa\to E_e)}{dE_e}\right)$$

Summary

- Migdal effect provides some of the strongest limits on sub-GeV dark matter
- Ongoing effort to observe Migdal effect in neutron scattering
- Requires improved theory – beyond dipole approximation
  Currently finalising predictions for MIGDAL experiment
- Public code for Migdal probabilities coming soon...