Thermodynamics of graviton condensate

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Graviton Condensate (Dvali and Gomez)

- Black holes (BH) could be understood as a graviton condensate at the critical point of a quantum phase transition.
- Thermodynamics properties such the entropy of a BH can be understood using tools from condensed matter physics.
- Hawking radiation would be explained due to quantum depletion of the gravitons from the condensate.
- Whole black holes physics can be explained in terms of just one number N, the number of "off-shell" gravitons contained in the BEC.
- The strength of graviton-graviton interaction which is measured by a dimensionless coupling α as follows $\alpha \sim \frac{L_P^2}{\lambda^2}$, λ is the characteristic wavelength of the graviton. L_P is Planck length.
- The number of gravitons is given by $N \sim \frac{M r_g}{\hbar}$ Where r_g is the gravitational radius. For maximal N the wavelength is such that $r_g = \lambda$, then one has that $N\alpha \sim 1$. Black holes must always satisfy this critical condition. Then: $T_h \sim \frac{1}{\sqrt{N}}, \quad S_{bh} \sim N$

Geometrical Model of Graviton Condensate(J.A., D. Espriu and L. Gabbanelli)

- The background spacetime is given, and over it, there is a Bose-Einstein graviton condensate.
- Besides, it is possible to generalize the graviton condensate to include charge and to apply the mathematical construction of the graviton condensate to the case of De Sitter spacetime.
- The authors of the geometrical graviton condensate model have not explicitly described the thermodynamics of the black hole. If a black hole can be described by a graviton condensate, shouldn't we associate the black hole with a pressure and volume term like any other condensate?
- The main purpose of this work is to show that it is possible to describe in a consistent way black hole thermodynamics in the graviton condensate model if we use an alternative approach to black hole thermodynamics called the Horizon Thermodynamics approach.

THE GEOMETRICAL GRAVITON CONDENSATE MODEL(J.A., D. Espriu and L. Gabbanelli)

- The metric is split as follows: $g_{\mu\nu} = \tilde{g}_{\mu\nu} + h_{\mu\nu}$. Where $\tilde{g}_{\mu\nu}$ is the background metric and $h_{\mu\nu}$ the quantum fluctuation that describes the graviton condensate.
- The number of gravitons is proposed to be built from the quantum fluctuation $h_{\mu\nu}$ as follows

$$N = \int_0^{r_h} \eta dV, \quad \eta \equiv \frac{1}{2r_h} h_{\alpha\beta} h^{\alpha\beta} \tag{1}$$

  Where η is the number density of gravitons, r_h the event horizon radius and the differential volume is taken as $dV = r^2 \sin(\theta) dr d\theta d\phi$.
- In order to respect the underlying symmetry of GR, the simplest form to introduce a term related to the BEC, at the level of action, is $S_{BEC} = -\frac{1}{8}\int d^4x\sqrt{-\tilde{g}}\mu(x) h_{\alpha\beta} h^{\alpha\beta}$
  Where the scalar field $\mu(x)$ would represent the chemical potential of the BEC.

Graviton condensate Line element

$$ds^2_{BEC} = -\frac{1}{(1-B)}\left(1-\frac{2M}{r}\right)dt^2 + \frac{1}{(1-B)\left(1-\frac{2M}{r}\right)}dr^2 + r^2 d\Omega^2 \quad (2)$$

The nonzero mixed components of the effective energy-momentum tensor are given by $T^t{}_t = T^r{}_r = -\frac{B}{8\pi r^2}$ Here B is a constant only defined inside of the BH. Its magnitude is taken between 0 to 1, otherwise, it could change the sign between g_{tt} and g_{rr}. We call this line element, the BEC-Schwarzschild solution. The outside of the black hole is the standard Schwarzschild solution. Thus, the constant B is zero outside of the event horizon, otherwise the metric would not be asymptotically flat. Finally, we can compute the number of gravitons contained in the BEC $N = \frac{4\pi}{3}4M^2B^2 \Rightarrow M \sim \sqrt{N}$. Even more remarkable, we can write the Hawking Temperature and the entropy of the black hole in terms of the number of gravitons as follows $T_h \sim \frac{1}{\sqrt{N}}, \quad S_{bh} \sim N$

Critique of the geometrical model of graviton condensate

We can argue that this graviton condensate model has some problematic interpretation. To see this, we express the BEC-Schwarzschild solution in the Advanced Eddington-Finkelstein coordinates [EFC] as follows

$$ds^2_{BEC} = -\frac{1}{1-B}\left(1 - \frac{2M}{r}\right)du^2 + \frac{2}{1-B}dudr + r^2 d\Omega^2 \tag{3}$$

The original setting introduces a metric discontinuity precisely at the horizon. From the exterior solution described in the EFC we have $g_{ru}^{out} = 1$ at the horizon, but from the interior solution given by (3), one has $g_{ru}^{BEC} = \frac{1}{1-B}$, then $g_{ru}(r_h)^{out} \neq g_{ru}(r_h)^{BEC}$. The golden age of black holes in the 1970s began thanks to the discovery that the metric is not discontinuous at the event horizon. To avoid the discontinuity, we will accept that $B \neq 0$ for the whole spacetime. Therefore, the BEC-Schwarzschild solution is not asymptotically flat. In the context of Loop Quantum Gravity, a similar problem appears(Ashtekar:2020). Apparently, quantum corrections cannot live just inside the black hole.

HORIZON THERMODYNAMICS APPROACH(Padmanabhan)

In the particular case of spherical symmetry, the approach of HT is quite simple. Under this symmetry, the most important result is that we can identify the thermodynamic pressure of black hole as $P \equiv T^r{}_r|_{r_h}$, where r_h represents each horizon or roots of the lapse function. The first law of HT is given by $dU = T_h dS - PdV, T_h \equiv \frac{\kappa}{2\pi}, \quad S \equiv \frac{A}{4}$,
$V \equiv \frac{4\pi r_+^3}{3}, \quad P \equiv T^r{}_r|_{r_+}, \quad U \equiv \frac{r_+}{2}$ The internal energy U is the Misner-Sharp mass evaluated at r_+. The Misner-Sharp mass is a quasi-local definition of energy that, in spherical symmetry, is well-established and has useful properties (Hayward). In a vacuum solution, that is to say, for the Schwarzschild BH, the standard BH thermodynamics approach coincides with the HT approach. The pressure vanishes, and the internal energy is $U = M = E_k$ (E_k is the Komar mass). The volume described here is called the thermodynamic volume and coincides with the naive geometrical volume. T^μ_ν is the energy-momentum tensor of matter.

THERMODYNAMICS OF GRAVITON CONDENSATE

In the last slide, the construction behind of HT approach assumed that $g_{tt} = g_{rr}^{-1}$. However, this is not true in the line element for graviton condensate of equation (2). We will derive the HT first law for our case. We write the line element for static spherical symmetry solutions as follows

$$ds^2 = -f(r)dt^2 + \frac{1}{h(r)}dr^2 + r^2 d\Omega^2 \quad (4)$$

Using the radial component of the Einstein equation, we arrive at

$$G^r{}_r = 8\pi T^r{}_r \Rightarrow \frac{f(h-1) + rhf'}{r^2 f} = 8\pi T^r{}_r \Rightarrow h - 1 + \frac{h}{f} r f' = 8\pi T^r{}_r r^2$$

We must notice that $\frac{h(r)}{f(r)} = (1-B)^2$. So, it is well-defined to evaluate this fraction at $r = r_+$.

Taking $T^r{}_r|_{r_+} \equiv P$, and recalling that $f(r_+) = h(r_+) = 0$, we arrive at

$$\Rightarrow \frac{(1-B)^2}{2} r_+ f'(r_+) - \frac{1}{2} = 4\pi P r_+^2$$

Finally, multiplying the whole equation with dr_+, and reorganizing the differential in a suggesting way, we have the following expression

$$\Rightarrow (1-B)^2 \frac{f'(r_+)}{4\pi} d(\pi r_+^2) - d(r_+/2) = P d\left(\frac{4}{3}\pi r_+^3\right)$$

In this way, we recover the first law of HT, which is given by

$$dU = T_h dS - P dV, \quad S \equiv \frac{A}{4}, \quad V \equiv \frac{4\pi r_+^3}{3}$$

Where the internal energy and the Hawking temperature are given by

$$U \equiv \frac{r_+}{2} \Rightarrow U = M \tag{5}$$

$$T_h \equiv \frac{(1-B)^2 f'(r_+)}{2\pi} \Rightarrow T_h = \frac{(1-B)}{8\pi M} \tag{6}$$

We evaluate the pressure as the HT approach stated to do, which is given by

$$P \equiv T^r{}_r|_{r_+} \Rightarrow P = -\frac{B}{32\pi M^2} \tag{7}$$

We also notice the following interesting relation

$$P = -\frac{B}{3}\frac{M}{V} \tag{8}$$

This equation looks like the relation between pressure and energy density in the case of electromagnetic radiation. Finally, we can express each thermodynamic quantity in terms of N

$$S \sim N, \quad M \sim \sqrt{N} \quad T_h \sim \frac{1}{\sqrt{N}} \tag{9}$$

$$V \sim N^{3/2}, \quad P \sim \frac{1}{N} \tag{10}$$

THE FORMAL EQUIVALENCE WITH LETELIER BLACK HOLE

- Moving infinitesimally thin string that traces out a two-dimensional world sheet Σ, which is parameterized as follows $x^\mu = x^\mu(\lambda^a), \quad a = 0, 1.$ The Nambu-Goto action determines the motion of the string.
- Introduce $\Sigma^{\mu\nu} \equiv \epsilon^{ab} \frac{dx^\mu}{d\lambda^a} \frac{dx^\nu}{d\lambda^b}$ into the Nambu-Goto action to get $S_{NG} = D \int_\Sigma \sqrt{-\frac{1}{2}\Sigma_{\mu\nu}\Sigma^{\mu\nu}} d\lambda^0 d\lambda^1$ -> Energy momentumm tensor of a cloud of string
- Take spherical symmetry to solve the Einstein equation. The solution is called the Letelier black hole which has the following line element

$$ds^2 = -\left(1 - a - \frac{2m}{R}\right) dT^2 + \frac{1}{\left(1 - a - \frac{2m}{R}\right)} dR^2 + R^2 d\Omega^2 \quad (11)$$

Here a is an adimensional parameter related to the energy density of the cloud of string.

- Define $m = M(1-a)$ and obtain that

$$ds^2 = -\left(1 - a - \frac{2M(1-a)}{R}\right) dT^2 + \frac{1}{\left(1 - a - \frac{2M(1-a)}{R}\right)} dR^2 + R^2 d\Omega^2$$

In this expression, we can still recover the Schwarzschild solution demanding that $a = 0$, where obviously $m = M$. Introducing the following coordinate transformations $R = r$ and $T = \frac{t}{\sqrt{1-a}}$, we obtain

$$ds^2 = -\frac{1}{(1-a)}\left(1 - \frac{2M}{r}\right) dt^2 + \frac{1}{(1-a)\left(1 - \frac{2M}{r}\right)} dr^2 + r^2 d\Omega^2$$

which is the BEC-Schwarzschild BH. Is this a mere coincidence? In the Letelier spacetime, the energy-momentum tensor(EMT) is due to a cloud of strings. In the graviton condensate, the EMT is built from the quantum fluctuation of the background metric given by $h_{\mu\nu}$. Could it be possible that quantum fluctuations of metric are due to a cloud of strings? We will leave this possibility for later works.

KISELEV BLACK HOLE

The multi-components Kiselev black hole has the following lapse function

$$f(r) = 1 - \frac{2M}{r} - \sum_i \frac{C_i}{r^{3\omega_i+1}} \tag{12}$$

The corresponding EMT is

$$T^t{}_t = T^r{}_r = \sum_i \rho_i, \, T^\theta{}_\theta = T^\phi{}_\phi = -\frac{1}{2}\sum_i \rho_i(3\omega_i + 1), \quad \rho_i \equiv \frac{3C_i\omega_i}{8\pi r^{3(\omega_i+1)}} \tag{13}$$

We call the parameter ω_i the state parameter, and C_i the Kiselev charge. The Kiselev BH can parameterize the most famous black holes with static spherical symmetry. Of course, this idea works with the energy-momentum tensor as well.

BEC-KISELEV BLACK HOLE AND ITS THERMODYNAMICS

In these slides, we will generalize the discussion made previously of a Schwarzschild BEC. This time, the graviton condensate will include matter. To do this, we extend the BEC action as follows

$$S_{BEC} = -\frac{1}{8}\int d^4x\sqrt{-\tilde{g}}(\nu(x)h^2 + \mu(x)h_{\alpha\beta}h^{\alpha\beta}) \tag{14}$$

Using equation (13) as "matter" energy-momentum tensor $T^{matter}_{\alpha\beta}$, we have found the following line element

$$ds^2_{BEC} = -\frac{1}{(1-B)}\left(1-\frac{2M}{r}-\frac{C}{r^{3\omega+1}}\right)dt^2 + \frac{1}{(1-B)\left(1-\frac{2M}{r}-\frac{C}{r^{3\omega+1}}\right)}dr^2 + r^2d\Omega^2 \tag{15}$$

The line element (15) would describe a graviton condensate that is surrounded by different matter contents. We call this solution the BEC-Kiselev black hole. To study the thermodynamics of this BH, we start computing the number of gravitons , which results in

$$N = \frac{4\pi}{3}(r_+)^2 B^2 \Rightarrow N \sim S \tag{16}$$

As before, the number of gravitons is proportional to the entropy. The radial mixed component of the effective energy-momentum tensor is given by

$$T^r{}_r = -\frac{B}{8\pi r^2} + \frac{3C\omega(1-B)}{8\pi r^{3(\omega+1)}} \tag{17}$$

Evaluating at the horizon, we obtain the thermodynamic pressure as prescribed in the HT approach

$$P_{tot} = -\frac{B}{8\pi r_+^2} + \frac{3C\omega(1-B)}{8\pi r_+^{3(\omega+1)}} \tag{18}$$

We split the total pressure as $P_{tot} \equiv P_{vac} + P_{matt}$. Then, we have

$$P_{vac} \equiv -\frac{B}{8\pi r_+^2}, \quad P_{matt} \equiv \frac{3C\omega(1-B)}{8\pi r_+^{3(\omega+1)}} \tag{19}$$

The first term is the pressure related to the vacuum solution, and the second term is the pressure related to the matter content P_{matt}. While the Hawking temperature is

$$T_h = \frac{(1-B)}{4\pi}\left(\frac{2M}{r_+^2} + \frac{(3\omega+1)C}{r_+^{3\omega+2}}\right) \tag{20}$$

Therefore, the first law of HT can be written as
$dU = T_h dS - (P_{vac} + P_{mat})dV$
The entropy and volume have the usual definition: $S = \pi r_+^2$ and $V = \frac{4\pi r_+^3}{3}$. The internal energy is still $U = \frac{r_+}{2}$. We must emphasize that we cannot use the number of gravitons N to express each thermodynamic quantity due to the presence of matter, as we did before.

Conclusions

- The geometrical graviton condensation model of Alfaro, Espriu, and Gabbanelli succeeded in putting the qualitative ideas of Dvali and Gomez into geometric and quantitative terms. However, there was no thermodynamic study of the model. Using the Horizon Thermodynamics approach, we have completed this model by obtaining its thermodynamics including the volume and pressure terms.
- We have modified the original proposal by extending the parameter B to all space instead of being defined only inside the black hole. Thanks to this, we have established a formal equivalence between the Letelier black hole with the BEC-Schwarzchild line element.
- We extended the geometrical graviton condensate model in order to include different types of matter. We have obtained a new solution which we called the BEC-Kiselev black hole. When there are matter contents, we cannot express all the thermodynamic quantities in terms of the number of gravitons N. However, $S \sim N$.

THANK YOU